Linear Zeeman effect for collective modes in He^3-*A*

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A recent experiment in which the clapping-mode frequency in He^3-*A* (Ref. 1) was exactly measured is in good agreement with the Brusov-Popov theory.[2] The same method of functional integration is employed in the present paper to obtain a complete set of equations describing the collective excitations in He^3-*A* in an arbitrary magnetic field with allowance for attenuation of the collective excitations. The equations are solved for weak fields and zero momenta of the collective excitation. The linear Zeeman effect is obtained for the clapping and pair-breaking modes, viz., triple splitting of the modes which can be observed experimentally. The effect of the magnetic field on the attenuation of the collective excitations is also investigated.

1. INTRODUCTION

THE TWO MAIN METHODS OF INVESTIGATING SUPERFLUID He^3

The *A*-phase is probably one of the most interesting objects in superfluid He^3. It provides us with an example of an anisotropic quantum superfluid. The main properties of He^3-*A* are connected with the existence, on the Fermi surface, of two poles at which the gap in the single-particle spectrum vanishes. This leads, as Volovik noted,[3] to the existence in the system of chiral fermions, gauge fields, and *W*-bosons, the phenomenon of zero charge, and also to the attenuation of the collective modes even for zero momenta of the excitations[2] and to many other consequences.

There are two main theoretical approaches to studying superfluid He^3: the first method historically was the kinetic equation (KE) and the other method is the functional integration (FI) method. Each of these two methods has its own advantages and disadvantages. The main virtue of the FI method is, as a rule, a more accurate (in comparison with the KE method) calculation of the frequencies of the collective excitations. Thus, for example, in He^3-*B* the complete spectrum of the collective excitations (including the *pb*-modes) was first calculated by Brusov and Popov[4] by the FI method. A study of the stability of the Goldstone modes with respect to decay required the calculation of corrections $\sim k^4$ (where k are the momenta of the collective excitations) to the spectrum of collective modes (CM) and was realized using by the same method,[5] as was the above-mentioned calculation of the complete CM spectrum in He^3-*A* with attenuation taken into account.[2]

The main accomplishments of the KE method are the calculation of the coupling of zero sound with the collective modes. A beautiful example of this is the calculation by Koch and Wolfle[6] of the coupling between zero sound and the real squashing (rsq) mode, which exists as a result of the very small particle-hole asymmetry. The reason for such a situation is the following. The application of the FI method to the study of superfluid He^3 was developed by Brusov and Popov[7] especially for the investigation of the Bose spectrum. Therefore in this method integration over all of the Fermi degrees of freedom was carried out and Bose fields which describe the Cooper pairs of fermions on the Fermi surface were obtained. This led to further progress in the solution of problems associated with the CM eigenfrequencies than was possible with the KE method. However, this simplification did not make it possible to investigate the interaction between the Bose and Fermi degrees of freedom. In what follows we propose to perfect the formalism which we developed, extending the treatment to the Fermi degrees of freedom (forgoing integration over the "slow" Fermi fields).

The KE method, which considers the Bose and Fermi fields simultaneously, was up until now more complicated than the FI method and less successful in calculating the CM spectrum. From our point of view, both methods are equivalent. A good example which confirms this is Combescot's reproduction,[8] by the KE method, of the equations for the CM spectrum in He^3-*B* two years after Brusov and Popov[4] obtained them by the FI method. Therefore, those differences in the results of calculations of the CM spectrum in He^3-*A* about which we will be speaking are explained not only by the specific situation described above, but also by the difference in the physical mechanisms brought into consideration.

2. RECENT ULTRASOUND EXPERIMENTS IN He^3-*A* AND THEIR INTERPRETATION

An exact measurement of the frequency of the clapping (*cl*) mode in He^3-*A* in 1989[1] gave the value $E_{cl} = 1.15\Delta_0(T)$, which differs by 6% from the result $E_{cl} = 1.23\Delta_0(T)$ obtained by many authors (see, e.g., Ref. 9) by the KE method. The authors of Ref. 1, not knowing the results of Ref. 2, which give for the frequency of the *cl*-mode $E_{cl} = (1.17 \pm 0.01)\cdot\Delta_0(T)$ and are in good agreement with experiment (see Fig. 1), tried to explain this difference in terms of Fermi-liquid corrections and the effects of higher-order pairings, but without apparent success. The reason why the difference between the results obtained by the KE and FI methods is that the authors of the second method took into account the attenuation of the collective modes by the presence of zeros in the gap in the Fermi spectrum. This gives a nonzero value of the imaginary part of the CM energy

