Mass operator of the scalar superpartner of the electron in a supersymmetric QED in a constant external field

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The mass operator of the scalar superpartner of the electron in constant crossed fields is calculated. The probability for the process $\varphi \to e + \lambda$ and the radiative shift of the mass of the scalar are studied as functions of the parameter representing the spontaneous breaking of supersymmetry and also as functions of a dynamic parameter which determines the effect of the external field on the process under consideration. Incorporating the radiative corrections to the mass of the scalar electrons in the Fayet-Iliopoulos model of supersymmetric QED does not result in an equalization of the masses in the perturbation-theory approach.

INTRODUCTION

Thermal effects, a finite density, and the incorporation of an external field can all lead to a breaking of supersymmetry.[1-3] Since supersymmetry must be broken in realistic models, it is interesting to analyze the possible features of theories with a spontaneously broken supersymmetry in intense external fields. In Refs. 4 and 5, for example, the effective Lagrangian of the Fayet-Iliopoulos model of supersymmetric QED was calculated in the single-loop approximation. That calculation was carried out for a constant magnetic field. Kapustnikov[4] pointed out that incorporating quantum corrections to the Fayet-Iliopoulos mechanism in a constant magnetic field could result in a complete restoration of supersymmetry. This conclusion was reached in Ref. 4 on the basis that incorporating the vacuum quantum corrections could cause the energy of the ground state to vanish. As we know, in theories with a supersymmetry which is spontaneously broken in the tree approximation this energy is always positive. The question of equalizing the masses of the superpartners of the model was not examined in Ref. 4, however. Furthermore, although the energy of the vacuum state is zero in supersymmetric gauge theories, this circumstance does not by itself mean, to the best of our knowledge, that the vanishing of the ground-state energy due to radiative effects in external fields in theories with a spontaneously broken supersymmetry will equalize the mass spectrum of the particles of the model and restore supersymmetry.

In the present paper we calculate the mass operator of the scalar superpartner of the electron in the Fayet-Iliopoulos model of supersymmetric QED in constant crossed fields (Sec. 1). In Sec. 2 we examine how the probability for the process $\varphi \to e + \lambda$ and the radiative shift of the mass of the scalar depend on the parameter representing the spontaneous breaking of supersymmetry and also on the dynamic parameter which determines the effect of the external field on the process under consideration. The cases in which the mass of the scalar is larger than and smaller than the mass of the electron are treated separately. It follows from the results that the masses of the scalars are not equalized by radiative corrections in constant crossed fields in a perturbation-theory approach.

1. MASS OPERATOR OF THE SCALAR SUPERPARTNER OF THE ELECTRON IN CONSTANT CROSSED FIELDS

In the Fayet-Iliopoulos model, the Lagrangian of $N = 1$ supersymmetric QED which describes the interaction of a vector superfield V with two chiral superfields $\Phi_\pm$ is[6]

$$\mathscr{L} = \frac{1}{4}[WW|_{\theta\theta} + \overline{W}\overline{W}|_{\bar\theta\bar\theta}] + \Phi_+{}^+ e^{gV}\Phi_+|_{\theta\theta\bar\theta\bar\theta} + \Phi_-{}^+ e^{-gV}\Phi_-|_{\theta\theta\bar\theta\bar\theta} + m[\Phi_+\Phi_-|_{\theta\theta} + \Phi_+{}^+\Phi_-{}^+|_{\bar\theta\bar\theta}] + 2kV|_{\theta\theta\bar\theta\bar\theta}. \quad (1)$$

We consider the case $m^2 > (1/2)gk$ (k is the parameter of the spontaneous breaking of supersymmetry), in which the model contains two complex scalar fields φ_1 and φ_2, with masses $[m^2 + (1/2)gk]^{1/2}$ and $[m^2 - (1/2)gk]^{1/2}$; one massless vector field A_μ; and a massless spinor field λ, which describes Majorana photinos and two Weyl spinors ψ_+ and ψ_-. The latter form one Dirac spinor ψ, which describes an electron with a mass m and a charge $e = g/2$. In the case $m^2 > (1/2)gk$, the supersymmetry is broken spontaneously, and the gauge $U(1)$ symmetry of Lagrangian (1) is preserved. Using (1), we can write the supersymmetric contribution to the mass operator of the scalar electron in the component fields as follows:

