Effect of electron-electron interactions on the thermal conductivity of impure metals

D. V. Livanov, M. Yu. Reizer,[1] and A. V. Sergeev

Institute of Applied Physics, USSR Academy of Sciences

(Submitted 27 November 1990)
Zh. Eksp. Teor. Fiz. **99**, 1230–1235 (April 1991)

Using the quantum kinetic equation method, we calculate corrections to the thermal conductivity of an impure metal due to interference of the electron-electron and electron-impurity interactions. In contradiction to the conclusions of previous papers, we show that when these interactions are taken into account the Widemann-Franz law is violated at low temperatures.

1. INTRODUCTION

The theory of weak localization, which includes interference effects between interactions, has been successfully validated by its ability to explain a wide range of experimental facts associated with low-temperature charge transport in conductors with short electron mean free paths.[1] In recent times there has been additional interest in studying the effect of electron-electron interactions on the transport of heat by electrons in impure metals.

Central to this question is the issue of what operator to use for the thermal flux in a system of interacting electrons and how to incorporate it into various methods for calculating the kinetic coefficients. For the thermal conductivity, use of the Kubo–Greenwood method (i.e., linear response) leads to extremely laborious calculations. There are several reasons for this: first of all, in addition to terms that describe the transport of heat by the interacting particles, the operator for the thermal current also contains corrections from the interaction,[2,3] leading to a considerable increase in the number of diagrams. Secondly, when the kinetic coefficients are calculated, the results obtained when the electron energy in successive integrations goes into the temperature are accompanied by anomalously large terms in which the inverse scattering time of electrons by impurities (i.e., the "uncertainty" of the electron energy) appears instead of the temperature. The discussions that follow will show clearly that these anomalous terms must be retained when we include corrections to the thermal current operator from the electron-impurity interactions. However, retaining these terms turns out to be extremely laborious in practice, and in a number of papers they are ignored, leading to erroneous results.[4]

In view of these complications associated with the use of the Kubo–Greenwood method, the electron thermal conductivity of impure metals at low temperatures was investigated in Refs. 5 and 6 by using a generalization of the Ward identities and an analysis of the skeleton structure of the conductivity and thermal conductivity diagrams. In these papers the authors concluded that, despite the inapplicability of simple Fermi liquid descriptions of the electron subsystem, the Widemann–Franz law is obeyed as usual, and that the electron thermal conductivity can be obtained easily by taking into account modifications in the conductivity.[1]

In this paper we calculate the electron thermal conductivity directly, using the quantum kinetic equation method. As we show below, the operator for the thermal current can easily be used in a form that contains the electron frequency ε in place of the electron energy and corrections from the interactions, which then passes directly to the temperature so that anomalous terms do not arise. The results we obtain differ from those found in Refs. 5 and 6: the Widemann–Franz law is not obeyed, because the temperature and mean-free-path dependences we find for the thermal conductivity of an electron are closely related to singularities in the density of states for electrons[1] rather than the behavior of the conductivity.

The article is structured in the following way. In Sec. 2 we derive the thermal current operator for interacting electrons. In Sec. 3 we briefly describe our application of the quantum kinetic equation. The calculation of the thermal conductivity is carried out in Sec. 4. Finally, in the Conclusion we analyze the results we have obtained, and also discuss possible reasons why our results disagree with those of Ref. 5.

2. THERMAL CURRENT OPERATOR FOR INTERACTING ELECTRONS

The problem of finding the thermal current in systems with interactions has received many treatments in the literature. Following Refs. 7 and 8, we will consider the thermal current to be an energy current as defined within the Lagrangian formalism, i.e., in terms of an energy-momentum tensor.

