Excitation of electromagnetic fields in an inhomogeneous plasma by an anisotropic ionization pulse

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A hydrodynamic model is proposed for finding a self-consistent description of the space-time evolution of low-frequency electromagnetic fields in an inhomogeneous plasma with an anisotropic electron distribution. An ionization source and collisional dissipation are taken into account. This model is used to calculate the electromagnetic fields in a plasma slab formed in a medium after the passage of a pulse of ionizing radiation through it. The mechanism for the excitation of electromagnetic fields involves the combined effects of two factors: the inhomogeneity of the plasma density and the anisotropy of the electron energy distribution. Under certain conditions the energy of the electromagnetic field may be comparable in magnitude to the thermal energy of the plasma which is produced. The duration and spectrum of the electromagnetic pulse are evaluated.

1. INTRODUCTION

As pulses of hard electromagnetic radiation or particles pass through a medium, a plasma wake forms and electromagnetic waves are excited in the microwave and rf ranges, at frequencies well below the plasma frequency. This process was examined in Refs. 1 and 2 for the case of a pulse of γ radiation in the earth's atmosphere. In the present paper we wish to point out that the efficiency with which the energy of the ionizing pulse is converted into microwave radiation can be raised significantly. The idea is to make use of the fact that the plasma which is produced in the wake of the ionizing pulse is a nonequilibrium, anisotropic plasma and to arrange conditions for the conversion of the nonequilibrium plasma energy into electromagnetic radiation.

Deriving a complete self-consistent kinetic description of the excitation of low-frequency electromagnetic waves in an anisotropic plasma runs into serious difficulties and requires extensive numerical calculations. A simpler model of anisotropic rotational electron hydrodynamics was proposed in Refs. 3 for the purpose of describing self-consistent electromagnetic structures in plasmas with an anisotropic pressure. That model is based on a system of equations for the quasistatic magnetic field and the stress tensor. That model follows from the equations of the ten-moment approximation (the density, the velocity, and the stress tensor) and Maxwell's equations when the higher-order moments (the moments of third and higher orders) of the electron distribution are ignored. The theory derived in Refs. 3 has been used to study the excitation of electromagnetic fields as a result of the Weibel instability of an anisotropic, spatially homogeneous plasma.[4]

In an inhomogeneous plasma, the instability mechanism is joined by another mechanism which excites electromagnetic fields. This other mechanism stems from the anisotropic thermopower. It corresponds to an increase in the amplitude of the magnetic field which is linear in time, so if the inhomogeneity and the anisotropy are sufficiently pronounced this other mechanism may be more important than the instability mechanism, for which the increase in the fields with the time is exponential. In a plasma with an isotropic pressure, the thermoelectric effect stems from the presence of crossed gradients of the temperature and the density. In an anisotropic plasma, it would be sufficient for only one of these quantities to be inhomogeneous. For example, Aliev *et al.*[5] have examined the excitation of a magnetic field due to the existence of an angle between the anisotropy vector and the temperature gradient for the case of a plasma produced by a laser beam.

For a plasma produced by hard ionizing radiation, the electron density gradient is more important than the electron temperature gradient. Accordingly, in the present paper we study the excitation of electromagnetic fields as the result of the existence of an angle between the anisotropy vector and the density gradient. For this purpose we use the model of anisotropic rotational electron hydrodynamics, in which we include, in addition to the factors considered in Ref. 3, an external source of anisotropic ionization and electron collisions, which describe the magnetic viscosity, randomization of the pressure, and Joule heating of the plasma. To first order in the amplitude of the magnetic field which is excited, we analyze the excitation of low-frequency electromagnetic fields over times much shorter than the electron collision time. We do this for four characteristic density profiles, which represent, respectively, a plasma slab, a semi-infinite plasma with a diffuse boundary, an unbounded plasma with a modulated density, and a plasma with a linear density profile.

We will see that the electromagnetic field is excited most efficiently in the case of a fairly high density gradient with a length scale $\lesssim c/\omega_p$, where c is the velocity of light, and ω_p the plasma frequency. Over times shorter than the ionization time of the medium, the rate at which the electromagnetic field is excited is determined by the shape of the ionizing pulse, while after a long time the strength of the magnetic field increases linearly with the time. The spatial distributions of the magnetic field in a plasma slab and in the boundary region of a semi-infinite plasma are soliton-like, corresponding to the magnetic field of a system of two antiparallel currents. The magnetic field in a plasma with a sinusoidally modulated density is spatially periodic and (generally) anharmonic. In a collisionless plasma, the growth of a quasistatic electromagnetic field has the result that the energy of this field becomes comparable to the thermal energy of

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the electrons fairly quickly, if the electrons resulting from the ionization of the medium by the pulsed source have a sufficiently anisotropic energy. In this case the excitation of the electromagnetic field goes into the nonlinear regime, whose analysis lies outside the scope of the present paper.

