Self-focusing of relativistic electron bunches in a plasma

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The three-dimensional equilibrium of a modulated ultrarelativistic electron beam (or of a train of electron bunches) in a plasma is analyzed in the self-focusing case. The compression of a bunch by the field of the plasma wave excited by the beam is balanced by the gradient of the kinetic pressure in the bunch. Self-focusing occurs if the beam modulation frequency ω_m is lower than the plasma frequency ω_p. An analytic solution derived for arbitrary modulation frequencies $\omega_m < \omega_p$ shows that a deviation from the resonance $\omega_m \approx \omega_p$ results in an elongation of the bunches, without a change in the distance between the edges of neighboring bunches, $\delta z = \pi c/\omega_p$, or in the beam radius $r_0 \sim \delta z$.

A problem of current interest in plasma physics is the transport of intense electron beams over large distances through dense plasmas. One approach to the solution of this problem, proposed in Ref. 1, is to break the beam up into distinct bunches with a modulation frequency $\omega_m = 2\pi v/l$ (l and v are, respectively, the spatial period and velocity of the beam) lower than the plasma frequency of the plasma, ω_p. In this case the dielectric constant $\varepsilon = 1 - \omega_p^2/\omega_m^2$ is negative, and a radial self-focusing of the electron bunches occurs.

It was shown in Ref. 2 that under the condition $\varepsilon < 0$ it becomes possible to maintain equilibrium longitudinal dimensions of bunches in a situation in which the repulsive Coulomb force and the force associated with the gradient of the kinetic pressure are balanced by the field of the focusing wave mode. It was pointed out in Ref. 2 that transporting a beam in the form of distinct bunches has the advantage that the growth rate for the instability of satellite modes is lower than in the case of a solid beam.

It was shown in Refs. 3 and 4 that it is possible to simultaneously achieve radial focusing and phase focusing of electron beams in a plasma with a "negative" dielectric constant for the case of an azimuthally symmetric nonrelativistic beam under resonant conditions with $|\varepsilon| \ll 1$. A radial and longitudinal compression of the bunches is caused by the space-charge wave which is excited by the beam and which displaces plasma electrons from the beam volume. The ion potential wells which arise, and which are synchronized with the beam, capture the bunches and lead to a three-dimensional equilibrium of the beam with the wave.[3]

In the present paper, the nonrelativistic theory[1–4] of the self-focusing of a modulated electron beam in a plasma is generalized to relativistic energies, at which the emission by the beam in the plasma is not electrostatic, and the problem goes beyond the scope of the electrostatic approximation. The magnetic field of the wave weakens the Coulomb repulsion of the electrons in the bunches to an extent proportional to $\gamma^{-2} = 1 - v^2/c^2$ and expands the region of modulation frequencies $\omega_m < \omega_p$ which correspond to self-focusing, since the focusing polarization field does not depend on the relativistic factor.

An analytic solution of the problem is derived in the nonresonant case $\omega_m < \omega_p$ for ultrarelativistic energies, $\gamma^{-2} \ll 1$, at which the nonlinear wave equation for the self-consistent potential simplifies, and variables can be separated. It thus becomes possible to treat the radial and longitudinal equilibria of the electron beams independently.

When we go to relativistic energies, the growth rate of the beam instability[5] decreases in proportion to γ^{-1}, and there is a corresponding increase in the distance over which the beam can be transported in the plasma.

1. NONLINEAR EQUATION FOR THE SELF-CONSISTENT POTENTIAL

The emission by an electron beam with a density ρ and a velocity $\mathbf{v}$ in a dense (linear) plasma is described by a system of inhomogeneous wave equations for the potentials $\mathbf{A}$ and φ:

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right)\left[\Delta - \frac{1}{c^2}\left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right)\right]\varphi = 4\pi e \frac{\partial^2 \rho}{\partial t^2},$$
$$\left[\Delta - \frac{1}{c^2}\left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right)\right]\mathbf{A} = 4\pi e \rho \mathbf{v}/c. \quad (1)$$

These potentials satisfy the gauge condition

$$\frac{\partial}{\partial t}\operatorname{div}\mathbf{A} + \frac{1}{c}\left(\frac{\partial^2}{\partial t^2} + \omega_p^2\right)\varphi = 0. \quad (2)$$

