Pinning of proton vortex filaments in superfluid neutron star cores

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We show that proton vortices, which are associated with remnant and generated pulsar magnetic fields, are pinned in superfluid neutron star cores. An asymmetric energy release at the surface of the superfluid core occurs as the result of the motion of the pinned vortices; this leads to the formation of two localized pulsar radio-emission sources. We study the evolution of the pulsar magnetic fields. We find that the magnetic field component at right angles to the rotational axis disappears exponentially with a characteristic time $\tau = 2p/\dot{p}$, where p is the pulsar period.

1. INTRODUCTION

Pulsars are rapidly rotating neutron stars with strong magnetic fields, $B \sim 10^{12}$ G. Their main observational manifestations are the emission of very regular short pulses in the radio band. The problems of the pulsar radio-emission, the dynamics of their rotation, and the evolution of their magnetic fields are of great interest and are at the present time the subject of many studies (see the surveys in Refs. 1 to 4).

A neutron star is essentially a low-temperature object. The characteristic temperatures in many-body theory, such as the Fermi temperature of the neutrons or the interaction energy of the lattice of nuclei are on the order of 10^{10}–10^{12} K (1–100 MeV) which is considerably higher than the temperature of the interiors of neutron stars, $T \sim 10^8$ K.

The measured neutron star masses in binary systems are $M \sim 1.4 M_\odot$. Their radii lie in the range of 8 to 16 km, depending on the equation of state of cold nuclear matter. The outer crust (or *Ae* phase) is a lattice of ionized nuclei, immersed in a degenerate electron liquid. When the density increases inverse β-decay occurs, due to the capture of high-energy electrons by the nuclei:

$$e^- + (A, z) \to \nu + (A, z-1),$$

leading to neutronization of the nuclei. For densities ρ larger than or equal to 4.3×10^{11} g·cm^{-3} (inner crust or *An* phase) free neutrons appear in the system; they pair in $^1S_0$ states forming a superfluid neutron liquid coexisting with the lattice of the neutron-enriched heavy nuclei.

When the density is further increased a first-order phase transition occurs at a density of the order of nuclear matter density, $\rho_{\text{nucl}} \approx 2.8 \times 10^{14}$ g·cm^{-3}: the heavy nuclei dissolve in the neutron liquid and a mixture of quantum liquids is formed in the star, consisting of superfluid neutrons, superconducting protons, and normal relativistic electrons. This system is the main component of a neutron star—its superfluid core or *npe* phase. In the *npe* phase the neutrons and the protons are, respectively, paired in $^3P_2$ and $^1S_0$ states; the values of the energy gaps are of order 0.1 MeV. Since the proton density, which is equal to the electron density, is only a few percent of the density of the system there are no neutron-proton pairs due to the large difference in their Fermi energies.

We note that in the densest regions of the superfluid core ($\rho \gtrsim 10^{15}$ g·cm^{-3}) exotic forms of matter, such as pion and kaon condensates or a quark-gluon plasma, can be formed (see Ref. 5). According to the standard model of stellar structure, the superfluid core for a star with a mass of the order of $1.4 M_\odot$ has a radius on the order of 10 km, and the thickness of the crust is on the order of 1 km.

It was first noted in Ref. 6 that as a result of the rotation of the crust, even for negligibly small angular velocities a dense lattice of quantum vortices appears in the superfluid neutron liquid. The neutron vortices are arranged parallel to the rotational axis and form a triangular lattice with a distance between the vortices equal to $a = 1.075(\varkappa/2\Omega)^{1/2}$, where $\varkappa = \hbar/2m_n$ is the circulation quantum, m_n the neutron mass, and Ω the pulsar rotational velocity. A stationary rotation of the star corresponds to a uniform distribution of neutron vortices with a density equal to $n_n = 2\Omega/\varkappa \approx 6.3 \times 10^3 p^{-1}$ cm^{-2} where $p = 2\pi/\Omega$ is the pulsar period.

