Effective dielectric permittivity of electrons in nonuniform semiconductor alloys

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We discuss the propagation of infrared radiation in nonuniform semiconductor alloys (e.g., $Al_xGa_{1-x}As$, $Cd_xHg_{1-x}Te$, $Pb_xSn_{1-x}Te$, etc.). Because of unscreened fluctuations in the free carrier effective mass there is an additional mechanism for absorption; for $\omega < v_F/l_c$ (where v_F is the Fermi velocity and l_c is the correlation length) this causes the spectral dependence of the absorption coefficient α_ω to be slower than the ordinary dependence $\alpha_\omega \propto \omega^{-2}$ (specifically, $\alpha_\omega \propto$ const or $\propto \omega^{-1}$). These fluctuations also lead to an additional mechanism for broadening of resonance lines associated with magneto-optic effects, so that the shapes of these lines are found to depend on the classical magnetic field. Because of the nonuniformity in the dielectric permittivity the longitudinal fluctuations of the infrared radiation are also damped by free carriers in the spectral region $\omega \sim v_F/l_c$ according to the Landau mechanism.

1. INTRODUCTION

The study of electron kinetic phenomena in substitutional semiconductor alloys of the form $A_x\bar{A}_{1-x}B$ (for example, $Al_xGa_{1-x}As$, $Cd_xHg_{1-x}Te$, $Pb_xSn_{1-x}Te$ etc.) is widespread nowadays. However, simultaneous control of the quantity $\delta_r = x_r - \langle x_r \rangle$ and the scale of spatial fluctuations (determined by the correlation length l_c) has not been carried out with the result that the influence of the latter on the electron kinetics has not been studied. This is connected with the fact that direct methods (e.g., x-ray structure, etc.) of investigating δ_r are usually applicable only for large-scale fluctuations in which l_c is much larger than the kinetic lengths. A method of determining, compositional fluctuation parameters based on characteristics of these electron kinetic phenomena that is convenient over a wide interval of variations of l_c require comparison between the experimental data and some theory that describes the influence of these fluctuations on the kinetic coefficients. Such a theory has not been developed even for the simple case of small amplitude nonuniformities (in this paper we will discuss only this case). The fact is that variation in the alloy composition leads to changes not only in the position of the band extrema (i.e., the potential energy of the charge carriers), but also in other characteristics of the material: the effective mass m_r, the static and high-frequency dielectric permittivities ϵ_r and $\varkappa_r$, etc. Fluctuations in these parameters determine the behavior of the kinetic coefficients for $l_c < r_0$ (where r_0 is the screening radius), i.e., where small fluctuations in the potential energy are effectively screened out.

We will show that fluctuations in the kinetic energy are screened only "on an average." When the fluctuation scale l_c is larger than the de Broglie length and r_0 we write the energy of an electron as follows:

$$E_{pr} = p^2/2m_r + \varepsilon_c(\mathbf{r}) + e\varphi_r. \tag{1}$$

In Eq. (1) the potential energy of the bottom of the conduction band $\varepsilon_c(\mathbf{r})$ appears, as well as the screening potential φ_r determined from the Poisson equation

$$\nabla \epsilon_r \nabla \varphi_r = \frac{8\pi e}{V} \sum_{\mathbf{p}} [f(E_{pr}) - \langle f(E_{pr}) \rangle], \tag{2}$$

where $f(E)$ is the equilibrium distribution function, V is the normalized volume, and $\langle ... \rangle$ denotes an average over fluctuations. Limiting ourselves to the case of small changes in composition, we first linearize the charge density [i.e., the right-hand side of Eq. (2)] with respect to $\varepsilon_c(\mathbf{r}) + e\varphi_r$ and with respect to fluctuations in the kinetic energy. As a result, for $l_c \gg r_0$, the fluctuations $\varepsilon_c(\mathbf{r})$ are effectively screened [in (3) we omit contributions that are small with respect to $(r_0/l_c)^2$], while fluctuations in the kinetic energy are screened only "on the average" [the average energy $\bar{\varepsilon}$ appears in (3), which equals $\frac{3}{2}T$ for nondegenerate and ε_F for degenerate electrons]; then in place of (1) we obtain the expression

$$E_{pr} = \frac{p^2}{2m} + \alpha(\bar{\varepsilon} - \varepsilon_p)\delta_r - W_0 r_0^2 \nabla \frac{\epsilon_r}{\epsilon} \nabla \delta_r, \tag{3}$$

in which $\varepsilon_p = p^2/2m$, $m = \langle m_r \rangle$; here the rate of change of the effective mass with composition (α) is determined by the relation $\alpha\delta_r \equiv (m_r - m)/m$, while $W_0 = \varepsilon_c(\mathbf{r})/\delta_r$.

From this we see that alloys with composition fluctuations may exhibit phenomena that are not encountered in the usual impurity disordered systems.[1] Even in the regime of small-wavelength disorder, where the static potential, which is caused by the impurities, is completely screened, the kinetic properties of these materials vary because of unscreened fluctuations in the mass (for example, new mechanisms arise for optical nonlinearity and scattering of infrared radiation). In this paper we will consider the contribution of free carriers (for definiteness we will discuss electrons) to the effective dielectric permittivity of the semiconductor alloy. For the high-frequency approximation under discussion here the problem reduces to averaging solutions to the system of collisionless kinetic and wave equations.