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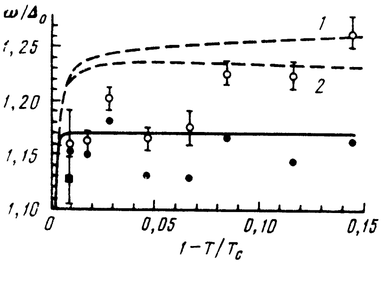

FIG. 1. Normalized frequency of the clapping mode. ○ and ●—experimental data from Ref. 1 for $A = 2.03$ (○) and 2.64 (●).
$\Delta_0(T) = \Delta_0(0)\ \text{th}\ [(\pi/A)\{\Delta c/c)(T_c/T - 1)\}^{1/2}]$.
Dashed lines correspond to the equation
$\omega_{cl}/\Delta_0(T) = 1.23[1 - (0.005 - 0.106x_3^{-1} - 0.052F_2^s)\{\Delta_0(T)/k_BT\}]$,
obtained by the kinetic equation method for $x_3^{-1} = 0$ (1) and -0.4 (2); 2—Brusov-Popov theory.[2]

[for the *cl*-mode we have $E_{cl} = (1.17 \pm 0.13)\Delta_0(T)$], which by virtue of the dispersion relations leads to a renormalization of the real part of the CM energy (frequency).

Thus, we can conclude that it is more important to take into account CM attenuations in the determination of the CM frequencies than the Fermi-liquid corrections or the effects of higher pairings. Since attempts[10,11] to use the latter to explain effects at the edge of the ultrasound absorption spectrum in He^3-*B* were also unsuccessful,[1]) one gets the impression that it is not so important to take them into account in superfluid He^3.

Another interesting fact in He^3-*A*, first noted in Ref. 2, is that in the weak coupling approximation the number of Goldstone modes (*gd*) is equal to nine as against five in real He^3-*A*. The existence of additional quasi-Goldstone spin-orbit modes is, as was first noted by Volovik,[3] a consequence of the presence of hidden symmetry. These four additional modes are an analog of massless *W*-bosons in the theory of the electroweak interaction. However, in He^3-*A* *W*-bosons acquire a mass thanks to close-coupling corrections[2] in contrast with the Weinberg-Salaam theory, in which the acquisition of mass by the *W*-bosons takes place due to the Higgs mechanism.[3]

In the present paper, using the FI method, we obtain the complete set of equations which describe the collective modes in He^3-*A* in an arbitrary magnetic field and for arbitrary momenta of the collective excitations (CE) in the region T_c-$T \sim T_c$ and solve them in the region of small fields and zero momenta of the CE. The influence of the magnetic field on the CE spectrum was studied in Refs. 14 and 15. In the Conclusion we compare our results with the results of Refs. 14 and 15, but three important differences are already clear: 1) the different number of *gd*-modes, 2) the more accurate calculation of the CM frequencies in the FI method, and 3) the possibility of using the method proposed here to study the influence of the magnetic field on the attenuation of the collective modes.

3. EQUATIONS FOR THE SPECTRUM OF COLLECTIVE EXCITATIONS IN He^3-*A* IN ARBITRARY MAGNETIC FIELDS

The investigated system is characterized by the hydrodynamic-action functional (HAF)

$$S_h = g^{-1}\sum_{p,i,a} c_{ia}^+(p)c_{ia}(p) + {}^1/_2 \ln\det\left[\frac{M(c,c^+)}{M(c^{(0)},c^{(0)+})}\right], \qquad (1)$$

where $c_{ia}^+(p)$ and $c_{ia}(p)$ are the Bose fields which describe the Cooper pairs of quasifermions at the Fermi surface, and M is an operator which depends on $c_{ia}(p)$ and the quasifermion parameters:

$$M = \begin{pmatrix} Z^{-1}(i\omega - \xi + \mu H\sigma_3)\delta_{p_1p_2}, & (\beta V)^{-1/2}(n_{1i} - n_{2i})c_{ia}(p_1+p_2)\sigma_a \\ -(\beta V)^{-1/2}(n_{1i} - n_{2i})c_{ia}^+(p_1+p_2)\sigma_a, & Z^{-1}(-i\omega + \xi + \mu H\sigma_3)\delta_{p_1p_2} \end{pmatrix}. \qquad (2)$$

