Influence of the linear magnetostriction on the orientation of phase boundaries in dysprosium orthoferrite

S. L. Gnatchenko, N. F. Kharchenko, K. Piotrowski,[1)] H. Szymczak,[1)] and R. Szymczak[1)]

Physicotechnical Institute of Low Temperatures, Academy of Sciences of the Ukrainian SSR, Kharkov

(Submitted 12 July 1990; resubmitted 3 October 1990)
Zh. Eksp. Teor. Fiz. **99**, 874–886 (March 1991)

A visual study of the polarization gave information on two-phase domain structures formed as a result of the first-order phase transition $\Gamma_{14}(F_zG_xG_y)\leftrightarrow\Gamma_4(F_zG_x)$ in a dysprosium orthoferrite plate cut at right-angles to the **c** axis and subjected to a field $\mathbf{H}=(0,H_y,H_z)$. The dependence of the orientation of the phase boundaries on the direction of the antiferromagnetic vector in coexisting magnetic phases was investigated. The orientations of the phase boundaries were governed by magnetostriction stresses that appeared in the crystal in the two-phase state.

Elastic stresses appear in a magnetic material split into domains with walls which are not of the 180° type.[1,2] This is due to the difference between the magnetostrictive strains coexisting in magnetic states. The magnetostrictive stresses that appear in a magnetically inhomogeneous state may influence the orientation of domain walls in a crystal.[3,4] In the case of a two-phase domain structure formed in the course of field-induced spin-orientation phase transitions in ferromagnets the phase boundaries (domain walls) are not of the 180° type. Moreover, the field-induced first-order phase transitions in antiferromagnets are usually accompanied by a large abrupt change in magnetostrictive strains. We can therefore expect the magnetoelastic interactions to have a considerable influence on the formation of an equilibrium two-phase domain structure.

The influence of the magnetostriction on the domain structure of a crystal depends on the relationship between the magnetoelastic energy and the energies of the demagnetization fields. In the case of many-sublattice magnetics the magnetoelastic interactions may play a major role in the formation of a domain structure, since the magnetic moment of a sublattice, whose magnitude determines the magnetostriction energy, is usually much greater than the net magnetic moment governing the energy of the magnetizing fields.[5]

In contrast to the uniaxial ferromagnets, for which the influence of the magnetostrictive stresses on the orientation of domain walls in a magnetic field had been studied in Refs. 3 and 4, in the case of a two-phase structure in antiferromagnets the magnetostriction linear in respect of the field can have a considerable influence on the orientation of domain walls. This linear magnetostriction may give rise to a dependence of the orientation of phase boundaries on the direction of the antiferromagnetic vectors in the coexisting phases. In the case of dysprosium orthoferrite the linear magnetostriction is comparable in magnitude with the quadratic magnetostriction in the field inducing the $\Gamma_{14}\leftrightarrow\Gamma_4$ phase transition[6] and we found that the orientation of phase boundaries changed as a result of a change in the direction of the antiferromagnetic vector in the antiferromagnetic (AFM) or weakly ferromagnetic (WFM) phases.[7]

We employed visual magnetooptic observations of two-phase domain structures formed as a result of the $\Gamma_{14}\leftrightarrow\Gamma_4$ phase transition in a field $\mathbf{H}=(0,H_y,H_z)$, to study the orientation of phase boundaries in a $DyFeO_3$ plate, cut at right-angles to the **c** axis, as a function of the direction of the AFM vectors in the coexisting phases. We analyzed the magnetostrictive stresses which appeared as a result of formation of a magnetically inhomogeneous AFM + WFM state and determined the influence of these stresses on the orientations of phase boundaries. The results of our analysis were compared with the results of visual observations of two-phase domain structures.

1. VISUAL POLARIZED-LIGHT INVESTIGATION OF TWO-PHASE DOMAIN BOUNDARIES

A magnetic field $\mathbf{H}=(0,H_y,H_z)$ induces a first-order phase transition from the AFM to the WFM state in $DyFeO_3$ at temperatures below the Morin point. The transition-inducing field H_t is a function of the temperature of a crystal and of the orientation of the external field relative to the crystallographic axes. Figure 1 shows the H–T phase diagrams obtained by visual observations of the coexisting phases of $DyFeO_3$ for different directions of **H** in the *bc* plane of a crystal. The curves in this diagram represent lines of the first-order phase transition $\Gamma_{14}(F_zG_x\cdot G_y)\leftrightarrow\Gamma_4(F_zG_x)$ ($\Gamma_1(G_y)\leftrightarrow\Gamma_4(F_zG_x)$ if $\mathbf{H}\|\mathbf{b}$) and they separate regions of existence of the AFM and WFM phases in the H–T plane.

Such two-phase AFM + WFM domain structures formed as a result of the $\Gamma_{14}\leftrightarrow\Gamma_4$ phase transition were observed visually in polarized light incident on a $DyFeO_3$ plate cut at right-angles to the **c** axis. The thickness of this plate was about 40 μm. The Morin temperature of the sample was $T_M=51.6$ K. The contrast between the domains in $DyFeO_3$ observed in polarized light was governed by the angle of rotation φ of the polarization ellipse, which depended on the Faraday rotation and on the linear birefringence. When the plane of polarization of the incident light coincided with the principal plane of the optical indicatrix of the crystal, the angle of rotation of the polarization ellipse could be determined from the relationship[8]

$$\varphi=\theta(\mathbf{F},\mathbf{G})(1-\cos\delta)+2\rho(\mathbf{F},\mathbf{G})\sin\delta/\delta, \qquad (1)$$

where θ is the angle of rotation of the optical indicatrix about the direction of propagation of light and is governed by the linear magnetooptic effect; ρ is the Faraday rotation; δ is the shift between the optical phases due to the linear birefringence; **F** and **G** are the ferromagnetic and antiferromagnetic

 0038-5646/91/030485-07$03.00

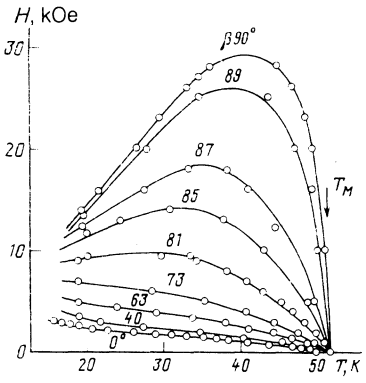

FIG. 1. Phase (H–T) diagrams of $DyFeO_3$ obtained for different orientations of **H** in the bc plane of the crystal.

vectors. In the $\mathbf{k}\|\mathbf{c}$ geometry the angle θ could be found from the relationship

$$\theta = q_1 F_z G_y + q_2 G_x G_y, \tag{2}$$

where q_1 and q_2 are the magnetooptic constants. This angle was $\theta = 0$ for the Γ_1 and Γ_4 states. The sign of Γ_{14} in the G_y state was a function of the sign of the projection of G_y, so that a contrast appeared between the AFM states $\Gamma_{14}^{+}(F_z^{+}G_x^{+}G_y^{+})$ and $\Gamma_{14}^{-}(F_z^{+}\cdot G_x^{+}G_y^{-})$, and it was possible to observe visually the AFM domains in polarized light. The contrast between the AFM and WFM domains, and also between the WFM domains $\Gamma_4^{+}(F_z^{+}G_x^{+})$ and $\Gamma_4^{-}(F_z^{-}G_x^{-})$ was created by the Faraday effect. Allowing for the ferromagnetic and antiferromagnetic contributions to the Faraday effect,[9-11] we could express in the $\mathbf{k}\|\mathbf{c}$ case the dependence of ρ on the orientations of the magnetic vectors **F** and **G** in the form

$$\rho = fF_z + qG_x, \tag{3}$$

where f and q are the magnetooptic constants. The optical phase shift δ was governed primarily by the natural birefringence, which was much stronger than the magnetic birefringence. We could therefore ignore the dependence of δ on the magnetic order and assume it to be constant.