Let us pause to discuss some qualitative features of the problem under discussion. First of all, during the propagation of long-wavelength electromagnetic perturbations (i.e., $\lambda_r \gg l_c$, where λ_r is the wavelength of the incident radiation) in a random nonuniform medium the fluctuations in $\varkappa_r$ give rise to short-wavelength (i.e., with wave vector $\sim l_c^{-1}$) scattering of the radiation;[2] the longitudinal component of this radiation is damped by free carriers according to the Landau mechanism when the condition $\omega \sim v_F/l_c$ is fulfilled (i.e., not the more restrictive requirement $\omega \sim v_F/\lambda_r$, where v_F is the Fermi velocity of the degenerate carriers). Secondly, due

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to fluctuations in the effective mass m_r long-wavelength components of the electromagnetic radiation are absorbed. This is clear from the equation of motion

$$\frac{d\mathbf{p}_t}{dt}=\mathbf{F}, \quad \frac{d\mathbf{r}_t}{dt}=\frac{\mathbf{p}_t}{m(\mathbf{r}_t)} \qquad (4)$$

for an electron that is subject to forces $\mathbf{F}$ caused by fluctuations in the composition and by external fields [$\mathbf{p}_t$, $\mathbf{r}_t$ are the momentum and position of particles at the instant t; see Eq. (7) below]. In a uniform electric field $\mathbf{F}=e\mathbf{E}\cos\omega t$ and for $m_r \neq$const, a current $e\mathbf{p}_t/\mathbf{m}(\mathbf{r}_t)$ is induced that contains a contribution $\propto \cos\omega t$, which also gives rise to absorption of radiation. Finally, the distinctive features of the dynamics of particles with fluctuating mass described by expressions (3) and (4) are also reflected in the magneto-optic properties of the electron gas. Because of fluctuations in m_r the cyclotron resonance line is found to broaden, with a width that depends on the magnetic field $\mathbf{H}$ in the classical-field region.

Below, in Sec. 2, we present general expressions that determine the linear response of free carriers in the semiconductor alloy. We then calculate the conductivity tensor averaged with respect to small fluctuations (Sec. 3) and discuss the propagation of electromagnetic radiation in the spectral region $\omega \sim v_F/l_c$ (Sec. 4) as well as the cyclotron resonance line shape (Sec. 5). In the Conclusion we discuss the possibility of observing the predicted structure in the effective dielectric permittivity of free carriers experimentally, while the Appendix gives criteria for neglecting the effect of fluctuations on the force in Eq. (4).

2. LINEAR RESPONSE TO HIGH-FREQUENCY ELECTROMAGNETIC PERTURBATIONS

Let us describe the propagation of an electromagnetic wave of frequency ω in a doped semiconductor alloy with compositional fluctuations by calculating the current density induced by the field $\mathbf{E}_{r\omega}$

$$\mathbf{j}_{r\omega}=\frac{2e}{V}\sum_{\mathbf{p}}\mathbf{v}_{\mathbf{p}r}\left[-\frac{df(E_{\mathbf{p}r})}{dE_{\mathbf{p}r}}\right]\chi_{\mathbf{p}r\omega}, \quad \mathbf{v}_{\mathbf{p}r}=\frac{\partial E_{\mathbf{p}r}}{\partial\mathbf{p}}. \qquad (5)$$

The function $\chi_{\mathbf{p}r\omega}$ is found by linearizing the kinetic equation (for a discussion of this equation, see Refs. 3 and 4)

$$\left\{-i\omega+\mathbf{v}_{\mathbf{p}r}\nabla-(\nabla E_{\mathbf{p}r})\frac{\partial}{\partial\mathbf{p}}+\frac{e}{c}[\mathbf{v}_{\mathbf{p}r}\mathbf{H}]\frac{\partial}{\partial\mathbf{p}}\right\}\chi_{\mathbf{p}r\omega}$$
$$-I_c(\chi|\mathbf{p}r)=e\mathbf{E}_{r\omega}\mathbf{v}_{\mathbf{p}r}, \qquad (6)$$

in the high-frequency ($\omega\tau_0 \gg 1$) approximation we can make the replacement $\tau_c^{-1}\to +0$ (where τ_0 is the relaxation time) for the collision integral I_c in this equation.

This corresponds to a dynamic description of the motion of electrons over length scales on the order of the size of the fluctuations l_c, which are considered to be small compared to the momentum relaxation length determined by the collision integral I_c (or the time to traverse a fluctuation $\tau_c=l_c/v_F \ll \tau_0$).

The solution to Eq. (6) is obtained by multiplying it by m_r/m, which causes the fluctuating frequency $\omega_r=\omega m_r/m$ to appear in this equation. In the equation of motion (4) we introduce the averaged effective mass, and also the sum of the average Lorentz force and a random force due to fluctuations in the dispersion law (3):

$$\mathbf{F}_{\mathbf{p}r}=\frac{e}{mc}[\mathbf{pH}]-\frac{m_r}{m}\nabla E_{\mathbf{p}r}. \qquad (7)$$

Using the path $\mathbf{p}_t$, $\mathbf{r}_t$ determined from this equation (with the initial conditions $\mathbf{p}_{t=0}=\mathbf{p}$, $\mathbf{r}_{t=0}=\mathbf{r}$), let us write the solution to Eq. (3) in the form

$$\chi_{\mathbf{p}r\omega}=\frac{e}{m}\int_{-\infty}^{0}dt\exp\left[\frac{t}{\tau_0}-i\int_0^t dt'\,\omega(\mathbf{r}_{t'})\right](\mathbf{E}_{\mathbf{r}(t)\omega}\mathbf{p}_t). \qquad (8)$$