Taking collisions into account, we study the excitation of quasistatic electromagnetic fields in the two limiting cases $L^2 \gg c^2/\omega_p^2$ (slightly inhomogeneous plasma) and $L^2 \ll c^2/\omega_p^2$ (highly inhomogeneous plasma), where L is a length scale of the field variation. We study the space-time distribution of the magnetic field generated by a pulsed source which acts over a time scale much shorter than the electron-collision time. The general expressions are illustrated in the particular examples of a plasma slab and a density well. The results derived below are applied to a fully ionized plasma, for which the collision rate (the rate of electron-ion collisions) is proportional to the electron density, and also to a weakly ionized plasma, with a collision rate (in this case, the rate of electron-neutral collisions) proportional to the atomic density. If the electron collision rate is sufficiently high, the magnetic-field energy is small in comparison with the anisotropic thermal energy of the electrons. In this case the linear approximation is sufficient (a nonlinear analysis is not required).

2. EQUATIONS OF DISSIPATIVE ANISOTROPIC ROTATIONAL HYDRODYNAMICS WITH A SOURCE

We start from a kinetic equation for f, the electron distribution function, in which we incorporate the collision integral $J_c[f]$ and an ionization source with a strength $Q(\mathbf{r}, t)$:

$$\frac{\partial f}{\partial t}+\mathbf{v}\nabla f+\frac{e}{m}\left(\mathbf{E}+\frac{1}{c}[\mathbf{vB}]\right)\frac{\partial f}{\partial \mathbf{v}}=J_c[f]+Q(N_a-n)f_0\equiv J. \tag{2.1}$$

Here e, m, and n are the charge, mass, and density of the electrons; c is the velocity of light; the last term on the right side of (2.1) describes the ionization of a medium with atomic density n_a; $N_a = zn_a$; and z is the degree of ionization of the atoms. We assume that the electrons resulting from the ionization have an anisotropic energy distribution (in the direction of the unit vector $\mathbf{h}$), which is described by a function $f_0(\mathbf{v})$ which depends on $v_\parallel^2 = (\mathbf{hv})^2$ and $v_\perp = [\mathbf{hv}]^2$ and which satisfies the conditions

$$\int d\mathbf{v}\, f_0=1, \qquad \int d\mathbf{v}\, \mathbf{v} f_0=0,$$
$$m\int d\mathbf{v}\, v_i v_j f_0=T_{ij}\equiv T_\parallel h_i h_j+T_\perp(\delta_{ij}-h_i h_j). \tag{2.2}$$

Multiplying Eq. (2.1) by 1, $\mathbf{v}$, and mv_iv_i, integrating over the velocity $\mathbf{v}$, and using the ten-moment approximation, we find the following system of equations for the electron density, the average velocity $\mathbf{u}$, and the components P_{ij} of the stress tensor P:

$$\frac{\partial n}{\partial t}+\operatorname{div} n\mathbf{u}=J_0, \tag{2.3}$$

$$n\left(\frac{\partial u}{\partial t}+(\mathbf{u}\nabla)\mathbf{u}\right)+\frac{1}{m}\nabla P-\frac{en}{m}\mathbf{E}-n[\mathbf{u}\boldsymbol{\Omega}]=\mathbf{J}_1, \tag{2.4}$$

$$\frac{\partial P}{\partial t}+(\mathbf{u}\nabla)P+\{(P\nabla)\mathbf{u}\}-\{[P\boldsymbol{\Omega}]\}=J_2. \tag{2.5}$$

Here $\boldsymbol{\Omega} = e\mathbf{B}/mc$ is the electron gyrofrequency, and the moments of the electron distribution function are given by

$$n=\int d\mathbf{v}\, f, \quad \mathbf{u}=\int d\mathbf{v}\, \mathbf{v} f, \quad P=m\int d\mathbf{v}\, (\mathbf{v}-\mathbf{u})(\mathbf{v}-\mathbf{u})f.$$