The experimental method used in the observation of two-phase domain structures in $DyFeO_3$ using oblique fields was described in Refs. 12 and 13. An important part of the apparatus used to study these structures in fields $\mathbf{H} = (0,H_y,H_z)$, was a system of two solenoids, which were employed to rotate a field in a plane perpendicular to the surface of the investigated plate. Moreover, equilibrium domain structures were induced by a "magnetic jolt" with an alternating magnetic field, which was gradually reduced from a value close to the saturation field down to zero.

Figures 2–4 reproduce photographs of two-phase domain structures formed as a result of the $\Gamma_{14}\leftrightarrow\Gamma_4$ phase transition in a field $\mathbf{H} = (0,H_y,H_z)$. In the case of the $\Gamma_1\leftrightarrow\Gamma_4$ transition in a field $\mathbf{H}\|\mathbf{b}$ and the $\Gamma_{14}\leftrightarrow\Gamma_4$ transition in a field $\mathbf{H} = (0,H_y,H_z)$ when $H_z < H_z^{\rm sat}$, where $H_z^{\rm sat}$ is the saturation field of the WFM phase, the demagnetizing fields in the plate were insufficient to form a periodic two-phase domain structure. It was found that in this case $H_z^{\rm sat} \approx 4\pi N_{zz} m_z^{\rm WFM} \approx 200$ Oe, where N_{zz} and $m_z^{\rm WFM}$ are the z components of the tensor of the demagnetization factors N and of the magnetic moment of the WFM phase. It is clear from Fig. 2 that the transition induced an incoherent two-phase domain structure consisting of separate AFM and WFM phases. The demagnetizing fields split the WFM phase into WFM^{+} and WFM^{-} phases. Inside the WFM region there was a clear stripe domain structure. In the fields close to the saturation field of the WFM phase ($H_z \lesssim H_z^{\rm sat}$) a sample could contain separate regions with periodic AFM + WFM^{+} and WFM^{+} + WFM^{-} phases (Fig. 4b).

When such an incoherent two-phase domain structure was formed, the range of its existence in terms of the field or temperature was governed not so much by the demagnetizing fields as by the internal inhomogeneities in a crystal. The field and temperature ranges of existence of the incoherent structure formed as a result of the phase transition in the investigated part of the sample were 0.4–0.7 kOe and 0.3–0.5 K, respectively. The line of the $\Gamma_1\leftrightarrow\Gamma_4$ phase transition in the field $\mathbf{H}\|\mathbf{b}$, plotted in Fig. 1, corresponds to the midpoints of these ranges.

For $H_z > H_c^{\rm sat}$ it was found that the $\Gamma_{14}\leftrightarrow\Gamma_4$ phase transition in the investigated $DyFeO_3$ plate produced a coherent two-phase domain structure of an intermediate magnetic state (Fig. 3). The field range of existence of this inter-

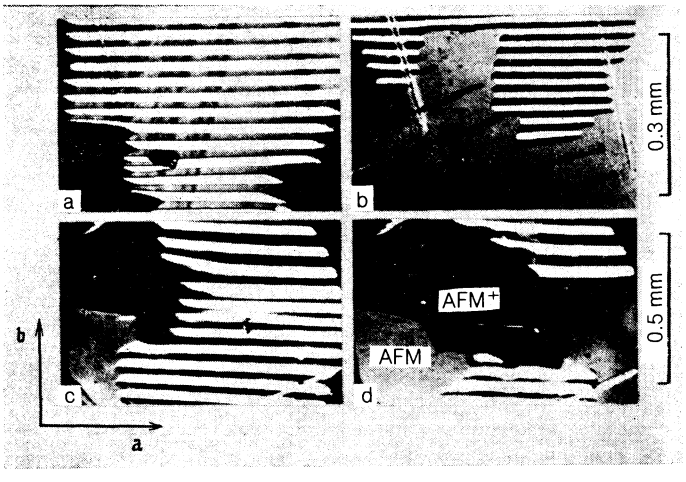

FIG. 2. Two-phase (AFM + WFM) domain structures formed in a $DyFeO_3$ plate cut at right-angles to the **c** axis and undergoing the $\Gamma_1(G_y)\leftrightarrow\Gamma_4(F_zG_x)$ phase transition in a field $\mathbf{H}\|\mathbf{b}$ and the $\Gamma_{14}(G_zG_xG_y)\leftrightarrow\Gamma_4(F_zG_x)$ phase transition in a field $\mathbf{H} = (0,H_y,H_z)$ when $H_z < H_2^{\rm sat}$. The scales are the same for a, b and c, d, respectively. a) $T \approx 16$ K, $H_y = 11.6$ kOe, $H_z = 0$, $\beta = 90°$. Here the coexisting phases are the AFM^{+} phase $\Gamma_1^{+}(G_y^{+})$ (shown black) and the WFM phases $\Gamma_4^{+}(F_x^{+}+G_x^{+})$ and $\Gamma_4^{-}(G_z^{-}G_x^{-})$ (white and gray). b) $T \approx 51$ K, $H_y = 12.8$ kOe, $H_z = 50$ Oe, $\beta \approx 89.8°$. The coexisting phases are the AFM^{+} phase $\Gamma_{14}^{+}(F_z^{+}G_x^{+}G_y^{+})$ (gray) and the WFM phases $\Gamma_4^{+}(F_z^{+}G_x^{+})$ and $\Gamma_4^{-}(F_z^{-}G_x^{-})$ (white and gray). c), d) $T \approx 8$ K, $H_y = 7.6$ kOe, $H_z \approx 110$ Oe, $\beta \approx 89.2°$. The coexisting phases are the AFM phases $\Gamma_{14}^{+}(F_z^{+}G_x^{+}G_y^{+})$ (black) and $\Gamma_{14}^{-}(F_z^{+}G_x^{+}G_y^{-})$ (gray), and the WFM phases $\Gamma_z^{+}(F_z^{+}G_x^{+})$ and $\Gamma_4^{-}(F_z^{-}G_x^{-})$ (white and black).

