Internal structure of vortices in exotic superconductors near the lower critical field

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We present an analytic approach to the description of the internal structure of vortices in hexagonal and tetragonal exotic superconductors subjected to magnetic fields close to H_{c1}, and use this approach to discuss arbitrary orientations of the magnetic field relative to the axes of the crystal. We study the structure of single-quantum singular vortices, and identify parameter regions in which these singular solutions are unstable and nonsingular vortices are energetically favored. We find the ranges of angle between the magnetic field and the crystal axes for which various types of vortices are allowed, and derive the angular dependence of the lower critical field H_{c1} that is a distinctive characteristic of superconductors with nontrivial pairing.

1. INTRODUCTION

Recent experimental and theoretical investigations of the mixed state in superconductors with nontrivial pairing have generated a great deal of interest. While nontrivial pairing is primarily associated with heavy-fermion superconductors, it may have some relevance to high-temperature superconductors as well. The magnetic properties of these latter systems are usually quite anisotropic, with distinctive angular dependences of the critical fields and vortex lattices with peculiar structure. The anisotropy of the upper critical field H_{c2} in superconductors with multicomponent order parameters has been discussed in a number of theoretical papers.[1–3]

There is also an undeniable interest in the investigation of features of the mixed state for low values of the field H, i.e., near the lower critical field H_{c1}. The structures of vortex lattices and vortex cores in this range of fields can be quite different from the corresponding structures in the large-H range. This suggests that magnetic phase transitions may be possible in the vortex lattices. Apparently such transitions have been observed experimentally in the superconductor UPt_3 at fields $H \sim 0.6\, H_{c2}$.[4–6]

A number of theoretical explanations for these experiments have been proposed in the literature.[3,7–9] One of these theories (Ref. 9) involves a phase transition that converts axisymmetric singular vortices to nonsingular vortices via axial symmetry breaking. The phenomenological Ginzburg-Landau (GL) equations are solved numerically in this paper when the magnetic field is directed parallel to the anisotropy axis t of the system; the results show that the postulated axially nonsymmetric vortices can exist over a wide range of parameters. These vortices are nonsingular, i.e., at least one of the components of the superconducting order parameter is nonzero everywhere within the vortex. Nonaxisymmetric vortices have been proposed[10] previously in discussions of superfluidity in ^{3}He.

In this paper we use an analytic approach to analyze the core structure of nonaxisymmetric singular and nonsingular vortices in exotic superconductors with symmetry groups D_{6h} and D_{4h}. This approach allows us to treat arbitrary orientations of **H** relative to the crystal axes and to obtain the angular dependence of H_{c1}. Here we limit ourselves to cases where the energetically favored phase is superconducting with broken time-reversal invariance in the absence of a magnetic field. For the best-studied compound UPt_3, which has the symmetry group D_{6h}, there is experimental evidence for the existence of phases[11] which belong to superconducting classes that transform as two-dimensional representations of the groups D_4 and D_6.[12] These phases are characterized by the presence of a vector that behaves like a magnetic moment under the group transformations.[12] Orientations of this vector along the anisotropy axis and opposite it correspond to superconducting phases that are degenerate in the absence of a magnetic field. For applied magnetic fields near H_{c1} the degeneracy is lifted;[9] this leads to an unusual angular dependence of H_{c1}, which will be discussed below. The experimental observation of this dependence may be viewed as confirmation of the many-component nature of the order parameter in these systems.

2. DISTINCTIVE FEATURES OF THE VORTEX STRUCTURE IN HEXAGONAL SUPERCONDUCTORS

A) Ginzburg-Landau functional

Several experiments have shown that the superconductivity classes for UPt_3 involve order parameters that belong to two-dimensional representations of the group D_6,[11,13] and therefore transform as a two-component complex vector $\eta = (\eta_x, \eta_y)$. We write the GL functional in a magnetic field as follows:

$$F=\int (-a\eta_i\eta_i^*+\beta_1(\eta_i\eta_i^*)^2+\beta_2|\eta_i\eta_i|^2+K_1p_i^*\eta_j^*p_i\eta_j$$
$$+K_2p_i^*\eta_i^*p_j\eta_j \tag{1}$$
$$+K_3p_i^*\eta_j^*p_j\eta_i+K_4p_z^*\eta_i^*p_z\eta_i)dV,$$

$$\mathbf{p}=-i\hbar\nabla-(2e/c)\mathbf{A},\quad a=\alpha(T_c-T),$$
$$\beta_1>0,\ \beta_2>-\beta_1,\ K_1+K_2+K_3>|K_2|,\ K_1>|K_3|,$$
$$K_4>0. \tag{2}$$

In order for a phase with broken time-reversal invariance to occur in the absence of a magnetic field, we must have $\beta_2>0$. This corresponds to solutions with $\eta_\pm \sim (1, \pm i)$. We can introduce a vector $l = i[\eta\eta^*]$ which characterizes the "moment" of the uniform phase, and which is directed along the z-axis or opposite it for η_+ and η_- respectively. Let us consider solutions to the GL equation for a single vortex oriented at an angle γ to the anisotropy axis z, and rewrite (1) in terms of the functions

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$$\Psi_1=(\beta_1/a)^{1/2}(\eta_x-i\eta_y),\Psi_2=(\beta_1/a)^{1/2}(\eta_x+i\eta_y). \quad (3)$$

Let us also rotate the coordinate system x, y, z around the x axis by an angle γ such that for the new coordinates $\tilde{x}, \tilde{y}, \tilde{z}$ the $\tilde{z}$ axis is oriented parallel to the vortex axis, and carry out an additional scale transformation of the coordinates in the $\tilde{x}\tilde{y}$ plane:

$$\tilde{x}=\xi(1+C)^{1/2}\tilde{\tilde{x}},$$
$$\tilde{y}=\xi(1+C)^{1/2}\left[\cos^2\gamma+\frac{K_4\sin^2\gamma}{K_1(1+C)}\right]^{1/2}\tilde{\tilde{y}}, \quad (4)$$

where $\xi^2=h^2K_1/a$, $C=(K_2+K_3)/(2K_1)$. In the compounds under investigation the GL constant satisfies $\varkappa\gg 1$. This allows us to neglect the vector potential $\mathbf{A}$ in the GL equations while investigating the structure of the vortex core. We will also neglect quantities of order $(K_2-K_3)/(K_1\varkappa^2)$, for two reasons: not only do we have $\varkappa\gg 1$, but also

$$(K_2-K_3)/K_1\sim(T_c/\varepsilon_F)^2\ll 1.$$

This is due to approximate electron-hole symmetry at the Fermi surface.[3,12] We now write an expression for the free energy density in the range of distances $\rho=\tilde{\tilde{x}}^2+\tilde{\tilde{y}}^2\ll\varkappa$ from the vortex axis that includes the transformations described above:

$$\mathcal{F}=\mathcal{F}_0\xi^2(1+C)\left[\cos^2\gamma+\frac{K_4\sin^2\gamma}{K_1(1+C)}\right]^{1/2}$$
$$\times\Big\{-\Psi_1^*a^+a^-\Psi_1-\Psi_2^*a^+a^-\Psi_2$$
$$-\frac{C}{4(1+C)}(s^2\Psi_1^*a^+a^+\Psi_2+q^2\Psi_1^*a^-a^-\Psi_2+2sq\Psi_1^*a^+a^-\Psi_2$$
$$+s^2\Psi_1a^-a^-\Psi_2^*+q^2\Psi_1a^+a^+\Psi_2^*+2sq\Psi_1a^+a^-\Psi_2^*)$$
$$+\frac{1}{2}(|\Psi_1|^4+|\Psi_2|^4) \quad (5)$$
$$+(1+2b)|\Psi_1|^2|\Psi_2|^2-|\Psi_1|^2-|\Psi_2|^2\Big\},$$

$$\mathcal{F}_0=a^2/2\beta_1,\ b=\beta_2/\beta_1,$$
$$s=1-\cos\gamma\left[\cos^2\gamma+\frac{K_4\sin^2\gamma}{K_1(1+C)}\right]^{-1/2},\quad q=2-s, \quad (5a)$$
$$a^\pm=\frac{\partial}{\partial\tilde{\tilde{x}}}\pm i\frac{\partial}{\partial\tilde{\tilde{y}}}.$$

In calculating the energy of a vortex it is convenient to reserve for separate discussion the range of small distances ρ where both components Ψ_1 and Ψ_2 of the order parameter differ considerably from zero. At large distances, a solution that describes an isolated vortex must reduce to one that describes a uniform superconducting state, in which either Ψ_1 or Ψ_2 reduces to zero. Therefore, it follows from (5) that for large ρ we can use the usual GL theory with a single-component order parameter and an anisotropic mass tensor. For this reason, in what follows we will be interested primarily in those states of the system that are described by expression (5) and whose asymptotic form at large distances corresponds to the usual one-component vortex. For definiteness we will assume that the component Ψ_1 of the order parameter is nonzero for $\rho\gg 1$; then the vector $\mathbf{l}$ is directed along the z-axis for large ρ (we denote its value by $\mathbf{l}_\infty$).

B) Singular vortices oriented parallel to the anisotropy axis

For $\gamma=0$ and $\gamma=\pi$ solutions to the GL equations derived from expression (5) exist which describe an order parameter whose components are axisymmetric with respect to their magnitude:

$$\Psi_1=e^{im\theta}G_m(\rho),\quad \Psi_2=e^{in\theta}R_n(\rho); \quad (6)$$

here ρ, θ are polar coordinates in the $\tilde{\tilde{x}}\tilde{\tilde{y}}$ plane. The numbers m and n are connected by the relation $m+2=n$ for $\gamma=0$ and $m-2=n$ for $\gamma=\pi$. These relations follow immediately from symmetry considerations: because the GL functional (1) is invariant with respect to rotation through an arbitrary angle around the z-axis, we can assign a certain value of the projection of the moment on this axis to the solution (6). For $\gamma=0$ the moment of the component Ψ_1 is $m+1$, and the moment of Ψ_2 is $n-1$, since (3) and (6) imply that a rotation around the z axis through an angle φ gives rise to multiplication of these quantities by $\exp[i(m+1)\varphi]$ and $\exp[i(n-1)\varphi]$ respectively. The required connection between the numbers m and n is obtained from the requirement that the moments for Ψ_1 and Ψ_2 be equal. According to the assumptions made in the previous section, the function $G_m(\rho)$ is nonzero, while for positive b $R_n(\rho)$ reduces to zero outside the core. This implies that the vortex contains m magnetic flux quanta. We note that for $\gamma=0$ the case $m=-2$, $n=0$ corresponds to a two-quantum nonsingular vortex. The possible existence of such a vortex at high fields was investigated in Ref. 14. However, for sufficiently low fields and large $\varkappa$ it is apparently always more advantageous to have single-quantum vortices.

Let us set $m=-1$. Then the magnetic field $\mathbf{H}$ is oriented parallel and antiparallel to the vector $\mathbf{l}_\infty$ for $\gamma=0$ and $\gamma=\pi$ respectively. Depending on the orientation of $\mathbf{H}$ with respect to $\mathbf{l}_\infty$, two types of single-quantum vortex are possible. For $\mathbf{H}\uparrow\uparrow\mathbf{l}_\infty$ we have $n=1$, so that the phase circulation of component Ψ_2 in one circuit around the vortex axis equals 2π and $R_1\propto\rho$ as $\rho\to 0$. For $\mathbf{H}\uparrow\downarrow\mathbf{l}_\infty$ we have $n=-3$; in this case the phase circulation of component Ψ_2 equals -6π and $R_{-3}\propto\rho^3$ as $\rho\to 0$. Both kinds of axisymmetric vortices are singular, because the functions G_{-1}, R_1, and R_{-3} reduce to zero at $\rho=0$. Let us write the equations for R_1, G_{-1} in the region $\rho\ll\varkappa$ for $\gamma=0$:

$$-\left(R''_{1\rho\rho}+\frac{1}{\rho^2}R_{1\rho}'-\frac{1}{\rho^2}R_1\right)+R_1[(1+2b)G_{-1}^2-1]+R_1^3$$
$$=\frac{C}{1+C}\left(G''_{-1\rho\rho}+\frac{1}{\rho}G'_{-1\rho}-\frac{1}{\rho^2}G_{-1}\right), \quad (7)$$
$$-\left(G''_{-1\rho\rho}+\frac{1}{\rho}G'_{-1\rho}-\frac{1}{\rho^2}G_{-1}\right)+G_{-1}[(1+2b)R_1^2-1]+G_{-1}^3$$
$$=\frac{C}{1+C}\left(R''_{1\rho\rho}+\frac{1}{\rho}R_{1\rho}'-\frac{1}{\rho^2}R_1\right).$$

For $\gamma=\pi$ the equations for R_{-3} and G_{-1} have the form $(\rho\ll\varkappa)$

$$-\left(R''_{-3\rho\rho}+\frac{1}{\rho}R'_{-3\rho}-\frac{9}{\rho^2}R_{-3}\right)+R_{-3}[(1+2b)G_{-1}^2-1]+R_{-3}^3$$
$$=\frac{C}{1+C}\left(G''_{-1\rho\rho}-\frac{3}{\rho}G'_{-1\rho}+\frac{3}{\rho^2}G_{-1}\right), \quad (7a)$$

$$-\left(G''_{-1\rho\rho}+\frac{1}{\rho}G'_{-1\rho}-\frac{1}{\rho^2}G_{-1}\right)+G_{-1}[(1+2b)R_{-3}^2-1]+G_{-1}^3$$
$$=\frac{C}{1+C}\left(R''_{-3\rho\rho}+\frac{5}{\rho}R'_{-3\rho}+\frac{3}{\rho^2}R_{-3}\right).$$