$$M(x,x') = ig^2 \,\mathrm{Sp}\, R(L) S^{(e)}(x,x') L(R) S^M(x,x'), \quad (2)$$

where $S^{(e)}(x, x')$ and $S^M(x, x')$ are the Green's functions of the electron and the photino, and the operators $R(L)$ are projection operators:

$$\binom{R}{L} = \frac{1}{2}(1 \pm \gamma^5). \quad (3)$$

We have omitted from (2) the contribution of a diagram which stems from the four-boson interaction in (1):

$$\mathscr{L}(\varphi_i) = -{}^1/_2 g^2 (\varphi_i{}^+\varphi_i)^2. \quad (4)$$

This contribution is irrelevant to our calculation below of the radiative shift of the energy of a scalar electron in an external electromagnetic field.

When the other diagrams which arise, along with the single-loop electrodynamic correlation to the mass of the

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scalar particle, are taken into account jointly, the quadratic divergences cancel out completely in the supersymmetric QED.

We carry out the calculation in (2) in the $E_p(x)$ representation,[7] in which the mass operator is written in the form

$$M(p,p')=ig^2\int d^4x\,d^4x'\,\varphi_p^+(x)M(x,x')\varphi_{p'}(x'), \qquad (5)$$

and the Green's function of the electron is

$$S^{(e)}(x,x')=-\frac{1}{(2\pi)^4}\int\frac{d^4q}{q^2-m^2+i0}E_q(x)(q+m)\bar{E}_q(x'). \qquad (6)$$

The quantity $\varphi_p(x)$ in (2), (5), and (6) is the wave function of a scalar particle in the given constant crossed field, and the matrix $E_q(x)$ is[7]

$$E_q(x)=\left[1+\frac{g\hat{n}\hat{A}}{2np}\right]\exp(-iqx-i\eta), \qquad (7)$$

where

$$\eta(p,\varphi)=\int_0^\varphi\left[g\frac{pA(\varphi')}{np}-g^2\frac{A^2(\varphi')}{2np}\right]d\varphi', \qquad (8)$$

p is the 4-momentum of the scalar electron ($p^2=M^2$), and the Green's function of the Majorana photino is

$$S^M(x,x')=\langle 0|T\lambda(x)\bar{\lambda}(x')|0\rangle$$

$$=-\frac{1}{(2\pi)^4}\int\frac{\hat{\varkappa}}{\varkappa^2+i0}\exp[-i\varkappa(x-x')]d^4\varkappa. \qquad (9)$$

Carrying out the calculation in the special frame of reference with

$$n^\mu=(1,0,0,1),\quad \varphi=nx=x_0-x_3,\quad a^\mu=(0,-a,0,0), \qquad (10)$$

we find the following representation for the mass operator in (5):

$$M(p,p')=\pi^3\frac{g^2}{p_0}\int_0^\infty\frac{dt}{(t+1)^2}\int_0^\infty\frac{dv}{v}\left[\frac{1}{4}g^2a^2v^2t+m^2t-2i\frac{np}{v}\right.$$

$$\left.-2i(np)\delta(v)\right]\delta(p-p')$$

$$\times\exp\left\{-i\left[\frac{v}{2np}(m^2(t+1)-p^2)+\frac{e^2a^2}{24np}tv^3\right]\right\}. \qquad (11)$$

Because of the difference between the signs of the charges of the fields φ_1 and φ_2, their masses go off in different directions with respect to the electron mass upon spontaneous breaking of supersymmetry because of the Fayet-Iliopoulos $U(1)$-D term. Consequently, we will renormalize (11), calculate the radiative shift of the mass of the scalar, and calculate the probability for the process $\varphi\to e+\lambda$ in crossed fields separately for each case ($M_1>m$ and $M_2<m$).