The braces (curly brackets) mean that the corresponding tensors have been put in symmetric form: $\{A_{ij}\} = A_{ij} + A_{ji}$. The quantity $[P\boldsymbol{\Omega}]$ is a tensor with the components $[P\boldsymbol{\Omega}]_{ij} = e_{ikl}P_{jk}\Omega_l$, $(\nabla P)_i$ is a vector with the components $\nabla_j P_{ij}$, $\mathbf{gg}$ is a tensor with the components $g_i g_j$, and the right sides of Eqs. (2.3)–(2.5) can be expressed in terms of the right side of the kinetic equation (2.1) as follows:

$$J_0=\int d\mathbf{v}\, J[f], \quad \mathbf{J}_1=\int d\mathbf{v}\, (\mathbf{v}-\mathbf{u})J[f],$$
$$J_2=m\int d\mathbf{v}\, (\mathbf{v}-\mathbf{u})(\mathbf{v}-\mathbf{u})J[f]. \tag{2.6}$$

To find explicit expressions (2.6), we use the approximation

$$J_c=-\nu(f-\bar{f}), \quad \bar{f}=\int\frac{do_\mathbf{v}}{4\pi}f(\mathbf{v}),$$

where $\bar{f}(\mathbf{v})$ is the distribution function averaged over the angle of the vector $\mathbf{v}$. In addition, we assume that, in the simplest case, the electron collision rate ν is independent of $\mathbf{v}$.

Making use of the comments regarding the integrals in (2.6), we can write

$$J_0=Q(N_a-n), \quad \mathbf{J}_1=-\nu n\mathbf{u}-Q(N_a-n)\mathbf{u},$$
$$J_2=Q(T+m\mathbf{uu})-\nu\left(P-\frac{I}{3}\operatorname{Sp}P\right)+\nu mn\left(\mathbf{uu}+\frac{I}{3}\mathbf{u}^2\right), \tag{2.7}$$

where I is the unit tensor ($I_{ij} = \delta_{ij}$). The transport equations (2.3)–(2.5) are supplemented with Maxwell's equations, in which we ignore the displacement current:

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}=-\operatorname{rot}\mathbf{E}, \quad \operatorname{rot}\mathbf{B}=\frac{4\pi}{c}en\mathbf{u}. \tag{2.8}$$

We assume that the time scales of the variations of the electric and magnetic fields are much longer than the period of the electron plasma waves, and we assume that the length scale of these variations is much greater than the Debye length. Following Ref. 3, we ignore the potential component of the velocity in Eqs. (2.3)–(2.5). This approach corresponds to the model of an anisotropic rotational electron hydrodynamics.[1)] Equations (2.3)–(2.5), (2.8) then reduce to the following equations for the density, the quasistatic magnetic field $\mathbf{B}$, and the components of the stress tensor P:

$$\frac{\partial n}{\partial t}=Q(N_a-n), \tag{2.9}$$

$$\frac{\partial \boldsymbol{\Omega}}{\partial t}+c^2\operatorname{rot}\frac{1}{\omega_p^2}\operatorname{rot}\frac{\partial\boldsymbol{\Omega}}{\partial t}+\operatorname{rot}\frac{c^2}{4\pi\sigma}\operatorname{rot}\boldsymbol{\Omega}$$
$$=c^2\operatorname{rot}\frac{1}{\omega_p^2}\left[(\boldsymbol{\Omega}\nabla)\boldsymbol{\Omega}-c^2(\operatorname{rot}\boldsymbol{\Omega}\nabla)\frac{1}{\omega_p^2}\operatorname{rot}\boldsymbol{\Omega}\right]-\frac{1}{m}\operatorname{rot}\frac{1}{n}\nabla P, \tag{2.10}$$

$$\frac{\partial P}{\partial t}+\frac{c^2}{\omega_p^2}(\operatorname{rot}\boldsymbol{\Omega}\nabla)P+c^2\left\{(P\nabla)\frac{1}{\omega_p^2}\operatorname{rot}\boldsymbol{\Omega}\right\}-\{[P\boldsymbol{\Omega}]\}$$
$$=Q(N_a-n)\left[T+\frac{c^4}{\omega_p^4}\operatorname{rot}\boldsymbol{\Omega}\operatorname{rot}\boldsymbol{\Omega}\right]-\nu\left(P-\frac{I}{3}\operatorname{Sp}P\right)$$
$$+\nu mn\frac{c^4}{\omega_p^4}\left(\operatorname{rot}\boldsymbol{\Omega}\operatorname{rot}\boldsymbol{\Omega}+\frac{I}{3}\operatorname{rot}^2\boldsymbol{\Omega}\right), \tag{2.11}$$

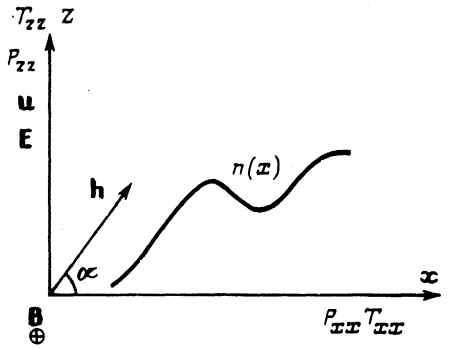

FIG. 1. Geometry of the problem.

where $\omega_p = (4\pi e^2 n/m)^{1/2}$ is the plasma frequency, and $\sigma = e^2 n/m\nu$ is the electrical conductivity of the plasma. With $\nu = 0$ and $Q = 0$ we find $n = \text{const}$ from (2.9), and Eqs. (2.10) and (2.11) become the equations derived for the model of anisotropic rotational electron hydrodynamics in Ref. 3.