Here $\xi = c_F(k - k_F)$, $n_i = k_i/k_F$, H is the magnetic field (we choose it to be directed along the z axis), $\beta = T^{-1}$, μ is the quasiparticle magnetic moment, σ_a ($a = 1, 2, 3$) are Pauli matrices, and $\omega = (2n + 1)T$ are the Fermi frequencies. The negative constant is proportional to the scattering amplitude of the two quasifermions near the Fermi surface assuming that this amplitude is equal to $g(k_1 - k_2, k_3 - k_4)$, where k_1 and k_2 are the momenta of the incoming fermions, and k_3 and k_4 are those of the outgoing ones. The functional (1) contains all the information on the physical properties of the system and describes, in particular, the CE spectrum. In the first approximation this spectrum is determined by the quadratic part of the HAF, which is obtained by the shift $c_{ia}(p) \to c_{ia}(p) + c_{ia}^{(0)}(p)$ in the Bose fields in the HAF. Here $c_{ia}^{(0)}(p)$ is the wave function of the condensate and is proportional to the order parameter (OP). After making this shift we can expand the functional

$$ {}^1/_2 \ln\det\left[\frac{M(c,c^+)}{M(c^{(0)},c^{(0)+})}\right] = {}^1/_2 \mathrm{Sp}\ln(1+\hat{G}\hat{U}) = -\sum_{n=1}^{\infty}\frac{1}{4n}\mathrm{Sp}(\hat{G}\hat{U})^{2n}$$

and keep only the term with $n = 1$. Here

$$M(c^{(0)}, c^{(0)+}) = \hat{G}^{-1}, \quad \hat{M}(c, c^+) = \hat{G}^{-1} + \hat{U}.$$

According to the ideas of Nasten'ka and Brusov,[16] to investigate the CE spectrum in a magnetic field we must take into account not only the additional HAF term associated with the magnetic field, but also the deformation of the order parameter caused by it, which in the present case is equal to[17]

$$c_{ia}(p)=c(\beta V)^{1/2}\delta_{p_0}(\delta_{a1}\alpha_+ + i\delta_{a2}\alpha_-)(\delta_{i1}+i\delta_{i2}). \quad (3)$$

Here

$$\alpha_\pm=\frac{\Delta_\uparrow\pm\Delta_\downarrow}{2\Delta},\quad \Delta_{\uparrow\downarrow}=\frac{N(0)(\tau\pm\eta h)}{2\beta_{245}},$$

$$\eta=\left(\frac{N'(0)}{N(0)}\right)T_c\ln\left(\frac{1{,}14\varepsilon_0}{T_c}\right),\quad h=\frac{\mu_0 H}{T_c},\quad \Delta=2cZ$$

is the gap in the homogeneous spectrum, determined by the following equation:

$$g^{-1}=-\frac{Z^2}{\beta V}(\alpha_+^2+\alpha_-^2)^{-1}$$

$$\times\sum_p\Big[\frac{(\alpha_++\alpha_-)^2\sin^2\theta}{\omega^2+(\xi+\mu H)^2+\Delta^2\sin^2\theta(\alpha_++\alpha_-)^2}+$$

$$+\frac{(\alpha_+-\alpha_-)^2\sin^2\theta}{\omega^2+(\xi+\mu H)^2+\Delta^2\sin^2\theta(\alpha_+-\alpha_-)^2}\Big]. \quad (4)$$

Substituting the expressions for $c(0)^+$ and $c(0)$ from Eq. (3) in Eq. (2), we obtain for G^{-1}

$$G^{-1}=\begin{pmatrix} Z^{-1}(i\omega-\xi+\mu H\sigma_3)I_+, & 2c(n_1+in_2)(\alpha_+\sigma_1+i\alpha_-\sigma_2)I_- \\ -2c(n_1-in_2)(\alpha_+\sigma_1-i\alpha_-\sigma_2)I_-, & Z^{-1}(-i\omega+\xi+\mu H\sigma_3)I_+\end{pmatrix}. \quad (5)$$