2. RADIATIVE SHIFT OF THE MASS; PROBABILITY FOR THE PROCESS $\varphi\to e+\lambda$

Case in which the mass of the scalar particle is greater than the electron mass. The mass operator in (11) is diagonal, and the amplitude for the elastic scattering of the scalar $M(p,a)$ on the mass shell ($p^2=M_1^2$) is found from

$$M(p,p')=(2\pi)^4\delta(p-p')M(p,a). \qquad (12)$$

According to the optical theorem, the imaginary part of the amplitude for elastic scattering is related to the total probability for the process $\varphi\to e+\lambda$ by

$$W=-2\,\mathrm{Im}\,M(p,a). \qquad (13)$$

Singling out the imaginary part of (11), which is finite and which does not require regularization, and using (12) and (13), we find the spectral distribution of the probability for the process:

$$\frac{dW}{dt}=\frac{g^2M_1^2}{8\pi p_0}\frac{1}{(t+1)^2}\left[\Delta\Phi_1(y)-2t\left(\frac{\chi}{t}\right)^{2/3}\Phi'(y)\right], \qquad (14)$$

where

$$\Phi_1(y)=\int_y^\infty\Phi(t)\,dt, \qquad (15)$$

and the argument of the Airy functions is

$$y=\left(\frac{t}{\chi}\right)^{2/3}\left[1-\Delta\frac{t+1}{t}\right]. \qquad (16)$$

In the Fayet-Iliopoulos model, the parameter $\Delta=1-\mu=1-m^2/M_1^2$ varies over the range

$$0<\Delta<1/2, \qquad (17)$$

and the case $\Delta=0$ corresponds to a vanishing value of the parameter k (which represents the spontaneous breaking of supersymmetry), if supersymmetry is not broken in the tree approximation. The effect of the external field on this process is determined by the known parameter

$$\chi=\frac{g}{M_1^3}[-(F_{\mu\nu}p^\nu)^2]^{1/2}=\frac{ga(np)}{M_1^3}. \qquad (18)$$

The argument of the Airy functions in (16) can take on both positive and negative values; this situation is typical of processes which also occur in the absence of the field.[7]

Using $0.5<\mu<1$, we find from (14) the following asymptotic expansions for the total probability for the process for small and large values of the parameter χ:

$$W=\begin{cases}\dfrac{g^2M_1^2\Delta^2}{8p_0}\left[1+\dfrac{6-16\Delta+6\Delta^2}{3\Delta^2}\chi^2\right], & \chi\ll 1,\\[2ex] \dfrac{g^2M_1^2}{36p_0}\Gamma\left(\dfrac{2}{3}\right)(3\chi)^{2/3}, & \chi\gg 1.\end{cases} \qquad (19)$$

The first term of the expansion of the probability for the process $\varphi\to e+\lambda$, which does not depend on χ, is the same in the case $\chi\ll 1$ as the result found in Ref. 8 for free decay. To find the real part of the mass operator, which is finite and which depends on the external field, we need to subtract from the real part of the mass operator in (11) its value in a zero field. As a result, the radiative shift of the mass of the scalar particle due to the loops of supersymmetric particles becomes

$$\Delta M_1=\frac{p_0}{M_1}\mathrm{Re}[M(p,a)-M(p,0)]=\frac{g^2M_1}{16\pi}\int_0^\infty\frac{dt}{(t+1)^2}$$

$$\times\left\{-2t\left(\frac{\chi}{t}\right)^{2/3}\Upsilon'+\Delta\int_y^\infty dt\left[\Upsilon(t)-\frac{1}{t}\right]\right\}, \qquad (20)$$

where

$$\Upsilon(y)=\int_0^\infty dt\sin\left(yt+\frac{t^3}{3}\right), \tag{21}$$

and the integral is to be understood in the principal-value sense. We find asymptotic expressions for (20) by a slightly modified version of the procedure of Ref. 9. We go over from the variable t to the variable

$$v=1/(t+1), \tag{22}$$

and we break up the range of integration into two parts, from 0 to μ and from μ to 1. In each part, the argument in (16) of the function $\Gamma(y)$ is of fixed sign. We then use the integral Mellin-Barnes representation for the digamma function:

$$\begin{pmatrix}\Upsilon(y)\\ \Upsilon(-y)\end{pmatrix}=\frac{3^{1/6}}{2y}\int_{s_0-i\infty}^{s_0+i\infty}\frac{ds}{2\pi i}\,y^{-3s}g^{-s}\Gamma\left(s+\frac{1}{3}\right)\times\Gamma\left(s+\frac{2}{3}\right)\begin{pmatrix}-\operatorname{ctg}\pi s\\ \dfrac{\cos 2\pi s}{\sin\pi s}\end{pmatrix}, \tag{23}$$