The Lagrangian of a system of electrons interacting among themselves and also with an impurity field has the form

$$L=i(\psi^+\psi_{,0}-\psi_{,0}^+\psi)-\frac{1}{2m}\psi_{,i}^+\psi_{,i}$$

$$-V(\mathbf{x},\mathbf{x}')\psi^+\psi'^+\psi'\psi-\psi^+\psi W(\mathbf{x}). \qquad (1)$$

Here $\psi=\psi(\mathbf{x},t)$ and $\psi'=\psi(\mathbf{x}',t')$ are the electron field operators: $\psi_{,0}=\partial\psi/\partial t$, $\psi_{,i}=\partial\psi/\partial x_i$, $i=1,2,3$; $V(\mathbf{x},\mathbf{x}')$ is the potential of the electron-electron interaction, while $W(\mathbf{x})$ is the potential of the impurity field:

$$W(\mathbf{x})=\sum_{j=1}^{N}U(\mathbf{x}-\mathbf{R}_j);$$

here $U(\mathbf{x})$ is the potential of an impurity center located at the point $\mathbf{x}=0$.

The operator for energy current $\mathbf{u}$ can be written from the energy momentum tensor T_μ^ν:

$$T_\mu^\nu=\frac{\partial L}{\partial\psi_{,\nu}}\psi_{,\mu}+\psi^+_{,\mu}\frac{\partial L}{\partial\psi^+_{,\nu}}-\delta_\mu^\nu L, \qquad (2)$$

0038-5646/91/040760-05$03.00

$$u^t = \int T_0^{\,t}\, d^3x\, d^3x' \tag{3}$$

(δ^{ν}_{μ} is the Kronecker symbol). Substituting (1) into (2) and (3), we obtain an expression for the energy flux operator:

$$\mathbf{u} = -\frac{1}{2m}\left(\frac{\partial}{\partial t}\nabla + \frac{\partial}{\partial t'}\nabla'\right)\psi'^{+}\psi|_{\mathbf{x}=\mathbf{x}', t=t'}. \tag{4}$$

The thermal current operator is conveniently expressed in terms of the energy current operator by using the chemical potential as a reference energy for the electrons:

$$\mathbf{q} = \mathbf{u} - \mu\mathbf{J}, \tag{5}$$

where **J** is the particle flux operator and μ is the chemical potential.

3. THE QUANTUM KINETIC EQUATION METHOD

In calculating corrections to the thermal conductivity we will make use of the quantum kinetic equation method, which was developed as an outgrowth of the Keldysh diagram technique. In this case we will follow Ref. 9, in which a systematic method was discussed for calculating the kinetic coefficients of an impure conductor. The quantum kinetic equation method developed in Ref. 9 has been applied previously in calculations of temperature-dependent corrections to impurity conductivity due to electron-electron[10] and electron-phonon[11] interactions; it has also been applied to the problem of phonon renormalization of the thermoelectric power of an impure metal.[12]

In the Keldysh technique the Green's function and self-energy of an electron, as well as the electron-electron interaction potential, are represented by matrices:

$$\hat{G} = \begin{pmatrix} 0 & G^A \\ G^R & G^C \end{pmatrix}, \quad \hat{\Sigma} = \begin{pmatrix} \Sigma^C & \Sigma^R \\ \Sigma^A & 0 \end{pmatrix}, \quad \hat{V} = \begin{pmatrix} 0 & V^A \\ V^R & V^C \end{pmatrix}. \tag{6}$$

Without taking into account electron-electron interactions, the Green's function of an electron averaged over the positions of the impurities equals

$$G_0^{\,R}(\mathbf{p}, \varepsilon) = (\varepsilon - \xi_{\mathbf{p}} + i/2\tau_\varepsilon)^{-1} = [G_0^{\,A}(\mathbf{p}, \varepsilon)]^*, \quad \xi_{\mathbf{p}} = (p^2 - p_F^{\,2})/2m, \tag{7}$$

where τ_ε is the momentum relaxation time of an electron with frequency ε due to scattering by impurities, and p_F is the Fermi momentum. In using the kinetic equation we must include the effects of electron-electron interactions and the nonequilibrium nature of the system due to external perturbations. To first order in the inhomogeneity, the electron Green's function G^c has the form

$$G^C(\mathbf{p}, \varepsilon) = s(\mathbf{p}, \varepsilon)\ (G^A - G^R) + {}^1/_2 i\{S_0(\varepsilon), G^A + G^R\}, \tag{8}$$

where $s(\mathbf{p}, \varepsilon)$ plays the role of a distribution function. In equilibrium we have $s(\mathbf{p},\varepsilon) = S_0(\varepsilon) = -\tanh(\varepsilon/2T)$. In the presence of a temperature gradient, the Poisson brackets can be written in the following way:

$$\{A, B\} = \nabla T\left(\frac{\partial A}{\partial T}\frac{\partial B}{\partial \mathbf{p}} - \frac{\partial A}{\partial \mathbf{p}}\frac{\partial B}{\partial T}\right). \tag{9}$$

