Dynamics of a plasma compressed in a magnetic field by an exploding liner

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The dynamics of the magnetic "cumulation" initiated by an exploding liner is studied numerically. A simple "force" condition is proposed for estimating the extent of the magnetic field compression. The predictions of this estimate agree satisfactorily with the results of the numerical calculations. The particular compression regimes considered, with an incomplete transfer of energy to the magnetic field and with prolonged confinement of the buffer plasma, may occur in experiments on magnetic cumulation by means of plasma liners if an appreciable fraction of the initial liner energy is thermal.

INTRODUCTION

One possibility for producing ultrastrong pulsed magnetic fields is to compress a plasma with a magnetic field by means of a comparatively cool, dense shell (liner) which is accelerated to a high velocity by laser beams, particle beams, intense currents, or otherwise (Refs. 1 and 2, for example). Numerical calculations[3] on magnetic "cumulation" by means of a thin cylindrical liner have shown that, in agreement with estimates, the kinetic energy of the liner has converted completely into the magnetic-field energy of the buffer plasma by the time of cumulation, and the effects of the compressibility of the liner itself are negligible.

That is not always the case. For example, when a short but powerful laser pulse is applied to a liner, or when it causes breakdown of a gas, a large fraction of the energy which is introduced may be expended on the internal energy of the liner, rather than on its kinetic energy. In the course of an explosive process of this sort, the cumulation of the buffer plasma is accompanied by the simultaneous dispersal of the exploding liner, with the result that the efficiency of the energy transfer from the liner to the magnetic field is sharply reduced. We will present some estimates of the plasma parameter values which could be reached under such conditions. We will also report numerical calculations on the corresponding time-dependent one-fluid $1D$ plane MHD problem. These results support our estimates.

1. STATEMENT OF THE PROBLEM AND ESTIMATE OF THE MAXIMUM DEGREE OF COMPRESSION

A buffer plasma with a density ρ_{b0}, at rest, occupies the half-space $x \geqslant 0$. A heated, immobile, plane plasma slab of thickness $h_0 \ll l_0$ and density $\rho_0 \gg \rho_{b0}$, with a total energy per unit area E_0, is produced in this buffer plasma at the initial time, at a distance l_0 from an adiabatic wall (or symmetry plane). The initial magnetic field H_0 in the plasma is uniform and is oriented parallel to the slab. The initial pressure and the specific internal energy of the buffer plasma are assumed to be negligible. For simplicity we discuss the case in which the effective adiabatic indices of the buffer plasma and of the plasma slab are both equal to γ.

We focus on two critical parameters of the problem: the dimensionless energy and the dimensionless mass of the expanding plasma slab (the liner):

$$\varepsilon = E_0/(H_0^2 l_0/8\pi) \gg 1, \quad \mu = \rho_0 h_0/(\rho_{b0} l_0) \gg 1. \tag{1}$$

To estimate the maximum degree of compression of the buffer plasma,

$$\eta_{max} \approx \rho_{b\,max}/\rho_{b0} \approx l_0/l_{min},$$

where $\rho_{b\,\max}$ is the maximum average density, and $l_{\min}$ is the minimum dimension of the plug of compressed buffer plasma, we use a "force" condition: The characteristic pressure in the liner is equal to the characteristic pressure of the buffer plasma, $p_L = p_b$. As p_L we adopt the pressure behind the front of the shock wave which would propagate through the liner if the latter were reflected from a rigid wall in the absence of a magnetic field:

$$p_L = \frac{\gamma+1}{2}\frac{E_0}{l_0}. \tag{2}$$

The total pressure in the buffer plasma is the sum of the magnetic pressure and the plasma pressure: $p_b = p_M + p_G$. If magnetic flux is conserved (if the magnetic field is frozen in the plasma), we can write

$$p_M = H_0^2 \eta_{max}^2/8\pi. \tag{3}$$

The expanding plasma liner generates a shock wave in the buffer plasma (under our assumptions, this is a strong shock wave). This wave is subsequently damped, undergoing a series of reflections from the symmetry plane and the liner boundary (a plasma "piston"). It degenerates into an acoustic perturbation. Ignoring the increase in the entropy in the series of weakening reflected shock waves, and assuming that the compression of the buffer plasma from the state behind the front of the first shock wave reflected from the symmetry plane to the state with the maximum compression is an adiabatic process, we find