Perturbations of (8) due to the small random force are found to be unimportant (for an estimate see the Appendix) compared to fluctuations of ω_r encountered by an electron during its drift along a rectilinear or oscillating path in the magnetic field. Therefore, the randomly nonuniform quantities enter into $\mathbf{j}_{r\omega}$ through $f(E_{\mathbf{p}r})$ and ω_r, so that after averaging with respect to fluctuations we obtain the following expression for the conductivity tensor $\sigma_{\mu\nu}$ ($\mathbf{k}$, ω) (which determines the usual coupling between the Fourier components of the current density $\mathbf{j}_{\mathbf{k}\omega}$ and the field $\mathbf{E}_{\mathbf{k}\omega}$):

$$\sigma_{\mu\nu}(\mathbf{k},\omega)=\frac{e^2}{m}\frac{2}{V}\sum_{\mathbf{p}}\int_{-\infty}^{0}dt\exp\left(\frac{t}{\tau_0}+i\mathbf{k}\mathbf{r}_t\right)$$
$$\times\left\langle\left[-\frac{df(E_{\mathbf{p}r})}{dp_\mu}\right]\exp\left[-i\int_0^t dt'\,\omega(\mathbf{r}_{t'})\right]\right\rangle p_\nu(t), \qquad (9)$$

for estimation purposes we will assume that the correlator $\langle\delta_r\delta_{r'}\rangle$ that appears in this expression as a result of averaging is Gaussian. In calculating $\mathbf{E}_{\mathbf{k}\omega}$ we can omit the contribution of fluctuations to the current density (5) compared to fluctuations in the dielectric permittivity of the lattice $\delta\varkappa_r=\varkappa_r-\varkappa$, $\varkappa=\langle\varkappa_r\rangle$, as long as we are far from the plasma resonance. As a result, the propagation of electromagnetic radiation is described by the wave equation

$$[\mathbf{k}[\mathbf{k}\mathbf{E}_{\mathbf{k}\omega}]]+\left(\frac{\omega}{c}\right)^2\left[\varkappa+i\frac{4\pi}{\omega}\hat{\sigma}(\mathbf{k},\omega)\right]\mathbf{E}_{\mathbf{k}\omega}$$
$$+\left(\frac{\omega}{c}\right)^2\frac{1}{V}\sum_{\mathbf{k}'}\delta\varkappa_{\mathbf{k}-\mathbf{k}'}\mathbf{E}_{\mathbf{k}'\omega}=0, \qquad (10)$$

which will be discussed in Sec. 4 for the case of small fluctuations after we calculate the tensor (9).

3. SPECTRAL DEPENDENCE OF THE ABSORPTION COEFFICIENT

Limiting ourselves to the case of a strongly-degenerate electron gas and writing the Fermi distribution in the form

$$\theta(\varepsilon_F-E_{\mathbf{p}r})=\frac{1}{2\pi}\int_{-\infty}^{\varepsilon_F}d\mu\int_{-\infty}^{\infty}ds\exp[-is(\mu-E_{\mathbf{p}r})], \qquad (11)$$

we will use the following result in averaging (9) for the random functional in the exponent:[5]

$$\left\langle\exp\left(\int d\mathbf{R}F_{\mathbf{R}}\delta_{\mathbf{R}}\right)\right\rangle$$
$$=\exp\left[\frac{1}{2}\int d\mathbf{R}'\int d\mathbf{R}''F_{\mathbf{R}'}W(|\mathbf{R}'-\mathbf{R}''|)F_{\mathbf{R}''}\right], \qquad (12)$$

where $W(R)=\bar{\delta}^2\exp[-(R/l_c)^2]$ is the Gaussian correlator and $\bar{\delta}$ is the mean-square amplitude of the fluctuations. From this we see that (9) is transformed to the form

$$\sigma_{\mu\nu}(\mathbf{k},\omega)=\left(\frac{e}{m}\right)^2\frac{2}{V}\sum_{\mathbf{p}}p_\mu\int_{-\infty}^{0}dt\,p_\nu(t)\times\exp\left[\frac{t}{\tau_0}-i\omega t+i\mathbf{k}\mathbf{r}_t-(\alpha\omega)^2\Phi(t)\right](\varepsilon_F-\varepsilon),\qquad(13)$$

$$\Delta(\varepsilon_F-\varepsilon)=-\frac{d}{d\varepsilon}\frac{1}{\pi^{1/2}}\int_{\zeta^*}^{\infty}d\zeta\exp\left[\left(i\zeta-\frac{\alpha\omega\chi_{\varepsilon t}}{2\Psi_\varepsilon^{1/2}}\right)^2\right],\qquad(14)$$

where we have introduced the variable of integration $\zeta=(\mu-\varepsilon)/\Psi_\varepsilon^{1/2}$, $\zeta^*=(\varepsilon-\varepsilon_F)/2\Psi_\varepsilon^{1/2}$ and the function

$$\Phi(t)=\frac{1}{2}\int_0^t dt'\int_0^t dt''\,W(|\mathbf{r}_{t-t'}-\mathbf{r}_{t-t''}|),\qquad(15)$$

determined by the contribution of fluctuations to the frequency $\omega(\mathbf{r})$. This function cuts off the integral (13) for large times, which leads to the absorption discussed below. The fluctuations in the distribution of carriers in (5), and also the intermixing of fluctuations in the distribution function and frequency in (14), are caused by the factors Ψ_c and $\chi_{\varepsilon t}$:

$$\Psi_\varepsilon=\frac{\delta^2}{2}\left[\alpha^2(\varepsilon-\varepsilon_F)^2+12\alpha(\varepsilon_F-\varepsilon)W_0\left(\frac{r_0}{l_c}\right)^2+60W_0^2\left(\frac{r_0}{l_c}\right)^4\right],\qquad(16)$$

$$\chi_{\varepsilon t}=\int_0^t dt'\,W(|\mathbf{r}_{t-t'}-\mathbf{r}|)\left\{\alpha(\varepsilon_F-\varepsilon)-2W_0\left(\frac{r_0}{l_c}\right)^2\times\left[2\left(\frac{\mathbf{r}_{t-t'}-\mathbf{r}}{l_c}\right)^2-3\right]\right\}.$$

For $\alpha\varepsilon_F\ll W_0(r_0/l_c)^2$ the smearing of the Fermi distribution given by (14) is estimated by the quantity $\Delta\varepsilon=4\cdot15^{1/2}W_0(r_0/l_c)^2$, and for $\varepsilon-\varepsilon_F>\Delta\varepsilon$ the integral in (14) is exponentially small; for the case $\varepsilon_F-\varepsilon>\Delta\varepsilon$ it is close to unity, so that $\Delta(\varepsilon_F-\varepsilon)$ can be replaced by a δ function. In the opposite case $\alpha\varepsilon_F\gg W_0(r_0/l_c)^2$ we will use the relation $2\Psi_\varepsilon^{1/2}\approx2^{1/2}\bar{\delta}|\varepsilon_F-\varepsilon|$; then the lower limit of the integral (14) equals sign $(\varepsilon_F-\varepsilon)/2^{1/2}\bar{\delta}$, and once again $\Delta(\varepsilon_F-\varepsilon)$ can be replaced by a δ function.

