Nonstationary double layer in electron emission into a dielectric medium

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The behavior of the electric double layer produced in the nonstationary electron emission from metals into a dielectric medium was studied. The parameters of the electron cloud and their time evolution were determined. The efficiency of conversion of the double layer energy into electromagnetic radiation was studied.

Recently, a considerable theoretical and experimental effort has been focused on the formation and evolution of double electron layers.[1-10] The interest in this problem is related to the effect of these double layers on the physical processes occurring in plasma,[2-4] solid state electronics,[1] laser photoemission,[5] and in the generation of spontaneous electric and magnetic fields.[6-9]

The present theoretical work is focused on the time-dependent problem of the behavior of the double electric layer formed in the surface electron emission from metals into dielectric materials. The motion of the emitted electrons is described by their drift in the self-consistent electric field and by diffusion. Unlike the problems studied in the references cited above (with the exception of Ref. 9), this problem is significantly nonstationary. This makes it possible to study the conversion of the energy spent in the formation of the double layer into radiative energy. Reference 8 solved the time-dependent problem of dipole emission from the layer formed during short-pulse photoemission into vacuum. The same formalism was also applied to the time-dependent processes in MDM-structures. So, for example, Lampert and Mark[1] calculated the resistance of the dielectric to an alternating current, and studied the distribution of high-amplitude charge pulses. Unlike the work of Lampert and Mark,[1] in which a closed circuit with an external source was treated, and the total current was determined by the difference between the electron ("drift" + "diffusion") currents and the displacement current, in the present work the displacement current cancels out the sum of the two electron current components.

Let us consider a plane semi-infinite layer of a uniform dielectric medium ($x>0$) interfacing a metallic surface ($x=0$), and an electron current $J_e(t)$ emitted from the metal surface, due, for example, to a photoemission process. The system of equations that describes the electron distribution and the electric field inside the dielectric is

$$E_t=4\pi eJ/\varepsilon, \quad E_x=-4\pi en/\varepsilon,$$
$$\varphi_x=-E, \quad J=-\mu enE-\mu kTn_x, \tag{1}$$

where E is the electric field intensity, φ is the electric potential, n is the electron density, J is the electron current, ε is the dielectric permittivity ($\varepsilon=$ constant), μ is the mobility, and μkT is the diffusion coefficient.

Within the range of validity of the diffusion approximation, in which $J(t)\ll J_e(t)$ holds and the characteristic emission time is much larger than the electron collision time, the boundary conditions for the system (1) are determined by the given density of emitted electrons on the boundary and by the normalization of the potential:

$$n(x=0,\ t)=J_e(t)/\langle u\rangle, \ \varphi(x=0,\ t)=0, \quad E(x=\infty,\ t)=0, \tag{2}$$

where $\langle u\rangle$ is the average velocity of the emitted electrons. Initially, the field intensity and the electron density inside the material are equal to zero.

Equations (1) and (2) determine completely the time-dependent fields $E(x,t)$ and densities $n(x,t)$ when $J_e(t)$, $\langle u\rangle$, and the parameters ε, μ, and T are given. It is convenient to introduce dimensionless functions and variables. Let n_0 be an arbitrary electron density describing the density of emitted electrons. All the other quantities are then determined as follows:

$$t_0=\omega_0^{-1}=(4\pi e^2n_0/m_e)^{-1/2}, \quad x_0=(\mu kTt_0)^{1/2},$$
$$E_0=4\pi ex_0n_0/\varepsilon, \quad \varphi_0=x_0E_0. \tag{3}$$