Together with the system of neutron vortex filaments there exist in the superfluid core of a neutron star two classes of proton vortices: vortices associated with the remnant pulsar magnetic field,[7] and vortices caused by the pulsar generated magnetic fields.[8,9] In general, the first make an arbitrary angle with the rotational axis, while the second are parallel to it. The pulsar magnetic field in the superfluid core of a neutron star thus has a differentiated nonuniform structure due to the existence of independent systems of proton vortices. Outside the superfluid core, even at distances of the order of the penetration depth of the magnetic field, $\lambda \sim 10^{-11}$ cm, the magnetic field is a superposition of the generated and the remnant magnetic fields. As a result, to a first approximation the magnetic field in the crust and the pulsar magnetosphere has a dipole structure and a direction which, in general, is not the same as that of the rotational axis.

The possibility of pinning (sticking) of the proton vortices associated with the remnant magnetic field to the neutron vortices was considered first of all in Ref. 10. If a generated magnetic field is present the pinning of the proton-proton vortex filaments may turn out to be important. The aim of the present paper is a study of the pinning of the two classes of vortices and a consideration of some astrophysical consequences following from this.

2. PROTON VORTEX DISTRIBUTION AND PINNING

When a supernova with a typical "frozen-in" surface magnetic field of 100 G collapses adiabatically the magnetic flux is conserved and as a result the neutron star produced in the supernova outburst acquires a magnetic field B_0 of about

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10^{12} G.[11] When the star cools down the neutron and proton liquids make a transition to the superfluid state at critical temperatures of $T_{cn} \approx 10^{10}$ K and $T_{cp} \approx 5 \times 10^9$ K, respectively.

The time needed to eject the magnetic flux from the core of the star until a superfluid proton condensate is formed in it is determined by its conductivity in the normal state. According to the estimates of Ref. 7 the high conductivity, $\sigma \sim 10^{29}$ s^{-1}, which is caused by relativistic electrons, gives rise to a characteristic flux ejection time on the order of 10^8 years. This is much longer than the time t of about 10^3 years needed for the star to cool down to the critical temperature T_{cp} of the superfluid transition. The flux of the remnant magnetic field in the core of a neutron star is thus conserved up to the transition of the protons into a superfluid state.

Since the proton condensate is a superconductor of the second kind (we have $\lambda / \xi_p \sim 10 > 2^{-1/2}$, where λ is the penetration depth of the magnetic field and ξ_p the coherence length of the superconducting proton liquid), the magnetic field is localized in regions with a radius equal to $\lambda \sim 10^{-11}$ cm: in the superfluid core there appears a uniform lattice of proton vortices, each of which carries a magnetic flux quantum $\Phi_0 = 2.07 \times 10^{-7}$ G·cm^2. The proton vortices form a triangular two-dimensional lattice with a distance between the vortices equal to $d_0 \approx 4.9 \times 10^{-10} B_0^{-1/2}$ cm. In general, one may expect that the magnetic moment $\mathbf{M}_R$ of the remnant magnetic field is at an arbitrary angle to the rotational axis of the neutron star (i.e., to $\mathbf{\Omega}$). However, an analysis of the stable configuration of the remnant magnetic field[12] shows that the magnetic moment $\mathbf{M}_R$ is oriented at an angle to $\mathbf{\Omega}$ which is close to 90°. In what follows we shall assume for the sake of clarity that $\mathbf{M}_R$ is perpendicular to $\mathbf{\Omega}$. The generalization of our results to the case of an arbitrary angle is obvious.

The entrainment of the protons by the neutrons generated in the superfluid core, as well as the remnant magnetic field, a strong inhomogeneous field with a magnetic moment $\mathbf{M}_G$, which is parallel to $\mathbf{\Omega}$, is generated.[13] It is zero in most ($\geqslant 80\%$) of the volume of the superfluid core, while near the cores of the neutron vortices it is on the order of 10^{14} to 10^{15} G. The magnetic field generation by the drag forces leads to the formation of a dense network of proton vortices localized within a radius

$$r_1 = b(\xi_p/\lambda)^{1/(3|k|)} \approx 1{,}45 \cdot 10^{-3} p^{1/2} \text{ cm} \tag{1}$$

around each neutron vortex. Here $|k| \approx 0.5$ is the coefficient for the drag of protons by neutrons, b the radius of the neutron vortex, and p the rotational period of the pulsar. If we assume that the average magnetic field induction around a neutron vortex is[8] $\bar{B} = k\Phi_0/4\pi\lambda^2$ the average density of proton vortices associated with each neutron vortex is equal to $\bar{n}_{GP} = \bar{B}/\Phi_0 \approx 4 \times 10^{20}$ cm^{-2}.