The solutions to Eqs. (7) and (7a) depend sensitively on the two parameters b and $\varepsilon=C/(1+C)$. Because these equations cannot be solved exactly, we cannot determine R_1, G_{-1}, and R_{-3}; however, we can approximate these functions for all regions of variation of ρ. Solutions to (7) and (7a) can be obtained by the method of successive approximations with respect to ε. To lowest order in ε only the function G_{-1} is different from zero. To first order in ε the functions R_1 and R_{-3} appear in Eqs. (7) and (7a) respectively. The correction to G_{-1} is of order ε^2. In what follows, we will assume that $\varepsilon\ll 1$. This approximation is completely justified, since the condition (2) and the previously mentioned condition $K_2\approx K_3$ imply that $|\varepsilon|<0.5$. By including terms of first order in ε in the solutions of Eqs. (7) and (7a), we can show that $G_{-1}(\rho)$ coincides with the magnitude of the order parameter for a vortex in a normal superconductor. In this case the quantity $G_{-1}(\rho)$ is close to unity outside the core (i.e., for $\rho>2$). Therefore, for $\rho>2$ Eqs. (7) and (7a) can be written approximately in the form

$$-\left(R''_{1\rho\rho}+\frac{1}{\rho}R_{1\rho}'-\frac{1}{\rho^2}R_1\right)+2bR_1=-\frac{C}{(1+C)\rho^2},$$
$$-\left(R''_{-3\rho\rho}+\frac{1}{\rho}R'_{-3\rho}-\frac{9}{\rho^2}R_{-3}\right)+2bR_{-3}=\frac{3C}{(1+C)\rho^2}. \qquad (8)$$

For $2<\rho\ll(2b)^{-1/2}$ we have $R_1\approx -C/(1+C)$, and for $\rho\gg(2b)^{-1/2}$ the function $R_1(\rho)$ falls off like ρ^{-2}. Matching the asymptotic forms for large and small ρ, we can obtain the following approximation for R_1 over the entire interval of ρ (for $\rho\ll\varkappa$):

$$R_1^{(1)}(\rho)=\begin{cases}-\dfrac{C\rho}{2(1+C)(1+8b)}, & \rho<2\\[2mm] -\dfrac{C}{(1+C)(1+2b\rho^2)}, & \rho>2\end{cases}. \qquad (9)$$

Analogously, for R_{-3} we obtain

$$R_{-3}^{(1)}(\rho)=\begin{cases}\dfrac{3C\rho^3}{8(1+C)(9+8b)}, & \rho<2\\[2mm] \dfrac{3C}{(1+C)(9+2b\rho^2)}, & \rho>2\end{cases}. \qquad (10)$$

Thus, the structure of a singular vortex oriented parallel to the anisotropy axis is approximately described to first order in ε by Eqs. (6), (9), and (10). For values of b that are not too small, corrections to $G_{-1}^{(0)}$ that are second order in ε do not change the asymptotic behavior of G_{-1}, either for $\rho\to 0$ or for large ρ. In the calculations that follow we will use the simplest approximation for G_{-1}, i.e., $\tanh(\rho/2)$; in particular, we use this form to calculate the free energy.

C) Nonsingular vortices oriented parallel to the anisotropy axis

When the system is subjected to a magnetic field **H** parallel to the z-axis, the axisymmetric solutions (6) with $m=\pm 1$ that we obtained above are not energetically favorable for all admissible values of the parameters b and C. The numerical calculations carried out in Ref. 9 have shown this for the case of a field **H** parallel to the z-axis. In reality, within the core of a vortex the curve along which the component Ψ_1 of the order parameter vanishes need not coincide with the corresponding curve for Ψ_2, and there may be several such curves. This implies the presence of a nonsingular vortex with broken axial symmetry. The configuration that we find in this case may be regarded as a bound state of two or more vortices, if by the term "vortex" we mean any curve along which some component of the order parameter has zero absolute value and a specific phase circulation when one circuit is made around this curve. At large distances this vortex structure has an asymptotic behavior that corresponds to a single quantum vortex.

The structure of a nonsingular vortex for $\mathbf{H}\uparrow\uparrow\mathbf{l}_\infty$ differs significantly from the corresponding structure for $\mathbf{H}\uparrow\downarrow\mathbf{l}_\infty$. For $\mathbf{H}\uparrow\uparrow\mathbf{l}_\infty$, certain values of the parameters b and C cause the core to become unstable with respect to a displacement which separates curves along which the components Ψ_1 and Ψ_2 vanish. For the orientation $\mathbf{H}\uparrow\downarrow\mathbf{l}_\infty$ this instability takes the form of the decay of a component-Ψ_2 vortex with -6π phase circulation around its axis into three component-Ψ_2 vortices with circulation -2π.

Let us first consider the case $\mathbf{H}\uparrow\uparrow\mathbf{l}_\infty$. We can describe the displacement of the zero-component curves in the simplest possible way by choosing the following trial functions:

$$\Psi_1=G_0(\rho)+G_{-1}(\rho)e^{-i\theta},$$
$$\Psi_2=R_0(\rho)+R_1(\rho)e^{i\theta}, \qquad (11)$$

where $G_0(\rho)$ and $R_0(\rho)$ do not reduce to zero at the coordinate origin. The functions G_0, R_0, and R_1 fall off with increasing distance from the axis of the vortex over a characteristic length ρ^*, which increases as b increases, while for $b\to 0$ we have $\rho^*\propto b^{-1/2}$. The primary contribution from those terms of the free energy that depend on these functions comes from distances $\rho\lesssim\rho^*$. Therefore, for $\rho^*\ll\varkappa$ we can use expression (5). Let us substitute (11) into (5) and perform the angular integration over θ. It is necessary to determine the region of parameters b and C within which the functions G_0, R_0 are nonzero. To do this, we will separate out that portion of the free energy that depends on G_0, R_0, saving only terms that are quadratic in these functions:

$$\Delta F=2\pi\mathcal{F}_0\xi^2(1+C)\int_0^\infty\rho\,d\rho\{-|R_0|^2+|R_0|^2[(1+2b)G_{-1}^2+2R_1^2]$$
$$+|R_{0\rho}'|^2+|G_{0\rho}'|^2+|G_0|^2(2G_{-1}^2-1)$$
$$+(1+2b)R_1G_{-1}(R_0^*G_0^*+R_0G_0)\}. \qquad (12)$$

Note that for the special case $C=0$, $R_1=0$, variation of the functional (12) leads to an equation for the function R_0 which formally coincides with the Schroedinger equation for a particle in a potential well:

$$-\Delta R_0+(1+2b)G_{-1}^2R_0=R_0. \qquad (13)$$

The function R_0 will be nonzero only when the lowest energy level of a particle in the potential $(1+2b)G_{-1}^2$ satisfies the condition $E_{\min}(b)<1$. Let us use the approximation

$G_{-1} \approx \tanh(\rho/2)$ to calculate $E_{min}(b)$, while for R_0 we choose the trial function

$$R_0 = A \exp\{-\rho^2/\lambda_a^2\},$$

where λ_a is a variational parameter. Minimizing the corresponding energy functional, we obtain

$$\lambda_a = \frac{16}{3}\left(\frac{2}{\pi}\right)^{1/2} \frac{1}{1+2b}, \quad E_{min}(b) = \frac{2}{\lambda_a^2} + \frac{1+2b}{8}\left(\frac{\pi}{2}\right)^{1/2} \lambda_a. \tag{14}$$

