Transport cross section for small-angle scattering

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It is shown that the only condition on the applicability of classical mechanics in a calculation of the transport cross section is that the de Broglie wavelength be small in comparison with the size of the scatterer. The classical expression for the transport cross section is also valid for diffractive small-angle scattering (even in the Born approximation), even though the differential cross section in this case is completely different from its classical value.

Classical mechanics is valid for describing potential scattering under the conditions (1) $\lambda \ll a$ and (2) $U \gg \hbar v/a$, where λ is the de Broglie wavelength, a is the characteristic size of the scatterer, U is the characteristic value of the potential energy, and v is the velocity of the scattered particle.[1] The second of these conditions means that the typical value of the classical scattering angle is far larger than the diffraction angle λ/a.

Below we show that this second condition need not hold in a derivation of the transport cross section. In other words, provided that the condition $\lambda \ll a$ holds, it is always possible to calculate the transport cross section from the expressions of classical mechanics, even in the region $U \lesssim \hbar v/a$, where the scattering is diffractive, and the differential cross section is greatly different from the classical cross section. The transport cross section is found from the classical expression even in the "anticlassical" case $U \ll \hbar v/a$, where the Born approximation can be used.

For $\lambda \ll a$, condition 2 must hold except in the case $U \ll E$, where E is the energy of the particle. In this case, the scattering is known to be a small-angle scattering, and the eikonal approximation can be used. Under the condition $U \ll \hbar v/a$, that approximation becomes the Born approximation, while for $E \gg U \gg \hbar v/a$ it generates the results of classical mechanics. We will show below that a calculation of the transport cross section in the eikonal approximation leads to the result which follows from classical mechanics.

The scattering amplitude in the eikonal approximation is[1]

$$f = \frac{ik}{2\pi} \int d^2\rho\, e^{-i\mathbf{q}\boldsymbol{\rho}} F(\boldsymbol{\rho}), \qquad (1)$$

$$F(\boldsymbol{\rho}) = 1 - \exp\left[-\frac{i}{\hbar v} \int_{-\infty}^{+\infty} dz\, U(r) \right], \qquad (2)$$

where $r = (\rho^2 + z^2)^{1/2}$, $\mathbf{k}$ is the wave vector of the incident particle (this vector is parallel to the z axis), and $\hbar\mathbf{q}$ is the momentum transfer. The vector $\mathbf{q}$ is perpendicular to $\mathbf{k}$ and determines the scattering angle: $\theta = q/k \ll 1$.

The transport cross section is given by

$$\sigma_{tr} = \frac{1}{2k^4} \int d^2q\, q^2 |f|^2, \qquad (3)$$

which can be rewritten as follows, with the help of (1) and (2):

$$\sigma_{tr} = \frac{1}{2k^2} \int \frac{d^2q}{(2\pi)^2} \int d^2\rho_1 F(\boldsymbol{\rho}_1) \frac{\partial}{\partial \boldsymbol{\rho}_1} \times \exp(-i\mathbf{q}\boldsymbol{\rho}_1) \int d^2\rho_2 F^*(\boldsymbol{\rho}_2) \frac{\partial}{\partial \boldsymbol{\rho}_2} \exp(i\mathbf{q}\boldsymbol{\rho}_2). \qquad (4)$$

Integrating over $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ by parts, and then integrating over $\mathbf{q}$ yields $\delta(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2)$. We finally find

$$\sigma_{tr} = \frac{1}{8E^2} \int d^2\rho \left(\int_{-\infty}^{+\infty} dz \frac{\partial U}{\partial \boldsymbol{\rho}} \right)^2 = \int d^2\rho \frac{\theta_{cl}{}^2(\rho)}{2}, \qquad (5)$$

where $\theta_{cl}(\rho)$ is the scattering angle calculated for an impact parameter ρ from classical mechanics under the condition $U \ll E$ (Ref. 2).

The transport cross section is thus determined by its classical value over the entire range of applicability of the eikonal approximation. Again, we wish to stress that the expression for the differential cross section in the region $U \lesssim \hbar v/a$ is very different from the classical expression.

To illustrate the situation we consider the scattering of a particle by a small potential well under the conditions $\lambda \ll a$ and $U \ll \hbar v/a$ (the Born approximation). The characteristic value of the differential cross section σ_d at small angles is on the order of $a^2(mUa^2/\hbar^2)^2$, and at the angles θ at which most of the scattering occurs this characteristic cross section is of order λ/a. The transport cross section can be estimated from

$$\sigma_{tr} \sim \sigma_d \theta^4 \sim a^2 (U/E)^2.$$

In classical mechanics with $U \ll E$, on the other hand, the differential cross section is $\sigma_d \sim a^2(E/U)^2$, and the angular region $\theta \sim U/E \ll \lambda/a$ is important. We thus see that estimates of the transport cross section in the Born approximation and from classical mechanics lead to results which are of the same order of magnitude. In fact, these results agree quite accurately, despite the fact that the differential cross sections are completely different in both magnitude and angular dependence.

In summary, the only condition on the applicability of classical mechanics in a calculation of the transport cross section is that the de Broglie wavelength be small in comparison with the size of the scatterer. The general result

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which we have derived has been established for specific particular cases in several studies: for the scattering of electrons by complex atoms,[3] for the scattering of two-dimensional electrons by a distant Coulomb center,[4] and for the nonpotential scattering of two-dimensional electrons by an Abrikosov vortex.[5]

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