In the superfluid core of a neutron star there thus exist two classes of proton vortices which are perpendicular to one another: the vortices associated with the remnant magnetic field (referred to below as the *RP* vortices) distributed uniformly in the whole volume of the stellar core and the vortices caused by magnetic field generation (referred to below as the *GP* vortices) which are localized as clusters around the neutron vortices. Outside the superfluid core the magnetic field is a dipole field and is a superposition of the generated and the remnant magnetic fields.

If the vortices intersect they may get fixed to one another (pinning), since in that case the energy of the system consisting of two vortices is decreased due to the gain in energy by the condensation of the intersected volume. In the case of neutron-proton vortices the pinning energy for a single combination can be estimated to be[4]

$$\varepsilon_{PIN}^{(np)} \approx \frac{\Delta_p^2}{E_{F_p}^2} \frac{\Delta_n^2}{E_{F_n}} (\xi_n^2 \xi_p) n \lesssim 0{,}1 \text{ MeV}, \tag{2}$$

where Δ, E_F, and ξ denote, respectively, the energy gap, the Fermi energy, and the coherence length of the superfluid liquid, the indices p and n refer to the proton and the neutron condensates, respectively, and n is the density of the system.

Since the neutron vortices are surrounded by a dense network of *GP* vortices the most effective pinning in the superfluid core of a neutron star is that of *GP* and *RP* vortices. In that case we have

$$\varepsilon_{PIN}^{(pp)} \approx \varepsilon_c(0) \xi_p^3 = \frac{3}{8} \frac{\Delta_p^2}{E_{F_p}} \xi_p^3 n. \tag{3}$$

Here $\varepsilon_c(0)$ is the condensation energy density at $T = 0$.

In Fig. 1 we show the pinning energy (3) as function of the density of the superfluid core. The values of the energy gap Δ_p of the protons which are paired in $^1S_0$ states are taken from Ref. 14. The proton-proton vortex filament pinning energy in the bulk of the superfluid core is thus higher than the corresponding energy for the neutron-proton vortex lines. Using an order-of-magnitude estimate for the pinning energy ($\varepsilon_{PIN}^{(pp)} \sim 1$ MeV) we can estimate the pinning force acting on a single combination:

$$F \approx \varepsilon/\xi_p \sim 10^6 \text{ dyn} .$$

Since the number of pinning centers per unit *RP* vortex is on the order of 10^9, a huge force, $f_{PIN} \sim 10^9$ dyn, acts on a unit vortex length. Hence, the *RP* vortices are tightly bound to the system of the *GP* neutron vortices.

We note also that the pinning does not change the magnetic energy of a system of two intersecting vortices. The magnetic field in the vicinity of a pinning center in a volume equal to about λ^3 is a superposition of the fields of the intersecting vortices, and in that region the circulation current undergoes restructuring to agree with the resulting field. Since the intervortex distance d_0 is approximately 10λ the

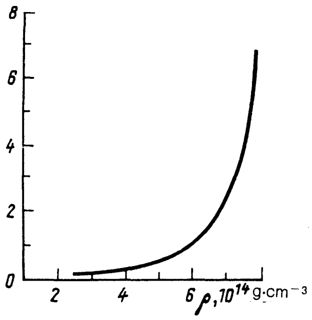

FIG. 1. Proton vortex pinning energy as function of the density of the superfluid core of the neutron star.

field in the bulk of the superfluid core has a differentiated structure and changes its direction only in a small volume ($\lambda^3 \ll d_0^3$) at the sites where the vortices intersect so as to agree with the resulting field.

