Effect of an electric field on collective excitations in He^3-B

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The effect of an electric field on collective excitations in the superfluid B phase of He^3 is studied by functional integration. It is found that the electric field causes a threefold splitting of the real squashing (rsq), squashing (sq), and pairbreaking (pb) modes. The effect found is very similar to the dispersion-induced splitting of collective modes which has been seen previously for the rsq mode. Specifically, a threefold splitting of the sq and rsq modes is also seen in the latter case. The frequencies of these modes fall in the order $E_0 > E_1 > E_2$. In addition, the ratios of the distances between the $J_z = 0$ branches and the branches with $|J_z| = 1.2$ for the rsq and sq modes are the same (1:4) as in the case of the dispersion-induced splitting. Fields of order $5\cdot 10^5$–$5\cdot 10^6$ W/cm (depending on the pressure) are required for observing the electric-field-induced splitting of the spectrum of collective modes. For this reason, the predicted splitting should be observable in part of the phase diagram (at pressures which are not too high). The electric field does not alter the spectrum of Goldstone (gd) modes, simply changing their velocity.

INTRODUCTION

Although the magnetic dipole interaction in the superfluid phases of He^3 is small ($E_D \sim 10^{-7}$ K), it leads to several interesting effects. Examples are a frequency shift of the transverse nuclear magnetic resonance (NMR) in the A phase and a longitudinal NMR in the A and B phases at frequencies $\Omega_A \approx \Omega_B$. In the B phase, this interaction is also involved in the appearance of a gap of order Ω_B in the spectrum of longitudinal spin waves, $E \approx c_F k\cdot 5^{1/2}$, and also the oscillator strength for nonphonon spin modes, $E = (8/5)^{1/2}\Delta$, which tend toward zero if the dipole interaction is ignored.[1]

In the absence of an external electric field, the electric dipole interaction in He^3 is zero, since the He^3 atoms do not have an electric dipole moment. An external electric field polarizes the He^3 atoms, and their dipole moments begin to interact with each other. This interaction, like the interaction of the magnetic dipoles, leads to several interesting effects. For example, Delrieu[2] and Maki[3] have shown that an electric field causes an orientational effect in the anisotropic A phase: The orbital angular momentum of the Cooper pairs becomes aligned perpendicular to the field in fields on the order of 10^4 W/cm. The Fermi-liquid corrections[4] increase the field to 10^5–10^6 W/cm, depending on the pressure.

Brusov and Popov[5] have studied the effect of an electric field on collective excitations in the superfluid A and B phases of He^3 by a functional-integration method. Constructing a hydrodynamic action functional, and using it to calculate the Bose spectrum of the system, they reached the conclusion[5] that an electric field does not alter the spectrum of nonphonon modes (which have a gap at a zero excitation momentum). It alters only the acoustic mode, whose velocity falls off in directions other than the field direction. Brusov later showed[6] that this conclusion was reached because the electric-field-induced deformation of the order parameter was ignored in Ref. 5.

In the present paper we examine the effect of an electric field on collective excitations in He^3-B. We show that incorporating the field-induced deformation of the order parameter leads to a substantial restructuring of the spectrum of collective modes: All the nonphonon modes (sq, rsq, and pb) undergo a threefold splitting in an electric field. We estimate the electric field which would be required for an experimental observation of this splitting.

1. HYDRODYNAMIC-ACTION FUNCTIONAL OF SUPERFLUID He^3 IN AN ELECTRIC FIELD

The action functional of He^3 in an external electric field is

$$S = \int d\tau\, d^3x \sum_s (\bar\chi_s(x)\,\partial_\tau\chi_s(x) - (2m)^{-2}\nabla\bar\chi_s(x)\,\nabla\chi_s(x) + (\mu_0+\alpha_0E^2)\bar\chi_s(x)\chi_s(x)) - {}^1/_2\int d\tau\, d^3x\, d^3y\, U_E(\mathbf{x}-\mathbf{y}) \times\sum_{ss'}\bar\chi_s(x)\bar\chi_{s'}(y)\chi_{s'}(y)\chi_s(x), \tag{1.1}$$

where

$$U_E(\mathbf{x}-\mathbf{y}) = U(\mathbf{x}-\mathbf{y}) + \alpha_0^2(E^2r^{-3} - 3(\mathbf{Er})^2r^{-5}) \tag{1.2}$$

is a binary-interaction potential with an admixture which describes the interaction of the induced dipole moments of the He^3 atoms in the electric field E; α_0 and μ_0 are the "seed" values of the susceptibility and the chemical potential, respectively; $\bar\chi_s(x)$ and $\chi_s(x)$ are anticommuting Fermi fields; $x = (\mathbf{x},\tau)$, $\mathbf{r} = \mathbf{x} - \mathbf{y}$; and $r = |\mathbf{r}|$.

Taking a functional integral over the "fast" Fermi fields $\bar\chi_{s1}(x)$ and $\chi_{s1}(x)$, which are given by

$$\chi_{s1}(x) = \chi_s(x) - \chi_{s0}(x), \tag{1.3}$$

$$\chi_{s0}(x) = \chi_{s0}(\mathbf{x},\tau) = (\beta V)^{-1/2}\sum_{\omega_F,k<|k-k_F|} a_s(p)\exp(i(\omega_F\tau + \mathbf{kx})), \tag{1.4}$$

where $p = (\mathbf{k},\omega)$ and $\omega_F = (2n+1)\pi T$, we find the functional $\exp\tilde S(\bar\chi_{s0}(x), \chi_{s0}(x))$. It depends on the "slow" Fer-

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mi fields $\bar{\chi}_{s0}(x)$ and $\chi_{s0}(x)$. The momenta of the slow Fermi fields are concentrated in a narrow layer $k<|k-k_F|$ near the Fermi sphere.

