# Interaction of relativistic particles with intense interference optical fields

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New mechanisms for the channeling, collimation, bunching, and modulation of relativistic charged-particle beams are described. Some new methods are proposed for amplifying and exciting electromagnetic radiation by means of relativistic beams. Some involve Doppler frequency conversion. Several phenomena are predicted: parametric resonance in a beam of relativistic particles, bunching of particles in an interference field, and the generation of harmonics and of sum and difference frequencies in relativistic beams.

## 1. INTRODUCTION

We carry out a theoretical investigation of the interaction of relativistic charged-particle beams with intense interference optical fields. This analysis leads, as we will see, to the suggestion of new methods for laser control of the motion of relativistic particles. It also reveals some new possibilities for exciting and amplifying electromagnetic radiation. The possible occurrence of some fundamentally new phenomena is predicted. Examples are parametric resonance in a beam of relativistic particles, bunching of particles in an interference field, and the generation of harmonics and of sum and difference frequencies by relativistic channeled particles.

The interaction of intense laser pulses with matter has a number of features not found in the interaction of moderate-intensity pulses. At a field intensity of order $10^{12}$ W/cm$^2$, for example, the energy of the interaction of an electron with a wave field reaches a level on the order of the intraatomic energy, so conventional perturbation theory has to be abandoned. In fields with intensities on the order of $10^{14}$ W/cm$^2$, tunneling becomes the predominant mechanism for the ionization of atoms. At field intensities on the order of $10^{18}$ W/cm$^2$, the electron oscillation velocity in the wave approaches the velocity of light, so relativistic effects become important.

For high-intensity pulses, an atom thus ceases to be a resonant system, since the electron oscillation amplitude in the wave becomes larger than the radii of the electron shells of the atom. The development of methods for the effective nonlinear conversion of ultrastrong laser pulses or for the amplification of these pulses thus requires a search for new nonlinear or resonant media. It turns out that a free relativistic electron, interacting with the field of a spatially nonuniform wave, can play the role of such a medium.

As we know, the interaction of a free electron, atom, or microscopic particle with the spatially nonuniform field of an intense laser wave leads to the appearance of a ponderomotive force. This force is the key to numerous methods for controlling the motion of particles[1,2] and for bunching particles in free-electron lasers.[3] This force has a significant effect on the atomic ionization spectra[4] and is in fact capable of causing self-organization processes.[5] Under certain conditions, the ponderomotive force can also cause substantial changes in the way an electron interacts with an electromagnetic wave. For example, an electron propagating along the line of nodes of an interference field can be captured into a channel, if the energy of the transverse motion of the electron is lower than the height of the potential barrier set up by the ponderomotive forces. In this case the electron executes bounded periodic oscillations between two neighboring antinodes of the interference field, and its response differs substantially from that in a plane wave. From the quantum-mechanical point of view, the explanation is that in a channeling regime a particle acquires discrete transverse-motion levels.[6,7] A channeled electron is thus equivalent to a relativistic atom. It can undergo radiative (spontaneous or induced) transitions between transverse-motion levels. The scattering of electromagnetic radiation accompanied by a change in the vibrational state of electron gives rise to sum and difference frequencies in the scattered field, while the nonlinearity of the response gives rise to harmonics.

As the field intensity is raised, forces which depend on the phase of the interference field become significant. These forces lead to bunching of the electrons in an initially uniform electron beam. This bunching is of considerable interest for the observation of coherent radiation. In this paper we take a theoretical look at these effects.

## 2. RELATIVISTIC ELECTRON IN AN INTERFERENCE FIELD

We consider the motion of a relativistic electron through an interference electromagnetic field. For definiteness we will be discussing electron beams, but this approach does not limit the generality of the discussion, since it is an elementary matter to make the switch to any other charged particle. We first consider the case in which the interference field is a transverse electric field. In this case the electric and magnetic fields can be written (Fig. 1)

$$\begin{aligned}\mathbf{E}&=\mathrm{Re}\{\mathbf{E}_1\exp(i\varkappa_\perp x+i\varkappa_\parallel z-i\omega t)+\mathbf{E}_2\exp(-i\varkappa_\perp x+i\varkappa_\parallel z-i\omega t)\}\\&=\mathbf{E}_0\cos(\varkappa_\perp x)\cos(\varkappa_\parallel z-\omega t),\\\mathbf{H}&=\mathbf{H}_\perp\cos(\varkappa_\perp x)\cos(\varkappa_\parallel z-\omega t)\\&\quad-\mathbf{H}_\parallel\sin(\varkappa_\perp x)\sin(\varkappa_\parallel z-\omega t),\end{aligned}\tag{1}$$