Inverting G^{-1}, for

$$G=\begin{pmatrix} G_{11} & G_{12}\\ G_{21} & G_{22}\end{pmatrix}$$

we obtain

$$G_{11}|=\begin{pmatrix}\frac{a^++b}{d_1} & 0\\ 0 & \frac{a^+-b}{d_2}\end{pmatrix}I_+,\quad G_{12}=\begin{pmatrix}0 & \frac{q_1-iq_2}{d_1}\\ \frac{q_1+iq_2}{d_2} & 0\end{pmatrix}I_-,$$

$$G_{21}=\begin{pmatrix}0 & \frac{-q_1^++iq_2^+}{d_2}\\ \frac{-q_1^+-iq_2^+}{d_1} & 0\end{pmatrix}I_-,\quad G_{22}=\begin{pmatrix}\frac{-a^++b}{d_2} & 0\\ 0 & \frac{-a^+-b}{d_1}\end{pmatrix}I_+,$$

$$I_+=\delta_{p_1,p_2},\quad I_-=\delta_{p_1+p_2,0},$$

where

$$a=Z^{-1}(i\omega-\xi),\quad b=Z^{-1}\mu H,\quad q_{1,2}=Z^{-1}\Delta(n_1+in_2)\alpha_{+,-},$$

$$d_{1,2}=Z^{-2}(\omega^2+(\xi\mp\mu H)^2+\Delta^2\sin^2\theta(\alpha_+\pm\alpha_-)^2).$$

Using the expression obtained for $\hat G$ and the following expression for $\hat U$

$$\hat U=(\beta V)^{-1/2}\begin{pmatrix}0, & (n_{1i}-n_{2i})c_{ia}(p_1+p_2)\sigma_a\\ -(n_{1i}-n_{2i})c_{ia}^+(p_1+p_2)\sigma_a, & 0\end{pmatrix}$$

we obtain the quadratic part of the HAF in the form

$$S_h=\frac{1}{g}\sum_{p,i,a}c_{ia}^+c_{ia}+\frac{Z^2}{\beta V}$$

$$\times\sum_{p_1+p_2=p}n_{1i}n_{2i}\{[-\Delta^2(n_1+in_2)^2(-\partial_3c_{i3}^+c_{j3}^++$$

$$+\partial_1(c_{i1}^+c_{j1}^+-c_{i2}^+c_{j2}^+)+i\partial_2(c_{i1}^+c_{j2}^++c_{i2}^+c_{j1}^+))+\text{э. c.}]+$$

$$+D_1(c_{i1}^+c_{j1}+c_{i1}c_{j1}^++c_{i2}^+c_{j2}+c_{i2}c_{j2}^+)$$

$$+D_2i(c_{i2}^+c_{j1}+c_{i1}c_{j2}^+-c_{i2}c_{j1}^+-c_{i1}^+c_{j2})+$$

$$+D_3(c_{i3}^+c_{j3}+c_{i3}c_{j3}^+)\}. \quad (6)$$

Here

$$\partial_{1,2}=\frac{(\alpha_++\alpha_-)^2}{d_1(p_1)d_1(p_2)}\pm\frac{(\alpha_+-\alpha_-)^2}{d_2(p_1)d_2(p_2)},$$

$$\partial_3=(\alpha_1-\alpha_2)\left[\frac{1}{d_1(p_2)d_2(p_1)}+\frac{1}{d_1(p_1)d_2(p_2)}\right],$$

$$D_{1,2}=\frac{(a^+(p_1)+b)(a^+(p_2)+b)}{d_1(p_1)d_1(p_2)}\pm\frac{(a^+(p_1)-b)(a^+(p_2)-b)}{d_2(p_1)d_2(p_2)},$$

$$D_3=\frac{(a^+(p_1)+b)(a^+(p_2)-b)}{d_1(p_1)d_2(p_2)}+\frac{(a^+(p_1)-b)(a^+(p_2)+b)}{d_2(p_1)d_1(p_2)}. \quad (7)$$