We direct the z axis of the cylindrical coordinate system along the beam velocity, and we consider an azimuthally symmetric steady-state solution:

$$\psi = \varphi - \frac{v}{c}A_z, \quad A_r = A_\varphi = 0,$$
$$\psi = \psi(r, z'), \quad z' = z - vt. \quad (3)$$

Switching to the function ψ in Eqs. (1), we obtain the equation

$$\left(\frac{\partial^2}{\partial z'^2} + \frac{\omega_p^2}{v^2}\right)\left(\Delta_r + \frac{1}{\gamma^2}\frac{\partial^2}{\partial z'^2} - \frac{\omega_p^2}{c^2}\right)\psi$$
$$= 4\pi e\left(\frac{1}{\gamma^2}\frac{\partial^2}{\partial z'^2} - \frac{\omega_p^2}{c^2}\right)\rho, \quad (4)$$

where Δ_r is the radial part of the Laplacian.

We find the functional dependence $\rho(\psi)$ by the trapped-particle method,[6] taking the electron distribution function in the bunches to be, in the frame of reference of the beam,

$$f(v') = \rho_0'\left(\frac{m}{\pi e \varphi_m'}\right)^{1/2}\left(\frac{2e\varphi'}{m} - v'^2\right)^{1/2}\theta\left(\frac{2e\varphi'}{m} - v'^2\right), \quad (5)$$

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where ρ_0' is the electron density at the bottom of the potential well, ψ_m'. The discontinuous function on the right side of (5),

$$\theta(\varphi')=\begin{cases}1, & \varphi'>0,\\ 0, & \varphi'<0,\end{cases}$$

reflects the condition for capture of the electrons by the wave.

Integrating (5) over velocity, we find the density of the trapped particles as a function of the potential in the frame of reference of the beam:

$$\rho'=\rho_0'\left(\frac{\varphi'}{\varphi_m'}\right)^2\theta(\varphi'). \quad (6)$$

A transformation to the frame of reference of the plasma is made with the help of a Lorentz transformation:

$$\rho=\rho_0\left(\frac{\psi}{\psi_m}\right)^2\theta(\psi), \quad \varphi'=\gamma\psi, \quad \varphi_m'=\gamma\psi_m, \quad \rho_0=\gamma\rho_0'. \quad (7)$$

Equation (4) simplifies substantially in the ultrarelativistic limit $\gamma^2\gg 1$, at which term of order $\gamma^{-2}\ll 1$ can be discarded:

$$\left(\frac{\partial^2}{\partial\zeta^2}+1\right)(\Delta_\xi-1)y=-qy^2\theta(y),$$

$$y=\psi/\psi_m, \quad \xi=\omega_p r/c, \quad \zeta=\omega_p z'/c, \quad q=4\pi e\rho_0 c^2/\psi_m\omega_p^2. \quad (8)$$

The boundary conditions on Eq. (8), which reflect the conditions for self-focusing of the bunches, are

$$\partial y(0,\zeta)/\partial\xi=0, \quad y(\infty,\zeta)=0,$$
$$y(0,0)=1, \quad \partial y(0,0)/\partial\zeta=0. \quad (9)$$

The point $\xi=\zeta=0$ corresponds to the center of a bunch, where the potential reaches its maximum with respect to both variables.

2. RADIAL SELF-FOCUSING OF A BEAM

Separation of variables can be carried out in the nonlinear partial differential equation (8):

$$y(\xi,\zeta)=q^{-1}R(\xi)Z(\zeta). \quad (10)$$

In this case the radial potential profile is described by

$$\xi^{-1}(\xi R')'=R-R^2,$$
$$R'(0)=0, \quad R(\infty)=0, \quad (11)$$

and the boundary conditions in (11) are a consequence of (9). The prime means the derivative with respect to the variable ξ.