Equations (2.9)–(2.11) constitute the foundation of a dissipative anisotropic rotational electron hydrodynamics. They generalize the model proposed previously by incorporative variations in the plasma, particle collisions, and the source causing anisotropic ionization of the medium. This source determines the rate of increase of the electron density [the right side of (2.9)] and the anisotropy of the particle energy [the first term on the right side of (2.11)]. Incorporating collisional dissipation determines the magnetic viscosity $c^2/4\pi\sigma$ [in Eq. (2.10)], the randomization of the pressure [the second term on the right side of (2.11)], and anisotropic Joule heating of the plasma by rotational currents [the last term on the right side of (2.11)].

We will discuss the 1*D* case, in which the vector **B** has only a y component, and the tensors P and T have only xx, xz, and zz components, which depend only on x. We assume that the anisotropy vector **h** of the source makes an angle α with the x axis; we thus have $T_{xz} \neq 0$ (the geometry is shown in Fig. 1). Equations (2.10) and (2.11) can then be rewritten as

$$\frac{\partial\Omega}{\partial t} - c^2\frac{\partial}{\partial x}\frac{1}{\omega_p^2}\frac{\partial^2\Omega}{\partial x\partial t} - c^2\frac{\partial}{\partial x}\frac{\nu}{\omega_p^2}\frac{\partial\Omega}{\partial x} = \frac{1}{m}\frac{\partial}{\partial x}\frac{1}{n}\frac{\partial P_{xz}}{\partial x}, \tag{2.12}$$

$$\begin{aligned}\frac{\partial P_{zz}}{\partial t} &= 2P_{xz}\left(\Omega - c^2\frac{\partial}{\partial x}\frac{1}{\omega_p^2}\frac{\partial\Omega}{\partial x}\right) \\ &\quad - \frac{\nu}{2}(P_{zz}-P_{xx}) + \frac{3}{2}\nu m n\frac{c^4}{\omega_p^4}\left(\frac{\partial\Omega}{\partial x}\right)^2 \\ &\quad + Q(N_a-n)\left[T_{zz} + m\frac{c^4}{\omega_p^4}\left(\frac{\partial\Omega}{\partial x}\right)^2\right],\end{aligned} \tag{2.13}$$

$$\begin{aligned}\frac{\partial P_{xx}}{\partial t} &= -2P_{xz}\Omega + \frac{\nu}{2}(P_{zz}-P_{xx}) \\ &\quad + \frac{1}{2}\nu m n\frac{c^4}{\omega_p^4}\left(\frac{\partial\Omega}{\partial x}\right)^2 + Q(N_a-n)T_{xx},\end{aligned} \tag{2.14}$$

$$\frac{\partial P_{xz}}{\partial t} = (P_{xx}-P_{zz})\Omega - c^2 P_{xx}\frac{\partial}{\partial x}\frac{1}{\omega_p^2}\frac{\partial\Omega}{\partial x} - \nu P_{xz} + Q(N_a-n)T_{xz}. \tag{2.15}$$

In (2.15) we ignore the terms which contain bilinear combinations of P and Ω. This simplification is legitimate for a sufficiently short time $t < \Omega^{-1}$, when we note that the values of the components are comparable, $P \sim N_a T$. On the other hand, the times under consideration here must not be so small in comparison with Q^{-1} that the medium is not ionized. Assuming that Q and T_{xz} are independent of x, we find

$$\frac{\partial\Omega}{\partial t} - c^2\frac{\partial}{\partial x}\frac{1}{\omega_p^2}\frac{\partial^2\Omega}{\partial x\partial t} - c^2\frac{\partial}{\partial x}\frac{\nu}{\omega_p^2}\frac{\partial\Omega}{\partial x} = G(x,t), \tag{2.16}$$

$$\begin{aligned}G(x,t) &= \frac{T_{xz}}{mS(t)}\int_0^t dt'\,\frac{dS(t')}{dt'}\frac{\partial}{\partial x}\frac{1}{N_a(x)}\frac{\partial}{\partial x} \\ &\quad \times N_a(x)\exp\left[\int_t^{t'} dt''\,\nu(x,t'')\right].\end{aligned} \tag{2.17}$$