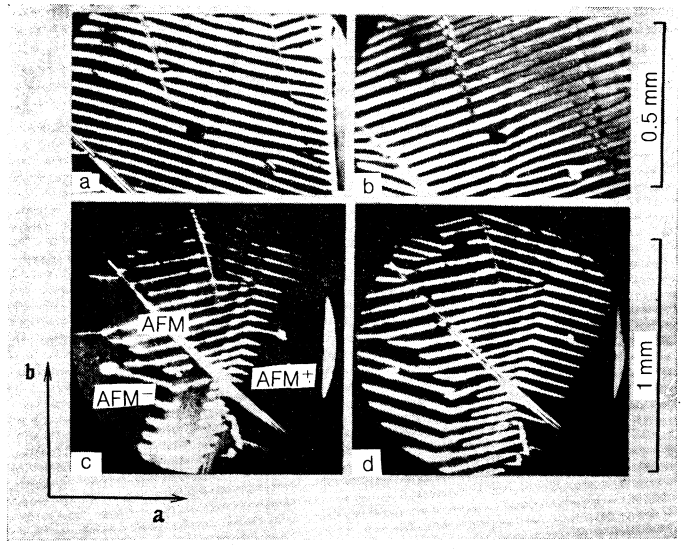

FIG. 3. Two-phase (AFM + WFM) domain structures formed in a $DyFeO_3$ plate as a result of the $\Gamma_{14}(F_zG_xG_y)\leftrightarrow\Gamma_4(F_zG_x)$ phase transition in a field $\mathbf{H}=(0,H_y,H_z)$ when $H_z>H_z^{sat}$. The scales are the same for a, b, and c, d, respectively. a) $T\approx 50$ K, $H_y=16$ kOe, $H_z=260$ Oe, $\beta\approx 89°$. The coexisting phases are the AFM$^+$ phase $\Gamma_{14}^+(F_z^+G_x^+G_y^+)$ (black) and the WFM$^+$ phase $\Gamma_4^+(F_z^+G_x^+)$ (white). b) $T\approx 50$ K, $H_y=16$ kOe, $H_z=-260$ Oe, $\beta\approx 91°$. The coexisting phases are the AFM$^+$ phase $\Gamma_{14}^+(F_z^-G_x^-G_y^+)$ (gray) and the WFM$^-$ phase $\Gamma_4^-(F_z^-G_x^-)$ (white). c), d) $T\approx 18$ K, $H_y=12.5$ kOe, $H_z=200$ Oe, $\beta\approx 89.1°$. The coexisting phases are the AFM phases $\Gamma_{14}^+(F_z^+G_x^+G_y^+)$ (black) and $\Gamma_{14}^-(F_z^+G_x^+G_y^-)$ (gray) and the WFM$^+$ phase $\Gamma_4^+(F_z^+G_x^+)$ (white).

mediate state was $\Delta\mathbf{H}=\mathbf{H}_2-\mathbf{H}_1$ and it was governed by the demagnetizing fields related to the abrupt change (jump) in the magnetic moment at the phase transition. The fields $\mathbf{H}_1$ and $\mathbf{H}_2$ could be found from the relationships[14]

$$\mathbf{H}_1=\mathbf{H}_t+4\pi\hat{N}\mathbf{m}_{\mathrm{AFM}},\quad \mathbf{H}_2=\mathbf{H}_t+4\pi\hat{N}\mathbf{m}_{\mathrm{WFM}}\,. \tag{4}$$

It should be pointed out that in oblique fields the different orientations of $\mathbf{m}$ and $\mathbf{H}$ altered not only the intensity, but also the direction of the internal field $\mathbf{H}_i=\mathbf{H}-4\pi\hat{N}\langle\mathbf{m}\rangle$ as the concentration of the coexisting phases varied. Since the transition field depended on the orientation of the magnetic field relative to the crystal axes, the range of existence of the intermediate state depended also on the change $\Delta\mathbf{H}_t$ in the transition field, i.e., $\Delta\mathbf{H}=4\pi\hat{N}(\mathbf{m}_{\mathrm{WFM}}-\mathbf{m}_{\mathrm{AFM}})+\Delta\mathbf{H}_t$.

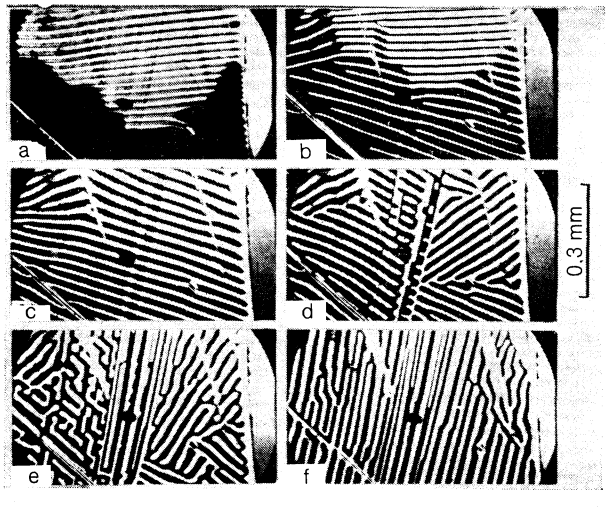

FIG. 4. Two-phase (AFM + WFM) domain structures formed in a $DyFeO_3$ plate as a result of the $\Gamma_{14}(F_zG_xG_y)\leftrightarrow\Gamma_4(F_zG_x)$ transition in a field $\mathbf{H}=(0,H_y,H_z)$ at $T\approx 51$ K: a), b) AFM phase $\Gamma_{14}(F_zG_xG_y)$ (black) and WFM phases $\Gamma_4^+(F_z^+G_z^+)$ and $\Gamma_4^-(F_z^-G_x^-)$ (white and gray); c)–f) AFM (black) and WFM$^+$ (white) phases. The photographs were obtained for the following values of H and β: a) $H=13.6$ kOe, $\beta=90°$ ($\mathbf{H}\|\mathbf{b}$); b) $H=12.8$ kOe, $\beta=89.6°$; c) $H=11.2$ kOe, $\beta=89.1°$; d) $H=8$ kOe, $\beta=88.8°$; e) $H=6.9$ kOe, $\beta=88°$; f) $H=0.28$ kOe, $\beta=0$.

Throughout the investigated temperature range the range of existence of the intermediate state in $DyFeO_3$ did not exceed an absolute value 300 Oe. The field-induced moment m_z^{AFM} was small, so that in the case of formation of a periodic two-phase domain structure as a result of the $\Gamma_{14}\leftrightarrow\Gamma_4$ transition, we could assume that $H_1\approx H_t$.