From this it follows that a seed of R_0 appears for $b < b^* \approx 0.37$, with characteristic radius $\lambda_a \approx 2.5$. Analogous considerations lead easily to the conclusion that for $C = 0$ no seed of the function G_0 appears for any value of $b > 0$. Thus, we have found that nonsingular but axisymmetric vortices can exist for $C = 0$ and $b < b^*$. By including terms of fourth order in R_0, it is not difficult to show that the amplitude of the component Ψ_2 on the axis of the vortex is a quantity of order $(b^* - b)^{1/2}$. For comparison, we give the numerical result found in Ref 9, i.e., $b^* = 0.24$ for $C \to 0$. For $C \neq 0$ a seed of R_0 appears against a preexisting background of R_1. This leads to a shift in the threshold value of b^* which depends on C, and also to the appearance of nonzero G_0. For R_1 we use expression (9), while for G_0 we take the trial function

$$G_0 = B \exp\{-\rho^2/\lambda_b^2\}.$$

Minimizing the energy (12) with respect to the parameters λ_a, λ_b, and B, we find the following relations between the amplitudes G_0 and R_0, along with the values of λ_b and $b_0^*(C)$:

$$B \approx \frac{1,8C}{1+C} A^*, \quad \lambda_a \approx \lambda_b \approx 2,5, \quad b_0^*(C) \approx 0,37 + \frac{0,7C^2}{(1+C)^2}. \tag{15}$$

The increase in the threshold value of $b_0^*(C)$ and its dependence on the sign of C agree with the results of Ref. 9. Above threshold $(b > b_0^*(C))$ we have an axisymmetric singular vortex. The order parameter in this vortex is unchanged by a rotation through an arbitrary angle φ. At $b = b_0^*(C)$ a spontaneous breaking of the axisymmetry occurs. In this case the position of the curves on which the component Ψ_2 vanishes are displaced by a distance $\rho_2 \sim (1 + C)(b_0^* - b)^{1/2}/C$ from the z-axis for b near threshold, while the curve on which the component Ψ_1 vanishes shifts by a distance $\rho_1 \sim C(b_0^* - b)^{1/2}/(1 + C)$ in the opposite direction. The vector $\mathbf{d}$ which connects the positions of the zeros on the xy plane can be oriented in an arbitrary fashion for an individual vortex. However, this degeneracy will be lifted by the interaction between vortex lines in a lattice.

Let us now consider the case $\mathbf{H} \uparrow\downarrow \mathbf{l}_\infty$ and determine its instability threshold with respect to breaking of axisymmetry and its core structure. We choose trial functions for Ψ_1 and Ψ_2 in the following way:

$$\Psi_1 = G_{-1}(\rho) e^{-i\theta}, \quad \Psi_2 = R_{-3}(\rho) e^{-3i\theta} + R_0(\rho). \tag{16}$$

In this case there is no displacement of the curves along which the component Ψ_1 vanishes. This corresponds to $G_0(\rho) = 0$, and is associated with the absence of an interaction between the harmonics R_0 and G_0 in this case. Carrying out calculations analogous to those presented above for $\gamma = 0$, in the small-ε approximation we obtain the following expression for the threshold value $b_\pi^*(C)$:

$$b_\pi^*(C) = 0,37 - \frac{0,28C^2}{(1+C)^2}. \tag{17}$$

Comparing (15) and (17), we see clearly that $b_\pi^*(C)$ depends on C somewhat more weakly than $b_0^*(C)$; more specifically, the value of b_π^* decreases with increasing C. This latter result contradicts Ref. 9, where it was found that b_π^* is independent of C. This disagreement is apparently connected with the omission of anisotropy in the function Ψ_1. Nevertheless, the structure of the core for the solution (16) coincides with that obtained in Ref. 9. At the same distance

$$\rho \sim (1+C)^{1/3} (b_\pi^* - b)^{1/6} / C^{1/3}$$

from the z-axis (and near the threshold b_π^* for b) we find three vortices for the component Ψ_2, which form an equilateral triangle. If we do not take into account neighboring component-Ψ_1 vortices, this state is degenerate with respect to rotations of the triangle. In this case, the nonaxisymmetric vortex preserves a symmetry with respect to the transformation $\exp(-i2\pi/3) C_3$, where C_3 is a rotation by an angle $2\pi/3$, as well as the transformation $U_2 R$, where R is the operation of complex conjugation and U_2 is a rotation through an angle π around one of the three axes located at an angle $2\pi/3$ to one another in the xy plane.

D) Singular vortices oriented at an angle to the anisotropy axis

A comparison of the results obtained in the previous sections with the results of numerical calculations carried out in Ref. 9 for the case of a field $\mathbf{H}$ parallel to the z-axis shows clearly that a satisfactory level of accuracy can be achieved in describing the structure of vortices in superconductors with nontrivial pairing without going beyond the simple approximate methods used here. Let us now consider the structure of a vortex which makes an arbitrary angle γ with the z-axis, a problem which has not yet been studied in the literature. We seek Ψ_2 in the form of an expansion in angular harmonics:

$$\Psi_2 = \sum_n R_n(\rho) e^{in\theta}. \tag{18}$$

For Ψ_1 we retain only the two harmonics G_{-1} and G_0 in the expansion. Again we will use the approximation $\varepsilon \ll 1$ in the calculation, and assume $G_{-1} \approx \tanh(\rho/2)$. Substituting (18) into the free energy functional, carrying out the angular integration over θ, and varying the the functional with respect to the harmonics R_n as before, we arrive at a system of equations for these harmonics. In this section we will investigate only the structure of the singular vortices; therefore, we will assume that R_0 and $G_0 = 0$. Furthermore, we will show that this scenario is indeed possible for sufficiently large values of b. For the three harmonics R_1, R_{-1}, and R_{-3} the component Ψ_1 plays the role of a source, whose amplitude is a quantity of order ε. The remaining R_n are proportional to higher powers of the parameter ε; therefore, in the approximation used here they will be neglected. These three harmonics can be approximated by the following expressions:

$$R_1(\rho,\gamma)=\frac{q^2}{4}R_1^{(1)}(\rho),$$
$$R_{-1}(\rho,\gamma)=\frac{2s}{q}R_1(\rho,\gamma), \tag{19}$$
$$R_{-3}(\rho,\gamma)=\frac{s^2}{4}R_{-3}^{(1)}(\rho).$$