$$p_G = C(\gamma)\frac{E_0\rho_{b0}}{h_0\rho_0}\eta_{max}^{\gamma}, \tag{4}$$

where

$$C(\gamma) = \frac{(\gamma+1)(3\gamma-1)}{\gamma-1}\left[\frac{(\gamma-1)^2}{\gamma(\gamma+1)}\right]^{\gamma}$$

is a constant. Using (1)–(4), we find the following dimensionless condition for estimating the maximum degree of compression, $\eta_{\max}$:

$${}^1/_2(\gamma+1)\varepsilon = \eta_{max}^2 + C(\gamma) M_A^2 \eta_{max}^{\gamma}. \tag{5}$$

Here $M_A = U_0/c_{A0} = (\varepsilon/\mu)^{1/2}$ is the characteristic Alfvén-

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Mach number of the plasma liner, U_0 is the characteristic liner velocity in the collapse stage, and $c_{A0} = H_0/(4\pi\rho_{b0})^{1/2}$ is the characteristic Alfvén velocity.

According to (5), the magnetic pressure exceeds the plasma pressure at a degree of compression

$$\eta_{max} > C(\gamma)^{1/(2-\gamma)} M_A^{2/(2-\gamma)}$$

(for $\gamma = \frac{5}{3}$ we would have $\eta_{max} > 4.1 \cdot 10^{-2} M_A^6$). Since we have $\eta_{max} \gg 1$ in cases of practical interest, it follows that in sub-Alfvénic compression regimes ($M_A < 1$) a magnetic mechanism operates to slow the liner. The maximum degree of compression,

$$\eta_{max} \approx [(\gamma+1)/2]^{1/2} \varepsilon^{1/2},$$

does not depend on the liner mass. For plasma-liner velocities above the Alfvén velocity, the gasdynamic slowing mechanism should be taken into account up to high degrees of compression of the buffer plasma (for purely gasdynamic slowing we would have

$$\eta_{max} \approx \left[\frac{(\gamma+1)}{2C(\gamma)} \mu\right]^{1/\gamma},$$

and this result would be independent of the liner energy).

At values $M_A \approx 1$, characteristic of the compression conditions analyzed in Ref. 3, the liner is slowed only by the magnetic field. The maximum degree of compression of the buffer plasma is $\eta_{max} \sim \varepsilon^{1/2}$ according to our force estimate (5), while an "energy" estimate[1,3] for an incompressible liner puts the maximum degree of compression at $\eta_{max} \sim \varepsilon$; i.e., the compression is considerably more effective. The reason for the discrepancy between these estimates is that in the problem under consideration here even an initially thin heated liner will expand during the ejection, and by the time of maximum compression of the buffer plasma this liner will become quite "thick." As a result, a significant fraction of its total energy cannot be transferred to the magnetic field; it is instead expended on a "self-heating" of the liner. Under these conditions, the energy estimate cannot be used. In general, the fraction of the total liner energy converted into energy of the buffer gas and of the magnetic field in it increases as the ratio of the liner thickness to the thickness of the buffer plasma at the time of its maximum compression decreases.

An optimum compression regime was discussed in Ref. 3. In that regime, the buffer plasma was compressed by the "piston" over most of the cumulation time, and the "explosion" of the plasma slab could occur only in the last stage, before the compression came to a stop (a similar situation may arise in experiments on magnetic cumulation by means of the plasma liners of Z-pinches; Refs. 4–6, for example). We will accordingly not make a direct quantitative comparison of these results with the calculations and estimates of Ref. 3.

2. RESULTS OF THE NUMERICAL CALCULATIONS

In this formulation of the problem, with the standard boundary conditions

$$u(0,t) = \frac{\partial H}{\partial x}(0,t) = 0,$$

we solved the system of magnetogasdynamic equations numerically by a completely conservative, implicit Lagrangian difference scheme like that described in Ref. 7. Effects stemming from magnetic-field diffusion were taken into account. We used the equation of state of an ideal gas with $\gamma = 5/3$ for the buffer plasma and for the liner plasma.