where $y>0$ and $-1/3<\mathrm{Re}\,s_0$. Each of the integrals which arises diverges separately, because of the well-known behavior of the function $\Gamma_1(x)$ (Ref. 7):

$$\Upsilon_1(x)\,|_{x\to0}\approx\ln x. \tag{24}$$

As in the dimensional-regularization procedure, we replace $y-\mu$ by $(y-\mu)^{1+\varepsilon}$ in each of the integrals. After evaluating the integrals and taking the limit $\varepsilon\to0$, we find the radiative shift of the mass of the scalar. In the case $\chi\ll\bar{\mu}=1-\mu$, in the logarithmic approximation, with $\ln(\bar{\mu}/\chi)\gg1$, we find

$$\Delta M_1=\frac{g^2M_1}{8\pi}\chi^2\left[C_1\ln\frac{\chi}{\mu^{1/2}}+2C_2\ln\frac{\chi\mu}{\bar{\mu}}\right], \tag{25}$$

where the coefficients C_1 and C_2 can be expressed in terms of hypergeometric functions of two arguments:[10]

$$\begin{aligned}C_1&=F_1(3,-2,2,3;\mu,b)-3F_1(3,-3,2,3;\mu,b)+{}^3/_4\bar{\mu}F_1(4,-2,3,4;\mu,b),\\ C_2&=-\frac{2}{9\mu}F_1(0,-1,2,3;w,z)+\frac{2}{3}\frac{\bar{\mu}}{\mu}F_1(0,-1,2,4;w,z)\\ &\quad-\frac{1}{9\mu}F_1(0,-2,3,3;w,z).\end{aligned} \tag{26}$$

In (26),

$$b=\frac{3\mu-1}{2},\quad w=\frac{\mu-1}{\mu},\quad z=\frac{3\mu-1}{3\mu}. \tag{27}$$

In the other limiting case, $\chi\gg1$, we find

$$\Delta M_1=-\frac{g^2M_1}{4\cdot3^{1/2}}\Upsilon'(0)\chi^{2/3},\quad \chi\gg1. \tag{28}$$

Case in which the mass of the scalar particle is smaller than the electron mass. In the case $M_2<m$, in contrast with the case $M_1>m$, discussed above, the process $\varphi\to e+\lambda$ is forbidden in the free case, and the mass operator in (11) is renormalized in the standard way:[7]

$$\bar{M}(p,p',a)=M(p,p',a)-M(p,p',0). \tag{29}$$

The probability for the process $\varphi\to e+\lambda$ and the radiative shift of the mass of a scalar electron are given (14) and (20), where it is necessary to make the substitution $M_1\to M_2$ everywhere. In this case the parameter Δ can take on only negative values or a value of zero:

$$-\infty<\Delta\equiv-\Delta_0=-\frac{1}{2}\frac{kg}{m^2-{}^1/_2kg}\leqslant0. \tag{30}$$

The argument of the Airy-Hardy function

$$f(y)=i\int_0^\infty dt\exp\left\{-i\left(yt+\frac{t^3}{3}\right)\right\}=\Upsilon(y)+i\Phi(y) \tag{31}$$

is nonnegative at arbitrary values of the spectral variable u. The asymptotic expansion of the probability for the process $\varphi\to e+\lambda$ with $\chi\ll\Delta_0\mu^{1/2}$ is found by the method of steepest descent. The leading term of this expansion is

$$W=\frac{g^2M_2{}^2\Delta_0\chi}{8\cdot3^{1/2}p_0(\mu)^{1/2}}\frac{1}{(1+\Delta_0/2\mu)^2}\exp\left[-\frac{3^{1/2}\Delta_0\mu^{1/2}}{\chi}\right]. \tag{32}$$

In the limit of large values of the parameter χ, we have, as expected [cf. (19)]

$$W=\frac{g^2M_2{}^2}{36p_0}\Gamma\left(\frac{2}{3}\right)(3\chi)^{2/3},\quad \chi\gg(1+\Delta_0)^{3/2}. \tag{33}$$