The functional $\tilde{S}$ in this model differs from S in (1.1) in that the seed values α_0 and μ_0 are replaced by their renormalized values α and μ. Another distinction is that the following replacements are made:

$$\bar{\chi}_s(x),\ \chi_s(x)\to\bar{\chi}_{s0}(x),\ \chi_{s0}(x),$$
$$U_E(\mathbf{x}-\mathbf{y})\to t_E(\mathbf{x}-\mathbf{y})\to t(\mathbf{x}-\mathbf{y})+\alpha_0^2(E^2r^{-3}-3(\mathbf{Er})^2r^{-5}). \tag{1.5}$$

In this model, the quantity $t(x-y)$ in the momentum representation,

$$t=g(\mathbf{n}_1-\mathbf{n}_2,\ \mathbf{n}_3-\mathbf{n}_4) \tag{1.6}$$

is the product of the negative constant g and the scalar product $(\mathbf{n}_1-\mathbf{n}_2,\mathbf{n}_3-\mathbf{n}_4)$, where $\mathbf{n}_i=\mathbf{k}_i/k_F$ $(i=1,2,3,4)$ are unit vectors proportional to the momenta of the particles near the Fermi sphere.

Following Ref. 4, we introduce the Bose field $c_{ia}(P)$, which describes Cooper pairs of fermions; we take the Gaussian integral over the new Bose field through the integral over the slow Fermi fields:

$$\int dc_{ia}^+(p)\,dc_{ia}(p)\exp\Big(\frac{1}{g}\sum_{p,i,a}c_{ia}^+(p)c_{ia}(p)\Big) \tag{1.7}$$

(this result is meaningful only if g is negative). We then introduce a field shift:

$$c_{i1}(p)\to c_{i1}(p)+\frac{g}{2}(\beta V)^{-1/2}\times\sum_{p_1+p_2=p}(n_{1i}-n_{2i})\,(a_+(p_2)a_+(p_1)-a_-(p_2)a_-(p_1)),$$
$$c_{i2}(p)\to c_{i2}(p)-\frac{g}{2}(\beta V)^{-1/2}\times\sum_{p_1+p_2=p}(n_{1i}-n_{2i})\,(a_+(p_2)a_+(p_1)+a_-(p_2)a_-(p_1)), \tag{1.8}$$
$$c_{i3}(p)\to c_{i3}(p)+\frac{g}{2}(\beta V)^{-1/2}\sum_{p_1+p_2=p}(n_{1i}-n_{2i})a_-(p_2)a_+(p_1).$$

Here $a_\pm(p)=a_s(p)$ are the Fourier coefficients of the fields $\chi_s(x)$. The shift in (1.8) annihilates the fourth-degree form in the slow Fermi fields $\bar{\chi}_{s0}$ and χ_{s0} which contains $t(x-y)$.

We now transform the expression for the dipole–dipole interaction, which can be written in the form

$$-\frac{\alpha}{2}\int d\tau\, d^3x\, d^3y\, \rho(x)\rho(y)\,(E^2r^{-3}-3(\mathbf{Er})^2r^{-5}), \tag{1.9}$$

where

$$\rho(x)=\sum_s\bar{\chi}_{s0}(x)\chi_{s0}(x).$$

In addition to this "classical long-range" term we need to consider the "contact" term, characteristic of quantum mechanics:

$$2\pi\alpha^2E^2\int d\tau\, d^3x\rho(x)\rho(x). \tag{1.10}$$

This term must be added to (1.9).

We now take the integral over the vector transverse Bose field $E(x)=E(\mathbf{x},\tau)$ through the functional integral:

$$\int\exp\Big\{-\int d\tau\, d^3x\frac{E^2(x)}{8\pi}\Big\}\prod_x\delta(\operatorname{div}\mathbf{E}(x))\prod_i dE_i(x). \tag{1.11}$$

The condition of transverse orientation is incorporated in the δ-function δ (div $\mathbf{E}(x)$). We now introduce the shift

$$\mathbf{E}(x)\to\mathbf{E}(x)+4\pi\alpha(\rho(x)\mathbf{E})_\perp, \tag{1.12}$$

where $(\rho(x)\mathbf{E})_\perp$ is the transverse part of the vector $\rho(x)\mathbf{E}$. After the shift (1.12), the expression for $E^2(x)/8\pi$ becomes

$$\frac{E^2(x)}{8\pi}+\alpha(\mathbf{E}(x),\mathbf{E})\rho(x)+2\pi\alpha^2((\rho(x),\mathbf{E})_\perp(\rho(x),\mathbf{E})_\perp). \tag{1.13}$$

We have omitted the subscript $\perp$ from the second term in (1.13), in $\rho(x)\mathbf{E}$, since a scalar product is formed from this vector and $\mathbf{E}(x)$.