Here $R_1^{(1)}(\rho)$, $R_{-3}^{(1)}(\rho)$ are the functions (9) and (10), which are solutions to the problem for $\gamma=0$ and $\gamma=\pi$ respectively. Expressions (19), (9), and (10) allow us to calculate the correction to the vortex energy associated with its core for arbitrary γ, and consequently to obtain the angular dependence of the upper critical field H_{c1} for a singular vortex:

$$H_{c1}=\frac{\phi_0}{4\pi\lambda^2\cos(\gamma-\varphi)}\left[\cos^2\gamma+\frac{K_4\sin^2\gamma}{K_1(1+C)}\right]^{1/2}\times\left[\ln\varkappa-\frac{C^2}{(1+C)^2}\Phi(\gamma)\right], \tag{20}$$

$$\Phi(\gamma)=\frac{s^4}{16}f_\pi+\frac{f_0}{4}\left(\frac{q^4}{4}+s^2q^2\right),$$
$$f_0=\frac{1}{2}\ln\left(1+\frac{1}{8b}\right)-\frac{0{,}3+8b}{(1+8b)^3},$$
$$f_\pi=\frac{1}{2}\ln\left(1+\frac{9}{8b}\right)-4{,}5\frac{51+56b}{(9+8b)^3},$$

where λ is the penetration depth of a magnetic field oriented along the anisotropy axis of the crystal, s and q are given by expression (5a), and φ is the angle the field $\mathbf{H}$ makes with the z-axis. For the case $C=0$ the expression for $H_{c1}(\gamma)$ coincides with the result obtained for normal superconductors with an anisotropic mass tensor[15] to logarithmic accuracy. For anisotropic superconductors φ need not coincide with γ. In order to determine γ we must investigate expression (20) at its minimum. Discarding terms $\sim\varepsilon^2$, we find the following condition:

$$K_1(1+C)\tan\varphi=K_4\tan\gamma. \tag{21}$$

For $\varphi=0$, $\varphi=\pi$ the direction of the field $\mathbf{H}$ is parallel to the vortex axis. For $C\neq0$ we find that the directions $\varphi=0$ and $\varphi=\pi$ are not equally justifiable. The function $\Phi(\gamma)$ is positive, and, e.g., for $b\approx0.4$ it increases monotonically as γ increases from zero to π. Consequently, $H_{c1}(\pi)<H_{c1}(0)$. For large $\varkappa$ and $b\approx0.4$ we have

$$\frac{H_{c1}(0)-H_{c1}(\pi)}{H_{c1}(0)}\approx\frac{0{,}4C^2}{(1+C)^2\ln\varkappa}. \tag{22}$$

The orientation of the vortex with respect to the magnetic field also changes. In particular, for $\varphi=\pi/2$

$$\gamma\approx\frac{\pi}{2}+\frac{C^2K_4}{(1+C)^3K_1\ln\varkappa}\Phi_\gamma'|_{\gamma=\pi/2}$$
$$\approx\frac{\pi}{2}+\frac{0{,}1C^2K_4^{1/2}}{(1+C)^2(1+C)^{1/2}K_1^{1/2}\ln\varkappa}. \tag{23}$$

However, when H_{c1} is calculated to order ε^2 we can use Eq. (21) once more. Let us now find the structure of a vortex for arbitrary γ. For small ρ the solution to (19) for Ψ_2 has the form

$$\Psi_2\approx\frac{3\varepsilon s^2}{32(9+8b)}\rho^3e^{-3i\theta}-\frac{\varepsilon q\rho}{8(1+8b)}(qe^{i\theta}+2se^{-i\theta}). \tag{24}$$

For $b^{-1/2}\ll\rho\ll\varkappa$ we have

$$\Psi_2\approx\frac{\varepsilon}{8b\rho^2}[3s^2e^{-3i\theta}-q(qe^{i\theta}+2se^{-i\theta})]. \tag{25}$$

For $\gamma\neq0$, $\gamma\neq\pi$ the component Ψ_2 simply changes sign under a rotation through an angle π in the xy plane; however, for arbitrary rotations there is no symmetry. For arbitrary γ several vortices of the component Ψ_2 appear in the core of the component-Ψ_1 vortex, and during a circuit around the centers of these vortices the phase circulation of Ψ_2 equals $2\pi n$ ($n<0$ if the circulation coincides in sign with the circulation of the phase of Ψ_1, and $n>0$ in the opposite case).

It follows from Eqs. (24) and (25) that in the range $0<\gamma<\gamma_1$, where γ_1 is determined from the relations

$$q(\gamma_1)=3s(\gamma_1),\qquad \gamma_1=\arctan\left[\frac{3K_1(1+C)}{K_4}\right]^{1/2},$$

there is one vortex of Ψ_2 in the core with $n=1$. For $\gamma>\gamma_1$ two more vortices of Ψ_2 appear for which $n=-1$. In this case, the curves along which the component Ψ_2 vanishes lie in the $\tilde{z}\tilde{y}$ plane. As γ increases these vortices approach the coordinate origin and merge with it for some γ_2, where

$$q(\gamma_2)=2s(\gamma_2),\qquad \gamma_2=\arctan\left[\frac{8K_1(1+C)}{K_4}\right]^{1/2}.$$

For $\gamma>\gamma_2$, we see that for $\rho=0$ a single vortex of Ψ_2 remains with $n=-1$. When γ becomes larger than $\gamma_3=\pi/2$ ($s(\gamma_3)=q(\gamma_3)$), two more vortices of Ψ_2 appear on the $\tilde{x}$ axis with $n=-1$, which for $\gamma=\pi$ migrate to the coordinate origin. In this case a vortex of Ψ_2 forms with $n=-3$.

The positions of the zeros of Ψ_2 are shown schematically in Fig. 1. The new vortices of the Ψ_2 component initially appear at a formally infinite distance from the $\tilde{z}$-axis,[1)] and then approach the origin as γ varies. Since expression (5) applies only in the region of distances $\rho\ll\varkappa$, this approach to describing the coordinates of Ψ_2-vortices is incorrect within narrow intervals near the angles γ_1 and γ_3, for which the Ψ_2-vortices are located at distances $\rho\gtrsim\varkappa$.

E) Nonsingular vortices oriented at an angle to the anisotropy axis

Let us now determine the instability threshold of a solution corresponding to a singular inclined vortex, and investigate the formation of nonsingular vortices. For this it is necessary to take into account the formation of seeds of $R_0(\rho)$ and $G_0(\rho)$ within the core. Just as for the orientations with $\gamma=0$, $\gamma=\pi$, which we investigated above, we use the trial functions