Expression (2.17) for the source G was found by solving Eq. (2.15). In the process we assumed that the ionization of the medium began at $t = 0$ and that the increase in the electron density is described by the following expression, in accordance with (2.9):

$$n(x,t) = N_a(x)S(t), \quad S(t) = 1 - \exp\left[-\int_0^t dt'\,Q(t')\right]. \tag{2.18}$$

Expression (2.18) for the electron density determines the coordinate and time dependence of the plasma frequency on the left side of Eq. (2.16). Since we have $T_{xz} = 1/2(T_\parallel - T_\perp)\sin 2\alpha$, where α is the angle between the anisotropy axis **h** and the x axis, we can assume $T_{xz} > 0$ for definiteness.

3. EXCITATION OF QUASISTATIC ELECTROMAGNETIC FIELDS IN A COLLISIONLESS PLANE-LAYER PLASMA

We first consider the case $\nu = 0$, which corresponds to short times—much shorter than the electron collision time scale. In this case we have

$$G(x,t) \equiv G(x) = \frac{T_{xz}}{m}\frac{\partial}{\partial x}\frac{1}{N_a(x)}\frac{\partial N_a}{\partial x}, \tag{3.1}$$

and Eq. (2.16) becomes

$$\frac{\partial}{\partial x}\frac{1}{N(x)}\frac{\partial\dot\Omega}{\partial x} - \frac{\omega_m^2}{c^2}S(t)\dot\Omega = -\omega_m^2\frac{v_T^2}{c^2}S(t)\frac{d^2}{dx^2}\ln N(x), \tag{3.2}$$

where $S(t)$ is determined by the shape of the ionizing pulse according to (2.18), and $v_T^2 = T_{xz}/m$. The quantity $N(x)$ in Eq. (3.2) is the density of atoms, normalized to its maximum value N_m ($N = N_a/N_m$), and $\omega_m^2 = \omega_p^2/NS = \text{const}$ is the maximum plasma frequency. For the analysis below, we will also write an equation for the rotational electric field $\mathscr{E}(x,t) = eE/m$, which is related to Ω by $\Omega = \partial\mathscr{E}/\partial x$ according to (2.8). Substituting it into (3.2), we find ($\omega_a^2 = 4\pi e^2 N_a/m$)

$$\frac{\partial^2\mathscr{E}}{\partial x^2} - \frac{\omega_a^2(x)}{c^2}S(t)\mathscr{E} = -\omega_a^2(x)\frac{v_T^2}{c^2}S(t)\frac{d\ln N_a}{dx} - \mathscr{E}_0 N_a, \tag{3.3}$$

where the constant of integration $\mathscr{E}_0(t)$ is to be determined from the boundary conditions.

It follows from (3.2) and (3.3) that the spatial distribution of the magnetic field is determined by the density profile $N_a(x)$. We will be looking at several characteristic density profiles. We first consider a plasma slab, setting

$$N(x)=1/\mathrm{ch}^2\,kx. \tag{3.4}$$

The boundary conditions on Eq. (3.2) are the conditions that the magnetic field and the current, i.e., Ω and Ω', be spatially bounded. Solving Eq. (3.2), we find

$$\Omega(x,t)=-2k^2\frac{v_T^2}{c^2}\frac{\omega_m^2}{\mathrm{ch}^2\,kx}\int_0^t\frac{S(t')dt'}{2k^2+(\omega_m^2/c^2)S(t')}. \tag{3.5}$$

After a sufficiently long time, greater than the length t_0 of the pulse $Q(t)$, with $S(t)\to S_0=\mathrm{const}$, the magnetic field increases linearly with time. The magnetic field reaches its highest value for thin slabs, with a density gradient satisfying $k\gtrsim\omega_m S_0^{1/2}/c$. If the source is sufficiently strong ($Qt_0\gg1$), we have $S_0\simeq1$; such values correspond to full ionization of the plasma. For $Qt_0\lesssim1$, the plasma is not fully ionized ($S_0<1$). The spatial distribution of the magnetic field, (3.5), reproduces the density profile (3.4). Figure 2 shows distributions of $B(x)$ and corresponding distribution of the current, $j(x)$, and of the electric field, $E(x)$.