where

$$\varkappa_\perp=\varkappa\sin\theta,\ \varkappa_\parallel=\varkappa\cos\theta,\ \varkappa=\omega/c,\ \mathbf{E}_1\|\mathbf{E}_2\|\mathbf{e}_y,$$
$$E_0=2E_1,\ H_\perp=2H_{1x}=-E_0\cos\theta,\ H_\parallel=2H_{1z}=E_0\sin\theta.$$

A relativistic electron enters the region of the interference field along the $z$ axis, which is also the propagation

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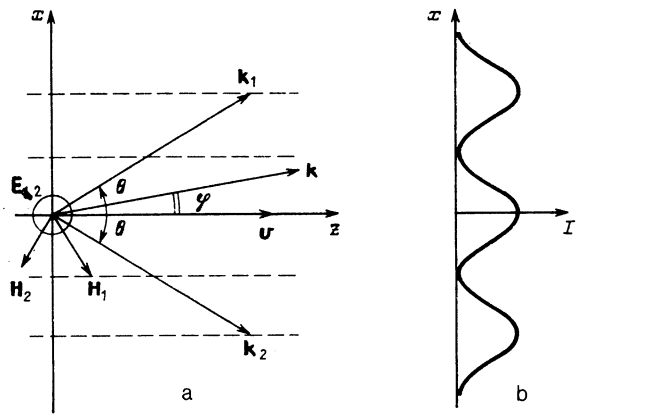


FIG. 1. a—Relative orientations of the wave vectors of the interfering waves ($\mathbf{k}_1, \mathbf{k}_2$), of the electron velocity ($\mathbf{v}$, $|\mathbf{v}| \approx c$), and of the wave vector ($\mathbf{k}$) of the wave emitted by the electron, along with the positions of the nodes and antinodes of the interference field (the dashed lines); b—the intensity of the interference field (the ponderomotive potential) as a function of the transverse coordinate $x$.

direction of an interference field with a phase velocity $v_{ph} = \omega/\varkappa_\parallel$. The equations of motion of the electron are

$$\frac{d}{dt}(m\dot{x}) = eE_0 \frac{\dot{y}}{c} \sin\theta \sin(\varkappa_\perp x) \sin(\varkappa_\parallel z - \omega t),$$

$$\frac{d}{dt}(m\dot{y}) = -eE_0\left(1 - \frac{\dot{z}}{c}\cos\theta\right)\cos(\varkappa_\perp x)\cos(\varkappa_\parallel z - \omega t) - eE_0 \frac{\dot{x}}{c}\sin\theta \sin(\varkappa_\perp x)\sin(\varkappa_\parallel z - \omega t), \tag{2}$$

$$\frac{d}{dt}(m\dot{z}) = -eE_0 \frac{\dot{y}}{c}\cos\theta\cos(\varkappa_\perp x)\cos(\varkappa_\parallel z - \omega t).$$

We assume in a first approximation that the coordinate $z$ on the right sides of Eqs. (2) is given by

$$z = z_0 + v_0 t. \tag{3}$$

We also assume that the velocity of the transverse motion is much lower than the velocity of the longitudinal motion: $|\dot{x}| \ll v_0$, $|\dot{y}| \ll v_0$. The first two equations then become

$$m\ddot{x} = eE_0 \frac{\dot{y}}{c}\sin\theta\sin(\varkappa_\perp x)\sin[\varkappa_\parallel z_0 - \omega(1-\beta\cos\theta)t],$$

$$m\ddot{y} = -eE_0(1-\beta\cos\theta)\cos(\varkappa_\perp x)\cos[\varkappa_\parallel z_0 - \omega(1-\beta\cos\theta)t] - eE_0\frac{\dot{x}}{c}\sin\theta\sin(\varkappa_\perp x)\sin[\varkappa_\parallel z_0 - \omega(1-\beta\cos\theta)t], \tag{4}$$

where $\beta = v_0/c$.