A solution of (11) in the form of a soliton with a maximum at the beam axis corresponds to radial self-focusing of a bunch. Since we cannot find an integral of this equation, we take the approach of Ref. 4 and examine the equation qualitatively, drawing on the mechanical analogy with the motion of a particle in a potential well.[7] Omitting the term $\xi^{-1}R'$, which acts as a friction force, from the left side of (11), and carrying out the integration, we find

$$\frac{R'^2}{2}+W(R)=C, \quad W(R)=\frac{R^3}{3}-\frac{R^2}{2}, \quad (12)$$

where the integration constant C corresponds to the total energy of the particle in the potential well $W(R)$. Figure 1 shows a plot of this function, whose zeros are at $R_1=0$ and $R_2=3/2$. For $C>0$, the motion is infinite, while for $C<0$ it is periodic. The case $C=0$ corresponds to the soliton solution.

If the friction force $\xi^{-1}R'$ in Eq. (11) is instead retained, the particle loses energy and falls to the bottom of the well. If the starting point in this case is sufficiently high, the particle will still have enough energy at the potential minimum for a return motion. Numerical integration shows that at $R_0=R(0)=2.392$ the particle first falls to the bottom of the well, then rises, and asymptotically approaches the point $R_1=0$. This case corresponds to the soliton solution of Eq. (11). Figure 2 shows a plot of the function $R(\xi)$ for this case.

3. LONGITUDINAL EQUILIBRIUM OF BUNCHES

An equation for the function $Z(\zeta)$ follows from (8)–(10) and describes the potential distribution along the beam:

$$Z''=-Z+Z^2\theta(Z),$$
$$Z'(0)=0, \quad Z(0)=qR_0^{-1}, \quad (13)$$

where the constant $R_0=2.392$ was determined in Sec. 2.

Pursuing the mechanical analogy,[7] we analyze the first integral of Eq. (13):

$$\frac{Z'^2}{2}+U(Z)=E, \quad U(Z)=\frac{Z^2}{2}-\frac{Z^3}{3}\theta(Z), \quad E=U(Z_0) \quad (14)$$

$[Z_0=Z(0)]$. It follows from the plot of the function $U(Z)$ in Fig. 3 that a physically meaningful periodic solution exists in the region

$$Z_{min}\leqslant Z\leqslant Z_0<1, \quad Z_{min}=-(2E)^{1/2}$$

and corresponds to motion of the particle between turning points Z_{min} and Z_0.

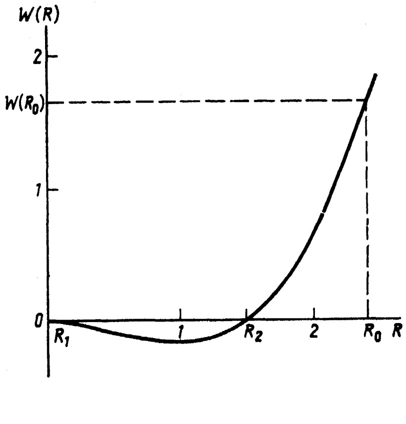

FIG. 1. The radial potential well $W(R)$.

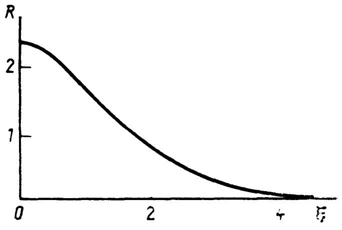

FIG. 2. Profile of the radial part of the potential, R, along the radial variable ξ.

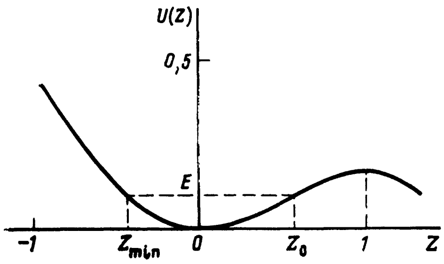

FIG. 3. Plot of the longitudinal potential well $U(Z)$.

Integrating Eq. (14), we find

$$\zeta=\int_{Z}^{Z_0}\frac{dZ}{[2(E-U)]^{1/2}}. \tag{15}$$

The period L is determined by Eq. (15): $L=2\zeta(Z_{\min})$. Allowing for the discontinuous function $\theta(Z)$, we can write this period in the form

$$L=\pi+2^{1/2}\int_{0}^{Z_0}\frac{dZ}{(E-Z^2/2+Z^3/3)^{1/2}}, \tag{16}$$

where the term π corresponds to the range of integration outside the bunch, and $Z_{\min}\leqslant Z\leqslant 0$.