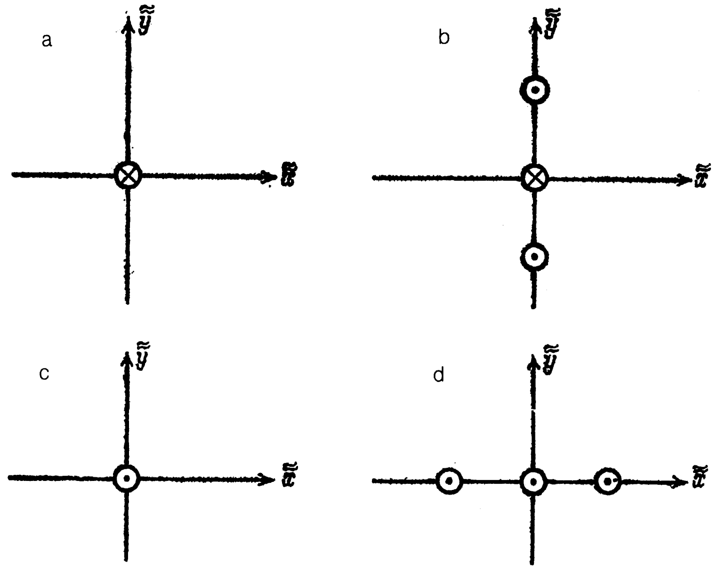

FIG. 1. Schematic spacing of vortices of the component Ψ_2 of the order parameter in the $\tilde{x}\tilde{y}$ plane for singular vortices in a hexagonal superconductor for various values of the inclination angle of the vortex axis γ to the anisotropy axis z: $\otimes$ is a component-Ψ_2 vortex with $n=1$; $\odot$ is a component-Ψ_2 vortex with $n=-1$; a—$0<\gamma<\gamma_1$; b—$\gamma_1<\gamma<\gamma_2$; c—$\gamma_2<\gamma<\gamma_3$; d—$\gamma_3<\gamma<\pi$.

$$R_0(\rho)=A\exp(-\rho^2/\lambda_a^2),$$
$$G_0(\rho)=B\exp(-\rho^2/\lambda_b^2). \qquad (26)$$

For sufficiently small b, the approximation $G_{-1}\approx\tanh(\rho/2)$ will be incorrect in the region where the nonsingular vortices exist. The amplitude R_0 increases as $(b^*-b)^{1/2}$, and as b increases away from threshold some Ψ_1 appears in the core. However, if we are interested only in the behavior of the solution near the threshold value b^*, this circumstance can be ignored. By varying the free energy functional with respect to the parameters λ_a, λ_b, and B, we obtain the following expression for that part of the free energy which depends quadratically on $A=\tilde{a}\exp(i\varphi_a)$ near the instability threshold:

$$\Delta F(A)\approx\mathcal{F}_0\xi^2(1+C)\left[\cos^2\gamma+\frac{K_4\sin^2\gamma}{K_1(1+C)}\right]^{1/2}\pi\tilde{a}^2$$
$$\times\left\{b-0{,}37+\frac{\varepsilon^2}{16}[0{,}17q^2(q^2+4s^2)+0{,}28s^4-0{,}9q^2(q^2+0{,}64s^2)\right.$$
$$\left.-1{,}1q^3s\cos 2\varphi_a\,\varphi\,]\right\}. \qquad (27)$$

We note that for $\gamma\neq 0$, $\gamma\neq\pi$ the degeneracy with respect to φ_a is lifted. The system possesses a minimum energy if $\varphi_a=0$ or $\varphi_a=\pi$. A consequence of this is the possibility that metastable spiral vortex structures can exist with a nonuniform distribution of $\varphi_a(\tilde{z})$ along the axis. For such a structure, the curves where components Ψ_1 and Ψ_2 are zero take the form of spirals which twist around the z axis. For $\varphi_a=0$ we can use Eq. (27) to find an expression for the threshold value $b^*(C,\gamma)$:

$$b^*(C,\gamma)=0{,}37-\varepsilon^2Q(\gamma),$$
$$Q(\gamma)={}^1/_{16}[0{,}17q^2(q^2+4s^2)+0{,}28s^4+0{,}28s^4+0{,}35q^3s$$
$$-0{,}9q^2(q+0{,}8s)^2]. \qquad (28)$$

For the parameter range $b<b^*(C,\gamma)$ a nonsingular vortex appears. Let us investigate the structure of its core. The zero of the component Ψ_1 below the threshold b^* is displaced along the $\tilde{x}$ axis by a distance

$$\rho\sim Cq(b^*-b)^{1/2}(q+s)/(1+C).$$

The vortex of the component Ψ_2 located at the coordinate origin is displaced in the opposite direction. The spacing of vortices of Ψ_2 for certain angles is shown in Fig. 2. To conclude this section, we note that for $b<b^*$ the angular dependence of the lower critical field can also change. Near threshold this correction to H_{c1} has the form

$$\delta H_{c1}\approx-\frac{\Phi_0[\cos^2\gamma+K_4\sin^2\gamma/K_1(1+C)]^{1/2}}{4\pi\lambda^2\cos(\gamma-\varphi)}0{,}2[b^*(C,\gamma)-b]^2. \qquad (29)$$

The appearance of nonsingular vortices is energetically more favorable for $\gamma=0$; therefore, for small b it is possible to have $H_{c1}(0)<H_{c1}(\pi)$, in contrast to the case of singular vortices.

3. VORTEX STRUCTURE FOR TETRAGONAL SUPERCONDUCTORS

An example of a compound with tetragonal symmetry which exhibits exotic superconductivity is $CeCu_2Si_2$. Recently the existence of nontrivial d-type pairing for high-temperature superconductors has been discussed in the literature.[16-18] In what follows we will investigate several features of the mixed state near H_{c1} for a tetragonal superconductor whose order parameter generates a two-dimensional representation of the group D_4. In this case a number of additional invariants appear in the free-energy functional compared to (1). Let us write out the correction to (1) that includes these invariants:

$$\Delta F=\int[\beta_3(|\eta_x|^4+|\eta_y|^4)+K_5(|p_x\eta_y|^2+|p_y\eta_x|^2)]dV. \qquad (30)$$

The total functional is now invariant only with respect to rotations by $\pi/2$ around the z-axis. Consequently, even when the vortex axis of the structure is oriented along z, only axially nonsymmetric solutions of the corresponding GL equations are possible. Let the axis of the vortex form an angle γ with the tetragonal axis of the crystal, and let the projection of this axis on the xy plane form an angle α with