Turning to an investigation of the spectral dependence of the absorption, let us continue to calculate (13) for $\mathbf{k}=0$ in the absence of a magnetic field. Neglecting random forces in the equations of motion (4) and (7) (for a discussion of this, see the Appendix), let us use a rectilinear path $\mathbf{p}_t=\text{const}$, $\mathbf{r}_t=\mathbf{r}+\mathbf{v}_\mathbf{p}t$. Within these approximations (15) contains only v_F, and the summation over momentum in (13) gives the electron concentration averaged over fluctuations. Introducing the time of flight across a fluctuation $\tau_c=l_c/v_F$ for an isotropic conductive σ_ω [here $\sigma_{\mu\nu}(\omega)=\sigma_\omega\delta_{\mu\nu}$] we obtain

$$\sigma_\omega=\frac{e^2n}{m}\tau_c\Psi(\omega\tau_c),\qquad\Psi(\Omega)=\int_0^\infty d\tau\exp[\Omega(i\tau-\gamma\Phi_\tau)],\qquad(17)$$

where the function $\Psi(\Omega)$ is given in dimensionless variables ($\Omega=\omega\tau_c$ is a dimensionless frequency, $\gamma=(\bar{\delta}\alpha)^2\Omega/2$), so that in place of (15) we have

$$\Phi_\tau=\int_0^\tau dx'\int_0^\tau dx''\exp[-(x'-x'')^2]\approx\begin{cases}\tau^2, & \tau\ll1\\ \pi^{1/2}\tau, & \tau\gg1\end{cases}.\qquad(18)$$

In deriving (17) we have also assumed $\tau_c\ll\tau_0$, i.e., we consider the usual mechanisms for absorption to be insignificant. For $\Omega\ll1$ the basic contribution to the integral (17) is given by the asymptotic form of (18) at large times, and we obtain the spectral dependence $\Psi(\Omega)\approx i/\Omega+\gamma\pi^{1/2}/\Omega$. For $\Omega\gg1$ we calculate $\Psi(\Omega)$ by the saddle-point method. The saddle point iz_γ is determined by the transcendental equation

$$1-2\gamma\int_0^{z_\gamma}dy\,e^{y^2}=0,\qquad(19)$$

where the integral that enters in here is tabulated in Ref. 6. Let us deform the contour of integration as shown in Fig. 1. In the integral along the portion $(0,z_\gamma)$ of the imaginary axis the basic contribution comes from the neighborhood of zero (up to terms of order Ω^{-2} this contribution equals i/Ω, i.e., it determines the plasma frequency), while the remaining contour contains half the contribution from the saddle point. In calculating the exponent in the exponential of the saddle-point contribution, we use (19) to obtain

$$\Phi_{iz_\gamma}=1-\exp(z_\gamma^2).\qquad(20)$$

From this we see that the correction to Im σ_ω arising from compositional fluctuations, which determines the plasma frequency, is small, and for the spectral dependence of the absorption coefficient arising from the fluctuation mechanism, introducing the usual relation

$$\alpha_\omega=\frac{4\pi}{c\varkappa^{1/2}}\operatorname{Re}\sigma_{\mu\mu}(0,\omega)=\Omega_p^2\tau_c\frac{\varkappa^{1/2}}{c}\Psi(\omega\tau_c),$$

we have

$$\alpha_\omega\approx\frac{\Omega_p}{c}(\pi\varkappa)^{1/2}\times\frac{\tau_c}{2}\begin{cases}(\bar{\delta}\alpha)^2, & \omega\tau_c\ll1,\\ \exp[\Omega\gamma(1-\exp z_\gamma^2)-z_\gamma/2]/(\Omega\gamma)^{1/2}, & \omega\tau_c\gg1,\end{cases}\qquad(21)$$

where Ω_p is the plasma frequency. These asymptotic forms of the spectral dependence are shown in Fig. 2, along with numerical calculations of the integral (17), for the intermediate spectral regions $\omega\sim\tau_c^{-1}$. For the limiting high-frequency case $\gamma\gg1$ we may assume $z_\gamma\approx\frac{1}{2}\gamma$ and write the

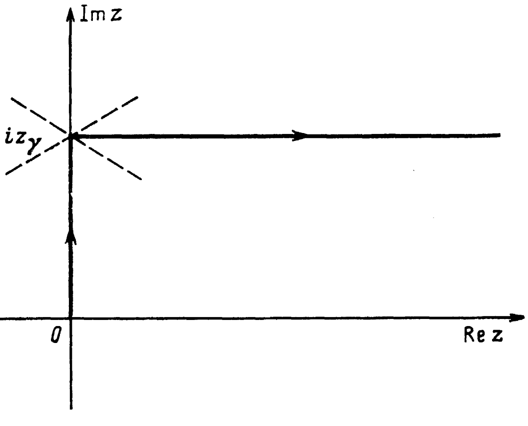

FIG. 1. Integration contour used to calculate (17).