The integral in (16) reduces to an incomplete elliptic integral of the first kind:[8]

$$L=\pi+2\varkappa^{-1}F(\varphi_0,k),\quad k^2=\frac{1}{2}\left(1+\frac{2Z_0+1}{4\varkappa^2}\right),$$
$$\varphi_0=\arcsin\left\{2\left[1+\left(\frac{9-6Z_0}{1+2Z_0}\right)^{1/2}\right]^{-1}\right\}^{1/2},$$
$$\varkappa=\frac{1}{2}\left[\left(1-\frac{2Z_0}{3}\right)(1+2Z_0)\right]^{1/4}. \tag{17}$$

Since the arguments of the function $F(\varphi_0,k)$ depend on $Z_0\propto\psi_m^{-1}$, Eq. (17) can be used to reconstruct the dependence of the depth of the potential well, ψ_m, on the dimensionless spatial period of the beam, $L=\omega_p l/c$. Figure 4 shows a plot of $\psi_m(L)$.

Similar manipulations lead to an expression for the function $Z(\zeta)$ in terms of elliptic functions:[8]

$$Z(\zeta)=Z_0\begin{cases}\left\{1-\frac{1}{2}\left[1+\left(\frac{9-6Z_0}{1+2Z_0}\right)^{1/2}\right]\operatorname{sn}^2(\varkappa\zeta,k)\right\}\operatorname{dn}^{-2}(\varkappa\zeta,k), & |\zeta-nL|\leqslant\zeta_0,\\ -\left(1-\frac{2}{3}Z_0\right)^{1/2}\cos(L/2-\zeta) & \zeta_0\leqslant|\zeta-nL|\leqslant L/2,\end{cases} \tag{18}$$

where $2\zeta_0=L-\pi$, and the point $\zeta_n=nL$ corresponds to the center of the bunch of index n. Without any loss of generality, we can set $n=0$ at this point; i.e., we can treat the case of a bunch which is centered at the origin of coordinates, $\zeta_L=0$. Figure 5 shows the shape of the electron bunches for various values of the parameter L.

The general expressions (17) and (18) simplify in limiting cases in which the asymptotic forms of the special functions can be used. In the limit $Z_0\to 0$, we use power series in Z_0,

$$\operatorname{sn}(\varkappa\zeta,k)\approx\sin(\zeta/2),\quad \operatorname{dn}(\varkappa\zeta,k)\approx 1,$$
$$L\approx 2\pi+\frac{4}{3}Z_0,\quad k^2\approx\frac{4}{3}Z_0,\quad \varkappa\approx\frac{1}{2}\left(1+\frac{2}{3}Z_0\right), \tag{19}$$

finding

$$Z(\zeta)=Z_0\cos\zeta,\quad \psi_m=\frac{2}{3\pi^2R_0}\frac{e\rho_0 l^2}{\delta},$$
$$\delta=L/2\pi-1\ll 1,\quad L=l\omega_p/c. \tag{20}$$

In the case $v=c$, expression (20) is the same as that found from the integral equation of Ref. 4 under resonant conditions, such that the beam modulation frequency $\omega_m=2\pi c/l$ is close to ω_p, the plasma frequency of the plasma.[1)]

Asymptotic expansions of (17) and (18) can also be derived in the case $\Delta=1-Z_0\ll 1$ (Ref. 8):

$$k^2=1-\frac{4\Delta}{3},\quad \varkappa=\frac{1}{2}\left(1+\frac{\Delta}{3}\right),\quad \varphi_0=\frac{\pi}{2}-\left(\frac{2\Delta}{3}\right)^{1/2},$$
$$\operatorname{dn}(\varkappa\zeta,k)=\left[1+\frac{\Delta}{6}(\zeta+\operatorname{sh}\zeta)\operatorname{th}\frac{\zeta}{2}\right]\operatorname{sech}\left(\frac{\zeta}{2}\right). \tag{21}$$

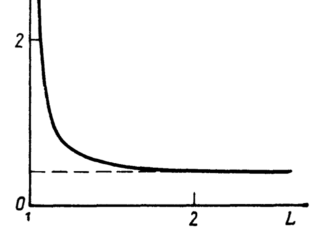

FIG. 4. The wave amplitude $\tilde\psi=e\psi_m\rho_{p0}/mc^2\rho_0$ versus the spatial period of the beam.