In the case $\Delta_0\mu^{1/2}\ll\chi\ll1$, we have $t\ll1$ in the region of importance in (14), and the first term of the expansion of the probability for the process $\varphi\to e+\lambda$, which corresponds to the case in which the supersymmetry is exact, is

$$W=\frac{g^2M_2{}^2}{8p_0}\chi^2. \tag{34}$$

We can also write several asymptotic expansions for the supersymmetric mass shift of a scalar electron in a constant crossed field. In the limit $\chi\ll\Delta_0\mu^{1/2}$, in which the argument of the Airy-Hardy function is $y\gg1$, the result can be expressed in terms of the hypergeometric function ${}_2F_1(a,b,c;z)$:

$$\Delta M_2=\frac{g^2M_2}{16\pi}\frac{1}{(1+\Delta_0)^2}\chi^2\left[2\,{}_2F_1\left(2,2,4;\frac{1}{1+\Delta_0}\right)+\frac{1}{18}\frac{1}{1+\Delta_0}{}_2F_1\left(3,3,5;\frac{1}{1+\Delta_0}\right)\right]. \tag{35}$$

In the case $\chi\gg(1+\Delta_0)^{3/2}$ we find a result which is the same as (28) aside from the replacement $M_1\to M_2$. In the limit $\chi\ll1$, the leading term of the expansion of the radiative shift of the mass of a scalar, (20), which corresponds to the case of unbroken supersymmetry, is

$$\Delta M_2=\frac{g^2M_2}{16\pi}\chi^2\left[2\ln\frac{1}{\chi}+2C+\ln2-5\right], \tag{36}$$

where C is Euler's constant. As was mentioned above, the complete shift of the mass of a scalar also includes a contribution from standard scalar electrodynamics:[7,11]

$$\Delta M=\frac{\alpha M}{6\pi}\int_0^\infty du\frac{5+7u+2u^2}{(u+1)^3}\left(\frac{\chi}{u}\right)^{2/3}f'(z). \tag{37}$$

For the real part of this contribution we have the following asymptotic expansions:[11]

$$\Delta M=\begin{cases}\dfrac{4\alpha M}{3\pi}\chi^2\left[\ln\dfrac{1}{\chi}+C+\dfrac{1}{2}\ln 3-\dfrac{15}{8}\right], & \chi\ll 1,\\ \dfrac{2\alpha M}{3^{5/3}}\Gamma\left(\dfrac{2}{3}\right)(3\chi)^{2/3}, & \chi\gg 1,\end{cases} \quad (38)$$

where α is the fine-structure constant.

It follows from (20) and (37) and from their asymptotic expressions that incorporating radiation effects in the Fayet-Iliopoulos model of supersymmetric QED in a constant external field does not equalize the masses of the scalars in a perturbation-theory approach.

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[1] V. K. Krivoshchekov, A. A. Slavnov, and L. O. Chekhov, Teor. Mat. Fiz. **72**, 12 (1987).
[2] S. Singh and P. N. Pandita, Phys. Rev. D **34**, 1859 (1986).
[3] P. A. Éminov, Yad. Fiz. **51**, 542 (1990) [Sov. J. Nucl. Phys. **51**, 346 (1990)].
[4] A. A. Kapustnikov, Yad. Fiz. **44**, 525 (1986) [Sov. J. Nucl. Phys. **44**, 338 (1986)].
[5] Th. Ohrnorb, Nucl. Phys. B **273**, 165 (1986).
[6] U. Wess and J. Begger, *Supersymmetry and Supergravity*, Princeton Univ., Princeton, New Jersey, 1983 (Russ. Transl. Mir, Moscow, 1986).
[7] V. I. Rutus, Trudy FIAN SSSR **111**, 5 (1979).
[8] G. Farrar and P. Fayet, Phys. Lett. B **89**, 191 (1980).
[9] A. V. Averin, A. V. Borisov, and A. I. Studenikin, Usp. Fiz. Nauk **50**, 1058 (1989) [Sov. Phys. Usp. **50**, 660 (1989)].
[10] A. P. Prudnikov, Yu. A. Brychev, and O. I. Marichev, *Integrals and Series: Additional Topics*, Nauka, Moscow, 1986. (Gordon and Breech, New York, 1986).
[11] V. N. Baĭer, V. M. Katkov, and V. S. Fadin, *Radiation by Relativistic Electrons*, Atomizdat, Moscow, 1973.

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