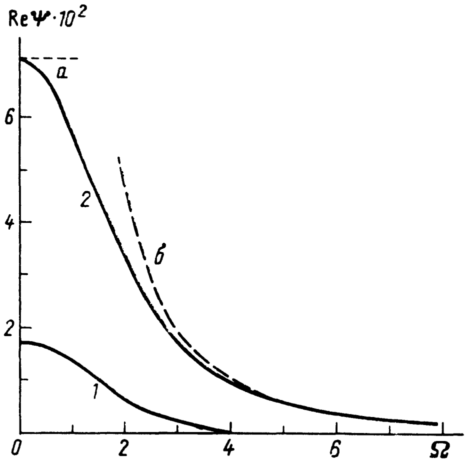

FIG. 2. The function $\mathrm{Re}\,\Psi(\Omega)$ that determines the spectral dependence of α_ω for $(\bar{\delta}\alpha)^2/2 = 0.1$ (curve *1*) and 0.2 (curve *2*); the dashed curve corresponds to the asymptotic form (21) (a—$\omega\tau_c \ll 1$, b—$\omega\tau_c \gg 1$).

simpler spectral dependence

$$\alpha_\omega \approx \frac{\Omega_p}{c\omega}(\pi\varkappa)^{1/4} f(2^{1/2}\bar{\delta}\alpha),$$

$$f(x) = x^{-1}\exp(-x^{-2}), \quad \omega\tau_c \gg \frac{2}{(\bar{\delta}\alpha)^2}. \tag{22}$$

However, in this particular frequency range the fluctuation mechanism is found to be ineffective because $\bar{\delta} \ll 1$; in this case (22) does not depend on l_c, since a total shutdown of the fluctuation mechanism occurs once l_c has increased to the point where the condition $\tau_c \ll \tau_0$ given above is violated [i.e., when l_c is comparable to the mean-free path, at which point we must treat the electron motion as diffusive rather than collisionless as assumed in (18)].

Let us compare the mechanism proposed here with the standard results for Drude absorption for $\omega\tau_0 \gg 1$, where in place of (21) we obtain

$$\alpha_\omega = \Omega_p^2 \varkappa^{1/2}/c\omega^2\tau_0.$$

Since (21) falls off exponentially at high frequencies, this contribution is superposed on the collisional mechanisms for $\omega\tau_0 \sim 1$, where the following condition is fulfilled

$$\tau_c/\tau \ll (\bar{\delta}\alpha)^2/2. \tag{23}$$

Therefore, the high-frequency asymptotic form [determined by relations (22) or (21)] goes over to the expression for the Drude absorption earlier than our discussion of the solution (6) in the diffusion regime would require. Let us also note that the region of applicability of the low-frequency asymptotic form (21) is bounded for small ω due to the necessity of including a contribution from the random forces entering into (17) (see the inequality in the Appendix).

4. LANDAU DAMPING IN A SEMICONDUCTOR WITH FLUCTUATIONS IN ITS DIELECTRIC PERMITTIVITY

In the spectral region $\omega \sim v_F/l_c$ a peak is superposed on the background absorption determined by expression (21). This peak is caused by the effective damping of the fluctuating component of the field due to the Landau mechanism. In describing this situation we will use the standard expressions[7] for the longitudinal and transverse conductivities $\sigma_{l,t}(k,\omega)$ obtained from Eq. (11) without including the contribution of fluctuations [the small shift and the smearing of the threshold $\omega < k(2\varepsilon_F m)^{1/2}$ due to the factors in (16) are not important].

The electric field of the radiation $\mathbf{E}_{\mathbf{k}\omega}$ contains an average component $\langle \mathbf{E}_{\mathbf{k}\omega} \rangle$ determined by Eq. (10) and a fluctuating component $\delta\mathbf{E}_{\mathbf{k}\omega}$. From a second-order calculation in $\delta\varkappa$ we find for the fluctuating field[2]

$$\delta\mathbf{E}_{\mathbf{k}\omega} = \frac{1}{k^2 V}\sum_{\mathbf{k}'} \delta\varkappa_{\mathbf{k}-\mathbf{k}'}\left\{\frac{[\mathbf{k}[\mathbf{k}\langle\mathbf{E}_{\mathbf{k}'\omega}\rangle]]}{\varkappa-(ck/\omega)^2+i\cdot 4\pi\sigma_t(k,\omega)/\omega} - \frac{\mathbf{k}(\mathbf{k}\langle\mathbf{E}_{\mathbf{k}'\omega}\rangle)}{\varkappa+i\cdot 4\pi\sigma_l(k,\omega)/\omega}\right\}. \tag{24}$$

In this expression the first term in the curly brackets describes the transverse scattered field; the electronic contribution σ_t to this term leads only to a shift in the pole away from the real axis. In the second term, inclusion of the electron contribution σ_l is fundamental, because it describes damping of the longitudinal component of the fluctuating field according to the Landau mechanism. The effective dielectric permittivity is obtained from (10) and (24) in the usual way[8] and its transverse component is given by the expression

$$\varkappa_t(k,\omega) = \varkappa + i\cdot 4\pi\sigma_t(k,\omega)/\omega - \chi^2[Q_S(k,\omega)+Q_L(k,\omega)],$$

$$\mathrm{Im}\,Q_S(k,\omega) = \frac{\pi}{2V}\sum_{\mathbf{k}'}\langle\delta_{\mathbf{k}}\delta_{\mathbf{k}'}\rangle\delta\left(\varkappa-\left(\frac{ck'}{\omega}\right)^2\right)\left[1+\frac{(\mathbf{k}\mathbf{k}')^2}{k^2k'^2}\right], \tag{25}$$

$$\mathrm{Im}\,Q_L(k,\omega) = \frac{2\pi}{\varkappa^2 V}\sum_{\mathbf{k}'}\langle\delta_{\mathbf{k}}\delta_{\mathbf{k}'}\rangle\left[1-\frac{(\mathbf{k}\mathbf{k}')^2}{k^2k'^2}\right]\frac{\sigma_l(k',\omega)}{\omega}$$