Apart from the mutual pinning of vortices pinning of proton vortices to inhomogeneities at the phase transition surface between the superfluid core and the inner crust of the neutron star also occurs. This leads to some deflection of the vortices when they move. It is difficult to determine the magnitude of the pinning or the glide coefficient of a vortex, since the structure of the transition layer has hardly been studied. However, taking account of the huge energy of tension per unit length of the vortex,

$$T_p \sim \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 \ln\frac{\lambda}{\xi_p} \sim 10^3 \text{ dyn}, \tag{4}$$

in a first approximation we can take the vortex to have a rectilinear structure with a small deflection at the phase transition surface to enable it to leave at right angles to the spherical surface.[3]

In what follows we shall consider the motion of vortices averaged over large time scales t of more than about 10 years. This makes it possible to completely neglect the effects of jumps in the pulsar rotational velocities and the generation of temporal noise in the arrival of the radio-pulses, connected with the pinning of the vortices at the boundary of the superfluid core, or the pinning of neutron vortices to the lattice of nuclei in the inner crust of the neutron star.

3. ASYMMETRIC ENERGY RELEASE IN THE SUPERFLUID CORE OF A NEUTRON STAR

In its main features the dynamics of pulsar rotation is similar to the rotation of superfluid helium II in laboratory experiments.[15] Any change in the rotational velocity of the star is accompanied by the radial motion of the neutron vortex structure: inwards in the case of acceleration and outwards in the case of deceleration. Denoting the neutron vortex density by $n(r,t)$ and their radial velocity by $\mathbf{v}_r(r,t)$, we can write the equation of continuity for the vortex filament density in the form

$$\frac{\partial n(r,t)}{\partial t} + \operatorname{div}[n(r,t)\mathbf{v}_r(r,t)] = 0, \tag{5}$$

where r is the distance between the vortex and the rotational axis $\mathbf{\Omega}$ in the plane perpendicular to $\mathbf{\Omega}$. In the superfluid core the relaxation time between the superfluid and the normal components is of the order of 10^{-14} s (Ref. 16), i.e., the superfluid liquid rotates and the normal component practically rigidly.

Ignoring the occurrence of sudden jumps in the rotational velocities of some pulsars, which is justified when we consider the dynamics of a pulsar on time scales $t \gtrsim 10$ year, we neglect the r dependence of n and Ω in what follows. In that case, substituting the Feynman relation $n(t) = 2\Omega/\varkappa$ into Eq. (5) we find for the radial velocity of a neutron vortex

$$v_r(r,t) = -\frac{\dot{\Omega}}{2\Omega} r. \tag{6}$$

According to observations the rotational velocity of pulsars decreases; hence this process is accompanied by the motion of neutron vortices to the boundary between the superfluid core and the neutron star crust with a velocity v_r given by Eq. (6). The cluster of *GP* vortices which is tightly fixed to the neutron vortex will also move with velocity v_r. The length of the *GP* vortices in the superfluid core will then decrease and the energy contained in these vortices will be released at the boundary between the core and the crust. Since the *RP* vortices are pinned to the *GP* vortices they will also take part in the motion and the energy release. Since the *RP* vortices are perpendicular to the *GP* vortices, the energy release will be asymmetric: at the surface of the superfluid core there occur two localized spots with an enhanced energy release intensity. A calculation of the intensity I_1 of the energy release connected with the motion of the *GP* vortices was given in Ref. 17. Noting that the energy per unit length of a proton vortex is equal to

$$\varepsilon_V = (\Phi_0/4\pi\lambda)^2 \ln \lambda/\xi_p,$$

we find

$$dI_1 = \frac{|k|}{4\pi}\left(\frac{\xi_p}{\lambda}\right)^{2/(3|k|)} \varepsilon_V \frac{|\dot{\Omega}|}{\Omega} R^3 \sin^3\vartheta \, d\vartheta \, d\varphi, \tag{7}$$

where the angles ϑ and φ are defined in Fig. 2 and where R is the radius of the superfluid core.