Transforming to the real and imaginary parts of the order parameter with the help of the formulas $u_{ia}=\operatorname{Re}c_{ia}$ and $v_{ia}=\operatorname{Im}c_{ia}$, we obtain by diagonalizing the quadratic form (6) the following canonical form:

$$S_h=\left(g^{-1}+\frac{2Z^2}{\beta V}\sum_{p_1+p_2=p} D_3\cos^2\theta\right)(u_{33}^2+v_{33}^2)$$

$$+\left[g^{-1}+\frac{2Z^2}{\beta V}\sum_{p_1+p_2=p}(D_1+D_2)\cos^2\theta\right][(u_{31}+v_{32})^2+(u_{32}+v_{31})^2]$$

$$+\left[g^{-1}+\frac{2Z^2}{\beta V}\sum_{p_1+p_2=p}(D_1-D_2)\cos^2\theta\right][(u_{31}-v_{32})^2+(u_{32}-v_{31})^2]$$

$$+\left[g^{-1}+\frac{2Z^2}{\beta V}\sum_{p_1+p_2=p}D_3\sin^2\theta\right][(v_{13}+u_{23})^2+(u_{13}-v_{23})^2]$$

$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum_{p_1+p_2=p}(D_3-\partial_3\Delta^2\sin^2\theta)\sin^2\theta\right](u_{13}+v_{23})^2$$

$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum_{p_1+p_2=p}(D_3+\partial_3\Delta^2\sin^2\theta)\sin^2\theta\right](u_{13}-v_{23})^2$$

$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum_{p_1+p_2=p}(D_1+D_2)\sin^2\theta\right][(u_{12}+v_{11}+u_{21}+v_{22})^2$$

$$+(u_{11}+v_{12}-u_{22}-v_{21})^2]+$$

$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum_{p_1+p_2=p}(D_1-D_2)\sin^2\theta\right][(u_{12}-v_{11}-u_{21}+v_{22})^2 \tag{8}$$

$$+(u_{11}-v_{12}+u_{22}-v_{21})^2]\cdot$$

$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum_{p_1+p_2=p}(D_1+D_2-(\partial_1+\partial_2)\Delta^2\sin^2\theta)\sin^2\theta\right]$$

$$\times(u_{12}+v_{11}-u_{21}-v_{22})^2+$$

$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum_{p_1+p_2=p}(D_1-D_2-(\partial_1-\partial_2)\Delta^2\sin^2\theta)\sin^2\theta\right]$$

$$\times(u_{12}-v_{11}+u_{21}-v_{22})^2+$$

$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum_{p_1+p_2=p}(D_1+D_2+(\partial_1+\partial_2)\Delta^2\sin^2\theta)\sin^2\theta\right]$$

$$\times(u_{11}+v_{12}+u_{22}+v_{21})^2\cdot$$

$$+\left[g^{-1}+\frac{Z^2}{\beta V}\sum_{p_1+p_2=p}(D_1-D_2+(\partial_1-\partial_2)\Delta^2\sin^2\theta)\sin^2\theta\right]$$

$$\times(u_{11}-v_{12}-u_{22}+v_{21})^2.$$

The equation $\det Q=0$, where Q is the matrix of the quadratic part of the HAF (8), gives 18 equations which completely describe the 18 collective modes of the order parameter in He^3-A in an arbitrary magnetic field for arbitrary momenta of the collective excitations.

4. THE LINEAR ZEEMAN EFFECT FOR THE *cl*- AND *pb*-MODES

We will consider the case of small magnetic fields and calculate the linear corrections to the CM spectrum. Assuming also that the CM momenta are equal to zero ($k=0$) and retaining only terms of first order in the field, we obtain the following 18 equations:

$$\text{I)}\quad \int_0^\pi(1-(1+2c)I)\cos^2\theta\sin^2\theta\,d\theta=0,\qquad (u_{33},v_{33}),$$

$$\int_0^\pi(1-(1+2c)I)\cos^2\theta\sin^2\theta\,d\theta\pm\mu H\int_0^\pi$$

$$\times\frac{4c}{1+4c}(1+2cI)\cos^2\theta\sin\theta\,d\theta=0,$$

$$(u_{31}-v_{32},\ u_{32}-v_{31}),\qquad (u_{31}+v_{32},\ u_{32}+v_{31}),$$

$$\text{II)}\quad\int_0^\pi I\sin^3\theta\,d\theta=0,\qquad (u_{23}-v_{13}),$$

$$\int_0^\pi I\sin^3\theta\,d\theta\pm\mu H\int_0^\pi\frac{4c}{1+4c}(2-I)\sin^3\theta\,d\theta=0,$$