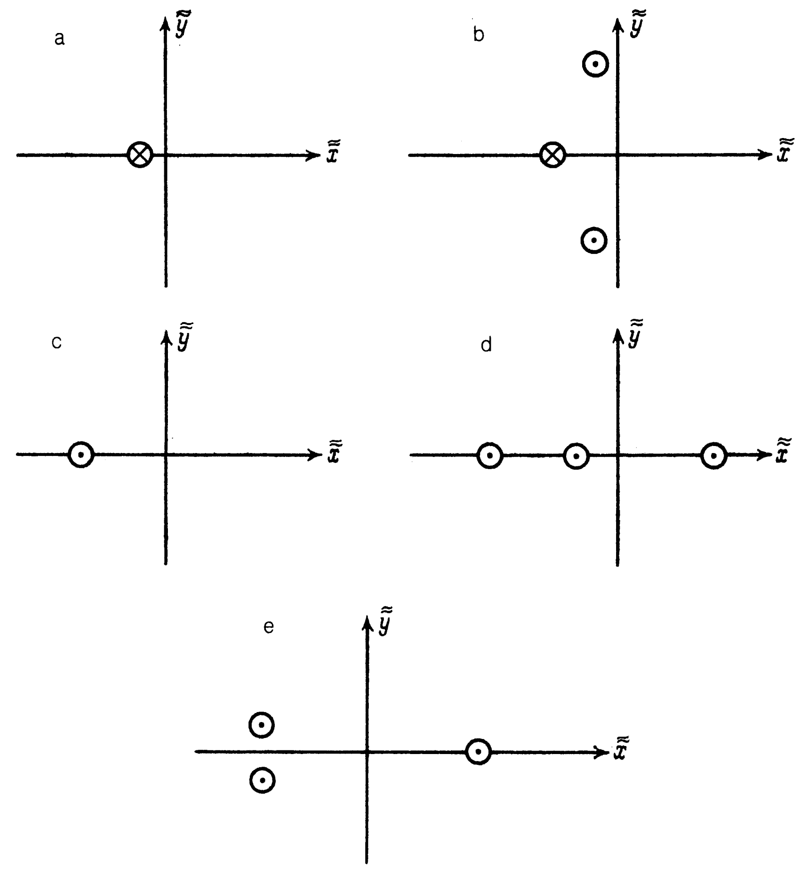

FIG. 2. Schematic positions of component-Ψ_2 vortices in the $\tilde{\tilde{x}}\tilde{\tilde{y}}$ plane for nonsingular vortices in hexagonal superconductors for various values of γ: a—$0<\gamma<\gamma_1$; b—$\gamma_1<\gamma<\gamma_2$; c—$\gamma_2<\gamma<\gamma_3$; d—$\gamma_3<\gamma<\gamma_4$; e—$\gamma_4<\gamma<\pi$. Near the threshold b^* we have $\pi-\gamma_4\sim[K_1(1+C)/K_4]^{1/2}(b^*-b)^{1/6}/\varepsilon^{1/3}$.

the x-axis. We will choose a system of coordinates $\tilde{x},\tilde{y},\tilde{z}$ such that the $\tilde{z}$ axis is directed along the vortex axis, and perform the scale transformation

$$\tilde{x}=\xi(1+C-k/2)^{1/2}\tilde{\tilde{x}},$$
$$\tilde{y}=\xi\left(1+C-\frac{k}{2}\right)^{1/2}\left[\cos^2\gamma+\frac{K_4\sin^2\gamma}{\tilde{K}_1(1+C-k/2)}\right]^{1/2}\tilde{\tilde{y}},\qquad(31)$$

where $\tilde{K}_1=K_1+K_5$, $C=K_2/\tilde{K}_1$, $k=K_5/\tilde{K}_1$, and $\xi^2=\hbar^2\tilde{K}_1/a$. Note that for a tetragonal superconductor the phase with broken time-reversal symmetry in the absence of a magnetic field is favored when $\beta_2>0$ and $\beta_3>-2\beta_2$. In what follows we will limit our considerations to these parameter relations. Taking into account (3) and the condition $\varkappa\gg1$, we can write out the free energy density near the vortex core:

$$\mathcal{F}=\mathcal{F}_0\xi^2\left(1+C-\frac{k}{2}\right)\left[\cos^2\gamma+\frac{K_4\sin^2\gamma}{\tilde{K}_1(1+C-k/2)}\right]^{1/2}$$
$$\times\Big\{-\Psi_1^*a^+a^-\Psi_1$$
$$-\Psi_2^*a^+a^-\Psi_2-\frac{1}{4}\left(\varepsilon_1-\frac{\varepsilon_2}{2}\right)[e^{-2i\alpha}\Psi_1^*(qa^-+sa^+)^2\Psi_2$$
$$+e^{2i\alpha}\Psi_1(qa^++sa^-)^2\Psi_2^*]+\frac{\varepsilon_2}{8}[e^{2i\alpha}\Psi_1^*(qa^++sa^-)^2\Psi_2$$
$$+e^{-2i\alpha}\Psi_1(qa^-+sa^+)^2\Psi_2^*]+\frac{1}{2}(|\Psi_1|^4+|\Psi_2|^4)$$
$$+(1+2b+\tilde{b})|\Psi_1|^2|\Psi_2|^2$$
$$+\frac{\tilde{b}}{2}(\Psi_1^2\Psi_2^{*2}+\Psi_1^{*2}\Psi_2^2)-|\Psi_1|^2-|\Psi_2|^2\Big\},\qquad(32)$$

$$\mathcal{F}_0=\frac{a^2}{2\beta_1+\beta_3},\quad b=\frac{\beta_2}{\beta_1+\beta_3/2},\quad \tilde{b}=\frac{\beta_3}{2\beta_1+\beta_3},$$

$$s=1-\cos\gamma\left[\cos^2\gamma+\frac{K_4\sin^2\gamma}{\tilde{K}_1(1+C-k/2)}\right]^{-1/2},\quad q=2-s,$$

$$a^\pm=\frac{\partial}{\partial\tilde{\tilde{x}}}\pm i\frac{\partial}{\partial\tilde{\tilde{y}}},\quad \varepsilon_1=\frac{C}{1+C-k/2},\quad \varepsilon_2=\frac{k}{2(1+C-k/2)}.$$

From the condition that the gradient terms in the GL functional must be positive definite, it follows that the values of ε_1 and ε_2 cannot exceed unity. We will assume $\varepsilon_1\ll1$, $\varepsilon_2\ll1$, which allows us as before to treat the component Ψ_1 of the order parameter as fixed, and to seek Ψ_2 as a small correction. As in the case of hexagonal symmetry discussed above, for tetragonal superconductors with $\varepsilon_{1,2}\neq0$ a component-Ψ_2 vortex appears in the core of a component-Ψ_1 vortex. Let us first consider singular vortex structures corresponding to solutions of the form (18); in the approximation under consideration here, there exist only the harmonics R_3, R_{-1}, and R_1, as before. However, for $\tilde{b}\neq0$ the equations for R_1 and R_{-3} are found to be coupled. In what follows, we consider the case $\tilde{b}\ll b$, for which the interaction of these harmonics can be neglected. In this case we can obtain the following approximate solution:

$$R_{-1}=sq\frac{1+C}{2C}[(\varepsilon_1-\varepsilon_2)\cos 2\alpha+i\varepsilon_1\sin 2\alpha]R_1^{(1)}(\rho),$$

$$R_1=\frac{1+C}{4C}\left[q^2\left(\varepsilon_1-\frac{\varepsilon_2}{2}\right)e^{2i\alpha}-\frac{\varepsilon_2}{2}s^2e^{-2i\alpha}\right]R_1^{(1)}(\rho),$$

$$R_{-3}=\frac{1+C}{4C}\left[s^2\left(\varepsilon_1-\frac{\varepsilon_2}{2}\right)e^{2i\alpha}-\frac{\varepsilon_2}{2}q^2e^{-2i\alpha}\right]R_{-3}^{(1)}(\rho),\qquad(33)$$

where $R_1^{(1)}$, $R_{-3}^{(1)}$ are determined by expressions (9) and (10). For the orientations $\gamma=0$, $\gamma=\pi$ the component Ψ_2 is symmetric relative to the transformation C_4. In this case a single component-Ψ_2 vortex is located on the z-axis with index $n=1$. Furthermore, four more vortices of Ψ_2 appear within the core with $n=-1$; these vortices intersect the $\tilde{x}\tilde{y}$ plane at the vertices of the square whose center is at the coordinate origin. For $\gamma=0$ the curve along which Ψ_2 vanishes is located at a distance