We assumed that the plasma conductivity σ remained constant. The actual conductivity depends on the temperature T, but our calculations and estimates show that in cases of practical interest the stage of buffer-plasma compression should arise under conditions similar to those for a frozen-in magnetic field, with a low loss of magnetic flux and an insignificant temperature dependence $\sigma(T)$. The simplifying assumption $\sigma =$ const is not of fundamental importance in the difference scheme which we used. It does lead to a universal result, which is not tied to specific physical characteristics of the plasma. In the calculations, we modeled the relative influence of effects stemming from magnetic-field diffusion in the plasma expansion stage by varying the value of σ. In a numerical analysis of specific magnetic-cumulation systems, this simplification can easily be withdrawn.

In the calculations, all quantities were put in dimensionless form through the use of characteristic values of the principal parameters, l_0, ρ_{b0}, and H_0:

$$u_0 = \left(\frac{H_0^2}{\rho_{b0}}\right)^{1/2}, \quad t_0 = \frac{l_0}{u_0}, \quad p_0 = H_0^2, \quad \sigma_0 = \frac{c^2}{u_0 l_0},$$

where c is the velocity of light. The dimensionless quantities are used, without any additional notation, in all the figures presented below.

Let us examine some results of the calculation version with $\varepsilon = 2.5 \cdot 10^3$, $\mu = 2.5 \cdot 10^2$ (i.e., $M_A = 10^{1/2}$), $\sigma = 10^3$, and an initial liner thickness $h_0 = 0.1$.

Figure 1 shows the time (t) evolution of the coordinate of the discontinuity (the liner boundary), the position of the shock wave (at the beginning of the fast MHD shock wave) which propagates through the buffer plasma and which undergoes a series of successive reflections from the wall (the symmetry plane) and the contact discontinuity, and the position of the most intense shock wave that goes into the interior of the liner after the "collapse." The velocity of the plasma piston (the liner boundary) reaches $U \sim 1.3$ early in the process and then remains essentially constant until the times of maximum compression. In accordance with the initial

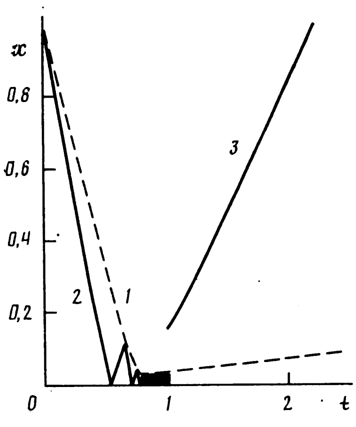

FIG. 1. 1—Trajectory of the discontinuity; 2—trajectory of the shock wave in the buffer plasma; 3—trajectory of the shock wave in the liner $\varepsilon = 2.5 \cdot 10^3$, $\mu = 2.5 \cdot 10^2$, $\sigma = 10^3$.

data of the problem, the liner also expands in the direction opposite the wall, but the trajectories of the corresponding waves are not shown in Fig. 1. The calculated Alfvén-Mach number, $M_A = (4\pi)^{1/2}U \sim 4.6$, is slightly above the corresponding parameter which was used in the estimate (5), $M_A \sim 3.2$. The latter value was determined from the average expansion velocity U_0, since an approximately linear velocity profile is established in the liner plasma during the nearly inertial stage of the collapse, and the relation $U > U_0$ holds.

After the cumulation, the velocity of the opposite expansion of the liner boundary (the expansion velocity of the buffer plasma) decreases sharply, by a factor of more than 30 in this particular version of the calculations. In the calculations of Ref. 3, in contrast, where no effects of a liner compressibility were manifested, the compression velocity and the expansion velocity of the buffer plasma were essentially the same. The reason why the process exhibits this dynamics is that the cumulation of the buffer plasma results from slowing of only the outer (boundary) layers of the liner in the case of a strong explosive expansion of the liner. After cumulation, most of the mass of the liner plasma continues to move by inertia toward the wall, retarding the expansion of the buffer plasma.

