Vortex rings and dissipation in type-II superconductors

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The subject of this paper is the mechanism which leads to destruction of type-II superconductivity by an external current of density j. It is shown that at zero temperature superconductivity disappears as soon as the Landau criterion is violated, which corresponds to thresholdless creation of closed vortex loops whose size is on the order of the correlation length. At nonzero temperature T dissipationless current flow is in fact impossible, even though the linear resistance vanishes. The mechanism for dissipation in this case is thermally-activated creation of closed vortex rings oriented perpendicular to the direction of current flow, followed by expansion and merging of these loops. For $j \ll j_c (\ln k)/k$ (where j_c corresponds to the Landau criterion and k is the Ginzburg-Landau parameter) the rate of dissipation is proportional to $j^{7/3}\exp(-\mathrm{const}/Tj)$; for $j_c(\ln k)/k \leqslant j \ll j_c$ and $j_c - j \ll j_c$, to exponential accuracy the dissipation rate behaves like $\exp[-\mathrm{const}\cdot \ln^2(j_c/j)/Tj]$ and $\exp[-\mathrm{const}(j_c - j)/Tj_c]$, respectively. A smooth resistive transition is predicted for materials with wide fluctuation regions, as well as effects analogous to the AC Josephson effect.

Nowadays there is general agreement that the most striking experimental characteristic of the transition of matter to the superconducting state is the possibility of stationary dissipationless current flow, which manifests itself in a conductor as the absence of electrical resistance (see, e.g., Ref. 1 and the papers cited therein). The goal of this paper is to show that this point of view is not entirely correct, using the example of a type-II superconductor in zero magnetic field. The correct statement is that although the linear resistance of a conductor in the superconducting state in fact vanishes at zero temperature, a state of dissipationless current flow is impossible. Although the amount of dissipation is extremely small in the majority of practical cases, the very fact that it exists is of extreme importance.

The physical origin of this dissipation can be traced to the following phenomenon. When a current flows in a superconductor, it turns out to be energetically advantageous to produce closed vortex rings lying in planes perpendicular to the direction of current flow. However, a state with a vortex ring is separated from the irrotational state by a potential barrier associated with the intrinsic energy of the vortex. At zero temperature this barrier disappears only at a certain critical current density given by the Landau criterion.[1] At nonzero temperature the situation changes, since thermally-activated creation of rings becomes possible. Rings whose radii exceed a certain critical value will expand until they merge with other rings which have preceded them in the same plane perpendicular to the current. This process of birth and merging of vortex rings is repeated periodically, leading to a nonzero average electric field intensity directed along the current, and consequently to finite dissipation. An analogous mechanism for the appearance of dissipation in superfluids was investigated in Refs. 2 and 3, and applied to extended Josephson junctions in the recent paper Ref. 4. In contrast to previously-investigated cases,[2,3] vortices in type-II superconductors have a finite interaction radius; this fact allows us to find the dependence of the electric field intensity on current up to a numerical factor of order unity by using analogies with the growth of a crystal surface. The resulting dependence is valid for small external currents (corresponding to conditions which will be described below).[5]

1. Assume that a superconductor carrying an external current density j contains an infinite rectilinear Abrikosov vortex. The external current exerts the following force per unit length on this vortex:[1]

$$\mathbf{f} = \frac{\Phi_0}{c}[\mathbf{jb}], \tag{1}$$

where $\Phi_0 = \pi\hbar c/e$ is the quantum of magnetic flux ($\hbar$ is Planck's constant, c is the velocity of light, and e is the electron charge), and $\mathbf{b}$ is a unit vector along the vortex axis. From this relation it is clear that the force with largest absolute value acts on a vortex oriented perpendicular to j. Therefore, we will consider closed vortex rings of radius $r \gg \xi$ (where ξ is the coherence length) lying in planes perpendicular to the direction of current flow. In this geometry $\mathbf{b}$ in Eq. (1) has the sense of a tangent vector which determines the orientation of the magnetic force lines. Depending on its direction, the radial force exerted by the external current on a ring can act either to compress the ring until it disappears or cause it to expand to $r = \infty$. In the latter case it is obvious that the formation of a ring becomes energetically advantageous. In what follows we will deal only with this favorably oriented type of ring.