We now consider the integral

$$-\int d\tau\, d^3x\, 2\pi\alpha^2((\rho(x),\mathbf{E})_\perp(\rho(x),\mathbf{E})_\perp)$$
$$=-2\pi\alpha^2\int d\tau\, d^3x\, d^3y\,(E_i\rho(x)E_j\rho(y))\,\delta_{ij}^{tr}(\mathbf{x}-\mathbf{y}), \tag{1.14}$$

where

$$\delta_{ij}^{tr}(\mathbf{x}-\mathbf{y})=(2\pi)^{-3}\int e^{i(\mathbf{k},\mathbf{x}-\mathbf{k})}\Big(\delta_{ij}-\frac{k_ik_j}{\mathbf{k}^2}\Big)d^3k \tag{1.15}$$

is a "transverse δ-function." Using

$$-(2\pi)^{-3}\int e^{i(\mathbf{k},\mathbf{x}-\mathbf{k})}\frac{k_ik_j}{k^2}d^3k=\partial_i\partial_j(2\pi)^{-3}\int e^{i(\mathbf{k},\mathbf{x}-\mathbf{k})}\frac{1}{k^2}d^3k$$
$$=\partial_i\partial_j(4\pi r)^{-1}=(4\pi)^{-1}(\delta_{ij}r^{-3}-3r_ir_jr^{-5}),$$

we can put (1.14) in the form

$$-2\pi\alpha^2E^2\int d\tau\, d^3x\,\rho(x)\rho(x)$$
$$+\frac{\alpha^2}{2}\int d\tau\, d^3x\, d^3y\,\rho(x)\rho(y)\,(E^2r^{-3}-3(\mathbf{Er})^2r^{-5}), \tag{1.16}$$

which cancels out with (1.9) + (1.10). As a result of this manipulation of the expression (1.8), along with (1.12), we eliminate the terms of fourth degree in the slow Fermi fields, and the integral over these fields becomes Gaussian. After we evaluate the Gaussian integral, we find a hydrodynamic (effective) action functional:

$$S_h(c_{ia}^+(p),c_{ia}(p),\mathbf{E}(-p))=\frac{1}{g}\sum_{p,i,a}c_{ia}^+(p)c_{ia}(p)$$
$$-(8\pi)^{-1}\sum(\mathbf{E}(p)\mathbf{E}(-p))$$
$$+{}^1/_2\ln\det\frac{\hat{M}(c_{ia}^+(p),c_{ia}(p)\mathbf{E}(p))}{\hat{M}(0,0,0)}, \tag{1.17}$$

where

$$\hat{M}_{p_1p_2}=\begin{bmatrix} Z^{-1}(i\omega_1-\xi_1+\alpha E^2)\delta_{p_1p_2}+ & \frac{n_{1i}-n_{2i}}{(\beta V)^{1/2}}\sigma_a c_{ia}(p_1+p_2) \\ +\frac{\alpha}{(\beta V)^{1/2}}(\mathbf{E},\mathbf{E}(p_1-p_2)) & \\ -\frac{n_{1i}-n_{2i}}{(\beta V)^{1/2}}\sigma_a c^+_{ia}(p_1+p_2) & Z^{-1}(-i\omega_1+\xi_1-\alpha E^2)\delta_{p_1p_2}- \\ & -\frac{\alpha}{(\beta V)^{1/2}}(\mathbf{E},\mathbf{E}(p_2-p_1)) \end{bmatrix}. \tag{1.18}$$

Here $\xi = c_F(k-k_F)$, c_F and k_F are the Fermi velocity and Fermi momentum, respectively, σ_a are two-dimensional Pauli matrices, $\omega_i=\omega_F=(2n+1)\pi T$, $\beta = T^{-1}$, V is the volume of the system, $\mathbf{n}_i=\mathbf{k}_i/k_F$, and Z is a normalization constant.

To calculate the Bose spectrum we use a method developed previously (see Ref. 7 and the bibliography there). In the functional S_h we introduce a field shift by the condensate function $c_{ia}^{(0)}(p)$:

$$c_{ia}(p)\to c_{ia}(p)+c_{ia}^{(0)}(p). \tag{1.19}$$

We then separate from S_h a quadratic form in the field $E(p)$ and in the fields $c_{ia}^+(p)$ and $c_{ia}(p)$—fluctuations of the original fields around their condensate values $c_{ia}^{+(0)}$ and $c_{ia}^{(0)}$. In a first approximation, the Bose spectrum is determined by the equation $\det Q=0$, where Q is a matrix of quadratic form. The quadratic form Q depends on the condensate function $c_{ia}^{(0)}(p)$, which differs from phase to phase.

2. SPECTRUM IN THE ABSENCE OF A DEFORMATION OF THE ORDER PARAMETER

We turn now to the calculation of the spectrum of collective excitation in He^3-B. As a first step, we will go through this calculation without consideration of the deformation of the order parameter; this deformation will be taken into account later (Sec. 3).