The two-phase domain structures that formed in the field $\mathbf{H}=(0,\ H_y,\ H_z)$ when the angle β between $\mathbf{H}$ and the $\mathbf{c}$ axis was close to 90° exhibited a clear regularity of the orientations of the phase boundaries. We reproduced in Fig. 2 photographs of the two-phase structures observed for $\mathbf{H}\|\mathbf{b}$ (a) and also using a field $\mathbf{H}=(0,\ H_y,\ H_z)$ on condition that $H_z<H_z^{sat}$ (Figs. 2b–2d). It is worth noting the sawtooth configuration of the boundary between the AFM and (WFM$^+$ + WFM$^-$) states. The domain walls between the WFM domains $\Gamma_4^+(F_z^+G_x^+)$ and $\Gamma_4^-(F_z^-G_x^-$ were in the *ac* plane of the crystal. The phase boundaries were also in a plane perpendicular to the surface of the plate, but reoriented at some angle ψ to the *ac* plane. The orientation of the phase boundaries depended on the nature of the adjacent domains: the phase boundaries between the $\Gamma_{14}^+(F_z^+G_x^+G_y^+)$ [$\Gamma_1^+(G_y^+)$ if $\mathbf{H}\|\mathbf{b}$] and $\Gamma_4^+(F_z^+G_x^+)$ domains were oriented at an angle $+\psi$, whereas the phase boundaries between $\Gamma_4^+(F_z^+G_x^+G_y^+)$ and $\Gamma_4^-(F_z^-G_x^-)$ domains were oriented at an angle $-\psi$ to the *ac* plane (Figs. 2a and 2b). The direction of the tilt of the phase boundary from the *ac* plane also changed to opposite when the AFM state $\Gamma_{14}^+(F_z^+G_x^+G_y^+)$ was replaced with the $\Gamma_{14}^-(F_z^+G_x^+G_y^-)$ state. It is clear from Figs. 2c and 2d that the phase boundaries between the $\Gamma_{14}^+(F_z^+G_x^+G_y^+)$ and $\Gamma_4^+(F_z^+G_x^+)$ domains, as well as between the $\Gamma_{14}^-(F_z^+G_x^+G_y^-)$ and $\Gamma_4^-(F_z^-G_x^-)$ domains were oriented at an angle $+\psi$, whereas the phase boundaries between the $\Gamma_{14}^-(F_z^+G_x^+G_y^-)$ and $\Gamma_4^+(F_z^+G_x^+)$ domains, and also between the $\Gamma_{14}^+(F_z^+G_y^+G_y^+)$ and $\Gamma_4^-(F_z^-G_x^-)$ domains were oriented at an angle $-\psi$ to the *ac* plane of the crystal.

The angle ψ was difficult to determine accurately for the domain structures in Fig. 2 because the rectilinear sections of the phase boundary were short. Rough measurements indicated that the absolute values of the tilt of the phase boundaries in one or the other direction from the *ac* plane were

the same. In the case of the domain structures shown in Fig. 2a the absolute values of the angle ψ were within 30–40°, whereas for the domain structures in Figs. 2b–2d they were within the range 40–50°. It should be noted that the phase boundaries between the individual WFM $^+$ domains of the resultant intermediate state (Fig. 2b) made an angle of about 20° with the *ac* plane of the crystal.

The relationships governing the orientations of the phase boundaries in the coherent two-phase domain structure of the intermediate magnetic state (Fig. 3) were the same as in the above case of the incoherent structure. However, the absolute values of the angle ψ for the coherent two-phase structure were between 13 and 19°. The value of ψ for the resultant coherent structure was determined more accurately.

A reduction of the field component H_y or of the angle β (Fig. 4) increased the influence of the elastic stresses associated with the inhomogeneity of a crystal on the orientations of the phase boundaries (Figs. 4d and 4e). In the field **H**‖**c** (Fig. 4f) the phase boundary orientation was no longer a function of the directions of the magnetic vectors **F** and **G** of the coexisting phases.

2. ANALYSIS OF THE INFLUENCE OF THE MAGNETOSTRICTIVE STRESSES ON THE PHASE-BOUNDARY ORIENTATIONS

The experimentally observed dependence of the phase boundary orientations on the direction of the AFM vectors in the coexisting magnetic phases was attributed by us to the magnetostrictive stresses that appeared in the magnetically inhomogeneous state because of the linear magnetostriction. We shall now analyze the influence of these stresses on the phase boundary orientations. We shall consider a two-phase stripe domain structure with the phase boundaries perpendicular to the surface of the plate and oriented in a plane making an angle ψ with the *ac* plane of the crystal. We shall consider only the case when the concentrations of the coexisting phases are the same. The equilibrium magnetostrictive strains are different in the homogeneous AFM and WFM states. Therefore in the magnetically inhomogeneous state, as a result of the elastic interactions between the domains, the strains of the coexisting phases deviate from the equilibrium in the homogeneous state. Magnetostrictive stresses appear then in a crystal and the energy of the crystal rises. This increase in the energy of the crystal in the two-phase state as a result of the elastic stresses can be found from the relationship

$$\Delta\Phi = {}^1/_2(\sigma_{ij}^{(1)}\varepsilon_{ij}^{(1)} + \sigma_{ij}^{(2)}\varepsilon_{ij}^{(2)}), \tag{5}$$

where $\sigma_{ij}^{(k)}$ represents the magnetostrictive stresses and $\varepsilon_{ij}^{(k)} = \xi_{ij}^{(k)} - u_{ij}^{(k)}$ represents the deviations of the strains from the equilibrium values; $\xi_{ij}^{(k)}$ are the strains in the AFM and WFM phases in the magnetically inhomogeneous states; $u_{ij}^{(k)}$ are the equilibrium strains in the homogeneous state. The AFM phase corresponds to $k = 1$ and the WFM phase to $k = 2$.

We shall consider the components of the tensors $\sigma_{ij}^{(k)}$ and $\xi_{ij}^{(k)}$ in a coordinate system linked to a phase boundary and rotated by an angle ψ about the **c** axis relative to the orthorhombic axes of the investigated crystal (Fig. 5). The components of the tensor of the magnetostrictive stresses

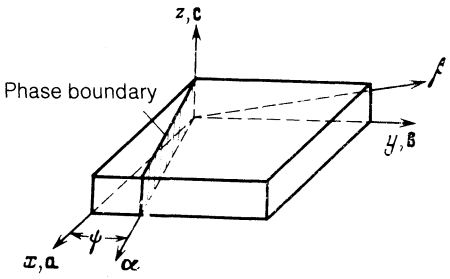

FIG. 5. Position of a phase boundary in a $DyFeO_3$ plate.

normal to the phase boundary plane vanish, because of the equilibrium in an elastic body,[1] i.e.,

$$\sigma_{\beta\beta}^{(k)} = \sigma_{\beta z}^{(k)} = \sigma_{\alpha\beta}^{(k)} = 0. \tag{6}$$

The components of the strain tensor ξ_{ij} tangential to the phase boundary are independent of the coordinate normal to this plane, in accordance with the condition of continuity of an elastic medium.[1] We can therefore write

$$\xi_{\alpha\alpha}^{(1)} = \xi_{\alpha\alpha}^{(2)} = \xi_{\alpha\alpha}, \quad \xi_{zz}^{(1)} = \xi_{zz}^{(2)} = \xi_{zz}, \quad \xi_{\alpha z}^{(1)} = \xi_{\alpha z}^{(2)} = \xi_{\alpha z}. \tag{7}$$

Using the relationships (6) and (7), we find that $\varepsilon_{ij}^{(k)} = \xi_{ij} - u_{ij}^{(k)}$ for $i, j = \alpha, z$ and $\varepsilon_{\beta\beta}^{(k)} = \varepsilon_{\beta z}^{(k)} = \varepsilon_{\alpha\beta}^{(k)} = 0$.