In dimensionless form, Eqs. (1) and (2) become

$$\varphi_{xt}=\gamma_0nE+n_x, \quad E_x=-n,$$
$$\varphi_x=-E, \quad \gamma_0=m_e\omega_0^2x_0^2/\varepsilon kT, \tag{4}$$

where all quantities E, n, φ, x, t, and γ_0 are dimensionless. The significance of the parameter γ_0 is the following. If the thermalization time of the emitted electrons is large compared to the time of change of $J_e(t)$, which may be the case for gases of sufficiently low density, then kT represents the characteristic emission temperature. Consequently, we have $\gamma_0\approx\varepsilon^{-1}t_ct_0^{-1}$, where t_c is the collision time of the emitted electrons, determined by their average velocity $\langle u\rangle$. Alternatively, in the case of crystal dielectrics, the electrons start to diffuse with a thermal velocity $v_c^T<\langle u\rangle$, and kT represents the temperature of the medium. In this case, however, the range of emission currents is such that heating of the dielectric can be neglected.

Let us consider first the stationary case, in which the effect of the way the emitted electrons move on the space-time structure of the double layer can be understood. Assuming

$$J_e(t)=J_{e0}=\text{const}, \quad J=-\gamma_0nE-n_x=0,$$

one obtains

$$n_x=-\gamma_0nE, \quad E_x=-n, \quad n(0)=J_{e0}/n_0\langle u\rangle. \tag{5}$$

It is convenient to express the solutions of Eqs. (5) in dimensional form as follows:

$$E=\frac{E_{max}}{1+x/L_E}, \quad n=\frac{n_{max}}{(1+x/L_E)^2}, \quad Q=\frac{E_{max}}{4\pi},$$
$$\varphi=-E_{max}L_E\ln(1+x/L_E),$$

where

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$$L_E=x_0\left(\frac{2n_0\langle u\rangle}{J_{e0}\gamma_0}\right)^{1/2}\approx\varepsilon^{1/2}L_D,\quad L_D=\langle u\rangle/\omega_0,$$

L_D is the Debye radius of the electron plasma,

$$E_{max}=E_0(2n_0\langle u\rangle/J_{e0}\gamma_0)^{1/2}\sim 4\pi n_e eL_D/\varepsilon^{1/2},$$

Q is the total charge, and $n_{\max}=J_{e0}/\langle u\rangle$.

The time required for the stationary regime to develop, t_y, is determined from the condition

$$L_E\approx\varepsilon^{1/2}L_D\sim(\mu kTt_y)^{1/2}\sim(\langle u\rangle^2 t_c t_y)^{1/2},\quad t_y\sim\varepsilon t_0(t_0/t_c).$$

Thus, unlike the case of emission into vacuum,[5] in this case the field inside the layer decreases by a factor of $\varepsilon^{1/2}$ and the thickness of the layer increases by a factor of $\varepsilon^{1/2}$, i.e., the potential difference determined by the energy of the emitted electrons does not depend on ε. The time $t_y\sim t_0\varepsilon(t_0/t_c)\gg\varepsilon^{1/2}t_0$ (since $L_E\approx\varepsilon^{1/2}L_D\gg\langle u\rangle t_c$ and $t_0/t_c\gg\varepsilon^{-1/2}$) increases.

Numerical calculations for photoemission of electrons from a metal with work function $W=5$ eV, quantum efficiency $\sim 10^{-2}$, and photon energy ~ 10 eV give the following results:

$$J_{e0}=6\cdot10^{19}\ \text{cm}^{-2}\cdot\text{s}^{-1},\quad n_0=6\cdot10^{11}\text{cm}^{-3},\quad t_0=2{,}5\cdot10^{-11}\ \text{s}.$$

For $\mu kT\approx 10^4$ cm$^2\cdot$s^{-1}, one obtains

$$x_0=5\cdot10^{-4}\text{ cm},\quad E_0=5\cdot10^2\varepsilon^{-1}\text{W}\cdot\text{cm}^{-1},\quad \gamma_0=5\cdot10^{-2}\varepsilon^{-1},$$
$$L_E\approx 3{,}2\cdot10^{-3}\varepsilon^{1/2}\text{cm},\quad E_{max}=3{,}2\cdot10^3\varepsilon^{-1}\text{W}\cdot\text{cm}^{-1}.$$