We approximate a semi-infinite plasma by the density profile corresponding to an isothermal rarefaction wave:

$$N(x)=\begin{cases}e^{kx}, & x\leqslant0\\ 1, & x>0\end{cases}.$$

There is no source in Eq. (3.3) for $x>0$ in this case, and we have $\mathscr{E}_0=0$, since there are no fields as $x\to\infty$. Solving Eq. (3.3) in the regions $x<0$ and $x>0$, and imposing the conditions that the electric and magnetic fields be continuous at $x=0$, we find

$$\Omega(x,t)=-\omega_m v_T^2\frac{k}{c}\int_0^t\frac{S^{1/2}(t')dt'}{I_0(\xi_0)+I_1(\xi_0)}\times\begin{cases}I_1(\xi_0)\exp(-\xi_0kx/2), & x\geqslant0\\ I_1(\xi)\exp(kx/2), & x<0\end{cases}, \tag{3.6}$$

where

$$\xi=\frac{2\omega_m}{kc}S^{1/2}(t')\exp\left(\frac{kx}{2}\right),$$

$\xi_0=\xi(x=0)$, and I_0 and I_1 are modified Bessel functions. Qualitatively, the magnetic field distribution in (3.6) and the corresponding current distribution correspond to the preceding case (Fig. 3). The magnetic field reaches a maximum in the transition layer ($x=0$). As in the case of a slab of finite thickness, the magnetic field is generated most efficiently under the condition $k\gtrsim\omega_m/c$, which corresponds to a fairly rapid change in the density at the boundary.

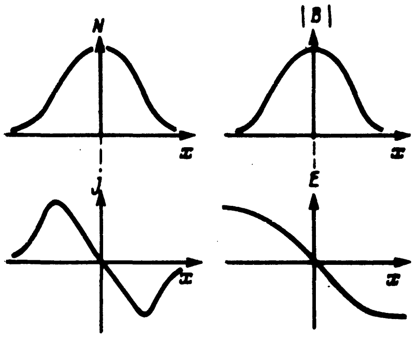

FIG. 2. Distributions of the magnetic field, the electric field, and the current for a slab of collisionless plasma.

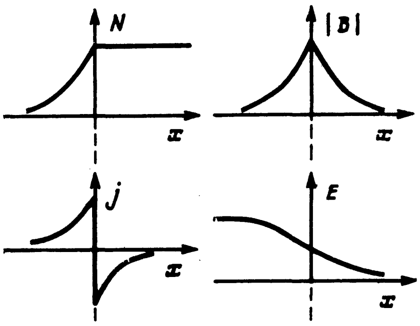

FIG. 3. Distributions of the magnetic field, the electric field, and the current in a semi-infinite collisionless plasma.

In the asymptotic regime ($t\gg t_0$), with a strong source $S_0\approx1$ and a large density gradient $k\gtrsim\omega_m/c$, Eqs. (3.4) and (3.6) yield the estimate

$$\frac{B^2}{4\pi N_m T_{xz}}\sim\frac{v_T^2}{c^2}(\omega_m t)^2. \tag{3.7}$$

According to (3.7), the magnetic-field energy becomes comparable to the thermal energy of the electrons over a time $t\sim\omega_m^{-1}(c/v_T)$. At such times, however, the plasma dynamics should be described self-consistently by means of the nonlinear equations (2.12)–(2.15). The linear regime of the excitation of quasistatic electromagnetic fields, in which we are interested here, corresponds to time $t\ll\omega_m^{-1}(c/v_T)$.

As another example we consider a stratified structure, which we model with a density distribution

$$N(x)=\frac{1+\varepsilon\cos kx}{1+\varepsilon},\quad 0<\varepsilon<1. \tag{3.8}$$

In this case the equation for the rotational electric field, (3.3), becomes a Mathieu equation. Introducing the dimensionless coordinate $z=(x\omega_m/c)\cdot[S(t)/(1+\varepsilon)]^{1/2}$ and the dimensionless field $g=(kv_T^2\varepsilon)^{-1}\mathscr{E}$, we find

$$g''-(1+\varepsilon\cos k_0z)g=\sin k_0z,\quad k_0=\frac{kc}{\omega_m}\left[\frac{1+\varepsilon}{S(t)}\right]^{1/2}. \tag{3.9}$$

A spatially bounded solution of Eq. (3.9) can be written in series form:

$$g=-\sum_{n=1}^{\infty}G_n\sin k_0nz. \tag{3.10}$$

The coefficients of this series are found from the chain of algebraic equations

$$\begin{gathered}(1+k_0^2)G_1+\frac{\varepsilon}{2}G_2=1,\\ (1+k_0^2n^2)G_n+\frac{\varepsilon}{2}(G_{n+1}+G_{n-1})=0,\quad n\geqslant2.\end{gathered} \tag{3.11}$$