The kinetic equation for the electron distribution function linearized with respect to ∇T has the form

$$\mathbf{v}\nabla T\frac{\partial S_0(\varepsilon)}{\partial \varepsilon}\frac{\varepsilon}{T} = I_{e-imp} + I_{e-e}, \tag{10}$$

where **v** is the electron velocity; the collision integrals on the right side of Eq. (10) are associated with electron-impurity and electron-electron interactions respectively. Each collision integral can be expressed in terms of the self-energy part corresponding to it by using the relations

$$I(s) = I^0(s) + \delta I(s), \quad I^0 = -i[\Sigma^C - s(\Sigma^A - \Sigma^R)],$$
$$\delta I = -i[\delta\Sigma^C - S_0(\delta\Sigma^A - \delta\Sigma^R)] + {}^1/_2\{\Sigma^A + \Sigma^R, S_0\}, \tag{11}$$

where the symbol δ without a label corresponds to corrections in the form of Poisson brackets.

Assuming that the primary mechanism for relaxation of the electron momentum is scattering by impurities, we will solve the kinetic equation iteratively: $s = S_0 + \varphi_0 + \varphi_1$. When the electron-electron interactions are not included, the nonequilibrium correction to the distribution function equals

$$\varphi_0(\mathbf{p}, \varepsilon) = \tau_\varepsilon \mathbf{v}\nabla T\frac{\partial S_0(\varepsilon)}{\partial \varepsilon}\frac{\varepsilon}{T}. \tag{12}$$

To first-order in perturbation theory with respect to the interaction we have

$$\varphi_1(\mathbf{p}, \varepsilon) = \tau_\varepsilon[I_{e-e}(S_0 + \varphi_0) + \delta_{int}I_{e-imp}(S_0 + \varphi_0)], \tag{13}$$

where $\delta_{\text{int}} I_{e-\text{imp}}$ is a correction to the impurity collision integral due to renormalization of the electron density of states:

$$\delta_{int}I_{e-imp}(\mathbf{p}, \varepsilon)$$
$$= \frac{2}{\pi\nu_\varepsilon\tau_\varepsilon}\int\frac{d^3k}{(2\pi)^3}[s(\mathbf{k}, \varepsilon) - s(\mathbf{p}, \varepsilon)]\,\mathrm{Im}[\delta_{int}G^A(\mathbf{k}, \varepsilon)]; \tag{14}$$

here

$$\delta_{int}G^A = (G_0^{\,A})^2\Sigma^{A}_{e-e}(S_0 + \varphi_0) \tag{15}$$

is a correction to the Green's function of an electron due to electron-electron interactions. The renormalized density of states equals

$$\nu_\varepsilon = \frac{2}{\pi}\int\frac{d^3p}{(2\pi)^3}\,\mathrm{Im}\,G_0^{\,A}(\mathbf{p}, \varepsilon) = \frac{m}{\pi^2}[2m(\varepsilon + \varepsilon_F)]^{1/2}. \tag{16}$$