$$(u_{11}+v_{12}+u_{22}+v_{21}),\qquad (u_{11}-v_{12}-u_{22}+v_{21}),$$

$$\text{III)}\quad\int_0^\pi(1+2c)I\sin^3\theta\,d\theta=0,\qquad (u_{23}+v_{13},\ u_{13}-v_{23}),$$

$$\int_0^\pi(1+2c)I\sin^3\theta\,d\theta\pm\mu H\int_0^\pi\frac{4c}{1+4c}(1+2cI)\sin^3\theta\,d\theta=0,$$

$$(u_{12}+v_{11}+u_{21}+v_{22},\ u_{11}+v_{12}-u_{22}-v_{21}),$$

$$(u_{12}-v_{11}-u_{21}+v_{22},\ u_{11}-v_{12}+u_{22}-v_{21}),$$

$$\text{IV)}\quad\int_0^\pi(1+4c)I\sin^2\theta\,d\theta=0,\qquad (u_{13}+v_{23}),$$

$$\int_0^\pi(1+4c)I\sin^3\theta\,d\theta\pm\mu H\int_0^\pi 4cI\sin^3\theta\,d\theta=0,$$

where

$$(u_{12}+v_{11}-u_{21}-v_{22}),\qquad (u_{12}-v_{11}+u_{21}-v_{22}),$$

$$I=\frac{1}{(1+4c)^{1/2}}\ln\frac{1+(1+4c)^{1/2}}{1-(1+4c)^{1/2}},\qquad c=\frac{\Delta^2\alpha_+^2\sin^2\theta}{\omega^2}.$$

We have four groups of equations, each of which contains three or six equations. Groups I and II describe the *gd*-modes. To obtain the frequencies of these modes it is necessary to take into account corrections quadratic in the field. In Ref. 2, investigating the question of the influence of the magnetic field on the number of *gd*-modes, we arrived at the conclusion that in a magnetic field three of the nine modes become nonphonon modes owing to the appearance of a gap $\sim\mu H$ in their spectrum. Equation groups III and IV describe the *cl*- and *pb*-modes. If we write these equations as $F_0(E)\pm\mu HF_1(E)=0$ and then seek E in the form $E=E_0\pm\mu HE_1$, we obtain

$$E=E_0\pm\mu HE_0\left(-1+\frac{1}{1-{}^3/_4F_1(E_0)}\right).$$

As was noted above, the values of E_0 for the *cl*- and *pb*-

modes were obtained by Brusov and Popov a long time ago.[2] Using these results for E_0, we obtain the following expressions for the energies of the *cl*- and *pb*-modes in weak magnetic fields:

$$cl: E_0=(1{,}17-i\cdot 0{,}13)\Delta_0,$$
$$E_{1,2}=(1{,}17\pm\mu H\cdot 1{,}70)\Delta_0-i\cdot(0{,}13\mp\mu H\cdot 1{,}20)\Delta_0,$$
$$pb: E_0=(1{,}96-i\cdot 0{,}31)\Delta_0,$$
$$E_{1,2}=(1{,}96\pm\mu H\cdot 2{,}04)\Delta_0-i\cdot(0{,}31\pm\mu H\cdot 0{,}06)\Delta_0.$$

Thus, in weak magnetic fields we have a trilinear (in the field) splitting of the spectrum of the *cl*- and *pb*-modes, i.e., we have obtained the linear Zeeman effect for these modes. The magnetic field completely lifts the degeneracy of the *pb*-modes (of which there are three) and only partially the degeneracy of the *cl*-modes (of which there are six), each of which remains doubly degenerate.

Note that the magnetic field alters both the real part of the CM energy and the imaginary, i.e., it alters both the frequencies of the collective excitations and their attenuation even in the linear approximation. In this case the attenuation of some modes grows, and of others decreases. Note that the frequencies and the attenuation of one *pb*-mode and two *cl*-modes in the linear approximation are not changed.