This effect is characteristic of the case of a thick plasma liner, with which we are dealing here. This effect is seen at any value of M_A; i.e., it does not depend on which mechanism (the magnetic mechanism or the gasdynamic mechanism) dominates the slowing. This effect means that the basic parameter values are maintained in the buffer plasma for a time longer than in the case of a thin, incompressible liner. These main parameters are the density, the pressure, and the magnetic field; they are maintained at a level close to their maximum values (if the diffusion of the magnetic field out of the buffer plasma is only slight).

The typical average value of the maximum degree of compression of the buffer plasma in this version of the calculations is $\eta_{\max} \sim 43$, essentially the same as the result estimated from (5), $\eta_{\max} \sim 41$ (when only the magnetic slowing mechanism is taken into account, this estimate yields $\eta_{\max} \sim 58$; when only the gasdynamic mechanism is taken into account, this estimate yields $\eta_{\max} \sim 62$; i.e., the two mechanisms are equally effective and should be taken into account together).

The particular value adopted for the conductivity σ leads to a situation in which the magnetic field is frozen in the buffer plasma essentially entirely in the stage of compression of this plasma. At the beginning of the compression, the magnetic Reynolds number

$$\mathrm{Re}_M \sim (4\pi)^{1/2}\sigma M_A/\eta \sim 10^4$$

is very large. By the time at which the buffer plasma begins to expand, after the cumulation, the characteristic value of this parameter estimated from the average expansion velocity and the maximum degree of compression $\eta_{\max}$ remains fairly high, $\mathrm{Re}_M \sim 10$, and effects stemming from magnetic-field diffusion are weak.

Figure 2 shows the time evolution of the magnetic field H_0 at the origin of coordinates (at the symmetry plane or the wall). The maximum field $H^0_{\max} \sim 45$ corresponds to the frozen-in condition, $H^0_{\max} \approx \eta_{\max}$. The jumps on the $H^0(t)$ curve are associated with the reflection of shock waves from

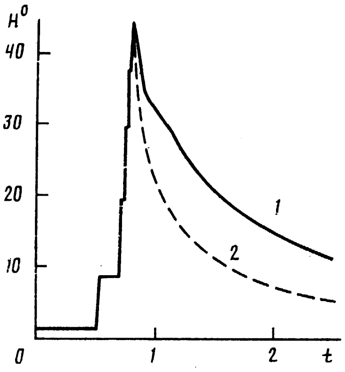

FIG. 2. Magnetic-field pulse at the origin of coordinates. 1—$\varepsilon = 2.5\cdot10^3$, $\mu = 2.5\cdot10^2$, $\sigma = 10^3$; 2—the same, but $\sigma = 50$.

the symmetry plane. Also shown in this figure, by the dashed line, is a corresponding $H^0(t)$ curve found in a calculation version which was the same except that the conductivity was reduced by a factor of 20 (i.e., to $\sigma = 50$). In this version of the calculations, the maximum and minimum characteristic magnetic Reynolds numbers are ~ 500 and ~ 0.5, respectively. A comparison of the $H^0(t)$ curves shows that the decrease in the conductivity has no effect during the compression stage: The maximum values $H^0_{\max}$ are essentially the same. During the subsequent expansion, a diffusion of magnetic field out of the buffer plasma occurs for the smaller value of σ and reduces the efficiency of the magnetic cumulation.

Figure 3 illustrates the energy balance in the buffer plasma for this version of the calculations. Shown here is the time evolution of the total energies in the buffer plasma: the internal energy, the kinetic energy, and the magnetic-field energy, all normalized to E_0. Since the relation $M_A > 1$ holds in this version of the calculations, a relatively large amount of energy is expanded on heating the buffer plasma. By the time of maximum compression, only about 8% of the total liner energy E_0 has been transferred to the buffer plasma and the magnetic field in this plasma (half the liner energy is "carried off" in the direction opposite the wall as the liner expands).

When the liner energy is reduced by a factor of 25 from that in the first version, while the parameters are otherwise left the same (i.e., with $\varepsilon = 100$ and $M_A \approx 0.63$), the liner is slowed by essentially the magnetic field alone. The calculated value of the maximum degree of compression is $\eta_{\max} \sim 15$, while the estimated value is

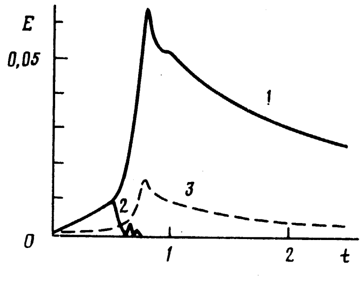

FIG. 3. Energy balance in the buffer plasma. 1—Internal energy; 2—kinetic energy; 3—magnetic-field energy ($\varepsilon = 2.5\cdot10^3$, $\mu = 2.5\cdot10^2$, $\sigma = 10^3$).