We shall find the components of the strains $u_{ij}^{(1)}$ and $u_{ij}^{(2)}$ for the homogeneous AFM and WFM states. The dependence of the magnetostrictive strains in a many-sublattice magnetic crystal on the intensity of the magnetic field can be represented in the form

$$u_{ij} = u_{ij0} + p_{ijm}H_m + b_{ijml}H_mH_l + \ldots \tag{8}$$

The tensor u_{ij0} describes the spontaneous strains due to the magnetic ordering in the absence of the field. The axial *c*-tensor p_{ijm} symmetric in *i* and *j* describes the inverse piezomagnetic effect or the linear magnetostriction, whereas the polar *i*-tensor b_{ijml} symmetric in the pairs of indices *ij* and *rs* represents the quadratic magnetostriction. The tensor p_{ijm} is the inverse of the tensor of the magnetic piezoelectric moduli the matrices of which are given in Ref. 15. It should also be noted that the tensor p_{ijm} has exactly the same symmetry as the magnetooptic tensor q_{ijm} describing the linear magnetooptic effect. The matrices of the tensor q_{ijm} are given in Ref. 16 for all the magnetic classes that can be described by the Shubnikov groups. Dysprosium orthoferrite in the AFM state Γ_1 is described by the magnetic point symmetry group *mmm*, whereas in the WFM state Γ_4 it is described by the point group *m'm'm*. We shall use the following generally accepted index contraction:

ij	*xx*	*yy*	*zz*	*yz*	*xz*	*xy*
μ	1	2	3	4	5	6

Then, the matrices of the tensor p_{ijm} for the Γ_1 and Γ_4 states can be represented respectively in the form

$$p_{ijm}^{(1)} \sim p_{\mu m}^{(1)} \sim \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \odot & \cdot & \cdot \\ \cdot & \odot & \cdot \\ \cdot & \cdot & \odot \end{pmatrix}, \quad p_{ijm}^{(2)} \sim p_{\mu m}^{(2)} \sim \begin{pmatrix} \cdot & \cdot & \odot \\ \cdot & \cdot & \odot \\ \cdot & \cdot & \odot \\ \cdot & \odot & \cdot \\ \odot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}. \tag{9}$$

The nonzero components are identified by circles in the above matrix. The matrix of the tensor b_{ijml} is the same for the Γ_1 and Γ_4 states:

$$b^{(1)}_{ijml} \sim b^{(2)}_{ijml} \sim b_{\mu\nu} \sim \begin{pmatrix} \odot & \odot & \odot & \cdot & \cdot & \cdot \\ \odot & \odot & \odot & \cdot & \cdot & \cdot \\ \odot & \odot & \odot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \odot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \odot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \odot \end{pmatrix}. \tag{10}$$

In addition to the dependence (8) of the magnetostrictive strains on the external field, we shall consider also their dependence on the orientations of the magnetic vectors. In the two-sublattice approximation, retaining only terms up to the second order, this dependence can be represented by

$$u_{ij} = B^{GG}_{ijml} G_m G_l + B^{FG}_{ijml} F_m G_l + B^{FF}_{ijml} F_m F_l + \ldots \tag{11}$$

The matrices of the tensors B^{GG}_{ijml} and B^{FF}_{ijml} symmetric in the pairs of the indices ij and ml are the same and identical with the matrix (10). The matrices of the tensor B^{FG}_{ijml} symmetric only in the first pair of indices are given in Ref. 17. The antiferromagnetic ordering of the iron ions in orthoferrites is of the $2_z^- 2_x^-$ type.[18] The matrix of the B^{FG}_{ijml} tensor subjected to the usual contraction of the indices $ij = \mu$ and $lm = \nu$, in accordance with the rules

lm	xx	yy	zz	yz	xz	xy	zy	zx	yx
ν	1	2	3	4	5	6	7	8	9 ,

can be represented in the form

$$B^{(1)}_{i,ml} \sim B^{(2)}_{i,ml} \sim B_{\mu\nu} \sim \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \odot & \cdot & \cdot & \odot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \odot & \cdot & \cdot & \odot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \odot & \cdot & \cdot & \odot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \odot & \cdot & \cdot & \odot \\ \odot & \odot & \odot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \odot & \cdot & \cdot & \odot & \cdot & \cdot \end{pmatrix}. \tag{12}$$

Using the above tensor matrices, we can find the components of the tensor of the magnetostrictive strains in a field $\mathbf{H} = (0,\ H_y,\ H_z)$. In the case of the Γ_1 state transforming in a field $\mathbf{H} = (0,\ H_y,\ H_z)$ into Γ_{14}, we have

$$\begin{aligned} u^{(1)}_{xx} &= u^{(1)}_{xx0} + b^{(1)}_{12} H_y^2 + b^{(1)}_{13} H_z^2 = B^{GG}_{11} G_x^2 \\ &\quad + B^{GG}_{12} G_y^2 + B^{FF}_{12} F_y^2 + B^{FF}_{13} F_z^2 + B^{FG}_{18} F_z G_x, \\ u^{(1)}_{yy} &= u^{(1)}_{yy0} + b^{(1)}_{22} H_y^2 + b^{(1)}_{23} H_z^2 = B^{GG}_{21} G_x^2 \\ &\quad + B^{GG}_{22} G_y^2 + B^{FF}_{22} F_y^2 + B^{FF}_{23} F_z^2 + B^{FG}_{28} F_z G_x, \\ u^{(1)}_{zz} &= u^{(1)}_{zz0} + b^{(1)}_{32} H_y^2 + b^{(1)}_{33} H_z^2 = B^{GG}_{31} G_x^2 \\ &\quad + B^{GG}_{32} G_y^2 + B^{FF}_{32} F_y^2 + B^{FF}_{33} F_z^2 + B^{FG}_{38} F_z G_x, \end{aligned} \tag{13}$$

$$\begin{aligned} u^{(1)}_{yz} &= b^{(1)}_{44} H_y H_z = B^{FF}_{44} F_y F_z + B^{FG}_{49} F_y G_x, \\ u^{(1)}_{xz} &= p^{(1)}_{52} H_y = B^{FG}_{52} F_y G_y, \\ u^{(1)}_{xy} &= p^{(1)}_{63} H_z = B^{GG}_{66} G_x G_y + B^{FG}_{67} F_z G_y, \end{aligned}$$