### 3. ELECTRON CHANNELING

We integrate the second of Eqs. (4) over time once, obtaining

$$\dot{y} = \frac{eE_0}{m\omega}\cos(\varkappa_\perp x)\sin[\varkappa_\parallel z_0 - \omega(1-\beta\cos\theta)t]. \tag{5}$$

Substituting (5) into the first equation of system (4), we find

$$m\ddot{x} = \frac{e^2 E_0^2}{4m\omega c}\sin\theta\sin(2\varkappa_\perp x) \times \{1 - \cos[2(\varkappa_\parallel z_0 - \omega(1-\beta\cos\theta)t]\}. \tag{6}$$

Of the two terms on the right side of Eq. (6), one does not depend on the time, while the other oscillates at the doubled frequency. If we omit the latter term, the equation which remains becomes the equation of the oscillations of a mathematical pendulum after the substitution $\xi = \varkappa_\perp x - \pi/2$:

$$\ddot{\xi} + \Omega^2 \sin\xi = 0, \tag{7}$$

where

$$\Omega = eE_0 \sin\theta / 2^{1/2} mc. \tag{8}$$

It follows from (6) and (7) that a force acts on the electron along the $x$ axis, in the direction perpendicular to the field polarization vector. This force tends to displace the electron into a region of minimum intensity of the interference field, i.e., to the field nodes $\varkappa_\perp x_n = \pi(2n+1)/2$. The potential well created by these forces is (Fig. 1b)

$$U(x) = U_0 \frac{1 + \cos(2\varkappa_\perp x)}{2}, \tag{9}$$

where

$$U_0 = e^2 E_0^2 / 4m\omega^2. \tag{10}$$

We see from (7) that the electrons whose transverse-motion energy is below the height $U_0$ of the potential well execute harmonic oscillations between the two nearest antinodes of the interference field:

$$\frac{p_\perp^2}{2m} = \frac{p^2}{2m}\eta^2 \leqslant U_0,$$

where $\eta$ is the angle between the $z$ axis and the momentum of the particle. One can introduce a critical channeling angle

$$\eta_c = \frac{p_{\perp\, max}}{p} = \left(\frac{2mU_0}{p^2}\right)^{1/2} = \frac{eE_0}{2^{1/2} m\omega c} = \frac{e}{m\omega c}\left(\frac{2\pi I}{c}\right)^{1/2}. \tag{11}$$

We will estimate $U_0$ in order of magnitude. Assuming $\lambda = 1\ \mu$m, and expressing $E_0$ in terms of the intensity $I$, we find

$$U_0[\text{eV}] \approx \frac{10^{-13}}{\gamma} I \left[\frac{\text{W}}{\text{cm}^2}\right], \tag{12}$$

i.e., $U_0 = 1$ eV at $\gamma = (1-\beta^2)^{-1/2} = 10^2$ and $I = 10^{15}$ W/cm$^2$. We see that the barrier height can reach several electron volts even in moderate fields.

### 4. RADIATION EMITTED BY CHANNELED PARTICLES

The oscillations of the particles in the potential (9) give rise to electromagnetic radiation which is polarized along the $x$ axis and whose frequency is determined by the Doppler law

$$\nu = \frac{\Omega}{1 - \beta\cos\varphi}, \tag{13}$$

where $\varphi$ is the angle between the photon emission direction and the velocity of the particle. In our case, the velocity of the particle is along the $z$ axis. Since the potential (9) is not harmonic, we find that harmonics $n\Omega$ are excited. When the second term on the right side of (6) is taken into account, electromagnetic radiation at the sum and difference frequencies also appears in the spectrum:

$$\nu_{2n}=\frac{2\omega(1-\beta\cos\theta)\pm n\Omega}{1-\beta\cos\varphi}. \tag{14}$$

It follows from Eq. (3) that radiation also appears at new frequencies with a polarization which is the same as that of the interference field. This radiation has a frequency

$$\nu_{1n}=\frac{\omega(1-\beta\cos\theta)\pm n\Omega}{1-\beta\cos\varphi}. \tag{15}$$

## 5. PARAMETRIC RESONANCE

The most interesting case is that in which the frequency $\Omega$ satisfies the parametric-resonance condition:

$$\Omega=\omega(1-\beta\cos\theta). \tag{16}$$

An approximate solution of the equation

$$\ddot{x}+\Omega^2\{1-\cos[2(\varkappa_{\parallel}z_0-\omega(1-\beta\cos\theta)t)]\}x=0$$

can be written as follows in this case:

$$x(t)\approx\frac{v_0}{\Omega}\sin\Omega t+\frac{v_0t}{4}\cos(\varkappa_{\parallel}z_0-\Omega t). \tag{17}$$