Expressing the thermal flux in terms of the electron Green's function with the help of (5), we obtain

$$\mathbf{q} = -\varkappa\nabla T = \int\frac{d^3p\, d\varepsilon}{(2\pi)^4}\varepsilon\mathbf{v}s(\mathbf{p}, \varepsilon)\,\mathrm{Im}\,G^A(\mathbf{p}, \varepsilon), \tag{17}$$

from which it is clear that the corrections to the thermal conductivity $\varkappa$ are associated both with corrections to the distribution function $\varphi_0(\mathbf{p},\varepsilon)$ (12) and with various corrections to the electron density of states:

$$\Delta\varkappa = -\frac{2}{|\nabla T|}\int\frac{d^3p\, d\varepsilon}{(2\pi)^4}\varepsilon(\mathbf{vn})\{\varphi_1\,\mathrm{Im}\,G_0^{\,A} + \varphi_0\,\mathrm{Im}[\delta_{int}G^A(S_0)]$$
$$+ S_0\,\mathrm{Im}[\delta_{int}G^A(\varphi_0)] + S_0\,\mathrm{Im}[\delta G^A(S_0)]\}, \tag{18}$$

where **n** is a unit vector directed along ∇T, and

$$\delta G^A = (G_0^{\,A})^2\delta\Sigma^{A}_{e-e}. \tag{19}$$

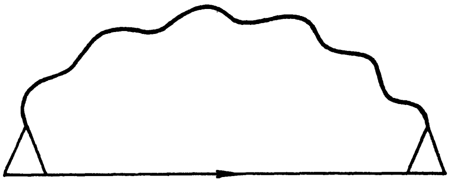

FIG. 1. Self-energy diagram of an electron.

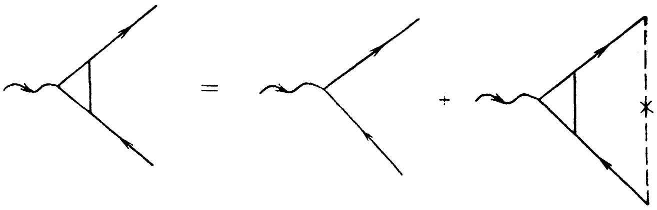

FIG. 2. Electron-electron interaction vertex.

4. CALCULATING THE CORRECTIONS TO THE THERMAL CONDUCTIVITY

The self-energy diagram for an electron including electron-electron and electron-impurity interactions Σ^A_{e-e} is shown in Fig. 1. The wavy lines denote the screened electron-electron interaction potential. The corresponding expression for the electron self-energy has the form

$$\Sigma^{ij}_{e-e}=i\int\frac{d^dq\,d\omega}{(2\pi)^{d+1}}V^{kl}(\mathbf{q},\omega)\Big[W^k_{ii'}G_{i'j'}(\mathbf{p}+\mathbf{q},\varepsilon+\omega)W^l_{j'j}$$

$$+\frac{i}{2}W^k_{ii'}\{G_{i'j'}(\mathbf{p}+\mathbf{q},\varepsilon+\omega),W^l_{j'j}\}$$

$$+\frac{i}{2}\{W^k_{ii'},G_{i'j'}(\mathbf{p}+\mathbf{q},\varepsilon+\omega)W^l_{j'j}\}\Big],\qquad(20)$$

where $\widehat{W}^1=\widehat{W}^1_0+\delta\widehat{W}^1$ should be regarded as the exact vertex for the electron-impurity interaction.

The vertex $\widehat{W}^k_0$ for electron-electron interactions renormalized by impurities (without including corrections in the form of Poisson brackets) is found within the ladder approximation by solving the following matrix equation,[10] corresponding to Fig. 2:

$$\widehat{W}_0{}^k=\hat{\omega}^k+\frac{1}{\pi\nu_\varepsilon\tau_\varepsilon}\int\frac{d^dp}{(2\pi)^d}\hat{\sigma}_x\hat{G}(\mathbf{p},\varepsilon)\widehat{W}_0{}^k\hat{G}(\mathbf{p}+\mathbf{q},\varepsilon+\omega)\hat{\sigma}_x;\qquad(21)$$

here $\hat{\omega}^k$ is the bare Coulomb vertex and $\hat{\sigma}_x$ is a Pauli matrix. To lowest order in $(p_Fl)^{-1}$, where $l=v_F\tau$, we obtain

$$W_{22}{}^1=\frac{1}{2^{1/2}(1-\zeta)},\quad W_{12}{}^1=\frac{S_0(\varepsilon)\zeta}{2^{1/2}(1-\zeta)},$$