In order to solve the time-dependent system of equations (1) and (2), one can write it as a single equation for $\varphi(x,t)$:

$$\varphi_{xt}=-\gamma_0\varphi_x\varphi_{xx}+\varphi_{xxx},\tag{6}$$

with the following boundary conditions:

$$\varphi(0,\ t)=0,\quad \varphi_{xx}(0,\ t)=n(t),\quad \varphi(x,\ 0)=0.\tag{6'}$$

The first integral of equation (6) has the form

$$\varphi_t=\varphi_{xx}-{}^1/_2\gamma_0\varphi_x{}^2+F(t),\tag{7}$$

where $F(t)$ is an arbitrary function of time. Further, by using the well known Cole–Hopf transformation[11]

$$\psi(x,\ t)=\exp[-{}^1/_2\gamma_0(\varphi-F)]\tag{8}$$

Eq. (7) is reduced to the linear thermal transport equation:

$$\psi_t-\psi_{xx}=0.\tag{9}$$

Since $F(t)$ can be specified to within a constant, let us assume $F(t=0)=0$, which, together with the initial conditions, gives

$$\psi(x,\ 0)=1,\quad \psi_x(x,\ 0)=0.\tag{10}$$

From the first equation in (6') one obtains

$$F(t)=(2/\gamma_0)\ln\ [\psi(0,\ t)],\tag{11}$$

and from the second

$$\psi_x{}^2(0,\ t)={}^1/_2[\psi_t{}^2(0,\ t)+\gamma_0 n(t)\psi^2(0,\ t)].\tag{12}$$

The function $\psi(x,t)$, which is a solution of the thermal transport equation (9), also satisfies the following equation:

$$\psi(0,t)=1-\int_0^t\psi_x(0,t-s)\frac{ds}{(\pi s)^{1/2}}.\tag{13}$$

Equations (12) and (13) determine two functions $\psi(0,t)=\psi_0(t)$ and $\psi_x(0,t)=\psi_{0x}(t)$ ($\psi_0(0)=1$, $\psi_{0x}(0)=0$). Finally, the solution of the general problem (9), (10), and (12) is reduced to the quadrature

$$\psi(x,t)=\int_0^t\psi_{0x}(t-s)\alpha(x,s)ds+\int_0^t\psi_0(t-s)\alpha_x(x,s)ds+\frac{1}{2}\Phi\left(\frac{x}{2t^{1/2}}\right)+\frac{1}{2},\tag{14}$$

where $\Phi(x)$ is the error function, and $\alpha(x,\ s)=\exp(x^2/4s)/2(\pi s)^{1/2}$.

Let us first consider the case in which the emission is switched on instantaneously:

$$n(t<0)=0,\quad n(t>0)=n^0=1.$$

Then, it is possible to calculate an approximate solution of Eqs. (12) and (13) for $t\to 0$ and $t\to\infty$:

$$\psi_0(t)=\exp(-\gamma_0 t/2)\approx 1-\gamma_0 t/2,\quad \psi_{0x}(t)=\gamma_0(t/\pi)^{1/2},\quad t<\pi/2\gamma_0,$$
$$\psi_0(t)=(2/\pi\gamma_0 t)^{1/2},\quad \psi_{0x}(t)=(\pi t)^{-1/2},\quad t>\pi/2\gamma_0.\tag{15}$$

The case of emission switched on instantaneously at time t_B, $n(t<t_B)=n^0$, $n(t>t_B)=0$, can be treated in a similar way. In this case, it is assumed that $t_B\gg\pi/2\gamma_0$. With $\tau=t-t_B$, the approximate solution of Equations (12) and (13) is:

$$\psi_0(\tau)=(2/\pi\gamma_0 t_B)^{1/2}\{1+{}^1/_2\gamma_0\tau[1-{}^4/_3(2\gamma_0\tau/\pi)^{1/2}]\},$$
$$\psi_{0x}(\tau)=[1-{}^4/_3(2\gamma_0\tau/\pi)^{1/2}]/(\pi t_B)^{1/2}.\tag{16}$$