Turning to a calculation of the energy release I_2 caused by the motion of the *RP* vortices we consider a surface parallel to the equatorial plane determined by choosing the angle ϑ. This is the vicinity in which the *RP* vortices are arranged parallel to one another while the *GP* vortices intersect them at right angles forming a hexagonal lattice (see Fig. 2). The velocity of the *RP* vortices is equal to

$$v_x = v_r \cos\varphi = \frac{|\dot{\Omega}|}{2\Omega} h, \tag{8}$$

where h is the distance of the vortex to the rotational axis of the star (Fig. 3). It is clear that although the *GP* vortices move radially with a velocity v_r, nonetheless each pinned *RP* vortex moves to the boundary of the superfluid core remain-

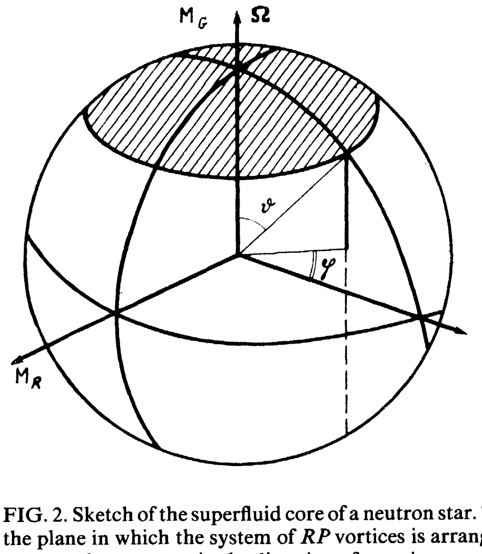

FIG. 2. Sketch of the superfluid core of a neutron star. The angle ϑ defines the plane in which the system of *RP* vortices is arranged. The maximum energy release occurs in the direction of an axis perpendicular to $\mathbf{M}_R$ and $\mathbf{\Omega}$.

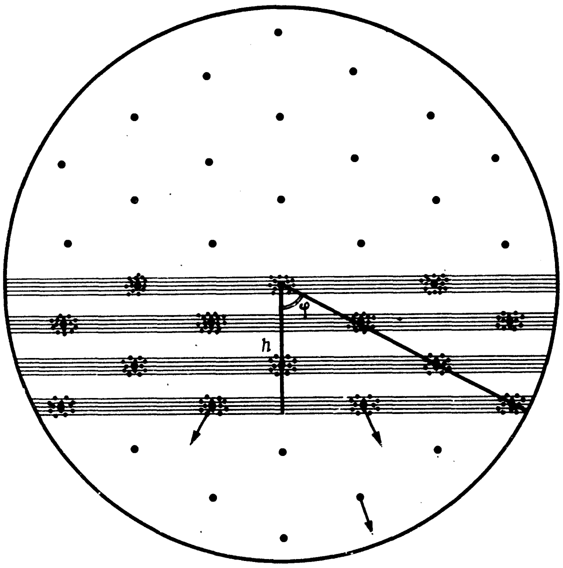

FIG. 3. Sketch of the intersection of the superfluid core of a neutron star by a plane perpendicular to the rotational axis. The points indicate neutron vortices and also the *GP* vortices, which are perpendicular to the plane of the figure, grouped around them. The arrows indicate the direction in which the neutron vortices and the *GP* vortices move. We also show in the figure the *RP* vortices which are pinned to the *GP* vortices; the *RP* vortices remain parallel to themselves and move in a direction perpendicular to the segment *h*.

ing parallel to itself (see Fig. 3). In other words, one can state that the radial motion of the *GP* vortices does not give rise to the appearance of a force which aims to change the initial rectilinear shape of the *RP* vortex. We note that if the initial *RP* vortex distribution is uniform, after a short time a characteristic structure is established which has been sketched in Fig. 3.