$$W_{21}{}^1=-\frac{S_0(\varepsilon+\omega)\zeta}{2^{1/2}(1-\zeta)},$$

$$W_{11}{}^1=\frac{1}{2^{1/2}}\left[\frac{1}{1-\zeta^*}-2S_0(\varepsilon)S_0(\varepsilon+\omega)\,\mathrm{Re}\frac{\zeta}{1-\zeta}\right],\qquad(22)$$

$$W_{22}{}^2=0,\quad W_{12}{}^2=W_{21}{}^1=\frac{1}{2^{1/2}},\quad W_{11}{}^2=\frac{[S_0(\varepsilon+\omega)-S_0(\varepsilon)]\zeta^*}{2^{1/2}(1-\zeta^*)},$$

where for $ql\ll1$, $\omega\tau\ll1$

$$\zeta=\frac{1}{\pi\nu_\varepsilon\tau_\varepsilon}\int\frac{d^dp}{(2\pi)^d}G^A(\mathbf{p},\varepsilon)G^R(\mathbf{p}+\mathbf{q},\varepsilon+\omega)=1+i\omega\tau-Dq^2\tau.\qquad(23)$$

Here d is the dimensionality of the electron system and $D=v_F^2\tau/d$ is the electron diffusion coefficient. According to (21), the equation for the vertex corrections $\widehat{W}^k_0$ due to the Poisson brackets has the form

$$\delta\widehat{W}^k=\frac{1}{\pi\nu_\varepsilon\tau_\varepsilon}\int\frac{d^dp}{(2\pi)^d}\hat{\sigma}_x\Big[\hat{G}(\mathbf{p},\varepsilon)\delta\widehat{W}^k\hat{G}(\mathbf{p}+\mathbf{q},\varepsilon+\omega)$$

$$+\frac{i}{2}\{\hat{G}(\mathbf{p},\varepsilon),\widehat{W}_0{}^k\}\hat{G}(\mathbf{p}+\mathbf{q},\varepsilon+\omega)$$

$$+\frac{i}{2}\{\hat{G}(\mathbf{p},\varepsilon)\widehat{W}_0{}^k,\hat{G}(\mathbf{p}+\mathbf{q},\varepsilon+\omega)\}\Big]\hat{\sigma}_x.\qquad(24)$$

Out of all the components of the vertex $\delta\widehat{W}^k_0$ we will calculate explicit expressions only for δW^1_{22}, i.e., the component with the strongest singularity (a squared diffusion pole). Use of this term gives the main contribution to the thermal conductivity:

$$\delta W_{22}{}^1=-\frac{i\zeta^*}{2^{1/2}(1-\zeta^*)^2}\frac{\partial_T{}^*}{\partial q}|\nabla T|$$

$$\times\left(\frac{\partial S_0(\varepsilon)}{\partial\varepsilon}\frac{\varepsilon}{T}+\frac{\partial S_0(\varepsilon+\omega)}{\partial\varepsilon}\frac{\varepsilon+\omega}{T}\right).\qquad(25)$$

The screened electron-electron interaction potential for the case of small momenta and energy transfer $ql\ll1$, $\omega\tau\ll1$ (but for $p_Fl\gg1$) has the form,[1]

$$V^R(\mathbf{q},\omega)=[V^A(\mathbf{q},\omega)]^*$$

$$=\begin{cases}4\pi e^2\left(q^2+\dfrac{k_3Dq^2}{-i\omega+Dq^2}\right)^{-1}, & d=3,\\[2ex] 2\pi e^2\left(|q|+\dfrac{k_2Dq^2}{-i\omega+Dq^2}\right)^{-1}, & d=2,\\[2ex] e^2\left(\ln^{-1}\dfrac{1}{q^2a^2}+\dfrac{e^2\nu_1Dq^2}{-i\omega+Dq^2}\right)^{-1}, & d=1,\end{cases}\qquad(26)$$

where $k_3^2=4\pi e^2\nu_3$, $k_2=2\pi e^2\nu_2$ (here $\nu_3=mp_F/\pi^2$, $\nu_2=m/2\pi$, $\nu_1=\nu_3a^2$), where a is a characteristic dimension of the sample. For an electronic system in equilibrium,

$$V^C(\mathbf{q},\omega)=-\mathrm{cth}\frac{\omega}{2T}[V^A(\mathbf{q},\omega)-V^R(\mathbf{q},\omega)].\qquad(27)$$