The amplitude of the stimulated oscillations increases linearly in time. The amplitude of the vector potential of the radiation field with polarization along the $x$ axis in the far zone is given by the expression[8]

$$A_x(\nu)\propto\sum_{i=1}^{N}\int_{-\infty}^{\infty}e\dot{x}_i(t)\exp[i(\nu t-\mathbf{k}\mathbf{r}_i(t))]dt$$
$$=\frac{et}{8}\sum_{i=1}^{N}v_{0i}\exp[i(\omega\cos\theta-\nu\cos\varphi)z_{0i}/c]\delta(\Omega-\nu(1-\beta\cos\varphi)), \tag{18}$$

where $r_i$ is the radius vector of particle $i$. At $\varphi=\theta$ we thus have a coherent growth of the field amplitude at the frequency of the waves which create the interference field:

$$\nu=\left.\frac{\omega(1-\beta\cos\theta)}{1-\beta\cos\varphi}\right|_{\varphi=\theta}=\omega.$$

Let us estimate the characteristic field intensity which would be required to satisfy the condition for parametric resonance, (16). Since

$$\Omega^2=\frac{2U_0}{m}\varkappa_{\perp}^2,$$

we find, using (12),

$$I\left[\frac{\mathrm{W}}{\mathrm{cm}^2}\right]=3\cdot10^{17}\left(\gamma\theta+\frac{1}{\gamma\theta}\right)^2. \tag{19}$$

We recall that this estimate refers to the case $\lambda=1\,\mu$m. With increasing $\lambda$, the intensity required falls off quadratically.

## 6. COLLIMATION OF ELECTRON BEAMS

We consider conditions far from those for a parametric resonance, (16). The motion of the electron is described in a first approximation by Eq. (7) in this case. In this equation, however, we have not taken account of the radiative-friction force which arises from the emission of electromagnetic radiation at a frequency $\Omega$ by the electron. When the term $2\gamma\dot{x}$, which corresponds to this force, is taken into account, the solution of Eq. (7) for the case of small oscillations is

$$x=x_0e^{-\gamma t}\cos\Omega t+\frac{v_0}{\Omega}e^{-\gamma t}\sin\Omega t+\frac{\pi(2n+1)}{2\varkappa_{\perp}}.$$

The amplitude of the electron oscillations and the velocity of the transverse motion thus decrease as time elapses. At $\gamma t\gg1$ we find $x\approx\pi(2n+1)/2\varkappa_{\perp}$, and the velocity of the transverse motion approaches zero.

Note that under the condition $\Omega<\omega(1-\beta\cos\theta)$ the amplitude of the high-frequency oscillations which stem from the second term in (6) is lower than the amplitude of the oscillations at the frequency $\Omega$. The amplitude of the high-frequency oscillations is determined, according to the equation

$$\ddot{x}=\frac{e^2E_0^2}{4m^2\omega c}\sin\theta\cos[2\omega(1-\beta\cos\theta)t], \tag{20}$$

by the expression

$$x_0(2\omega)=\frac{1}{2\varkappa_{\perp}}\frac{\Omega^2}{\omega^2(1-\beta\cos\theta)^2}. \tag{21}$$

Consequently,

$$x_0(2\omega)=x_0(\Omega)\frac{\Omega^2}{\omega^2(1-\beta\cos\theta)^2}.$$

In turn, the velocity of the high-frequency oscillations is given by

$$v_0(2\omega)\approx v_0(\Omega)\frac{\Omega}{\omega(1-\beta\cos\theta)}. \tag{22}$$

We thus have $v_0(2\omega)\ll v_0(\Omega)$ under the condition $\Omega\ll\omega(1-\beta\cos\theta)$.

When the radiative damping in the course of the oscillations of the electron in the interference field is taken into account, substantial suppression of the amplitude of the transverse oscillations thus occurs, and the electron beam undergoes (angular) collimation.