In the B phase the condensate function $c_{ia}^{(0)}(p)$ is

$$c_{ia}^{(0)}(p)=(\beta V)^{1/2}\delta_{p0}\delta_{ia}c, \tag{2.1}$$

where the constant c determines the density of the condensate. The quadratic part of functional S_h turns out to be

$$-(8\pi)^{-1}\sum_p(\mathbf{E}(p),\mathbf{E}(-p))$$

$$-{}^1/_2\sum_p(\mathbf{E},\mathbf{E}(p))(\mathbf{E},\mathbf{E}(-p))C(p)$$

$$-\sum_{p,i,a,j,b}(A_{ij}(p)c_{ia}^+(p)c_{ja}(p)+{}^1/_2B_{ijab}(p)(c_{ia}(p)c_{jb}(-p)$$

$$+c_{ia}^+(p)c_{jb}^+(-p)))$$

$$-\sum_{p,i,a}(\mathbf{E},\mathbf{E}(p))(c_{ia}(-p)-c_{ia}^+(p))D_{ia}(p). \tag{2.2}$$

Here

$$A_{ij}(p)=-\delta_{ij}g^{-1}\frac{4Z^2}{\beta V}\sum_{p_1+p_2=p}M_1^{-1}M_2^{-1}(i\omega_1+\xi_1)(i\omega_2+\xi_2)n_{1i}n_{2j},$$

$$B_{ijab}(p)=\frac{4Z^2\Delta^2}{\beta V}\sum_{p_1+p_2=p}M_1^{-1}M_2^{-1}(2n_{1a}n_{1b}-\delta_{ab})n_{1i}n_{2j},$$

$$C(p)=\frac{4Z^2\alpha^2}{\beta V}\sum_{p_1+p_2=p}M_1^{-1}M_2^{-1}((i\omega_1+\xi_1)(-i\omega_2+\xi_2)-\Delta^2),$$

$$D_{ia}(p)=\frac{2iZ^2\omega\alpha\Delta}{\beta V}\sum_{p_1+p_2=p}M_1^{-1}M_2^{-1}n_{1i}n_{1a}, \tag{2.3}$$

where

$$M_i=\omega_i^2+\xi_i^2+\Delta^2. \tag{2.4}$$

For small values of $p=(\mathbf{k},\omega)$ the function $D_{ia}(p)$ is proportional to $\omega\delta_{ia}$. As a result, only the variable

$$c_{ii}(-p)-c_{ii}(p)=2iv(p), \tag{2.5}$$

which corresponds to an acoustic mode, appears in the last term in (2.2), which describes the interaction of the collective modes with the electric field. In this approximation—without considering the deformation of the order parameter—the electric field thus affects only the acoustic mode of the collective excitations. It does not affect the other collective modes.

To find the acoustic spectrum of the system in an electric field, it is sufficient to set the following in (2.2):

$$c_{ia}(-p)=c_{ia}(p)=\delta_{ia}iv(p)=-c_{ia}^+(p)=-c_{ia}^+(-p). \tag{2.6}$$

We then replace (2.2) by

$$-(8\pi)^{-1}\sum_p(\mathbf{E}(p),\mathbf{E}(-p))$$

$$-{}^1/_2\sum_p(\mathbf{E},\mathbf{E}(p))(\mathbf{E},\mathbf{E}(-p))C(p)$$

$$-\sum_p v(p)[A_{ii}(p)-B_{ijij}(p)]-2i\sum_p(\mathbf{E},\mathbf{E}(p))v(p)D_{ii}(p). \tag{2.7}$$

For small $p=(\mathbf{k},\omega)$ we have (at $T=0$)

$$A_{ij}(p)-B_{ijij}(p)$$

$$=-3g^{-1}-\frac{4Z^2}{\beta V}\sum_{p_1+p_2=p}M_1^{-1}M_2^{-1}((i\omega_1+\xi_1)(-i\omega_2+\xi_2)$$

$$+\Delta^2)=\frac{4Z^2}{\beta V}\sum_{p_1+p_2=p}((\omega_1^2+\xi_1^2+\Delta^2)^{-1}$$

$$-\frac{(i\omega_1+\xi_1)(i\omega_2+\xi_2)+\Delta^2}{(\omega_1^2+\xi_1^2+\Delta^2)(\omega_2^2+\xi_2^2+\Delta^2)}\Bigg)$$

$$\approx\frac{Z^2k_F^2}{2\pi^2c_F\Delta^2}\left(\omega^2+\frac{c_F^2k^2}{3}\right),$$

$$C(p)\approx C(0)=\frac{2Z^2\alpha^2}{\beta V}\sum_p\frac{-\omega^2+\xi^2-\Delta^2}{(\omega^2+\xi^2+\Delta^2)^2}=-\frac{Z^2\alpha^2k_F^2}{\pi c_F},$$

$$D_{ii}(p)\approx\frac{2i\omega Z^2\alpha\Delta}{\beta V}\sum(\omega^2+\xi^2+\Delta^2)^{-2}=\frac{i\omega\alpha k_F^2Z^2}{2\pi^2\Delta c_F}. \tag{2.8}$$

Substituting (2.8) into (2.7), we find

$$-(8\pi)^{-1}\sum_p(\mathbf{E}(p),\mathbf{E}(-p))$$
$$+\sum_p(\mathbf{E},\mathbf{E}(p))(\mathbf{E},\mathbf{E}(-p))C(p)\frac{Z^2\alpha^2k_F^2}{\pi^2c_F}$$
$$-\sum_p\frac{4Z^2k_F^2}{2\pi^2c_F\Delta^2}\left(\omega^2+\frac{c_F^2k^2}{3}\right)+\sum_p(\mathbf{E},\mathbf{E}(p))v(p)\frac{\omega\alpha k_F^2Z^2}{2\pi^2\Delta c_F}. \tag{2.9}$$