As we have already noted, in calculating

$\Sigma_{e-e} = (\Sigma_{e-e})_0 + \delta\Sigma_{e-e}$ from Eq. (20) we obtain the primary contribution by using the component W^2_{11}. This is associated with the fact that when $ql \ll 1$ we need retain only the most singular term in q. The calculations show that terms containing the diffusion pole $(1-\zeta)^{-1} = \tau^{-1}(-i\omega + Dq^2)^{-1}$ raised to the highest possible power—i.e., cubic—are necessarily accompanied by an additional factor of q^2. From this it follows that, in addition to these terms, the leading-order contributions to the thermal conductivity are given by terms that contain the diffusion pole squared but without additional powers of q in the numerator. In taking into account this fact we obtain

$$(\Sigma^A_{e-e})_0 = -\frac{i}{2}\int\frac{d^dq\,d\omega}{(2\pi)^{d+1}}V^R(\mathbf{q},\omega)G^R(\mathbf{p}+\mathbf{q},\varepsilon+\omega)$$

$$\times\frac{\zeta}{(1-\zeta)^2}[S(\varepsilon+\omega)-S(\varepsilon)], \tag{28}$$

$$\delta\Sigma^A_{e-e} = \frac{|\nabla T|}{2}\int\frac{d^dq\,d\omega}{(2\pi)^{d+1}}V^R(\mathbf{q},\omega)$$

$$\times\left(\frac{\partial S_0(\varepsilon)}{\partial\varepsilon}\frac{\varepsilon}{T}+\frac{\partial S_0(\varepsilon+\omega)}{\partial\varepsilon}\frac{\varepsilon+\omega}{T}\right)$$

$$\times\left\{\frac{\zeta}{(1-\zeta)^3}\frac{\partial\zeta}{\partial q}G^R(\mathbf{q}+\mathbf{p},\varepsilon+\omega)\right.$$

$$\left.+\frac{\zeta}{(1-\zeta)^2}[G^R(\mathbf{p}+\mathbf{q},\varepsilon+\omega)]^2\left(\mathbf{v}+\frac{\mathbf{q}}{m},\mathbf{n}\right)\right\}. \tag{29}$$

Substituting Eqs. (12), (13), (28) and (29) into (18) and carrying out the subsequent integrations, we solve the problem as posed. Rather than writing out the contributions of all four terms in Eq. (18), we will just make the following comments. As we have already pointed out, the primary contributions to the thermal conductivity are expected to come both from terms that contain the diffusion pole as a cube and from terms with the diffusion pole squared but without additional powers of q in the numerator. In calculating the thermal conductivity according to Eq. (18) we encounter terms of first order only from the fourth term in (18) when we use the first-order terms in the curly brackets from (29). However, terms that contain the squared diffusion pole can in principle appear within all four terms of (18). In calculating the contributions of these terms it is found that the nonlocal part of the collision integral in the first term in (18) does not contribute to the thermal conductivity and we need include only the collision integral calculated using the equilibrium distribution function and corrections to φ_1 related to renormalization of the collision integral (14) by impurity interactions. Contributions of this type come from the second term in (18), and also from the fourth term in (18) due to the second term in the curly brackets of (29). Although the third term in (18) contains a double pole due to the squaring operation, it also contains additional powers of q in the numerator and therefore does not contribute to the thermal conductivity in leading order.