## 7. ELECTRON BUNCHING IN A RELATIVISTIC BEAM

In Sec. 2 we wrote out equations for the case of a transverse electric field, in which the polarization vectors of the interfering waves are perpendicular to the plane of the wave vectors of these waves. We now write equations for the case of a transverse magnetic field:

$$\frac{d}{dt}(m\dot{x})=-eE_0\left(\cos\theta-\frac{\dot{z}}{c}\right)\cos(\varkappa_\perp x)\cos(\varkappa_\parallel z-\omega t),$$

$$\frac{d}{dt}(m\dot{z})=eE_0\sin\theta\sin(\varkappa_\perp x)\sin(\varkappa_\parallel z-\omega t)$$
$$-eE_0\frac{\dot{x}}{c}\cos(\varkappa_\perp x)\cos(\varkappa_\parallel z-\omega t). \quad (23)$$

We have assumed that the wave vectors and the polarization vectors of the waves lie in the $xz$ plane. Again using the approximation (3), we find the following equation for the coordinate $x$.

$$m\ddot{x}=-eE_0(\cos\theta-\beta)\cos(\varkappa_\perp x)\cos[\varkappa_\parallel z_0-\omega(1-\beta\cos\theta)t]. \quad (24)$$

For the variable $\xi$, Eq. (24) can be rewritten as

$$\ddot{\xi}+\Omega^2\sin\xi\cos[\varkappa_\parallel z_0-\omega(1-\beta\cos\theta)t]=0, \quad (25)$$

where

$$\Omega^2=eE_0|\cos\theta-\beta|\varkappa_\perp/m. \quad (26)$$

As we mentioned in the preceding section, under the condition $\Omega\ll\omega(1-\beta\cos\theta)$ the amplitude of the high-frequency oscillations is much lower than the amplitude of the oscillations in the interference potential. We thus seek a solution of (25) in the form $\xi(t)=\xi_\Omega(t)+\delta\xi$, where $\xi_\Omega(t)$ is the slowly varying coordinate of the center of gravity of the high-frequency oscillations $\delta\xi(t)$. Writing $E_0(x)=E_0\cos(\varkappa_\perp x)$ as an expansion in a Fourier series,

$$E_0(x)=E_0(x_\Omega)+\delta x\left.\frac{\partial E_0}{\partial x}\right|_{x=x_\Omega}+\dots, \quad (27)$$

we can show without difficulty that the coordinate of the center of gravity of the high-frequency oscillations obeys the equation

$$m\ddot{x}_\Omega=-\frac{\partial U}{\partial x_\Omega}, \quad (28)$$

where

$$U(x)=\frac{e^2E_0^2}{4m\omega^2}\left(\frac{\cos\theta-\beta}{1-\beta\cos\theta}\right)^2\cos^2(\varkappa_\perp x). \quad (29)$$

The particle is thus in a potential well in the case $\Omega\ll\omega(1-\beta\cos\theta)$, as it was in the case of transverse oscillations. The only new feature is that both the high-frequency oscillations and the oscillations of their center of gravity occur along the same axis. The amplitude of the high-frequency oscillations is much lower than the amplitude of the oscillations in the channel.

We now consider the opposite limit, $\Omega\gg\omega\times(1-\beta\cos\theta)$. At times $t<\pi/[\omega(1-\beta\cos\theta)]$, Eq. (25) can be rewritten as

$$\ddot{\xi}+\Omega^2\cos(\varkappa_\parallel z_0)\sin\xi=0. \quad (30)$$

Consequently, while the stable equilibrium points on the intervals $-\pi/2+2\pi N<\varkappa_\parallel z_0<\pi/2+2\pi N$ are the points $x_0=\varkappa_\perp^{-1}(\pi/2+2\pi M)$, where $N$, and $M$ are integers, those on the intervals $\pi/2+2\pi N<\varkappa_\parallel z_0<3\pi/2+2\pi N$ are the points $x_0=\varkappa_\perp^{-1}(3\pi/2+2\pi M)$. Bunching of electrons thus occurs in an originally uniform electron beam. For $\theta\ll 1$, the period of the modulation along the $z$ axis is essentially the same as the wavelength $\lambda$.

Let us find the threshold field, which forms the boundary between these two regimes. From (26) we have

$$E_{\rm thr}=\frac{m\omega^2(1-\beta\cos\theta)^2}{e\varkappa_\perp|\cos\theta-\beta|}. \quad (31)$$

In particular, in the case $\gamma\theta>1$ we find the following estimate of the threshold intensity:

$$I\left[\frac{\rm W}{\rm cm^2}\right]=5\cdot10^{17}(\gamma\theta)^2. \quad (32)$$

Comparing with (19), we see that this quantity is the same as the field which is required for achieving parametric resonance. As in (19), in the case $\gamma\theta<1$ we need to replace $\gamma\theta$ in (32) by $(\gamma\theta)^{-1}$.