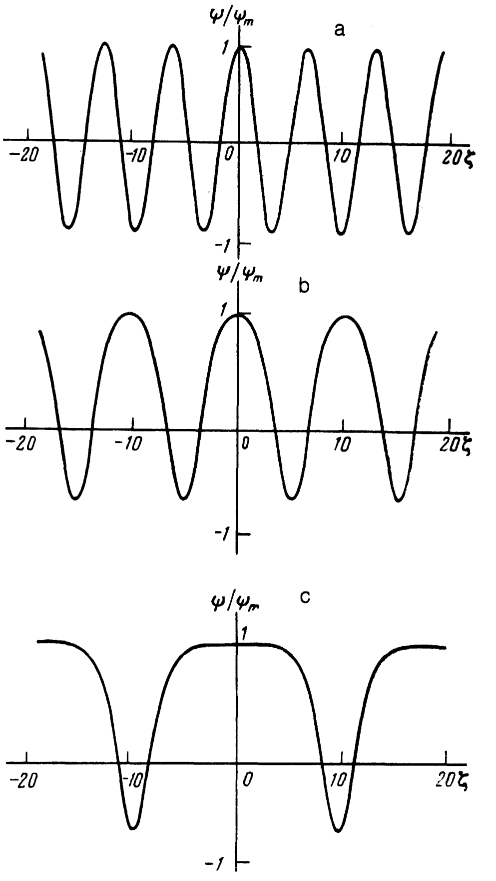

FIG. 5. Profile of the wave potential ψ/ψ_m at the beam axis ($\xi = 0$) for various values of the deviation $\delta = L/2\pi - 1$. a—$\delta = 0.036$; b—$\delta = 0.6$; c—$\delta = 2.1$.

Substituting (21) into (17), we find the period of the oscillations:

$$L=\pi+2\ln\left[\frac{12}{\Delta}(2-3^{1/2})\right]. \tag{22}$$

In going from (17) to (22) we made use of the asymptotic expression $K(k)\approx\ln(4/k_1)$, $k_1 = (1-k^2)^{1/2}$ for the complete elliptic integral. As the parameter Δ decreases the modulation period L increases, and the longitudinal dimensions of the bunches also increase. The distance between bunches, on the other hand, remains constant, equal to half the wavelength, $\pi c/\omega_p$ (Fig. 4).

In approximation (21) we find the following from expression (18):

$$Z(\zeta)=Z_0\begin{cases}1+2\Delta\left(1-\mathrm{dn}^{-2}\dfrac{\zeta}{2}\right), & |\zeta|\leqslant\zeta_0,\\ -3^{-1/2}\cos\left(\dfrac{L}{2}-\zeta\right), & \zeta_0\leqslant|\zeta|\leqslant L/2.\end{cases} \tag{23}$$

Restricting the discussion to the first term in the asymptotic expression for the function $\mathrm{dn}(\varkappa\zeta,k)$ in (21), $\mathrm{dn}(\varkappa\zeta,k)\approx\mathrm{dn}(\zeta/2)$, we find the potential distribution near the origin from (23):

$$Z(\zeta)=Z_0\left(1-2\Delta\,\mathrm{sh}^2\frac{\zeta}{2}\right),\quad |\zeta|\ll\zeta_0. \tag{24}$$

To simplify (23) at the boundary of a bunch, $\zeta\lesssim\zeta_0$, we switch to the new variable $\zeta = \zeta_0 + \zeta_1$ and use the relations

$$\mathrm{dn}\frac{\zeta_0}{2}=(2\Delta)^{1/2},\quad \mathrm{cn}\frac{\zeta_0}{2}=\left(\frac{2\Delta}{3}\right)^{1/2},\quad \mathrm{sn}\frac{\zeta_0}{2}=1.$$