In this case (of emission switched on and switched off instantaneously), the emitted radiation is treated in the dipole approximation. The dipole moment is given by the expression:

$$d(t)=\varphi(x,t)/4\pi\Big|_0^\infty=\varphi(\infty,t)/4\pi.$$

Calculation of the second derivative of the dipole moment in Equations (12) and (13) clearly shows that the main contribution to the radiation comes from switching off the emission. In this case, d_{tt} is given by:

$$d_{tt}(t)=(1/2\pi^2)(\pi\gamma_0/2)^{1/2}\omega_0{}^2\varphi_T(\omega_0\tau)^{-1/2},$$

and the radiation intensity by

$$P=(1/12\pi^3c^3)\gamma_0\omega_0{}^4\varphi_T{}^2S^2(\omega_0\tau)^{-1}\approx c\pi^4\varphi_T{}^2/6\gamma_0{}^2,$$

where $\varphi_T=kT/e$, and S is the area of the emitting surface, which in this case is approximately equal to $S\approx(\pi^2c/\gamma_0\omega_0)^2$. The total energy of the radiation pulse, for a square emission pulse of length approximately equal to $\pi/2\gamma_0\omega_0$, is

$$\mathscr{E}_{\text{rad}}=\int_{t_{c_1}}^{\pi/2\gamma_0\omega_0}P\,dt\approx(\pi^4c\varphi_T{}^2/6\omega_0\gamma_0{}^2)\ln[\pi e/2\gamma_0].$$

The radiation spectrum is centered within the frequency range $\Delta\omega=(2/\pi)\gamma_0\omega_0$. If the emission pulse is monitored

by a laser pulse, the energy conversion factor for the low-frequency dipole radiation is determined only by the energy of the laser photon, the work function of the irradiated surface (i.e. the emission temperature) and the quantum yield, and is equal to

$$\eta = \frac{\pi \langle u \rangle kT}{ch\nu} Y_\nu,$$

where Y_ν is the quantum yield. Assuming $h\nu = 10$ eV, $kT = 5$ eV, and $Y_\nu = 10^{-2}$ electrons/photon, the value $\eta = 10^{-4}$ is obtained, and for $q_0 = 10^4$ W/cm^2 and $t_c = 10^{-12}$ s, we have $\Delta\omega = 10^9\ \text{s}^{-1}$.

If the emission source contains a variable component:

$$n = 1 + n_1 \sin(\omega t),$$

the average radiation intensity with frequency equal to the modulation frequency is given by

$$I = \frac{(S\mu kTW_1)^2 \omega^4}{24\pi^2 c^3 \varepsilon (\omega + 2W_0)[\omega^{1/2} + (2W_0)^{1/2}]^2}$$

where

$$W_0 = (2\pi)^{3/2} e^2 \mu n_0 / \varepsilon, \quad W_1 = (2\pi)^{3/2} e^2 \mu n_1 / \varepsilon.$$

In conclusion, it should be noted that the relationships derived in this paper to within a numerical factor hold in general, since the physical mechanism for the motion of the emitted electrons is incorporated into these equations only through the dimensionless parameter γ_c. In fact, based on the results of Refs. 8 and 9, and of the present work, it is easy to demonstrate that $\gamma_0 \approx 1$ holds for emission into vacuum, $\gamma_0 \approx \omega_{\text{pl}}/\omega_0 < 1$ for emission into a collisionless plasma, and $\gamma_0 \approx \omega_0 t_c < 1$ for the diffusion motion of the electrons. In general, in this case, the efficiency of conversion of the laser energy into low-frequency radiation does not depend on the emission mechanism.

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