$$\rho_0\approx 4\left[\frac{|\varepsilon_1-\varepsilon_2/2|(9+8b)}{6|\varepsilon_2|(1+8b)}\right]^{1/2}\qquad(34)$$

from the origin. We note that (34) is valid for $\rho_0<2$. For $(\varepsilon_1-\varepsilon_2/2)/\varepsilon_2>0$ the sides of the square are oriented parallel to the x and y axes, while for $(\varepsilon_1-\varepsilon_2/2)/\varepsilon_2<0$ the diagonals of the square are oriented along these axes. For arbitrary angles α and γ vortices of the Ψ_2 component with index $n=-1$ are found at the vertices of a parallelogram whose center (i.e., the point where its diagonals intersect) is at the origin. Using Eqs. (9), (10), and (34), it is not difficult to obtain the following corrections to the lower critical field due to the core energy:

$$\delta H_{c1}=-\frac{\phi_0}{4\pi\lambda^2}\left[\cos^2\gamma+\frac{K_4\sin^2\gamma}{\tilde{K}_1(1+C-k/2)}\right]^{1/2}\cos^{-1}\vartheta\,\Phi(\gamma,\alpha),$$

$$\Phi(\gamma,\alpha)=\frac{f_0(b)s^2q^2}{4}[(\varepsilon_1-\varepsilon_2)^2\cos^2 2\alpha+\varepsilon_1^2\sin^2 2\alpha)]$$

$$+\frac{f_0(b)}{16}\left|q^2\left(\varepsilon_1-\frac{\varepsilon_2}{2}\right)e^{-2i\alpha}-\frac{\varepsilon_2}{2}s^2e^{2i\alpha}\right|^2$$

$$+\frac{f_\pi(b)}{16}\left|s^2\left(\varepsilon_1-\frac{\varepsilon_2}{2}\right)e^{-2i\alpha}-\frac{\varepsilon_2}{2}q^2e^{2i\alpha}\right|^2.\qquad(35)$$

Here $f_0(b)$ and $f_\pi(b)$ were defined above in (20), ϑ is the angle between the vectors $\mathbf{n}$ and $\mathbf{H}$, and $\mathbf{n}$ is a unit vector along the vortex $\tilde{z}$ axis. We note that for a tetragonal superconductor the energy of the vortex core depends considerably on the orientation of the vortex axis with respect to the crystal axes x and y. In particular this implies that for low fields the vortex axis need not lie in the plane σ formed by the field $\mathbf{H}$ and the z-axis. The angle between σ and the plane formed by the axes $\tilde{z}$ and z equals

$$\delta\approx\frac{\cos(\gamma-\varphi)}{\ln\varkappa\sin\gamma\sin\varphi}\Phi_\alpha'|_{\alpha=\alpha_H},\qquad(36)$$

where α_H is the angle between the x-axis and the projection of the vector $\mathbf{H}$ on the xy plane, and φ is the angle between $\mathbf{H}$ and the z-axis. Since we are calculating H_{c1} and δ to second-order accuracy in the small quantities ε_1 and ε_2, the angles γ and φ in (35) and (36) are connected by the relation

$$\tilde{K}_1(1+C-k/2)\tan\varphi=K_4\tan\gamma,\qquad(37)$$

while the angle ϑ in (35) is roughly equal to $\gamma-\varphi$. According to (35), a characteristic feature of tetragonal superconductors with nontrivial pairing is the periodic dependence of H_{c1} on the angle α between the crystallographic x-axis and the projection of the vortex axis on the xy plane. This period with respect to α is $\pi/2$. Furthermore, as in the hexagonal superconductors, the values of H_{c1} for $\gamma=0$ and $\gamma=\pi$ do not coincide. As we have already noted, this circumstance is associated with lifting of the degeneracy in energy of the superconducting phases $(1,i)$ and $(1,-i)$ in a magnetic field. In the region of small values of b singular vortices become unstable, and the zeros of components Ψ_1 and Ψ_2 shift with respect to one another. The approximate positions of the zeros of Ψ_2 for $\gamma=0$ are shown in Fig. 3. By including the harmonics R_0 and G_0, we can obtain an expression for the instability threshold. As $\varepsilon_{1,2}\to 0$ the region where these nonsingular vortices exist is given by the condition

$$b+\tilde{b}/2<b^*.\qquad(38)$$

Here $b^*=0.37$ if we use the results of the approximate approach presented above; $b^*=0.24$ according to the numerical calculations of Ref. 9. For $\gamma\neq 0$, $\gamma\neq\pi$, metastable spiral vortex structures are possible in tetragonal superconductors just as in hexagonal ones.

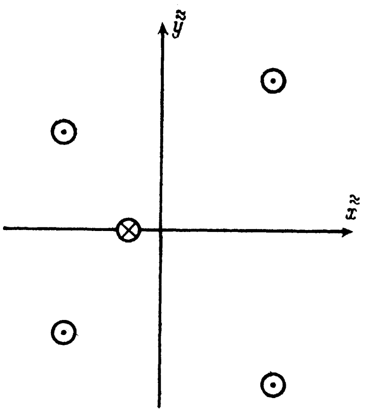

FIG. 3. Position of component-Ψ_2 vortices in the $\tilde{x}\tilde{y}$ plane for nonsingular vortices in tetragonal superconductors for $\gamma=0$ and positive real values of R_0.

CONCLUSION

In this paper we have discussed the structure of vortices in exotic superconducting phases with broken time-reversal invariance. We have considered the case of arbitrary orientation of a magnetic field with respect to the crystal axes and have obtained nonaxisymmetric solutions for an individual vortex. We have also calculated the angular dependences of H_{c1} for hexagonal and tetragonal superconductors. In this case the values of H_{c1} turn out to be different for orientations of the field parallel and antiparallel to the moment $\mathbf{l}$ of the superconducting phase. For arbitrary orientation of the vortex axis we have shown the possible existence of nonsingular single-quantum vortices, and have investigated their structure. As the magnetic field varies, transitions are possible from singular to nonsingular vortex lattices. We note that such transitions can take place both for hexagonal and for tetragonal superconductors for arbitrary orientations of the field relative to the crystal axes. There are strong indications that just this type of transition has been observed in UPt_3.[4-6]

In fields that are not parallel to the anisotropy axes, we have shown that it is possible for metastable spiral vortex structures to exist. The experimental observation of such features of the mixed state would confirm the existence of an exotic order parameter in the superconductors under discussion.

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[1)]The corresponding amplitude $|\Psi_2|$ is very small at such large distances [see (9), (10), (19)]. Therefore, the changes in the vortex structure mentioned here can actually take place only for sufficiently small $\rho \sim 1$. From this point of view the core of a single-quantum vortex under discussion here, as before, is a compact formation with characteristic size ξ.

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