The general formula from the theory of elliptic functions[8] can then be put in the form

$$\mathrm{dn}\frac{\zeta_0+\zeta_1}{2}=(2\Delta)^{1/2}\left(\mathrm{dn}\frac{\zeta_1}{2}-3^{-1/2}\,\mathrm{sn}\frac{\zeta_1}{2}\,\mathrm{cn}\frac{\zeta_1}{2}\right)\mathrm{cn}^{-2}\frac{\zeta_1}{2},$$
$$\mathrm{dn}\frac{\zeta_1}{2}=\mathrm{cn}\frac{\zeta_1}{2}=\mathrm{sech}\frac{\zeta_1}{2},\quad \mathrm{sh}\frac{\zeta_1}{2}=\mathrm{th}\frac{\zeta_1}{2}. \tag{25}$$

Substituting (25) into the first expression in (23). we find the potential distribution near the boundary of a bunch:

$$Z(\zeta)=1-\frac{3}{2}\,\mathrm{ch}^{-2}\left[\frac{\zeta_1}{2}-\mathrm{arcch}\left(\frac{3}{2}\right)^{1/2}\right]. \tag{26}$$

At the point $\zeta = \zeta_0$ ($\zeta_1 = 0$) the function in (26) and its derivative are the same as the second expression in (23).

The maximum value of the potential, found from the condition $Z_0 = 1$,

$$\psi_\infty=\frac{4\pi e\rho_0 c^2}{\omega_p{}^2 R_0}, \tag{27}$$

does not contain a resonant factor. It is an asymptotic function of $\psi_m(L)$ in the limit $L\to\infty$ (Fig. 4).

4. INDUCED CHARGE AND CURRENT OF THE PLASMA

To determine the electron current density $\mathbf{j}_p$ induced in the plasma by the beam,[9] we use equations which follow from (1) for the scalar potential φ and the vector potential A_z:

$$\varphi=-\frac{\partial^2\psi}{\partial\zeta^2},\quad A_z=-\left(\frac{\partial^2}{\partial\zeta^2}+1\right)\psi. \tag{28}$$

Substituting (28) into Maxwell's equations, we find

$$j_{pz}=\frac{\omega_p{}^2}{4\pi c}\psi,\quad j_{pr}=-\frac{\partial^2 j_{pz}}{\partial\xi\,\partial\zeta}, \tag{29}$$

where j_{pz} and j_{pr} are the longitudinal and radial projections of the vector $\mathbf{j}$.

To compare j_{pz} with the beam current density $j_b = -ec\rho$, we switch to the dimensionless functions[10]

$$j_{pz}=j_0RZ,\quad j_b=-j_0R^2Z^2\theta(Z),\quad j_0=\frac{ec\rho_0}{q^2}, \tag{30}$$

and we use the relations

$$\bar{R}=2\pi\int_0^\infty R\xi\,d\xi=2\pi\int_0^\infty R^2\xi\,d\xi,$$
$$\langle Z\rangle=L^{-1}\int_{-L/2}^{L/2}Z\,d\zeta=L^{-1}\int_{-L/2}^{L/2}Z^2\theta(Z)\,d\zeta, \tag{31}$$

which follow from Eqs. (11) and (13).

Integrating (30) over the cross section, we find the distributions of the plasma current and the beam current along the length of a bunch:

$$I_{pz}=I_0Z,\quad I_b=-I_0Z^2\theta(Z),\quad I_0=2\pi i_0c^2\bar{R}\omega_p{}^{-2} \tag{32}$$

(Fig. 6). The constant $\bar{R} = 9.9$ was found by numerical integration.

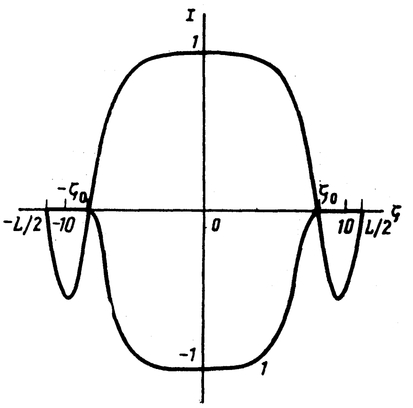

FIG. 6. Profiles of (1) the beam current and (2) the plasma current over a modulation period for the case $\delta = 2.1$.