whereas for the Γ_4 states we have

$$\begin{aligned} u^{(2)}_{xx} &= u^{(2)}_{xx0} + p^{(2)}_{13} H_z + b^{(2)}_{12} H_y^2 + b^{(2)}_{13} H_z^2 = B^{GG}_{11} G_x^2 \\ &\quad + B^{FF}_{12} F_y^2 + B^{FF}_{13} F_z^2 + B^{FG}_{18} F_z G_x, \\ u^{(2)}_{yy} &= u^{(2)}_{yy0} + p^{(2)}_{23} H_z + b^{(2)}_{22} H_y^2 + b^{(2)}_{23} H_z^2 = B^{GG}_{21} G_x^2 \\ &\quad + B^{FF}_{22} F_y^2 + B^{FF}_{23} F_z^2 + B^{FG}_{28} F_z G_x, \\ u^{(2)}_{zz} &= u^{(2)}_{zz0} + p^{(2)}_{33} H_z + b^{(2)}_{32} H_y^2 + b^{(2)}_{33} H_z^2 = B^{GG}_{31} G_x^2 \\ &\quad + B^{FF}_{32} F_y^2 + B^{FF}_{33} F_z^2 + B^{FG}_{38} F_z G_x, \end{aligned} \tag{14}$$

$$\begin{aligned} u^{(2)}_{yz} &= p^{(2)}_{42} H_y + b^{(2)}_{44} H_y H_z = B^{FF}_{44} F_y F_z + B^{FG}_{49} F_y G_x, \\ u^{(2)}_{xz} &= u^{(2)}_{xy} = 0. \end{aligned}$$

If the concentrations of the coexisting phases are equal, the strain components $\xi_{\alpha\alpha}$, ξ_{zz}, and $\xi_{\alpha z}$ in the magnetically inhomogeneous state can be regarded as equal to the average strains:[2]

$$\xi_{ij} = {}^1/_2 (u^{(1)}_{ij} + u^{(2)}_{ij}). \tag{15}$$

Then, the nonzero components of the tensor $\varepsilon^{(k)}_{ij}$, namely $\varepsilon^{(k)}_{\alpha\alpha}$, $\varepsilon^{(k)}_{zz}$, and $\varepsilon^{(k)}_{\alpha z}$ are described by the relationship

$$\varepsilon^{(1)}_{ij} = -\varepsilon^{(2)}_{ij} = {}^1/_2 (u^{(1)}_{ij} - u^{(2)}_{ij}) = \varepsilon_{ij}. \tag{16}$$

The tensor components ε_{ij} in the coordinate systems α, β, z and x, y, z are described by the expressions

$$\begin{aligned} \varepsilon_{\alpha\alpha} &= \varepsilon_{xx} \cos^2 \psi + \varepsilon_{xy} \sin 2\psi + \varepsilon_{yy} \sin^2 \psi, \\ \varepsilon_{\alpha z} &= \varepsilon_{xz} \cos \psi + \varepsilon_{yz} \sin \psi, \\ \varepsilon_{zz} &= \varepsilon_{zz}. \end{aligned} \tag{17}$$

The stresses due to deviation of the magnetostrictive strains from their equilibrium values are

$$\sigma^{(1)}_{ij} = -\sigma^{(2)}_{ij} = c_{ijlm} \varepsilon_{lm} = \sigma_{ij}. \tag{18}$$

Allowing for the orthorhombic symmetry of the investigated crystal, we can write down the components of the tensor σ_{ij} for the coordinate system x, y, z:

$$\begin{aligned} \sigma_{xx} &= c_{11}\varepsilon_{xx} + c_{12}\varepsilon_{yy} + c_{13}\varepsilon_{zz}, \\ \sigma_{yy} &= c_{21}\varepsilon_{xx} + c_{22}\varepsilon_{yy} + c_{23}\varepsilon_{zz}, \\ \sigma_{zz} &= c_{31}\varepsilon_{xx} + c_{32}\varepsilon_{yy} + c_{33}\varepsilon_{zz}, \\ \sigma_{yz} &= c_{44}\varepsilon_{yz}, \\ \sigma_{xz} &= c_{55}\varepsilon_{xz}, \\ \sigma_{xy} &= c_{66}\varepsilon_{xy}, \end{aligned} \tag{19}$$

whereas the nonzero components $\sigma_{\alpha\alpha}$, σ_{zz}, and $\sigma_{\alpha z}$ can be found from relationships similar to those given by the system of equations (17).

Allowing only for the nonzero components of the tensors ε_{ij} and σ_{ij}, and bearing in mind that the identical components of these tensors in the domains of the coexisting phases are equal in absolute magnitude and have opposite signs when the condition (15) is satisfied, we can write Eq. (5) for the increase in the magnetoelastic energy in the form

$$\Delta\Phi=\sigma_{\alpha z}\varepsilon_{\alpha z}+\sigma_{\alpha\alpha}\varepsilon_{\alpha\alpha}+\sigma_{zz}\varepsilon_{zz}. \quad (20)$$

It should be pointed out that in a ferromagnet with domain walls which are not of the 180° type the components $\varepsilon_{\alpha\alpha}$ and ε_{zz} and, consequently the strains $\sigma_{\alpha\alpha}$ and σ_{zz}, vanish in the domains.[4] These components do not vanish only in a domain wall. However, in the case of a two-phase domain structure it follows from the above analysis that $\varepsilon_{\alpha\alpha}, \varepsilon_{zz}, \sigma_{\alpha\alpha}$, and σ_{zz} are all nonzero. Therefore, the problem of finding the orientations of phase boundaries becomes more complex.

The angle ψ governing the orientation of a phase boundary in a crystal can be found by minimizing Eq. (20). The components σ_{zz} and ε_{zz} are independent of ψ and the term $\sigma_{zz}\varepsilon_{zz}$ can be omitted when $\Delta\Phi$ is minimized with respect to ψ. The experimental investigation of the magnetostriction of $DyFeO_3$ reported in Ref. 6 indicates that the abrupt changes in the strains $u_{xx}^{(1)}-u_{xx}^{(2)}$ and $u_{yy}^{(1)}-u_{yy}^{(2)}$ as a result of the $\Gamma_{14}\leftrightarrow\Gamma_4$ phase transition have similar magnitudes. Bearing this point in mind, we can assume that $\varepsilon_{xx}\approx\varepsilon_{yy}$. Then, since $\varepsilon_{xy}=0$ for $\mathbf{H}\|\mathbf{b}$, the relationships in the system (17) give $\varepsilon_{\alpha\alpha}\approx\varepsilon_{xx}\approx\varepsilon_{yy}$, i.e., $\varepsilon_{\alpha\alpha}$ is independent of ψ. Assuming that $\varepsilon_{\alpha\alpha}$ and, consequently, $\sigma_{\alpha\alpha}$ are quantities which are not greatly affected by the phase boundary orientation in a field $\mathbf{H}=(0,\ H_y,\ H_z)$, we can also omit the term $\sigma_{\alpha\alpha}\varepsilon_{\alpha\alpha}$ in Eq. (20) when $\Delta\Phi$ is minimized. We thus find that the orientation of a phase boundary is in this case influenced primarily by the shear component of the stress $\sigma_{\alpha z}$. Consequently, retaining in Eq. (20) only the ψ-dependent term $\sigma_{\alpha z}\varepsilon_{\alpha z}$, we can reduce the expression for $\Delta\Phi$ to

$$\Delta\Phi=\sigma_{xz}\varepsilon_{xz}\cos^2\psi+\sigma_{yz}\varepsilon_{yz}\sin^2\psi+{}^1/_2(\sigma_{yz}\varepsilon_{xz}+\sigma_{xz}\varepsilon_{yz})\sin 2\psi. \quad (21)$$