This expression shows that the following are dynamic variables:

$$v(p),\ (\mathbf{E}(p),\mathbf{E}_\perp)=a(p)E_\perp,\quad (\mathbf{E}(-p),\mathbf{E}_\perp)=a(-p)E_\perp, \tag{2.10}$$

where $E_\perp=E\sin\theta$ is the transverse component of the electric field **E** and is directed perpendicular to the excitation momentum **k**. In (2.9) we identify a quadratic form of variables with 4-momenta $\pm p$ with the matrix

$$\begin{pmatrix} a(p) & 0 & b(p)\\ 0 & a(p) & b(p)\\ -b(p) & -b(p) & c(p)\end{pmatrix}. \tag{2.11}$$

Here

$$a(p)=(4\pi)^{-1}-\frac{Z^2\alpha^2k_F^2}{\pi^2c_F},\qquad b(p)=\frac{\omega\alpha k_F^2Z^2}{2\pi^2\Delta c_F}E_\perp,$$

$$c(p)=\frac{Z^2k_F^2}{2\pi^2c_F\Delta^2}\left(\omega^2+\frac{c_F^2k^2}{3}\right). \tag{2.12}$$

The determinant of (2.11) is

$$a(p)(a(p)c(p)+2b^2(p)). \tag{2.13}$$

Using the replacement $i\omega\to E$, and equating the determinant to zero [this process reduces to equating the second factor in (2.13) to zero], we find the acoustic spectrum:

$$E(k)=\frac{c_F^2k^2}{3}\left(1-\frac{4Z^2\alpha^2k_F^2}{\pi c_F}E_\perp^2\right)$$
$$=\frac{c_F^2k^2}{3}\left(1-\frac{4Z^2\alpha^2k_F^2}{\pi c_F}E^2\sin^2\theta\right). \tag{2.14}$$

We see that when the electric field is applied the spectrum becomes anisotropic, and the sound velocity decreases in directions other than the field direction.

3. INCORPORATION OF A DEFORMATION OF THE ORDER PARAMETER; THREEFOLD SPLITTING OF THE SPECTRUM OF NONPHONON MODES

In the preceding section of this paper we found a fairly weak effect of an electric field on the spectrum of collective excitations. As we mentioned back in the Introduction, the reason for this result is that we have been ignoring the electric-field-induced deformation of the order parameter. In the present section of this paper we take this deformation into account. We show that this deformation changes the collective spectrum in He^3-*B* substantially: all the nonphonon modes undergo a threefold splitting.

The order parameter in He^3-*B* is

$$A_{ij}=\Delta(T)R_{ij}(\mathbf{n},\theta)e^{i\Phi}, \tag{3.1}$$

where $R_{ij}(\mathbf{n},\theta)$ is the rotation matrix describing the rotation through an angle θ, of the spin coordinate system with respect to the orbital coordinate system around the **n** axis. In the absence of a dipole interaction, the gap $\Delta(T)$ is isotropic, and the direction of the rotation axis **n** and the angle θ are both arbitrary. An electric field with an energy

$$F_E=-g_E(E_iA_{ki}A_{kj}E_j-{}^1/_3|E|^2A_{ki}A_{ki}) \tag{3.2}$$

fixes $\mathbf{n}\perp\mathbf{E}$. It leads to a deformation of the gap in the Fermi spectrum, increasing it along **E** and reducing it in the direction perpendicular to **E**:

$$\Delta_1^2=\Delta^2-(6\beta_{345})^{-1}g_EE^2,$$
$$\Delta_2^2=\Delta^2+(3\beta_{345})^{-1}g_EE^2. \tag{3.3}$$

The dipole interaction fixes the angle

$$\theta=\arccos\left[-\frac{\Delta_1}{2(\Delta_1+\Delta_2)}\right]\approx\arccos(-{}^1/_4). \tag{3.4}$$

When we incorporate both the orientating and deforming effects of the electric field, we find the following order parameter:

$$c_{ia}^{(0)}(p)\propto\begin{pmatrix}\Delta_1 & 0 & 0\\ 0 & \Delta_1\cos\theta & -\Delta_1\sin\theta\\ 0 & \Delta_2\cos\theta & \Delta_2\cos\theta\end{pmatrix}. \tag{3.5}$$

For simplicity in the discussion below we set $\theta=0$ in this order parameter at this point. In a first approximation the Bose spectrum is determined by the quadratic part, which is found through a shift $c_{ia}(p)\to c_{ia}(p)+c_{ia}^{(0)}(p)$ in S_h:

$$\sum_p(A_{ijab}(p)c_{ia}^+(p)c_{jb}(p)+{}^1/_2B_{ijab}(p)(c_{ia}(p)c_{jb}(-p)$$
$$+c_{ia}^+(p)c_{jb}^+(-p))). \tag{3.6}$$

The equation for the spectrum is $\det Q=0$, where Q is the quadratic-form matrix of (3.6). The tensor coefficients A_{ijab} and B_{ijab} are proportional to integrals (or sums) of products of the Green's functions of quasifermions:

$$A_{ijab}=\delta_{ab}\left\{\frac{\delta_{ij}}{g}+\frac{4Z^2}{\beta V}\sum_{p_1+p_2=p}n_{1i}n_{1j}\frac{(i\omega_1+\xi_1)(i\omega_2+\xi_2)}{(\omega_1^2+\xi_1^2+\Delta^2)(\omega_2^2+\xi_2^2+\Delta^2)}\right\}, \tag{3.7}$$

$$B_{ijab}=-\frac{4Z^2}{\beta V}\sum_{p_1+p_2=p}n_{1i}n_{1j}\frac{(2f_af_b-f_i^2\delta_{ab})}{(\omega_1^2+\xi_1^2+\Delta^2)(\omega_2^2+\xi_2^2+\Delta^2)}.$$