5. CONCLUSION

Let us compare the results obtained here with the results of Refs. 14 and 15. As was already mentioned, in addition to the difference in the number of *gd*-modes, our results are more accurate (by roughly 5%), since we have taken account of the influence of the attenuation of the collective modes on their frequencies. In addition, we have investigated the influence of the magnetic field on the attenuation of the collective excitations, which, naturally, is absent in Refs. 14 and 15.

In Ref. 14 some of the modes are not changed in a magnetic field, while the frequencies of others change from ω_1 to $(\omega_1^2+\Omega^2)^{1/2}$, where Ω is the effective Larmor frequency. Such a quadratic (in the field) frequency shift in small fields (instead of a linear shift) was obtained by the authors of Ref. 14 since they did not take account of the deformation of the gap in the Fermi spectrum. The authors of Ref. 15, taking account of the deformation of the gap, obtained a linear splitting of the frequencies of some of the modes

$$\omega_{i\sigma}\left(\frac{T}{T_{c\sigma}}\right)=\frac{T_{c\sigma}}{T_{c0}}\,\omega_i\left(\frac{T}{T_{c\sigma}}\right),\quad i=nfl,\ sfl,\ scl,\quad \sigma=\downarrow,\uparrow.$$

where *nfl* is the normal flapping mode, *sfl* is the super flapping mode, and *scl* is the super clapping mode, while the frequencies of other modes changed from ω_1 to $(\omega_1^2+\Omega^2)^{1/2}$, i.e., they remained invariant in the linear approximation.

Thus, the authors of Ref. 15, obtained as we did here a triple splitting of the collective mode spectrum in small fields, and in this sense the conclusions of both papers are similar. However, there exists an entire list of differences including qualitative ones. Thus, in Ref. 15 the collective mode spectrum in He^3-A in a magnetic field is obtained from the spectrum without a magnetic field by the substitution $\Delta\to\Delta\pm\mu H$, inasmuch as in the weak-coupling approximation, used in both works, the subsystems of fermions with spins aligned with and against the field are independent. Our result does not satisfy such a simple condition, and the reason for this in our opinion is the following. Tewordt and Schopohl[15] did not allow for the attenuation of the collective modes which takes place even in the case of zero momenta of the excitations and is connected with the vanishing of the gap at the poles. But the presence of attenuation, i.e., of an imaginary part of the energy (frequency) of the collective mode, renormalizes the real part of the energy of the collective mode. Since the attenuation is different for different modes, the above-mentioned condition for the spectrum to change in a magnetic field, as follows from our results when attenuation is taken into account, must be looked at more closely, i.e., for each collective mode separately and in the form $\Delta\to\Delta+\alpha_i\mu H$, where i stands for either *cl* or *pb*. But since attenuation (i.e., the imaginary parts) depends on the field, α_i can be different for the real and imaginary parts.

Another conclusion that can be drawn from an analysis of the results presented here is that the linear Zeeman effect for the *cl*- and *pb*-modes takes place due only to the deformation of the order parameter (or the particle-hole asymmetry) in the case in which the momenta of the excitations are zero: $k=0$. For nonzero momenta of the collective excitations there exists the fundamental possibility that the additional term in the HAF will, without deformation of the gap, lead to linear corrections to the frequencies of the collective modes.

The triple splitting of the spectra of the *cl*- and *pb*-modes, which we have obtained, can be observed in ultrasound experiments. Note that in the case of the *A*-phase (in contrast with the case of the *B*-phase) the deformation of the gap is linear in the field, which should make it possible to observe splitting of the spectrum of the collective modes equal in order of magnitude to the existence region of the A_1 phase in moderate fields.

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[1)] In 1983 Daniels *et al.*[11] detected resonant absorption of ultrasound in He^3-*B* near the absorption boundary in a magnetic field. At first their results were interpreted as the appearance of a new mode with $J=4$ (*f*-pairing was introduced) and $E=1.82\Delta_0$. However, in Schopohl and Tewordt[12] and Brusov and Popov[13] have shown for LT-17 that in reality what is happening is the excitation of the *pb*-mode, which has an energy $E=2\Delta_0$ in the absence of a magnetic field and exists in the presence of *p*-pairing.[4] Thus we see that the experiments in which Dobbs and Sanders participated consistently refute the idea of the importance of taking account of higher pairings in superfluid He^3, which up until the present time has been an active area of investigation.

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