#### 8. AXIAL CHANNELING IN NONDIFFRACTING LIGHT BEAMS

Cylindrical waves fall in the special category of fields for which a complete solution of the vector wave equation can be written out analytically.[9] The wave field is written in this case as the sum of two components, each of which can be expressed in terms of a function which is a solution of a scalar wave equation. The following qualify as these components:

a) a transverse electric wave,

$$\mathbf{E}_e=-\frac{1}{c}\frac{\partial}{\partial t}\,{\rm rot}\,\mathbf{\Pi}_e,\quad \mathbf{H}_e={\rm rot\,rot}\,\mathbf{\Pi}_e; \quad (33a)$$

b) a transverse magnetic wave,

$$\mathbf{E}_m={\rm rot\,rot}\,\mathbf{\Pi}_m,\quad \mathbf{H}_m=\frac{1}{c}\frac{\partial}{\partial t}\,{\rm rot}\,\mathbf{\Pi}_m. \quad (33b)$$

Here $\Pi=\{0,0,\psi\}$ is the Hertz vector, whose only nonvanishing projection is along the $z$ axis. This projection satisfies the scalar wave equation

$$\Delta\psi-\frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}=0. \quad (34)$$

Inside a uniform cylindrical region, the solution of this equation can be written in the following form (for example):

$$\psi_{n\varkappa_\parallel}(\mathbf{r},t)=J_n(\varkappa_\perp\rho)\exp[i(n\varphi+\varkappa_\parallel z-\omega t)]. \quad (35)$$

Here

$$\rho=(x^2+y^2)^{1/2},\quad \varphi={\rm arctg}(y/x),$$
$$\varkappa_\perp^2+\varkappa_\parallel^2=\varkappa^2=\omega^2/c^2, \quad (36)$$

and $J_n(x)$ is the Bessel function. We restrict the analysis to the axisymmetric case for simplicity. In this case we have[1)]

$$\psi(\mathbf{r},t)=J_0(\varkappa_\perp\rho)\exp[i(\varkappa_\parallel z-\omega t)]. \quad (37)$$

Using (33a), we can write the electric and magnetic fields in a transverse electric wave as

$$E_\rho=0,\quad H_\rho=A_0\varkappa_\parallel\varkappa_\perp J_1(\varkappa_\perp\rho)\sin\Phi,$$
$$E_\varphi=-A_0\varkappa\varkappa_\perp J_1(\varkappa_\perp\rho)\sin\Phi,\quad H_\varphi=0, \quad (38)$$
$$E_z=0,\quad H_z=A_0\varkappa_\perp^2J_0(\varkappa_\perp\rho)\cos\Phi,$$

where $\Phi = \varkappa_\parallel z - \omega t$, and $A_0$ is a normalization constant which can be expressed in terms of (for example) the energy of a pulse:

$$\mathscr{E} = \frac{1}{8\pi}\int_{V_0} (|\mathbf{E}|^2+|\mathbf{H}|^2)dV. \tag{39}$$

Here we have $V_0 = Sc\tau_p$, $S = \pi R^2$ is the cross-sectional area, and $\tau_p$ is the pulse length. Substituting (38) into (39), we find

$$\mathscr{E}=A_0^2c\tau_p\varkappa^2(\varkappa_\perp R)^2[J_0(\varkappa_\perp R)-J_2(\varkappa_\perp R)]^2/8\approx A_0^2c\tau_p\varkappa^2\varkappa_\perp R/\pi.$$

The amplitude $A_0$ can also be related to the pulse height:

$$I=\frac{\mathscr{E}}{S\tau_p}=A_0^2c\varkappa^2\frac{\varkappa_\perp}{\pi^2R}. \tag{40}$$

The equations of motion of a relativistic electron in the electromagnetic field (38) are

$$\begin{aligned} &m(\ddot\rho-\rho\dot\varphi^2)=-eA_0\frac{\rho\dot\varphi}{c}\varkappa_\perp^2J_0(\varkappa_\perp\rho)\cos(\varkappa_\parallel z-\omega t),\\ &\frac{m}{\rho}\frac{d}{dt}(\rho^2\dot\varphi)=eA_0\varkappa_\perp\left(\varkappa-\varkappa_\parallel\frac{\dot z}{c}\right)J_1(\varkappa_\perp\rho)\sin(\varkappa_\parallel z-\omega t)\\ &\qquad+eA_0\frac{\dot\rho}{c}\varkappa_\perp^2J_0(\varkappa_\perp\rho)\cos(\varkappa_\parallel z-\omega t),\\ &\frac{d}{dt}(m\dot z)=eA_0\frac{\rho\dot\varphi}{c}\varkappa_\parallel\varkappa_\perp J_1(\varkappa_\perp\rho)\sin(\varkappa_\parallel z-\omega t). \end{aligned} \tag{41}$$