Here

$$\Delta^2=\Delta^2(\theta)=\Delta_1^2(n_1^2+n_2^2)+\Delta_2^2 n_3^2$$

$$=\Delta^2+\Omega_0(\alpha_0+(n_1^2+n_2^2)(1-\alpha_0)), \qquad (3.8)$$

$$n_1^2+n_2^2=\sin^2\theta,$$

$$\mathbf{f}=(n_1\Delta_1;\ n_2\Delta_1\cos\theta+n_3\Delta_2\sin\theta;\ -n_2\Delta_1\sin\theta+n_3\Delta_2\cos\theta).$$

We turn now to the results of our calculations for zero momenta of the collective excitations ($\mathbf{k}=0$).

Working from the equation $\det Q=0$, evaluating the integrals over the frequencies and momenta of the quasifermions, and restricting the analysis to terms $\sim E^2$, since the corrections for the field are small in comparison with the frequencies of collective modes, we find the following equations for the spectrum (the corresponding variables are listed at the right; $v_{ia}=\operatorname{Im} c_{ia}$ and $u_{ia}=\operatorname{Re} c_{ia}$):

$$\int_0^1 (1-x^2)I(1+2c_1)\,dx=0, \qquad u_{21}+u_{12},\quad u_{11}-u_{22},$$

$$\int_0^1 I\{1+3x^2+4[(c_1(1-x^2))^{1/2}-2(c_2x^2)^{1/2}]^2\}\,dx=0,$$

$$u_{11}+u_{22}-2u_{33},$$

$$\int_0^1 I\{1+x^2+4[(c_2(1-x^2))^{1/2}+(c_1x^2)^{1/2}]^2\}\,dx=0,$$

$$u_{13}+u_{31},\quad u_{23}+u_{32},$$

$$\int_0^1 (1-x^2)I(1+4c-2c_1)\,dx=0, \qquad v_{21}+v_{12},\quad v_{11}-v_{21},$$

$$\int_0^1 I\{(1+3x^2)(1+4c)-4[(c_1(1-x^2))^{1/2}-2(c_2x^2)^{1/2}]^2\}\,dx=0,$$

$$v_{11}+v_{22}-2v_{33},$$

$$\int_0^1 I\{(1+x^2)(1+4c)-4[(c_2(1-x^2))^{1/2}+(c_1x^2)^{1/2}]^2\}\,dx=0,$$

$$v_{13}+v_{31},\quad v_{23}+v_{32},$$

$$\int_0^1 (1-x^2)I\,dx=0, \qquad u_{21}-u_{12},$$

$$\int_0^1 I\{1+x^2+4[(c_2(1-x^2))^{1/2}-(c_1x^2)^{1/2}]^2\}\,dx=0,$$

$$u_{13}-u_{31},\quad u_{23}-u_{32},$$

$$\int_0^1 I\{1+4c-4[(c_1(1-x^2))^{1/2}+(c_2x^2)^{1/2}]^2\}\,dx=0, \qquad v_{11}+v_{22}+v_{33},$$

$$\int_0^1 (1-x^2)(1+4c)I\,dx=0, \qquad v_{21}-v_{12},$$

$$\int_0^1 I\{(1+x^2)(1+4c)-4[(c_2(1-x^2))^{1/2}-(c_1x^2)^{1/2}]^2\}\,dx=0,$$

$$v_{13}-v_{31},\quad v_{23}-v_{32},$$

$$\int_0^1 I\{1+4[(c_1(1-x^2))^{1/2}+(c_2x^2)^{1/2}]^2\}\,dx=0, \qquad u_{11}+u_{22}+u_{33}.$$

Here

$$I=\frac{1}{(1+4c)^{1/2}}\ln\frac{1+(1+4c)^{1/2}}{1-(1+4c)^{1/2}},$$

$$c_n=\frac{\Delta_0^2+\alpha_n E^2}{\omega^2+(c_F(\mathbf{kn}))^2}[2-n+(2n-3)x^2], \quad n=1,2, \qquad c=c_1+c_2.$$

Results for k=0

1) All four *gd* modes (sound, a longitudinal spin wave, and two transverse spin waves) remain unperturbed.

2) There is a threefold splitting of the rsq modes:

$$E_0^2={}^8/_5\Delta^2+\Gamma_+E^2, \qquad u_{11}+u_{22}-2u_{33},$$
$$E_1^2={}^8/_5\Delta^2+{}^1/_2\Gamma_+E^2, \qquad u_{13}+u_{31}, \quad u_{33}+u_{32}, \qquad (3.9)$$
$$E_2^2={}^8/_5\Delta^2-\Gamma_+E^2, \qquad u_{12}+u_{21}, \quad u_{11}-u_{22}.$$

The subscript on the energy of a mode is equal to $|J_z|$;

$$\Gamma_+=\frac{8}{105}\left(3+\frac{6^{1/2}}{5(\pi-\operatorname{arctg}(6^{1/2}))}\right)\frac{g_E}{\beta_0}=0{,}25\frac{g_E}{\beta_0},$$

where $\beta_0=\beta_{345}$ in the weak-coupling approximation; and $u_{ij}=\operatorname{Re} c_{ij}$. The modes with projections $\pm J_z$ of the total angular momentum remain nondegenerate.