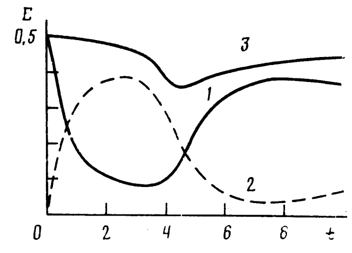

FIG. 4. Energy balance in the liner. 1—Internal energy; 2—kinetic energy; 3—the sum of the two ($\varepsilon = 10^2$, $\mu = 2.5 \cdot 10^2$, $\sigma = 10^3$).

$$\eta_{max} = [(\gamma+1)/2]^{1/2} \varepsilon^{1/2} \sim 12.$$

As in the case of compression at a velocity above the Alfvén velocity, the expansion velocity of the buffer plasma after the cumulation in this version of the calculations is far lower (by a factor of 17) than the collapse velocity.

Figure 4 shows the time evolution of the internal and kinetic energies (normalized to E_0) and of their sum in the half of the liner mass which is initially moving toward the wall. As the liner expands, we can clearly see conversion of a large fraction of its internal energy into kinetic energy and a subsequent conversion of kinetic energy, again primarily into internal energy: the self-heating of the liner mentioned earlier. At the time of maximum compression, $t \sim 4.5$, only about 15% of the total liner energy has been converted into magnetic energy in the buffer plasma. From this actual energy balance and from conservation of magnetic flux in the buffer plasma, we find the maximum degree of compression to be $\eta_{\max} \approx 0.15\varepsilon \sim 15$, i.e., the same as in the calculation. In Fig. 4 we also note the stage of liner expansion, which begins at $t \sim 8$–10.

In this version of the calculations, effects stemming from magnetic-field diffusion are rather weak. In the regime of magnetic slowing of the liner, with $\eta_{\max} \sim \varepsilon^{1/2}$ and with the help of the characteristic ratio (on the order of 10–20) of the compression velocity and the expansion velocity of the buffer plasma found in the calculations, we find that the condition for weak diffusion of magnetic field after the plasma cumulation, $\mathrm{Re}_M \gg 1$, can be rewritten as $\sigma \gg k\mu^{1/2}$, with an empirical coefficient $k \sim 10$.

If the liner mass is reduced by a factor of 10 from that in the first version (and if the parameter values are otherwise left the same), i.e., with $M_A = 10$, we find that the gasdynamic slowing dominates. The magnetic pressure in the buffer plasma at the time of maximum compression is about 4% of the plasma pressure, and the degree of compression itself is $\eta_{\max} \sim 17$ ($\eta_{\max} \sim 15$ according to the estimate), close to the maximum compression in the second version.

CONCLUSION

The problem of the compression of a buffer plasma with a magnetic field by an exploding liner thus differs in several important ways from the problem with a thin liner, whose initial energy is kinetic, even in the very simple formulation of the problem which we have discussed here (1*D* geometry, simplified equation of state, constant electrical conductivity of the plasma, and no effects of radiation, heat transfer, or departure of the temperature or the ionization from equilibrium). The main differences are the incomplete transfer of liner energy to the magnetic field and the decrease in the time over which the maximum parameter values are maintained, although the absolute values of these parameters are lower than in the case of a thin incompressible liner under otherwise equal conditions.

These distinctive features of the dynamics of plasma compression with a magnetic field, under conditions such that the initial thermal velocities of the expansion of the plasma liners are comparable to or greater than the inertial velocities of the liners, should be kept in mind in experimental studies of magnetic cumulation. If the degree to which energy of the external source is converted into kinetic energy of a "cold" liner is low (because of, for example, a substantial absorption of laser light by an erosion plasma), it may prove quite effective to use "hot" liners for magnetic cumulation, provided that the plasma instabilities which would develop are suppressed.

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