As a result, the Coulomb correction to the thermal conductivity has the form

$$\Delta\varkappa^d = \frac{\pi\nu\tau}{a^{3-d}}A_d\int\frac{d\varepsilon\,d\omega}{(2\pi)^2}S_0(\varepsilon)\frac{\partial S_0(\varepsilon+\omega)}{\partial\varepsilon}\frac{\varepsilon(\varepsilon+\omega)}{T}\int\frac{d^dq}{(2\pi)^d}$$

$$\times\left\{2v_F{}^2\tau^2\,\mathrm{Im}\frac{V^R(\mathbf{q},\omega)}{(1-\zeta)^2}-\left(\frac{\partial\zeta}{\partial q}\right)^2\mathrm{Im}\frac{V^R(\mathbf{q},\omega)}{(1-\zeta)^3}\right\},$$

$$A_d = \begin{cases} 2\pi/3, & d=3,\\ \pi/4, & d=2,\\ 1, & d=1.\end{cases} \tag{30}$$

$$\int d\varepsilon\left[\varepsilon(\varepsilon+\omega)S_0(\varepsilon)\frac{\partial S_0(\varepsilon+\omega)}{\partial\varepsilon}\right.$$

$$\left.+\varepsilon^2S_0(\varepsilon+\omega)\frac{\partial S_0(\varepsilon)}{\partial\varepsilon}\right] = -\omega^3\frac{d}{d\omega}\left(\mathrm{cth}\frac{\omega}{2T}\right). \tag{31}$$

In calculating the thermal conductivity we find that contributions containing the squared diffusion pole cancel out, and that the main contribution comes from a term proportional to the diffusion pole cubed. This cancellation takes place because the expression corresponding to (31) used to calculate the conductivity (i.e., without the electronic frequency in front of the distribution function) equals zero. In the three-dimensional case in the presence of interactions we find according to (26) that for $q \ll k$ and $q \sim (\omega/D)^{1/2}$ we have

$$\frac{Dq^2}{-i\omega+Dq^2}V^R(\mathbf{q},\omega)\approx\frac{1}{\nu_3}. \tag{32}$$

Carrying out the integration in (30) with respect to **q** and ω, we finally obtain

$$\Delta\varkappa^{3d} = \frac{5\cdot 6^{1/3}}{36\pi^2}\zeta\left(\frac{5}{2}\right)\Gamma\left(\frac{5}{2}\right)\frac{T^{3/2}}{v_F\tau^{1/2}}, \tag{33}$$

where $\zeta(x)$ is the Riemann zeta function and $\Gamma(x)$ is the gamma function.

Let us discuss the two-dimensional case in more detail. Here the contributions from the second term in (30) that contain the triple pole turn out to be of order T/a. The primary contribution to the thermal conductivity in this case is given by the first term in (30). We note that, just as in calculations of corrections to the electron density of states from electron-electron interactions,[1] the approximation (32) causes the integral with respect to **q** to diverge at its lower limit. In this case it is no longer permissible to neglect $|q|$ in the denominator of (26). Taking into account that the main contribution to the integration with respect to **q** comes from the region $\omega/Dk_2 \ll q \ll (\omega/D)^{1/2}$, by integrating the first term in (30) with respect to **q** we obtain to logarithmic accuracy

$$\mathrm{Im}\int\frac{d^2q}{(2\pi)^2}\frac{k_2}{|q|}\frac{1}{(-i\omega+D|q|k_2)(-i\omega+Dq^2)}$$

$$=\frac{1}{4\pi D\omega}\ln\frac{Dk_2{}^2}{\omega}. \tag{34}$$

Then the correction to the thermal conductivity in the two-dimensional case equals

$$\Delta\varkappa^{2d}=\frac{T}{12a}\ln\frac{Dk_2^2}{T}. \tag{35}$$

In the one-dimensional case a situation occurs that is analogous to the two-dimensional one: the primary contribution comes from the double pole in (30). In this case we cannot neglect the term $\ln^{-1}(1/q^2a^2)$ in the denominator of (26). As a result, we obtain to logarithmic accuracy

$$\Delta\varkappa^{1d}=-\frac{3\cdot 2^{1/2}}{4\pi^{5/2}}\zeta\left(\frac{3}{2}\right)\Gamma\left(\frac{3}{2}\right)\frac{v_F(\tau T)^{1/2}k_3}{a}\ln^{1/2}\frac{Dk_3^2}{T}. \tag{36}$$