3) There is a threefold splitting of the sq modes:

$$E_0^2={}^{12}/_5\Delta^2+\Gamma_-E^2, \qquad v_{11}+v_{22}-2v_{33},$$
$$E_1^2={}^{12}/_5\Delta^2+{}^1/_2\Gamma_-E^2, \qquad v_{13}+v_{31}, \quad v_{23}+v_{32}, \qquad (3.10)$$
$$E_2^2={}^{12}/_5\Delta^2-\Gamma_-E^2, \qquad v_{12}+v_{21}, \quad v_{11}-v_{22}.$$

where

$$\Gamma_-=\frac{2}{35}\left(1-\frac{2\cdot 6^{1/2}}{5\operatorname{arctg}(6^{1/2})}\right)\frac{g_E}{\beta_0}=0{,}016\frac{g_E}{\beta_0}, \qquad v_{ij}=\operatorname{Im} c_{ij}.$$

The threefold splitting of the rsq and sq modes is very reminiscent of the dispersion-induced splitting of these modes which has been predicted independently by Vdovin[7], Shivaram *et al.*,[9] and Brusov and Popov[10] and which has been observed experimentally by Daniels *et al.*[11] The frequencies fall in the order $E_0>E_1>E_2$. The ratio (1:4) of the differences between the frequencies of the branches with $J_z=0$ and $|J_z|=1{,}2$ is the same for the dispersion-induced splitting of the $J=2$ modes and for the splitting of these modes in an electric field.

4) There is a threefold splitting of pb modes:

$$\begin{aligned} E_{0+}^2&=4\Delta^2, \quad u_{11}+u_{22}+u_{33},\\ E_{\pm1}^2&=4\Delta^2+\Gamma_0E^2, \quad u_{13}-v_{31}, \quad v_{23}+v_{32},\\ E_0^2&=4\Delta^2-2\Gamma_0E^2, \quad v_{12}-v_{21}, \end{aligned} \tag{3.11}$$

where

$$\Gamma_0={}^2/_{15}\frac{g_E}{\beta_0}=0{,}133\frac{g_E}{\beta_0}.$$

The ratio of the frequency differences between the $E_{\pm1}$ mode and the E_0 and E_{0+} modes is 1:3. The energies of all the pb modes lie between $2\Delta_{max}=2(\Delta^2+\frac{5}{2}\Gamma_0E^2)^{1/2}$ and $2\Delta_{min}=2(\Delta^2-\frac{5}{4}\Gamma_0E^2)^{1/2}$. This result tells us that all three branches of the pb mode are moderately damped and could be observed in ultrasonic experiments, as resonances at the absorption edge, in addition to the absorption of zero sound as a result of pair-decay processes. This conclusion follows from the analogy with the case of the A phase,[10] where the gap $\Delta=\Delta_{max}\sin\theta$ is again anisotropic, and where excitations with energies below $2\Delta_{max}$ are moderately damped and can be observed as resonances.

The pb modes can thus be observed in an electric field, as resonances at the absorption edge, in the manner in which they have been observed by Daniels *et al.*[11] in a magnetic field.

We can compare the maximum splitting $\delta\omega_{max}$ for nonphonon modes. Using the formula

$$\omega=\omega_0\left(1+\frac{E^2\Gamma}{2\omega_0}\right),$$

where Γ is equal to any of Γ_0, Γ_+, Γ_-, we find the results

$$\begin{aligned} pb:&\quad \delta\omega_{max}={}^3/_2\frac{\Gamma_0E^2}{\omega_{pb}}={}^1/_5\frac{g_E}{\beta_0}\frac{E^2}{\omega_{pb}}=0{,}10\frac{g_E}{\beta_0}\frac{E^2}{\Delta},\\ sq:&\quad \delta\omega_{max}=\frac{\Gamma_-E^2}{\omega_{sq}}=0{,}016\frac{g_E}{\beta_0}\frac{E^2}{\omega_{sq}}=0{,}01\frac{g_E}{\beta_0}\frac{E^2}{\Delta},\\ rsq:&\quad \delta\omega_{max}=\frac{\Gamma_+E^2}{\omega_{rsq}}={}^1/_4\frac{g_E}{\beta_0}\frac{E^2}{\omega_{rsq}}=0{,}20\frac{g_E}{\beta_0}\frac{E^2}{\Delta}. \end{aligned} \tag{3.12}$$

We see that the maximum splitting ratios are

$$\delta\omega_{max}^{rsq}:\delta\omega_{max}^{pb}:\delta\omega_{max}^{sq}=20:10:1. \tag{3.13}$$

It would thus be easiest to observe the splitting of the spectrum in an electric field in the case of the rsq and pb modes.

The maximum splitting of the spectrum is evaluated in the Appendix.

CONCLUSION

An electric field causes a threefold splitting of the spectrum of all the nonphonon modes (rsq, sq, pb), leaving the Goldstone modes without a gap. The electric field strength which would be required for an observation of this splitting can be found from the following general arguments (see the Appendix for some more accurate estimates). Equating the dipole energy and the electric dipole energy, $g_D\approx g_EE^2$, we find $E\approx1.5\cdot10^4$ V/cm. In order to observe effects of an electric field in acoustic experiments, we would need fields stronger by a factor of $10^{1/2}$, i.e., $E\approx5\cdot10^4$ V/cm. The Fermi-liquid corrections (see the Introduction) increase these fields to $5\cdot10^5$–$5\cdot10^6$ V/cm (depending on the pressure). Since the critical field E_c in He^3 is [14] $E_c\approx2.7\cdot10^6$ V/cm, the threefold splitting of the spectrum of nonphonon modes could be seen in part of the phase diagram (at pressures which are not too high).