Again using the approximation (3), we can rewrite the first two equations in (41) as

$$\begin{aligned} &\ddot\rho-\rho\dot\varphi^2=-A_0\frac{e}{m}\frac{\rho\dot\varphi}{c}\varkappa_\perp^2J_0(\varkappa_\perp\rho)\cos[\varkappa_\parallel z_0-\omega(1-\beta\cos\theta)t],\\ &\frac{1}{\rho}\frac{d}{dt}(\rho^2\dot\varphi)\\ &=A_0\frac{e}{m}\varkappa_\perp\Big\{(\varkappa-\varkappa_\parallel\beta)J_1(\varkappa_\perp\rho)\sin[\varkappa_\parallel z_0-\omega(1-\beta\cos\theta)t]\\ &+\frac{\dot\rho}{c}\varkappa_\perp J_0(\varkappa_\perp\rho)\cos[\varkappa_\parallel z_0-\omega(1-\beta\cos\theta)t]\Big\}. \end{aligned} \tag{42}$$

Integrating the second equation in (42) over time once, we find

$$\dot\varphi=\frac{A_0e\varkappa_\perp}{mc}\frac{J_1(\varkappa_\perp\rho)}{\rho}\cos[\varkappa_\parallel z_0-\omega(1-\beta\cos\theta)t]. \tag{43}$$

Substituting this expression into the first equation of system (42), we find the following equation for $\rho$:

$$\ddot\rho+\frac{1}{2}\left(\frac{A_0e\varkappa_\perp}{mc}\right)^2\frac{dJ_1^2(\varkappa_\perp\rho)}{d\rho}\cos^2[\varkappa_\parallel z_0-\omega(1-\beta\cos\theta)t]=0. \tag{44}$$

For small oscillations we thus find the equation

$$\ddot\rho+\Omega^2\rho=0, \tag{45}$$

where

$$\Omega^2=\frac{1}{2}\left(\frac{A_0e\varkappa_\perp^2}{2mc}\right)^2=I\frac{\pi^2e^2\varkappa_\perp R}{8m^2c^3}\sin^2\theta. \tag{46}$$

Equation (44) is essentially the same as Eq. (6), so everything that we said in Secs. 3–6 also applies to the case of axial channeling. In particular, we find the following expression for the critical angle for axial channeling from (44) and (46):

$$\eta_c=\frac{\dot\rho_{\max}}{c}=\frac{e}{m\omega c}\pi J_1(\varkappa_\perp\rho_1)\left(\frac{\varkappa_\perp R}{2c}\right)^{1/2}I, \tag{47}$$

where $\rho_1$ is the first zero of the function $J_1'(\varkappa_\perp\rho)$. Comparing (47) with (11), we see that expressions (11) and (47) differ by a factor of $(\varkappa_\perp R)^{1/2}$. We need to recall, however, that pulses of different energies are required for producing beams of the same intensity in the cases of plane and cylindrical geometries.

Electron bunching can also occur in cylindrical beams. Here we would need to make use of a transverse magnetic cylindrical wave (as in Sec. 7).

## 9. INTERACTION WITH SURFACE WAVES

We showed above that the condition $\Omega\gtrsim\omega\times(1-\beta\cos\theta_{yzk})$ must be satisfied in order to observe parametric resonance and particle bunching. Since $\Omega$ increases with increasing intensity, we would need intense laser beams. However, it is possible to satisfy this inequality even at moderate beam intensities if we reduce the right side of the inequality. In particular, under the condition

$$1-\beta\cos\theta=1-\beta\varkappa_\parallel/\varkappa\approx 0, \tag{48}$$

it is possible to substantially relax the requirements on the intensity. The condition $\beta\cos\theta = 1$ means that the velocity of the particle is equal to the phase velocity of the wave. For motion in a dense medium, the particle velocity may exceed the phase velocity of the wave, as we know, but the mean free path of the electron would be substantially shorter. By leaving the electron beam in air, we can thus arrange a situation in which only the electromagnetic wave is obliged to propagate through a medium.