The gain in energy from forming a ring of radius r equals the negative of the work done by the external current as the ring expands from a microscopic size to r. We evaluate this quantity in the following way. The work done during an infinitely small radial expansion dr equals the product of the total force $2\pi r j \Phi_0/c$ acting on the ring and the displacement dr. Integrating this expression from ξ to r, we find that the energy gained by forming a ring of radius $r \gg \xi$ equals $\Phi_0 \pi r^2 j/c$. The cost in energy is related to the intrinsic energy of a Abrikosov vortex. Consequently, the total energy of a ring of radius r is

$$U(r) = 2\pi r \varepsilon(r) - \Phi_0 \pi r^2 j/c, \tag{2}$$

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where $\varepsilon(r)$ is the energy per unit length:[1]

$$\varepsilon(r)=\left(\frac{\Phi_0}{4\pi\lambda}\right)^2\begin{cases}\ln(r/\xi), & \xi\ll r\leq\lambda \quad (3a)\\ \ln(\lambda/\xi), & r>\lambda \quad (3b)\end{cases}.$$

Here λ is the London penetration depth of the magnetic field. The function $U(r)$ in (2) has a maximum for some $r=r_c$ whose position and height decrease with increasing j. The potential barrier disappears completely for microscopic values of r_c. However, relation (2) is meaningful only for $r\gg\xi$. Therefore, an order-of-magnitude estimate of the critical current density j_c can be obtained from (2) by setting $r_c\approx\xi$. Taking into account that $\varepsilon(\xi)\approx(\Phi_0/\lambda)^2$, we find the value

$$j_c\approx c\Phi_0/\lambda^2\xi, \qquad (4)$$

which is comparable to the current density required to destroy Cooper pairs.[1] In order to verify that this latter relation corresponds to the Landau criterion, it is convenient to forego the use of expressions from the microscopic theory (as was done in Ref. 1), and instead write (4) in a somewhat different form. Introducing the definitions

$$j=nev, \quad \lambda^2=mc^2/4\pi ne^2$$

(where n is the electron density, v their velocity, and m their mass)[1] and Φ_0, we obtain an expression for the critical velocity

$$v_c\approx\hbar/m\xi,$$

from which the electron charge and the velocity of light have been eliminated. If we now identify m with the mass of a superfluid atom and take into account that closed vortex rings of size ξ correspond to rotons, the equivalence of this relation to the Landau criterion becomes obvious.[6]

Thus, we conclude that at zero temperature superconductivity is destroyed when the current density is large enough to initiate thresholdless creation of closed vortex rings of size ξ.

2. Let us now assume that the temperature is nonzero, and that the current density j is such that the radius r_c for a critical ring greatly exceeds the London penetration depth λ for a magnetic field. In this case the tension per unit length ε of the ring does not depend on the scale of r and is given by expression (3b); therefore, we find for the critical radius r_c and energy U_c

$$r_c=\frac{\varepsilon c}{\Phi_0 j}, \quad U_c=\frac{\pi\varepsilon^2 c}{\Phi_0 j}=\frac{\varepsilon^2 e}{\hbar j}.$$

The inequality $r_c\gg\lambda$ is fulfilled under the condition

$$j\ll j_c\ln k/k,$$

where $k=\lambda/\xi$ is the Ginzburg-Landau parameter,[1] which is assumed to be large compared to unity.

Consider an arbitrary cross section of the sample perpendicular to the direction of the current flow. Vortex rings created in this cross section act to strongly decrease the probability of creating rings in neighboring cross sections when the distance between these cross-sections is smaller than λ. This is because all the rings created in this way have the same orientation; consequently, a strong repulsion arises between rings in parallel cross sections when the distance between them becomes smaller than the vortex interaction radius λ. Therefore, it is meaningful to divide a sample into parallel layers with thickness on the order of several λ within which the creation and expansion of vortex rings occur independently. This is similar to the filling-in of atomic layers on the growing surface of the crystal.[5] Each of the layers under discussion contains S/r_c^2 possible independent rings within an area S, each of which emerges with a probability per unit time

$$\frac{1}{\tau}=\frac{1}{\tau_0}\exp\left(-\frac{U_c}{T}\right),$$

where τ_0^{-1} is a certain microscopic frequency. After a time t the number of vortex rings is $(S/r_c^2)(t/\tau)$, and the average distance between them is