5. CONCLUSION

The main results of this paper are expressions for the electronic thermal conductivity in an impure metal (33), (34), and (36), in which we have taken into account interference between electron-electron and electron-impurity scattering processes. As already noted in the Introduction, questions have recently been raised in connection with thermal conductivity analyses based on generalized Ward identities, regarding the validity of the Wiedemann–Franz law when Coulomb interactions are taken into account. The temperature-dependent corrections to the conductivity based on electron-electron scattering have the form[1]

$$\begin{aligned}\Delta\sigma^{3d}&=-0{,}49e^2(T/D)^{1/2},\\ \Delta\sigma^{2d}&=0{,}05e^2\ln T\tau,\\ \Delta\sigma^{1d}&=0{,}03e^2(D/T)^{1/2}.\end{aligned} \tag{37}$$

A comparison of (33), (34), (36), and (37) shows that, in contradiction to the conclusions of Refs. 5 and 6, we find that the Wiedemann–Franz law is violated. This happens for the following reasons. The primary contribution to the expression for the conductivity is given by terms that contain the diffusion pole as a cube, while the terms with the pole squared cancel out. In calculating the thermal conductivity, terms with the squared diffusion pole give a contribution of the same order as terms with the cube in the three-dimensional case, but for cases of lower dimensionality these terms dominate. In calculations using the linear response method different diagrams correspond to terms with different powers of the diffusion pole. The disagreement between the results of this paper, which were obtained by direct calculations, and the conclusions of Refs. 5 and 6, is apparently connected with the fact that in their analysis of the skeleton structure of the diagrams these latter authors did not take into account the possibility of such cancellations.

The corrections we have calculated to the thermal conductivity due to electron-electron interactions are experimentally simpler to observe in low-dimensional samples. In the one-dimensional case the effect can be observed superposed on the electron conductivity connected with electron-impurity scattering by looking for a temperature dependence of the form $\Delta\varkappa^{1d}\propto T^{1/2}\ln^{1/2}T$. For samples with higher-dimensional interactions the corrections we have found are conveniently identified from their characteristic dependence on the electron mean free path.

[1)] Ohio State University, USA.

---

[1] B. L. Altshuler and A. G. Aronov, *Electron-Electron Interactions in Disordered Systems*, A. L. Efros and M. Pollak eds., p. 1. North-Holland, Amsterdam, Oxford, New York, Tokyo, 1985.

[2] V. Ambegaokar and L. Tewordt, Phys. Rev. **134**, 805 (1968).

[3] A. Villenkin and P. L. Taylor, Phys. Rev. B **18**, 5280 (1978).

[4] Y. Lu and B. Patton, Phys. Rev. B (in press) (1990).

[5] C. Castellani, C. Di Castro, G. Kotliar, and P. A. Lee, Phys. Rev. Lett. **59**, 477 (1987).

[6] C. Castellani, C. Di Castro, G. Kotliar, P. A. Lee *et al.*, Phys. Rev. B **37**, 9046 (1988).

[7] R. Balescu, *Equilibrium and Nonequilibrium Statistical Mechanics*, Vol. 2. Wiley, New York, 1975.

[8] J. S. Langer, Phys. Rev. **128**, 110 (1962).

[9] B. L. Al'tshuler, Zh. Eksp. Teor. Fiz. **75**, 1330 (1978) [Sov. Phys. JETP **48**, 670 (1978)].

[10] B. L. Al'tshuler and A. G. Aronov, Pis'ma Zh. Eksp. Teor. Fiz. **30**, 514 (1979) [Sov. Phys. JETP **30**, 482 (1979)].

[11] M. Yu. Reizer and A. V. Sergeev, Zh. Eksp. Teor. Fiz. **93**, 2191 (1987) [Sov. Phys. JETP **66**, 1250 (1987)].

[12] M. Yu. Reizer and A. V. Sergeev, Zh. Eksp. Teor. Fiz. **92**, 2291 (1987) [Sov. Phys. JETP **65**, 1291 (1987)].

Translated by Frank J. Crowne