If ε is sufficiently small ($\varepsilon\ll1$), or if $k_0^2\gg1$, then we find from (3.10) and (3.11)

$$g\approx-\frac{\sin k_0z}{1+k_0^2}+\frac{1}{2}\frac{\varepsilon}{1+k_0^2}\frac{\sin 2k_0z}{1+4k_0^2};$$

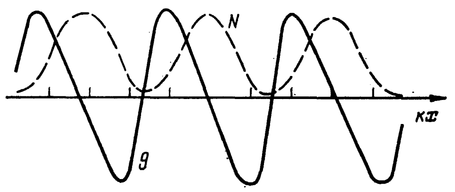

FIG. 4. Electric field distribution $g(x)$ (solid line) for a stratified plasma ($\varepsilon = 0.9$, $k_0 = 0.4$) and density distribution $N(x)$ (dashed line).

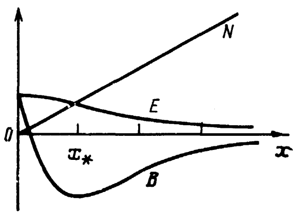

FIG. 5. Distributions of the magnetic and electric fields in a plasma with a linearly increasing density.

i.e., the magnetic field distribution is given by

$$\Omega = -\varepsilon k^2 v_T^2 \int_0^t \frac{dt'\, S(t')}{S(t') + k^2c^2/\omega_m^2} \times \left[\cos kx - \frac{\varepsilon \cos 2kx}{1 + 4k^2c^2/\omega_m^2 S(t')}\right]. \qquad (3.12)$$

Under the conditions $t \gg t_0$ and $S_0 \approx 1$ the strongest magnetic field is excited in the case of a short-wavelength ($k \gtrsim \omega_m/c$) density modulation. The field which is excited [see (3.10)] is periodic, with a spatial oscillation period equal to the period of the density modulation, (3.8). This field may reach the nonlinear level. Developing stratified structures with a period $\sim 2\pi c/\omega_m$ may prove to be an effective way to generate strong electromagnetic fields in large volumes. Figure 4 shows the electric field distribution in the case of a deep density modulation ($\varepsilon = 0.9$) with $k_0 = 0.4$.

Yet another example which we will discuss here is a linear density profile: $N_a(x) \propto x$ at $x \geqslant 0$ and $N_a = 0$ at $x < 0$. In this case the length scale of the plasma variations, $L = N_a/N_a' = x$, increases linearly along the coordinate. The condition that there is no electric field as $x \to \infty$ determines the constant $\mathscr{E}_0 = 0$ in Eq. (3.3). Introducing the length scale of the field variation,

$$\mathscr{K}^{-1} = \left[\frac{S(t)}{c^2}\frac{d\omega_a^2}{dx}\right]^{-1/3},$$

and the dimensionless coordinate $\xi = \mathscr{K}x$, we find an inhomogeneous Airy equation from (3.3):

$$\xi\mathscr{E} - \mathscr{E}'' = \mathscr{K} v_T^2. \qquad (3.13)$$

The following solution of Eq. (3.13) is bounded as $\xi \to \infty$ and satisfies the equality $\mathscr{E} = c\mathscr{K}\mathscr{E}'$, at $\xi = 0$ (this equality simulates the conditions at a plasma-vacuum interface, at which the magnetic field is equal to the electric field):

$$\mathscr{E} = \pi\mathscr{K} v_T^2 \Big\{ \mathrm{Bi}(\xi) \int_\xi^\infty d\xi' \, \mathrm{Ai}(\xi') + \mathrm{Ai}(\xi) \Big[\int_0^\xi d\xi' \, \mathrm{Bi}(\xi') - \frac{\mathscr{K}\,\mathrm{Bi}(0) - c\mathscr{K}^2\,\mathrm{Bi}'(0)}{\mathscr{K}\,\mathrm{Ai}(0) - c\mathscr{K}^2\,\mathrm{Ai}'(0)} \int_0^\infty d\xi' \, \mathrm{Ai}(\xi') \Big] \Big\}, \qquad (3.14)$$

where $\mathrm{Ai}(\xi)$ and $\mathrm{Bi}(\xi)$ are Airy functions. Figure 5 shows the spatial distributions of the electric field in (3.14) and of the corresponding magnetic field. The electric field falls off monotonically from the boundary into the plasma [$\mathscr{E}(x \to \infty) \sim 1/x$]. The magnetic field increases with distance from the boundary, goes through a maximum at the point $x_* \sim 1/\mathscr{K}$, at which the local length scale of the density variations, $L \simeq x_*$, is equal to the electromagnetic length scale $c/\omega_p(x_*)$, and decreases with distance into the plasma. A field configuration of this sort can be produced by a system of two oppositely directed currents [$j = (mc^2/4\pi e)\partial\Omega/\partial x$] flowing in the direction perpendicular to the density gradient and perpendicular to the magnetic field. We wish to stress that, as in the case $x \to \infty$ with a bounded plasma density, the magnetic and electric fields disappear as $x \to \infty$, despite the presence of a density gradient over the entire half-space $x > 0$. The reason is that the source of the fields is a quantity proportional to $d \ln N_a/dx$; i.e., this source is "turned off" as $x \to \infty$.