$$\delta\approx r_c(\tau/t)^{1/2}.$$

Under the action of the radial force (1) a ring must expand. Let us assume, following Ref. 1, that this expansion is hindered by a viscous force proportional to the velocity. Then the condition of mechanical equilibrium implies the following velocity of radial expansion

$$u=\Phi_0 j/\eta c,$$

where $\eta=\Phi_0 H_{c2}/\rho_n c^2$ is the coefficient of viscosity ($H_{c2}\approx\Phi_0/\xi^2$ is the upper critical field, and ρ_n is the specific electrical resistance in the normal state).[1] Two rings will merge when their radii $r=ut$ become equal to δ, i.e., when

$$t=\bar{t}=(r_c^2\tau/u^2)^{1/3}, \quad r=\delta=u\bar{t}=(ur_c^2\tau)^{1/3}.$$

This leads to the following expression for the total average rate of change of the phase difference of the order parameter on both sides of the layer of material under discussion:

$$\bar{\varphi}=\frac{2\pi}{\bar{t}}\approx\left(\frac{\xi^4\rho_n^2}{\varepsilon^2\tau_0}\right)^{1/3}j^{4/3}\exp\left(-\frac{\varepsilon^2 e}{3\hbar jT}\right). \qquad (5)$$

Using the Josephson relation $\bar{\varphi}=2eV/h$,[1] we can find the average voltage V applied across a layer of thickness several λ, and, consequently, the dependence of the average electric field intensity E on j:

$$E\approx\frac{\hbar}{e\lambda}\left(\frac{\xi^4\rho_n^2}{\varepsilon^2\tau_0}\right)^{1/3}j^{4/3}\exp\left(-\frac{\varepsilon^2 e}{3\hbar jT}\right). \qquad (6)$$

Note the nonanalytic character of the function $E(j\to 0)$ and the absence of a linear resistance. The amount of heat developed per unit time per unit volume of material is given by the derivative of Ej, and is found to be proportional to

$$j^{7/3}\exp(-\varepsilon^2 e/3\hbar jT).$$

For a quantitative estimate of this effect we introduce the nonlinear resistance $\rho(j)$ ($E=\rho(j)j$) and rewrite Eq. (6) in the form

$$\frac{\rho(j)}{\rho_n}\approx\frac{\hbar}{e\lambda}\left(\frac{\xi^4 j}{\varepsilon^2\tau_0\rho_n}\right)^{1/3}\exp\left(-\frac{\varepsilon^2 e}{3\hbar jT}\right).$$

At the limit of applicability of Eq. (6), i.e., for those current densities at which $r_c=\lambda$, the ratio $\rho(j)/\rho_n$ attains a value

$$\frac{\rho}{\rho_n}\approx\frac{\exp(-\pi\varepsilon\lambda/3T)}{k(k\ln k)^{1/3}}. \qquad (7)$$

In writing this latter ratio we have used the Drude-Lorentz formula[1] to estimate ρ_n, and have also assumed that the collision time of an electron is of order τ_0. If we pick the specific value $k = 10^2$ and values of T and λ that are typical of traditional low-temperature superconductors, i.e., $T = 10$ K, $\lambda = 10^{-5}$ cm, we find the ratio ρ/ρ_n is only $\sim\exp(-5\cdot 10^4)$, i.e., negligibly small. If we use values of T and λ which correspond to the high-temperature superconductors, i.e., $\lambda = 10^{-3}$ cm and $T = 100$ K, then $\rho/\rho_n \sim 10^{-15}$. Although this is an increase of many orders of magnitude, it remains a difficult quantity to measure experimentally. These estimates apply at temperatures far from the phase transition to the normal state. However, it was shown in Ref. 7 that within the temperature range in which fluctuations are important the parameters entering into relation (7) have values for which $\rho/\rho_n \sim 1$. From this it follows that ordinary low-temperature superconductors, whose fluctuation range is negligibly narrow, should exhibit as a function of temperature a sharp resistive transition with a discontinuous electrical resistivity, whereas this transition should be very much smoother in the new high-temperature materials with their rather broad fluctuation temperature ranges.