Minimization of the energy $\Delta\Phi(\psi)$ with respect to ψ yields the following expression for the angle ψ:

$$\operatorname{tg}2\psi=\frac{\sigma_{yz}\varepsilon_{xz}+\sigma_{xz}\varepsilon_{yz}}{\sigma_{xz}\varepsilon_{xz}-\sigma_{yz}\varepsilon_{yz}}=\frac{(c_{44}+c_{55})\varepsilon_{yz}\varepsilon_{xz}}{c_{55}\varepsilon_{xz}^2-c_{44}\varepsilon_{yz}^2}. \quad (22)$$

In the general case of a field $\mathbf{H}=(0,\ H_y,\ H_z)$, the components ε_{yz} and ε_{xz} are given by

$$\varepsilon_{yz}={}^1/_2\{-p_{42}^{(2)}H_y+(b_{44}^{(1)}-b_{44}^{(2)})H_yH_z\}$$
$$={}^1/_2\{B_{44}^{FF}(F_y^{(1)}F_z^{(1)}-F_y^{(2)}F_z^{(2)})+B_{49}^{FG}(F_y^{(1)}G_x^{(1)}-F_y^{(2)}G_x^{(2)})\},$$
$$\varepsilon_{xz}={}^1/_2p_{52}^{(1)}H_y={}^1/_2B_{52}^{FG}F_y^{(1)}G_y^{(1)}. \quad (23)$$

Substituting ε_{yz} and ε_{xz} into Eq. (22), we can calculate the angle of tilt ψ of the phase boundary from the *ac* plane.

3. DISCUSSION OF RESULTS AND CONCLUSIONS

The above analysis demonstrates that the appearance of the magnetostrictive stresses in a two-phase state may influence the orientation of phase boundaries in dysprosium orthoferrite. We shall now compare the results of this analysis with the experimental data obtained in a field $\mathbf{H}=(0,\ H_y,\ H_z)$ when $H_z>H_z^{\text{sat}}$. Figures 3a, 3b, and 3d reproduce photographs of two-phase periodic stripe domain structures in which the concentrations of the AFM and WFM phases were approximately the same. In this case the domain structures agreed with the model adopted in our analysis.

According to Eq. (23), in a field $\mathbf{H}=(0,\ H_y,\ H_z)$ the sign of the component ε_{xz} depends on the nature of the AFM state in a domain structure and is reversed when $G_y^{(1)}$ changes to $-G_y^{(1)}$. This reversal of the sign of ε_{xz} follows also from the fact that $p_{52}^{(1)}$ has opposite signs for the AFM^+ and AFM^- states. The component ε_{yz} is independent of the type of the AFM state in the domain structure, not only $b_{44}^{(1)}$, but also $F_y^{(1)}$, $F_z^{(1)}$, and $G_x^{(1)}$ are the same for both AFM domains. Reversal of the sign of the projection of H_z of the external field reverses the sign of the component ε_{yz}, since this reverses the signs of the projections of the ferromagnetic and antiferromagnetic vectors F_z and G_x in the two coexisting magnetic phases Γ_{14} and Γ_4. The components $p_{42}^{(2)}$ have opposite signs for the WFM^+ and WFM^- states, whereas $b_{42}^{(2)}$ are the same for both states. The component ε_{xz} is independent of the nature of the WFM state in the domain structure. Therefore, replacement of one of the magnetic states coexisting in a two-phase domain structure reverses the sign of one of the two components: ε_{xz} or ε_{yz}. It follows from Eq. (22) that reversal of the sign of one of the components ε_{yz} or ε_{xz} reverses also the sign of ψ, i.e., it reverses the direction of the tilt of a phase boundary from the *ac* plane of the crystal.

The experimental results indicated reversal of the sign of ψ on reversal of the sign of the projection H_z causing the replacement of the state $\Gamma_4(F_z^+G_x^+)$ with the state $\Gamma_4(F_z^-G_x^-)$ in the domain structure of the WFM state (Figs. 3a and 3b). Moreover, the phase boundary orientation depended on the type of the AFM state (Fig. 3). It is clear from this comparison that the experimentally observed relationships governing the orientations of the phase boundaries are in good agreement with the results of our analysis. It follows from Eqs. (22) and (23) that reversal of the sign of the projection H_y of the external field, which reverses the sign of F_y of both coexisting phases, does not affect the sign of ψ, because in this case the component ε_{yz} as well as the component ε_{xz} experiences reversal of the sign. This conclusion is also in agreement with the experimental data.

Our experiments have failed to reveal a dependence of the magnitude of the angle ψ on the external field intensity. In the range $H_z\gtrsim H_z^{\text{sat}}$ it was found that throughout the range of temperatures where the visual observations were made the value of ψ remained in the range 13–19° in spite of a strong dependence $\mathbf{H}_t(T)$ shown in Fig. 1. The small value of ψ is part of a considerable change in the transition field and could also be explained in the case under consideration. When the condition $H_z\gtrsim H_z^{\text{sat}}$ was satisfied, the z projection of the field did not vary greatly with temperature. On the other hand the change in the projection H_y due to a change in $\mathbf{H}_t(T)$ did not affect the value of the angle ψ, because—according to Eqs. (22) and (23)—ψ was independent of H_y. Therefore, in the case of a field $\mathbf{H}=(0,\ H_y,\ H_z)$ if $H_z\gtrsim H_z^{\text{sat}}$ the magnetostriction model accounted well for the experimentally observed relationships governing the orientations of the phase boundaries in $DyFeO_3$.

In the $\mathbf{H}\|\mathbf{b}$ case the expressions from (23) for ε_{yz} and ε_{xz} become simplified and, using Eq. (22), we can obtain the following expression for ψ:

$$\operatorname{tg} 2\psi = \frac{(c_{44}+c_{55})\,p_{42}^{(2)}\,p_{52}^{(1)}}{c_{44}(p_{42}^{(2)})^2 - c_{55}(p_{52}^{(1)})^2}$$

$$= \frac{(c_{44}+c_{55})B_{52}^{FG}F_y^{(1)}G_y^{(1)}F_y^{(2)}(B_{44}^{FF}F_z^{(2)}+B_{49}^{FG}G_x^{(2)})}{c_{44}(B_{52}^{FG}F_y^{(1)}G_y^{(1)})^2 - c_{55}(F_y^{(2)})^2(B_{44}^{FF}F_z^{(2)}+B_{49}^{FG}G_x^{(2)})^2}. \quad (24)$$

It follows from Eq. (24) that in a field $\mathbf{H}\|\mathbf{b}$ the sign of ψ is reversed on reversal of the sign of G_y in the AFM state. The same result is produced also by reversal of the sign of the projection G_x and, consequently, of the simultaneous reversal of the sign of the projection F_z in the WFM state. Moreover, it follows from Eq. (24) that the magnitude of the angle ψ is independent of the applied magnetic field.