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APPENDIX

Let us estimate the maximum splitting for the rsq modes [we can then use (3.13) to do the same for the sq and pb modes].

For the rsq mode we find from (3.12)

$$\delta\omega_{max}=0{,}2\frac{g_E}{\beta_0}\frac{E^2}{\Delta}, \tag{A1}$$

$$\Delta^2=(6\beta_{12}+2\beta_{345})^{-1}\alpha, \tag{A2}$$

$$\alpha={}^1/_3N(0)(1-T/T_c). \tag{A3}$$

Taking account of the temperature dependence of the gap,

$$\Delta=\Delta_0(1-T/T_c)^{1/2},$$

we find

$$\beta_0={}^1/_{15}N(0)\Delta_0^{-2}. \tag{A4}$$

From Ref. 12 we have

$$g_E=4\alpha^2(\gamma\hbar)^{-2}g_D, \tag{A5}$$

$$g_D=\frac{\pi}{10}\left[N(0)\gamma\hbar\ln\frac{1.13\hbar\omega}{k_BT_c}\right]^2, \tag{A6}$$

Here g_D is the dipole-interaction constant. From (A4) and (A5) we have

$$\frac{gE}{\beta_0\Delta}=6\pi\alpha^2N(0)\left[\ln\frac{1.13\hbar\omega}{k_BT_c}\right]^2\Delta_0. \tag{A7}$$

Here we have[12,13] $\alpha=2\cdot10^{-25}$ cm^3, $N(0)=(0.54$–$1.26)\cdot10^{38}$ erg$^{-1}\cdot$cm^{-3} at a zero pressure at the melting surface,

$$\frac{\hbar\omega}{k_B}\approx0{,}7\text{ K},\qquad \Delta_0=\Delta_{BCS}=1{,}76T_c.$$

The Fermi-liquid corrections have been ignored in (A7); they are taken into account below. Using (A7), we can estimate the maximum splitting of the rsq mode in an electric field.

1) If we ignore Fermi-liquid corrections we have

$$\delta\omega_{max}=0{,}2\frac{g_EE^2}{\beta_0\Delta}=1{,}2\pi\alpha^2N(0)\left[\ln\frac{1{,}13\hbar\omega}{k_BT_c}\right]^2\Delta_0E^2. \tag{A8}$$

We choose

$$E=10^5\,\text{V/cm}={}^1/_3\cdot 10^3\ \text{cgs/cm}\,, \qquad p=10\ \text{bar}. \tag{A9}$$

We then have

$$\Delta_0 = 1.76k_BT_c \approx 4.44\cdot 10^{-19}\ \text{erg},\quad N(0)=0{,}8\cdot 10^{38}\ \text{erg}^{-1}\cdot\text{cm}^{-3},\quad \ln\frac{1{,}13\hbar\omega}{k_BT_c}=6{,}08. \tag{A10}$$

Substituting (A9) and (A10) into (A8), we find

$$\delta\omega_{max}=3{,}3\ \text{kHz}.$$

This estimate of the splitting is close to the error level of ultrasonic experiments (5–10 kHz). Consequently, fields on the order of 10^5 V/cm would be required to observe the splitting of the spectrum of collective modes in acoustic experiments if the Fermi-liquid corrections are ignored.

2) We now take the Fermi-liquid corrections into account. Fomin *et al.*[4] have shown that the Fermi-liquid corrections reduce the energy associated with the electric field by a factor $\langle R^2\rangle = 4\cdot 10^{-3}$–$4.8\cdot 10^{-5}$ at zero pressure and on the melting surface, respectively.

For $\delta\omega_{\max}$ we thus have the formula

$$\delta\omega_{max}=1{,}2\pi\alpha^2N(0)\left[\ln\frac{1.13\hbar w}{k_BT_c}\right]^2\Delta_0. \tag{A11}$$

At critical fields $E=2.7\cdot 10^6$ V/cm we find the following estimates for the maximum splitting of the spectrum of rsq modes in an electric field:

$$\delta\omega_{\max}\approx 4.1\ \text{kHz},\quad p=0\ \text{bar},$$
$$\delta\omega_{\max}\approx 1.1\ \text{kHz},\quad p=10\ \text{bar}.$$

Taking into account the error level of the ultrasonic experiments, which we mentioned above (5–10 kHz for ultrasonic frequencies ~ 100 MHz), we conclude that it would be possible in principle to observe splitting of the rsq mode in fields close to the breakdown field and at low pressures.

We have found a fairly crude estimate of the splitting here, assigning several of the parameters values which are approximations in the Ginzburg-Landau region. If more-realistic values of the parameters were used in expression (A11), there might be an increase in $\delta\omega_{\max}$, and it might be simpler to actually observe the predicted splitting (and to observe it over a broader pressure range and in weaker fields).

An experimental study of the effect of an electric field on the spectrum of collective excitations in the superfluid *B* phase of He^3 is presently being carried out in J. B. Ketterson's laboratory at Northwestern University. We might point out in this connection that the first task is to study the splitting of the rsq mode at low pressures (near $p=0$) and thus at low temperatures and in fields close to the breakdown level.

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