in which the rate of change of the dielectric permittivity χ is included through the relation $\delta\varkappa_{\mathbf{k}} = \chi\delta_{\mathbf{k}}$, while the contributions $\mathrm{Im}\,Q_s$ and $\mathrm{Im}\,Q_L$ that arise from the first and second terms in the curly brackets of (24) respectively describe the damping of the average field $\langle\mathbf{E}_{\mathbf{k}\omega}\rangle$ due to scattering by fluctuations $\delta\varkappa$ and by electron Landau damping. The calculation of Q_L in the long-wavelength approximation $kl_c \ll 1$ gives

$$\mathrm{Im}\,Q_L(k,\omega) \approx \frac{\bar{\delta}^2}{\varkappa}\left(\frac{\Omega_p}{\omega}\right)^2 I\left(\frac{l_c\omega}{2v_F}\right), \quad I(x) = 2\pi^{1/2}x^3\,\mathrm{Ei}(-x^2), \tag{26}$$

where $I(x)$ is expressed in terms of the exponential integral function; a plot of this function is given in Fig. 3. The contribution of $\mathrm{Im}\,Q_s$ to (25) was described earlier[9] and in order of magnitude equals $\bar{\delta}^2(\omega l_c/c)^2$. Therefore, in the low-frequency spectral region $\omega l_c/v_F \sim 1$ (where the electronic contribution is largest) we obtain the estimate $\mathrm{Im}\,Q_s \sim \bar{\delta}^2(v_F/c)$, and the scattering processes are not important in this region. Comparison of the amplitude of the peak (26) with the plateau in the absorption coefficient determined by expression (21) shows that it is sharply defined if

$$\chi^2 I\left(\frac{l_c\omega}{2v_F}\right) > \alpha^2.$$

The analogous expression (25) for the longitudinal dielectric permittivity determines the dispersion of the plasma oscillations in the nonuniform semiconductor alloy. Its dis-

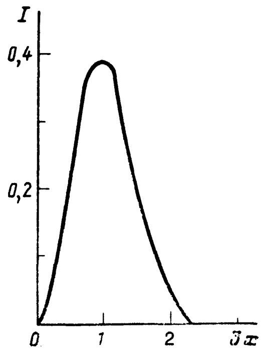

FIG. 3.

tinctive features [e.g., the change in the plasmon frequency for $kl_c \sim 1$ due to the dispersion $\varkappa_l(k, \omega)$] can be observed directly in Raman scattering of infrared radiation. A detailed discussion of this situation requires a special investigation.

5. CYCLOTRON RESONANCE LINE SHAPE

When the magnetic field **H** is parallel to the Z axis, Eq. (13) for the complex conductivity tensor contains paths $\mathbf{p}_t$, $\mathbf{r}_t$ that oscillate at the cyclotron frequency in a plane perpendicular to the magnetic field (we limit ourselves here to the case of strongly degenerate electrons).[7] If kv_F is small compared to the width of the resonance line obtained below we can neglect the spatial dispersion in (9). Following the procedures of Sec. 3, we can also omit the contribution of the factors of (16); then this description of the complex conductivity tensor differs from (17) only by an additional angular average over the Fermi surface (in what follows we put $\xi = \cos\vartheta$, where ϑ is the angle between **p** and **H**). We limit ourselves here to an investigation[1)] of the coefficient of cyclotron absorption $\alpha_\perp(\omega)$ (i.e., when the microwave radiation is polarized perpendicular to **H**); then in the resonance approximation $|\omega - \omega_c|/\omega \ll 1$ we obtain

$$\alpha_\perp(\omega) = \frac{\Omega_p^2\tau_c}{4c}\varkappa^{1/2}\int_0^1 d\xi\,(1-\xi^2)\int_{-\infty}^{\infty} dt\,\exp[i\Delta\Omega\tau - \gamma\Omega\Phi(\xi,t)], \tag{27}$$

where $\Delta\Omega = (\omega - \omega_c)\tau_c$ is the dimensionless frequency detuning. The cutoff factor $\Phi(\xi,t)$ due to frequency fluctuations, which in contrast to (15) and (18) depends on the angular variable ξ and the dimensionless cyclotron frequency $\Omega_c = \omega_c\tau_c$, is determined by the relation

$$\Phi(\xi,t) = \int_0^t dx' \int_0^t dx'' \exp\Big\{-(x'-x'')^2\xi^2 - \frac{2}{\Omega_c^2}[1-\cos(\Omega_c(x'-x''))](1-\xi^2)\Big\}. \tag{28}$$

In the high-frequency approximation Ω, $\Omega_c \gg 1$ we can neglect the small oscillatory contribution of the last term in the exponent (28), so that the cutoff factor arises from motion along the magnetic field. Introducing the variable of integration $\tau = t/\xi$, we obtain in place of (27)

$$\alpha_\perp(\omega) \approx \frac{\Omega_p^2\tau_c}{4c}\varkappa^{1/2}\int_0^1 d\xi\,\frac{1-\xi^2}{\xi}\int_{-\infty}^{\infty} d\tau\,\exp\Big[i\frac{\Delta\Omega\tau}{\xi} - \frac{\gamma\Omega}{\xi^2}\Phi_\tau\Big], \tag{29}$$

and the expression for the cutoff factor coincides with (18).