Let us consider the case in which an electron beam is directed along the surface of a medium in which an electromagnetic wave is in waveguide propagation. We assume that the angle of incidence on the interface is just slightly greater than the angle of total internal reflection. The electron beam then interacts with a nonuniform wave of the form

$$E=E_0e^{-\Gamma x}\cos(\varkappa_\parallel z-\omega t), \tag{49}$$

where

$$\varkappa_\perp=(\varkappa^2-\varkappa_\parallel^2)^{1/2}=i\Gamma,\quad \varkappa_\parallel=k_\parallel>\varkappa,$$

and $k_\parallel$ is the projection of the wave vector of the wave in the medium onto the interface. Since the refractive index of the medium satisfies $n>1$, the projection $k_\parallel$ may be greater than the magnitude of the wave vector in vacuum, $\varkappa=\omega/c$. Consider the case in which the polarization vector of the wave in the medium lies in plane of incidence. The equation for transverse oscillations of an electron moving at a relativistic velocity along the surface of the medium is

$$m\ddot x=-eE_0(\cos\theta-\beta)e^{-\Gamma x}\cos[\varkappa_\parallel vt_0+\omega(1-\beta\cos\theta)t]. \tag{50}$$

Here we have assumed that the coordinate of electron $i$ is given by $z_i = v(t - t_{0i})$ in a first approximation, where $t_{0i}$ is the time at which the electron intersects the boundary of the medium. Choosing the angle of incidence of the wave in the medium in accordance with

$$\varkappa_{\|}=k_{\|}=\varkappa/\beta, \tag{51}$$

we can put Eq. (50) in the form

$$m\ddot{x}=-eE_0\frac{1-\beta^2}{\beta}\cos(\varkappa_{\|}vt_0)e^{-\Gamma x}. \tag{52}$$

We see from this result that particles for which the condition $\cos(\varkappa_{\|}vt_0) > 0$ holds touch the surface of the medium and are lost from the beam. The particles for which the condition $\cos(\varkappa_{\|}vt_0) < 0$ holds, in contrast, move above the surface of the medium if the force due to the potential

$$U(x)=\frac{eE_0}{\Gamma}\frac{1-\beta^2}{\beta}(1-e^{-\Gamma x})$$

is greater than the electrical image force. We can thus modulate the density of a relativistic electron beam with a period $\Lambda = 2\pi/\varkappa_{\|}$ close to the wavelength in vacuum, $\lambda = 2\pi/\varkappa$.

By choosing the angle of incidence to be close to the value in (51), we can substantially lower the threshold intensities required for observing the parametric resonance. In this case the electron beam can move through a planar or cylindrical channel which is the central part of the waveguide structure for the wave which interacts with the beam.

## CONCLUSION

This study has shown that the interactions of relativistic beams with intense interference fields open up wide opportunities for controlling electron beams. These interactions present some new possibilities for collimation, channeling, bunching, and modulation of electron beams. Some new mechanisms for the excitation and amplification of electromagnetic radiation arise; some of these mechanisms make use of Doppler frequency up-conversion. In the case of parametric resonance, for example, we see from (18) that we could substantially raise the frequency of the electromagnetic radiation. With $\varphi = 0$, for example, we find from (18)

$$\nu=\frac{\omega(1-\beta\cos\theta)}{1-\beta}\approx\omega[1+(\gamma\theta)^2].$$

With $\theta = 10^{-1}$ rad and $\gamma = 10^2$ we thus find $\nu \approx \omega \cdot 10^2$. In other words, for forward emission we find a frequency increase by two orders of magnitude in comparison with the frequency of the applied electromagnetic radiation.

Our numerical estimates show that pulses of moderate intensity would be required for the observation of the effects discussed above. We should point out that channeling in interference fields has a significant advantage over channeling in a crystal, because there is absolutely no mechanism which would operate to dechannel the electrons. The distance over which the electron beam will interact with the electromagnetic pulse is determined by the process by which the velocities of the beam and the pulse become mismatched and by the dimensions of the interference region. At moderate intensities, this interaction distance might be on the order of a meter. There are thus grounds for expecting that these mechanisms for controlling relativistic electron beams and for exciting coherent radiation will soon find practical applications.

[1)] See Ref. 10 regarding experimental observation of nondiffracting beams.


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