If we denote the length of the *i*th *RP* vortex by l_i and the radius of the neighborhood defined by the angle ϑ by R_ϑ, and if we take into account that we have $l_i = 2R_\vartheta \sin\varphi$ and $d\varphi/dt = (|\dot\Omega|/2\Omega)\cot\varphi$, we have

$$\frac{dl_i}{dt} = \frac{|\dot\Omega|}{\Omega} R_\vartheta \cot\varphi \cos\varphi . \tag{9}$$

The energy release intensity due to the motion of the *RP* vortices in the neighborhood chosen by us will be equal to

$$\frac{dE_\varphi}{dt} = \varepsilon_V \frac{a}{d_0}\frac{dl_i}{dt} dN_\varphi, \tag{10}$$

and one notes easily that the number of vortices in an azimuthal angle range $d\varphi$ is

$$dN_\varphi = R_\vartheta \sin\varphi d\varphi / a.$$

Hence,

$$\frac{dE_\varphi}{dt} = \frac{\varepsilon_V}{d_0}\frac{|\dot\Omega|}{\Omega} R_\vartheta^2 \sin^2\vartheta \cos^2\varphi \, d\varphi. \tag{11}$$

When we sum over ϑ we note that the number of surfaces lying within a range $d\vartheta$ is

$$dN_\vartheta = R \sin\vartheta d\vartheta / d_0.$$

The total energy release intensity connected with the motion of the *RP* vortices is equal to

$$dI_2 = \frac{dE_2}{dt} dV = \frac{3^{1/2}}{2}\frac{B_0}{\Phi_0}\varepsilon_V \frac{|\dot\Omega|}{\Omega} R^3 \sin^3\vartheta \cos^2\varphi d\vartheta \, d\varphi , \tag{12}$$

where we have used the relation $d_0^2 = (2/3^{1/2})(\Phi_0/B_0)$. The total energy release intensity in the superfluid core of the neutron star is equal to

$$dI = dI_1 + dI_2 = \left[\frac{|k|}{4\pi\lambda^2}\left(\frac{\xi_p}{\lambda}\right)^{2/(3|k|)} + \frac{3^{1/2}}{2}\frac{B_0}{\Phi_0}\cos^2\varphi\right] \times \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 \left(\ln\frac{\lambda}{\xi_p}\right)\frac{|\dot\Omega|}{\Omega} R^3 \sin^3\vartheta \, d\vartheta \, d\varphi. \tag{13}$$

Sustituting typical values of the parameters, $\xi_p \approx 10^{-12}$ cm, $\lambda \approx 10^{-11}$ cm, $|k| \approx 0.5$, and $B_0 \approx 4 \times 10^{12}$ G, we have

$$dI = 1{,}2\cdot 10^{29}\frac{\dot p_{-15}}{p} R_6^3 (1+\cos^2\varphi)\sin^3\vartheta \, d\vartheta \, d\varphi, \tag{14}$$

where $\dot p_{-15} = 10^{15}\dot p$ and $R_6 = 10^{-6}R$. It follows from Eq. (14) that the intensity is a maximum in two localized regions, viz., where we have $\varphi \to 0$ and $\vartheta \to \pi/2$. For $\varphi = 0$ and $\vartheta = \pi/2$ we find the estimate

$$I_{max} \approx 2\cdot 10^{29}\frac{\dot p_{-15}}{p} R_6^3 \ \mathrm{erg\cdot s^{-1}} \tag{15}$$

for the maximum energy release intensity.

Thus, two active regions with an enhanced energy release intensity are formed because of the pinning of the proton vortices at the surface of the superfluid core of the neutron star. The electromagnetic part of the energy release must, according to Ref. 18, serve as a source for the pulsar radio emission. When the proton vortices move, a strongly inhomogeneous field in the shape of a vortex network appears in the normal crust of the neutron star, where the conductivity of the charged component, $\sigma_c \sim 10^{22}$ to 10^{23} s^{-1}, is much smaller than the conductivity of the superfluid core. Since the transverse magnetic field gradients are large, the magnetic field diffuses rapidly in the transverse direction. Characteristic frequencies for the decay of the proton vortex filaments are $\omega \sim 10^8$ s^{-1}. A region is thus formed for the generation of radio-frequency electromagnetic waves at the core-crust boundary of the neutron star. These waves can propagate in the crust of the neutron star in the form of Alfvén and magnetosonic waves and, according to Ref. 18, enter the circumstellar space without significant losses.