Let us estimate the integral with respect to τ in (29) at the maximum of the absorption peak $\Delta\Omega = 0$ for angles small and large in comparison with $(\gamma\Omega)^{1/2}$. This leads us to the following expression at the maximum of the absorption peak:

$$\alpha_{max} \approx \frac{\Omega_p^2\tau_c}{2c}\varkappa^{1/2}\begin{cases} \frac{1}{4}\pi^{1/2}\gamma\Omega + \text{const}, & (\gamma\Omega)^{1/2} \ll 1 \\ \frac{1}{3}(\pi/\gamma\Omega)^{1/2} + o(1/\gamma\Omega), & (\gamma\Omega)^{1/2} \gg 1 \end{cases}, \tag{30}$$

so that for $\bar{\delta} \to 0$ the cyclotron resonance line shape is controlled by collisions (since $\alpha_{\max}$ in (30) diverges), while for large compositional fluctuations (i.e., for $(\gamma\Omega)^{1/2} \gg 1$) the resonance is suppressed. An order-of-magnitude estimate of the width of the resonance line can be obtained by analyzing its moments[10] (since the peak is symmetric, only the even moments are found to be nonzero):

$$\alpha_{2m} = \int_{-\infty}^{\infty} d\Delta\Omega\,(\Delta\Omega)^{2m}\alpha_\perp \Big/ \int_{-\infty}^{\infty} d\Delta\Omega\,\alpha_\perp. \tag{31}$$

From (29) we obtain in the high-frequency approximation

$$\alpha_2 = \gamma\Omega,\quad \alpha_4 = 4\gamma\Omega/5 + 12(\gamma\Omega)^2, \ldots, \tag{32}$$

so that the half-width of the line is a quantity of order $(\gamma\Omega)^{1/2}$, while its shape is intermediate between Gaussian and Lorentzian, approaching the latter for small $\gamma\Omega$ (however, for $|\Delta\Omega| \gg 1$ an asymptotic estimate analogous to that presented in Sec. 3 gives exponentially decaying wings to the line). The results of numerical calculations of the two-dimensional integral in (29) are shown in Fig. 4. Like Eqs. (30) and (32), this plot shows the changes in the line shape (decreasing peak amplitude and broadening) as the classical magnetic field increases.

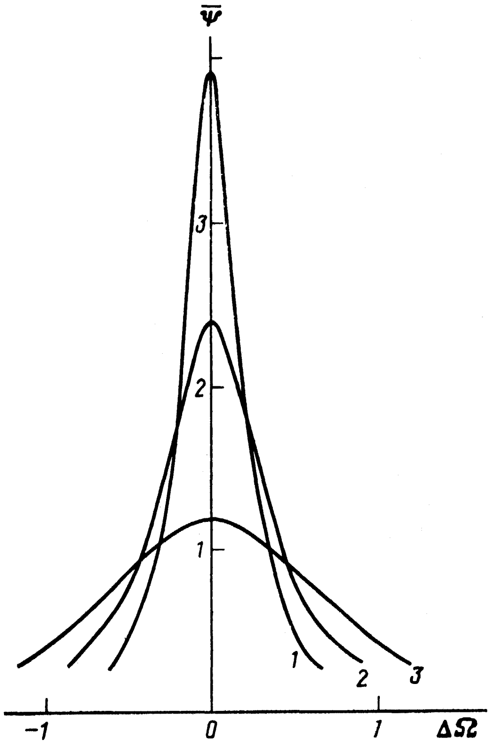

FIG. 4. Dimensionless function $\Psi(\Delta\Omega) = \alpha_\perp(\omega)4c/\Omega_p^2\tau_c\varkappa^{1/2}$ determined by Eq. (29) for $\gamma\Omega = 0.05$ (curve *1*), 0.1 (curve *2*), 0.3 (curve *3*).

6. CONCLUSION

The presence of unscreened long-wavelength kinetic-energy fluctuations and dielectric permittivity fluctuations in spatially-inhomogeneous substitutional semiconductor alloys qualitatively changes the character of the propagation of far-infrared radiation in these materials.[2)] The high-frequency asymptotic behavior of the absorption coefficients (20) and (22) is found to be slower than the usual dependence $\alpha_\omega \propto \omega^{-2}$ [note that for $\hbar\omega > \bar{\varepsilon}$ this dependence becomes sharper, since τ_0 now depends on ω; however, Eqs. (6) and (21) do not change in the quantum frequency limit], while for $\omega \sim v_F/l_c$ a peak appears in (26) caused by damping of longitudinal fluctuations in the radiation field due to the Landau mechanism. In addition, the line shapes of resonance magneto-optic effects begin to depend on the classical magnetic field. These features, along with the scatter in the magnitudes of these effects for different samples (or their dependence on the preparation technology), offer the possibility of experimentally identifying the mechanism described in this paper.

The quantitative estimates discussed in this paper for these effects in various materials are determined by the amplitude $\bar{\delta}$ and the correlation length l_c of the compositional fluctuations. These parameters were estimated in Ref. 4 for $Al_xGa_{1-x}As$ from experiments on the spin relaxation of electrons.[12] For these alloys the low-frequency asymptotic form (21) gives values of order 1 to 10 cm^{-1} for electron concentrations $\sim 10^{17}$ cm^{-3} (which already exceeds the Drude absorption at $\lambda_r \sim 10$ μm), whereas in the high-frequency region the fluctuation mechanism is ineffective because $\bar{\delta} \ll 1$. Note that for narrow-gap materials (e.g., for $Cd_xHg_{1-x}Te$) the effect is enhanced; however, it must be measured in the long-wavelength spectral region. At liquid-helium temperatures it is possible to observe broadening of the cyclotron resonance line in the microwave region $\lambda_r \sim 1$ mm, which is evidence that the mechanism discussed here operates in these materials.