3. The picture described above retains all of its qualitative features at high current densities, i.e., for

$$j_c \ln k/k \leqslant j \leqslant j_c.$$

However, in this case the radius of the critical ring satisfies the inequality

$$\lambda \leqslant r_c \ll \xi,$$

therefore, the processes of nucleation and diffusion of vortex rings cannot be considered independent of one another. In this case the tension per unit length in a ring is given by Eq. (3a), and the function $E(j)$ can be obtained only to exponential accuracy. For the critical radius r_c and energy U_c we find to logarithmic accuracy

$$r_c = \frac{\Phi_0 c}{(4\pi\lambda)^2 j} \ln\left[\frac{\Phi_0 c}{(4\pi\lambda)^2 j\xi}\right],$$

$$U_c = \frac{\pi\Phi_0^3 c}{(4\pi\lambda)^4 j} \ln^2\left[\frac{\Phi_0 c}{(4\pi\lambda)^2 j\xi}\right].$$

From this we obtain the electric field intensity to exponential accuracy:

$$E(j) \propto \exp\left\{-\text{const}\frac{\Phi_0^3 c}{\lambda^4 jT} \ln^2\left[\frac{\Phi_0 c}{\lambda^2 j\xi}\right]\right\}.$$

This relation can be used to recover the results of Refs. 2 and 3 regarding the braking force that appears in a superfluid moving with velocity $v \ll v_c$ due to fluctuation-induced creation of closed Feynman vortices. Actually, the properties of the latter are reproduced if we set the charge of an electron equal to zero in all the relations that apply to Abrikosov vortices. If we replace the parameters in this expression by those we defined earlier in our discussion of the Landau criterion, we conclude that the braking force and dissipation are proportional to

$$\exp\left\{-\text{const}\frac{\hbar^2 n\xi}{mT}\frac{v_c}{v}\ln^2\left(\frac{v_c}{v}\right)\right\}, \tag{8}$$

where n is the superfluid density and m has the sense of an atomic mass. We note that because of the nonanalytic character of the dependence on v the linear viscosity of the liquid is identically equal to zero. Near the temperature for the superfluid-to-normal liquid transition, this relation can be considerably simplified (recall that in liquid helium this region is extremely broad and is observable, e.g., in investigating the specific heat[6]), since, according to the Josephson relation,[8] the first of the factors under the exponent sign is a quantity of order unity.

Also noteworthy is the fact that although superfluidity in helium occurs at extremely low temperatures, the effect of fluctuation-induced creation of vortices is considerably more noticeable in liquid helium than it is in type-II superconductors. The reason for this is that the mass of a helium atom, which is many times larger than the electron mass, appears in the argument of the exponential in (8).

For current densities close to j_c, the energy barrier for creating a ring gradually disappears. In this region the corresponding dependence of the electric field intensity on the external current can have the following form (to exponential accuracy):

$$E(j) \propto \exp\left[-\text{const}\frac{\Phi_0^2\xi}{\lambda^2 T}\frac{j_c - j}{j_c}\right],$$

since expression (2) is linear in j. This same functional behavior of the dissipation and braking forces can be observed in superfluids if we replace j by v and $\Phi_0^2\xi/\lambda^2$ by $\hbar^2 n\xi/m$.

As already noted above, the mechanism leading to the appearance of dissipation consists of periodically repeated processes of thermally activated creation and subsequent expansion and merging of closed vortex rings. Consequently, a time-dependent oscillation will be superimposed on the average characteristics we obtained above with a characteristic frequency $\bar{\varphi}$ given in (5). This behavior recalls the AC Josephson effect,[1] and suggests that a number of phenomena that are characteristic of the latter may occur. The most obvious of these is the appearance of Shapiro steps[1] in the IV characteristic when an external AC field is applied, whose frequency ω satisfies the resonance condition $\bar{\varphi} = l\omega$ (where l is a whole number).

The distinctive feature of all the systems we have been discussing up to now is the possibility of creating vortex excitations. However, in type-I superconductors these vortices do not occur. Therefore, the question of whether superconductivity in the strict sense of the word exists in materials of this type still remains open.

In conclusion it should be noted that although the discussion given here has been rather general in character, its predictions should be observed most unequivocally in materials with high transition temperatures to the normal state and wide fluctuation regions; therefore, the most natural experimental systems to examine in order to see these effects are clearly the high-temperature superconductors.

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