It follows from what we have said that due to the pinning of the proton vortices active radio-emission regions are formed at the surface of the superfluid core of the neutron star in the equatorial region, which can lead to the pulsed radio-emission of the pulsar when the star rotates.

4. EVOLUTION OF THE PULSAR MAGNETIC FIELDS

We noticed above that in the superfluid core of a neutron star a dipole magnetic field is generated with a magnetic moment $\mathbf{M}_G$ of order 10^{30} G cm^3, which is parallel to the rotational axis. The average magnetic induction of this field is constant in time since it depends only on the microscopic parameters of the superfluid proton and neutron condensates. Indeed, although the radius b of the neutron vortices increases when the pulsar's rotational velocity diminishes, the ratio r_1/b, on which the induction of the generated field depends, stays constant. Hence, the volume occupied by the GP vortices in the superfluid core also remains unchanged with time.

On the other hand, the remnant magnetic field will fade away because of the ejection of the pinned RP vortices from the superfluid core into the neutron star crust. As a result the resulting pulsar field which is a superposition of the generated and the remnant fields, and also the angle α between the direction of this field and the rotational axis, will change. In the limit $\mathbf{M}_R \ll \mathbf{M}_G$, we find that $\alpha \to 0$.

We evaluate the characteristic time for the RP vortices to leave the superfluid core. Since the RP vortices are effectively coupled to the neutron vortices through the GP vortices, the characteristic time for their ejection is completely controlled by the motion of the neutron vortices, i.e., by the dynamics of the neutron star rotation. Solving the equation of continuity (5) for the neutron vortex density together with (6) we find

$$n(t)=n(0)\exp\left(-\frac{t}{\tau_1}\right), \tag{16}$$

where $n(t)$ and $n(0)$ are the neutron vortex densities at time t and at $t=0$, respectively, and $\tau_1 = \Omega/|\dot{\Omega}|$. Denoting the neutron vortex radii at times $t=0$ and t by $b(0)$ and $b(t)$ we get for the RP vortex density at time t

$$N(t)=\frac{2}{3^{1/2}d_0^2}\frac{b(0)}{b(t)}. \tag{17}$$

The magnitude of the remnant magnetic field at time t is then equal to

$$B(t)=\Phi_0 N(t)=B(0)\left[\frac{n(t)}{n(0)}\right]^{1/2}, \tag{18}$$

where $B(0) = 2\Phi_0/3^{1/2}d_0^2$ is the value of $B(t)$ at time t. Using (16) we finally find

$$B(t)=B(0)\exp\left(-\frac{t}{\tau_D}\right), \tag{19}$$

where $\tau_D = 2p/\dot{p}$ (p is the pulsar period).

The remnant magnetic field in the superfluid core of a neutron star thus decreases exponentially with a characteristic time $\tau_D \sim 10^6$–10^8 years. In the neutron-star crust, where the conductivity of the normal matter is much lower than the conductivity of the superfluid core, the characteristic fading time of a proton vortex is

$$\tau_c \approx \frac{4\sigma_c L^2}{\pi c^2} \sim 10^6 \text{ yr}, \tag{20}$$

where L is the crust thickness and σ_c its conductivity. For the main pulsar population we thus have $\tau_D \gtrsim \tau_c$ and the evolution of the magnetic field component perpendicular to the rotational axis is determined by the temporal behavior of the field in the superfluid core of the pulsar which is described by the relation (19). It is very important that the characteristic fading time of the remnant magnetic field is determined by the observable quantities p and $\dot{p}$ and is of the order of magnitude of the characteristic pulsar life time, $\tau_1 \sim p/\dot{p}$.

In conclusion we note that a statistical analysis of the observed distribution of the pulsars in period and in its derivative in a framework of a mechanism for the braking of the star by magnetic dipole radiation at the rotational frequency shows that the emitted magnetic field component decreases exponentially with a characteristic time of 5×10^6 yr.[19] This result is in good agreement with Eq. (19). Further analysis of the evolutionary tracks of pulsars, taking into account the evolution (19) of the remnant magnetic field, can give new information about the energy loss processes and the braking of pulsars.

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