APPENDIX

Inclusion of a random force in (7) causes the following correction to the paths resulting from the equation of motion (4) (where $\overline{\Delta\varepsilon} = \max[\Delta\varepsilon, T]$):

$$\delta\mathbf{p}_t = -\alpha\overline{\Delta\varepsilon}\int_0^t dt'\nabla\delta(\mathbf{r}_{t'}), \quad \delta\mathbf{r}_t = -\frac{\alpha\overline{\Delta\varepsilon}}{m}\int_0^t dt'\int_0^{t'} dt''\,\nabla\delta(\mathbf{r}_{t''}), \tag{A1}$$

and $\mathbf{r}_t = \mathbf{r} + \mathbf{v}_p t$; in a strong magnetic field the path r_t oscillates in the plane perpendicular to the z axis, and an oscillatory factor $\exp[\pm i(\omega - \omega_c)(t - t')]$ enters under the integral sign in (A1). The contribution $\delta\mathbf{p}_t$ in (9) is small, and we can neglect the random force if it changes the frequency fluctuations in the exponent only slightly. When the following condition is fulfilled

$$l_c^2 \gg \langle \delta r_t^2 \rangle \tag{A2}$$

we can expand $\delta(\mathbf{r}_t)$ in this exponent with respect to $\delta\mathbf{r}_t$; then an additional factor appears in (13) and (27):

$$\exp\left[-i\alpha\omega\int_0^t dt'\langle\nabla\delta(\mathbf{r}_{t'})\delta\mathbf{r}_{t'}\rangle\right], \tag{A3}$$

which does not change the character of the cutoff for the function (15), and should be compared to $\exp(-i\omega t)$ [or to $\exp[-i(\omega - \omega_c)t]$ in (27)]. From this we see that for the maximum times $t_{\max}$ that determine the cutoffs in (13) and (27) we have the inequality

$$\omega t_{max} \gg \frac{\alpha^2\overline{\Delta\varepsilon}}{m}\int_0^{t_{max}} dt\int_0^t dt'\int_0^{t'} dt''\langle\nabla\delta(\mathbf{r}_t)\nabla\delta(\mathbf{r}_{t''})\rangle, \tag{A4}$$

which translates into the condition $1 \gg (\bar{\delta}\alpha)^2\,\overline{\Delta\varepsilon}/\bar{\varepsilon}$; this condition does not limit the discussion given in this paper. In strong magnetic fields only the z component of the velocity enters from $\mathbf{r}_t$, which as before is estimated in terms of $\bar{\varepsilon}$, while the left-hand side of (A4) contains the difference $|\omega - \omega_c|$ which is on the order of the width of the cyclotron resonance line. In this case (A4) reduces to the condition $1 \gg \bar{\delta}\alpha\,\overline{\Delta\varepsilon}/\bar{\varepsilon}$, which is also fulfilled for small fluctuations. Due to the inequality (A2), the same limitations arise for the high-frequency region (where $\alpha_\omega \propto \omega^{-1}$). In the low-frequency region (where $\alpha_\omega \propto$ const), (A2) gives a more rigorous lower bound on the frequency integral:

$$v_F/l_c\omega < (\delta\alpha)^{2/3}(\bar{\varepsilon}/\overline{\Delta\varepsilon})^{1/3}.$$

However, for low frequencies the Drude absorption contribution dominates (see the inequality from Sec. 3) and fluctuations in the trajectory may be neglected if

$$(\delta\alpha)^{4/3}\omega\tau_0 < (\bar{\varepsilon}/\overline{\Delta\varepsilon})^{1/3}.$$

This condition is usually fulfilled because the spectral integral, which is where the random forces should enter into (7), is absent.

1) Because of the fluctuating cyclotron frequency, the absorption of the parallel magnetic field component of the radiation also changes; including the fluctuations of the dielectric permittivity will transform this component into a transverse resonance absorption for frequencies close to ω_c.

2) We note here an analogy between the effect of absorption due to fluctuations in the effective mass and the mechanism introduced in Ref. 11 for infrared transitions in a quantum well under the action of a radiation field parallel to the $2D$ layer caused by inhomogeneities in the velocity operator along the layer thickness.

1. B. I. Shklovskii and A. L. Éfros, *Electronic Properties of Doped Semiconductors*, Springer, New York (1984).
2. S. N. Rytov, Yu. A. Kravtsov, and V. I. Tatarskii, *Principles of Statistical Radiophysics*, Springer, New York (1987).
3. A. V. Marshak and C. M. van Vliet, Proc. IEEE **72**, 5 (1984).
4. F. T. Vas'ko and Yu. N. Soldatenko, Zh. Eksp. Teor. Fiz. **92**, 199 (1987) [Sov. Phys. JETP **65**, 112 (1987)].
5. V. L. Bonch-Bruevich, *Statistical Physics and Field Theory*, p. 337. Nauka, Moscow, 1973.
6. E. Jahnke, F. Emde, and F. Lösch, *Tables of Higher Functions*, McGraw-Hill, New York (1960).
7. E. M. Lifshits and L. P. Pitaevskii, *Physical Kinetics*. Nauka, Moscow, 1979 [Pergamon Press, Oxford, 1981].
8. L. D. Landau and E. M. Lifshits, *Electrodynamics of Continuous Media*. Nauka, Moscow, 1982 [Pergamon Press, Oxford, 1960 (transl. of 1st ed.)].
9. Y. A. Ryzhov and V. V. Tamoikin, Izv. vuzov Radiofiz. **13**, 356 (1970) [in Russian].
10. R. M. White, *Quantum Theory of Magnetism*, McGraw-Hill, New York (1970).
11. F. T. Vas'ko and Yu. N. Soldatenko, Fiz. Tekh. Poluprovodn. **20**, 920 (1986) [Sov. Phys. Semicond. **20**, 579 (1986)]; A. Ya. Shik, Fiz. Tekh. Poluprovodn. **22**, 1843 (1988) [Sov. Phys. Semicond. **22**, 1165 (1988)].
12. A. H. Clark, R. D. Byrnham, D. J. Chadi, and R. M. White, Phys. Rev. B **12**, 5758 (1975).

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