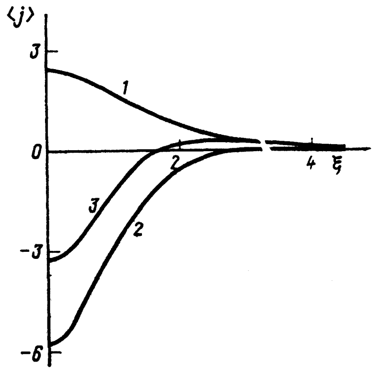

FIG. 7. Profiles of (1) the average current density of the plasma, (2) that of the beam, and (3) their sum along the radial variable ξ.

The radial profile of the current density,

$$\langle j_{pz}\rangle = j_0 R\langle Z\rangle, \quad \langle j_b\rangle = -j_0 R^2 \langle Z\rangle, \tag{33}$$

is shown in Fig. 7. The constant $\langle Z\rangle$ (the area under the curve over the integral $|\zeta| < L/2$) reduces to incomplete elliptic integrals:[8]

$$\langle Z\rangle = \frac{2}{L}\Big[3^{1/4}2^{1/2}Z_0(h/g)^{1/2}\frac{g+3^{1/2}h}{gh+3^{1/2}(Z_0-1/2)}F(\varphi_0,k) - 3^{3/4}(2gh)^{1/2}E(\varphi_0,k) - \left(\frac{3}{2}\right)^{1/2}(3^{1/2}g-h) - \left(\frac{2}{3}\right)^{1/2}Z_0 h\Big],$$

$$g = (Z_0 + 1/2)^{1/2}, \quad h = (3/2 - Z_0)^{1/2}. \tag{34}$$

From (29) and the continuity equation for the plasma electrons, we find the density of the induced charge to be

$$\rho_p = \frac{1}{4\pi e}\left(\Delta_r - \frac{\omega_p^2}{c^2}\right)\psi = -\frac{\rho_0}{q^2} Z R^2. \tag{35}$$

From (30), (35), and (31) we find

$$\langle I_{pz}\rangle = -\langle I_b\rangle, \quad \langle\rho_p\rangle = -\langle\rho_b\rangle, \tag{36}$$

so the plasma with the beam has charge neutrality and also current neutrality on the average over a period.

5. SUMMARY OF RESULTS

In previous studies[1-4] carried out in the electrostatic approximation, it was predicted that an electron beam broken up into bunches before entering a dense plasma would undergo self-focusing in the plasma. In the present study, it has been shown that this self-focusing occurs at relativistic energies. The electromagnetic radiation emitted by a relativistic beam in a plasma simultaneously creates longitudinal and radial potential wells for the electron bunches if the longitudinal dimension of these bunches is greater than the wavelength $\lambda_p = 2\pi c/\omega_p$.

The length of a bunch varies from $\lambda_p/2$ to $(L-\pi)c/\omega_p$ as the modulation frequency of the beam is reduced in the region $\omega_m < \omega_p$. However, the beam radius $r_0 \sim \lambda_p$ and the distance between bunches, $\delta Z' = \lambda_p/2$, are not changed.

As the deviation from resonance increases,[2)] for $L \gg \pi$, the potential well becomes shallower, and the temperature of the captured electrons does not exceed

$$T \approx e\psi_\infty \approx mc^2\rho_0 n_p^{-1}.$$

In contrast with Ref. 9, where the radius of the beam injected into the plasma was left as an adjustable parameter, and the system was not neutralized in terms of current, in the case at hand the parameters of the beam are reconciled with the field, and charge neutrality and current neutrality prevail on the average over a period of the beam.

We wish to thank P. V. Fomin for assistance in this study.

[1)] The condition under which the plasma is linear, $\tilde\rho_p \ll \rho_{p0}$ ($\tilde\rho_p$ and ρ_{p0} are the perturbed and background components of the electron density of the plasma), imposes a restriction on the wave amplitude: $\psi_m \ll mc^2/e$. This restriction corresponds to a low-density beam, $\rho_0 \ll \rho_{p0}\delta$.

[2)] The steady state, which we have been discussing in this paper, persists under the inequality $L \ll c\tau$, where $\tau = r_0^2/\lambda_p^2\nu$ is the time scale for diffusion of the magnetic field,[10] ν is the rate of electron-ion collisions in the plasma, and r_0 is the beam radius.

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