A comparison of the results of our analysis with the experimental data in the cases $\mathbf{H}\|\mathbf{b}$ and $\mathbf{H} = (0, H_y, H_z)$ if $H_z < H_z^{\text{sat}}$ is not quite correct because under these conditions we can expect incoherent two-phase domain structures in a $DyFeO_3$ plate cut at right-angles to the **c** axis. However, it should be pointed out that in the case of an incoherent two-phase structure with a toothed configuration of phase boundaries (Fig. 2) and in the case of a coherent stripe two-phase structure (Fig. 3) the main relationships governing the orientation of phase boundaries are the same, although there are significant differences in the value of the angle ψ. The relationships governing the orientation of phase boundaries in domain structures shown in Fig. 2 are in qualitative agreement with the results of an analysis of Eq. (24).

In the field $\mathbf{H}\|\mathbf{c}$ it follows from Eqs. (13) and (14) that $u_{yz}^{(1)} = u_{xz}^{(1)} = u_{yz}^{(2)} = u_{xz}^{(2)} = 0$. Therefore, in this case the magnetostrictive stresses do not influence the orientation of phase boundaries, which is confirmed by the experimental results. Therefore, in the $\mathbf{H}\|\mathbf{c}$ case an important role in the orientation of phase boundaries is played by other factors, particularly by the elastic stresses unrelated to the magnetic ordering but present always in real crystals. Visual observations of two-phase domain structures in a field $\mathbf{H}\|\mathbf{c}$ obtained for different samples suggest that in this case the phase boundary orientation is governed by the directions of the crystal growth bands. As **H** tilts away from the *b* axis and the component H_y decreases, the role of the nonmagnetic elastic stresses in the orientation of the phase boundaries becomes increasingly greater. It is clear from Figs. 4d and 4e that the orientation of the phase boundaries changes as H_y decreases. The phase boundaries rotate to that orientation which is expected for $\mathbf{H}\|\mathbf{c}$. Although the value of the angle ψ is independent of H_y for an ideal crystal, the magnetostrictive stresses and the associated increase in the magnetoelastic energy of a crystal are determined by this particular projection of the field. In the case of comparable values of the magnetostrictive and nonmagnetic elastic stresses the orientations of the phase boundaries are set by a compromise between the effects of the two mechanisms.

The good agreement between the results of a theoretical analysis of the influence of the magnetostriction on the orientation of the phase boundaries and the results of our experimental investigation of two-phase domain structures allows us to draw the conclusion that the magnetostriction mechanism is responsible for the orientation of the phase boundaries in $DyFeO_3$ when the fields have directions parallel to or close to the **b** axis of this crystal. The linear magnetostriction associated with the field-induced projection of the WFM moment $F_y = \chi_{yy}H_y$ has the dominant influence on the phase boundary orientation. The value of F_y in a field $\mathbf{H}\|\mathbf{b}$ can increase because of the magnetization of the subsystem of the rare-earth ions in the applied magnetic field. This rare-earth subsystem is in a paramagnetic state and the Ising axis of the Dy^{3+} ions is inclined $\approx 30°$ to the **b** axis in the *ab* plane of the crystal.[19]

The authors are deeply grateful to V. V. Eremenko for his interest in this investigation and encouragement, to D. A. Yablonskiĭ for a valuable discussion of the results and helpful comments, and to B. Krzymanski and P. P. Lebedev for their help in the experiments.

[1)] Institute of Physics, Polish Academy of Sciences, Warsaw.

[1] E. M. Lifshitz, Zh. Eksp. Teor. Fiz. **15**, 97 (1945).
[2] A. Hubert, *Theorie der Domänenwande in geordneten Medien*, Springer Verlag, Berlin (1974) [Lecture Notes in Physics, Vol. 26].
[3] A. Hubert, Phys. Status Solidi **22**, 709 (1967).
[4] A. A. Khalfina, M. A. Shamsutdinov, and M. M. Farztdinov, Fiz. Met. Metalloved. **61**, 820 (1986).
[5] V. G. Bar'yakhtar, A. N. Bogdanov, and D. A. Yablonskiĭ, Fiz. Tverd. Tela (Leningrad) **28**, 3333 (1986) [Sov. Phys. Solid State **28**, 1876 (1986)].
[6] A. K. Zvezdin, I. A. Zorin, A. M. Kadomtseva *et al.*, Zh. Eksp. Teor. Fiz. **88**, 1098 (1985) [Sov. Phys. JETP **61**, 645 (1985)].
[7] K. Piotrowski, H. Szymczak, R. Szymczak *et al.*, IEEE Trans. Magn. **MAG-24**, 1701 (1988).
[8] N. F. Kharchenko and S. L. Gnatchenko, Fiz. Nizk. Temp. **7**, 475 (1981) [Sov. J. Low. Temp. Phys. **7**, 234 (1981)].
[9] R. V. Pisarev, Zh. Eksp. Teor. Fiz. **58**, 1421 (1970) [Sov. Phys. JETP **31**, 761 (1970)].
[10] G. S. Krinchik and V. E. Zubov, Pis'ma Zh. Eksp. Teor. Fiz. **20**, 307 (1974) [JETP Lett. **20**, 137 (1974)].
[11] B. B. Krichevtsov, K. M. Mukimov, R. V. Pisarev, and M. M. Ruvinshteĭn, Pis'ma Zh. Eksp. Teor. Fiz. **34**, 399 (1981) [JETP Lett. **34**, 379 (1981)].
[12] S. L. Gnatchenko, N. F. Kharchenko, P. P. Lebedev *et al.*, Fiz. Nizk. Temp. **15**, 868 (1989) [Sov. J. Low. Temp. Phys. **15**, 480 (1989)].
[13] S. L. Gnatchenko, N. F. Kharchenko, P. P. Lebedev *et al.*, J. Magn. Magn. Mater. **81**, 125 (1989).
[14] V. G. Bar'yakhtar, A. N. Bogdanov, and D. A. Yablonskiĭ, Usp. Fiz. Nauk **156**, 47 (1988) [Sov. Phys. Usp. **31**, 810 (1988)].
[15] B. A. Tavger, Kristallografiya **3**, 342 (1958) [Sov. Phys. Crystallogr. **3**, 344 (1958)].
[16] V. V. Eremenko, N. F. Kharchenko, Yu. G. Litvinenko, and V. M. Naumenko, *Magnetooptics and Spectroscopy of Antiferromagnets* [in Russian], Naukova Dumka, Kiev (1989), p. 14.
[17] A. I. Mitsek and V. G. Shavrov, Fiz. Tverd. Tela (Leningrad) **6**, 210 (1964) [Sov. Phys. Solid State **6**, 167 (1964)].
[18] E. A. Turov, *Physical Properties of Magnetically Ordered Crystals*, Academic Press, New York (1965).
[19] G. Gorodetsky, B. Sharon, and S. Shtrikman, J. Appl. Phys. **39**, 1371 (1968).

Translated by A. Tybulewicz