The linear stage of the excitation of quasistatic magnetic fields in a homogeneous plasma with an anisotropic electron energy under conditions corresponding to the onset of a Weibel instability was studied in Refs. 7 and 8. In the course of that instability, the electromagnetic perturbations which are excited most efficiently are those which have a characteristic wave number $\sim \omega_p/c$, i.e., which have the same wavelength as in our case of a stimulated excitation of a magnetic field. If the electron pressure is sufficiently anisotropic ($T_\| \gg T_\perp$), the nonlinear effects come into play at times

$$t \sim \omega_p^{-1} \frac{c}{v_T} \ln \frac{(4\pi n T_\|)^{1/2}}{B(t=0)} \gg \frac{1}{\omega_p}\frac{c}{v_T},$$

when the magnetic-field energy rapidly reaches a value comparable to the thermal energy of the particles. [For example,[8] we would have $B^2_{\max}/4\pi n T_\| \approx 0.2$ for an anisotropy $T_\|/T_\perp = 25$, if the initial level of the magnetic fluctuations were $B(t=0)/(4\pi n T_\|)^{1/2} = 10^{-2}$.] It can thus be asserted that under the condition $T_{xz} \sim T_\|$ (provided that $\sin 2\alpha$ is not small in comparison with unity), the mechanism of a stimulated excitation of electromagnetic fields, which predicts that a nonlinear regime will be reached in a time $\sim \omega_p^{-1} c/v_T$, turns out to be more effective than the field excitation due to an electromagnetic instability which develops from an initial noise at the thermal level.

4. EXCITATION OF A QUASISTATIC ELECTROMAGNETIC FIELD PULSE IN A COLLISIONLESS PLASMA

It was shown above that over a time

$$t \sim \omega_m^{-1} \frac{c}{v_T} \left(1 + \frac{\omega_m^2 L^2}{c^2}\right),$$

where $L = 1/k$ is the length scale of the plasma density variations, the magnetic-field energy becomes comparable in order of magnitude to the thermal energy of the electrons, so nonlinear effects must be taken into account. In a real situation, however, this event would be preceded by the onset of collisionless-dissipation effects, which would suppress the excitation of the quasistatic electromagnetic field. In this case, a linear theory with collisions might prove sufficient for a comprehensive description of the space-time evolution of the magnetic field which is excited. This problem should be solved on the basis of Eq. (2.16) with the source (2.17) and relation (2.18). We restrict the discussion below to the two limiting cases $L^2 \gg c^2/\omega_p^2$ and $L^2 \ll c^2/\omega_p^2$.

In the first of these cases, $L^2 \gg c^2/\omega_p^2$, the equation for the magnetic field is

$$\frac{\partial\Omega}{\partial t}-\frac{\partial}{\partial x}a^2\frac{\partial\Omega}{\partial x}=G(x,t), \tag{4.1}$$

$$G(x,t)=\frac{v_T{}^2}{S(t)}\int\limits_0^t dt'\,Q(t')\frac{\partial}{\partial x}\frac{1}{N(x)}\frac{\partial}{\partial x}N(x)$$

$$\times\exp\Big[-\int\limits_0^{t'}Q(t'')\,dt''-\int\limits_{t'}^{t}\nu(x,t'')\,dt''\Big], \tag{4.2}$$

where $a^2 = c^2/4\pi\sigma \equiv \nu(x,t)\,c^2/\omega_p^2(x,t)$ is the diffusion coefficient. For definiteness, we relate $\nu(x,t)$ to either electron-ion collisions (a fully ionized plasma) or electron-neutral collisions (a weakly ionized plasma). In the former case, the collision rate is proportional to the electron density, $\nu/\omega_p^2 = \text{const}$, and the diffusion coefficient a^2 is constant. In the latter case the collision rate is proportional to the density of neutral atoms, and we have $\nu/\omega_p^2 \propto S^{-1}(t)$; i.e., the diffusion coefficient is determined by the shape of the ionizing pulse. In each case the diffusion coefficient is independent of the coordinate x, so we can immediately write out the solution of Eq. (4.1).

For a fully ionized plasma, the following solution of Eq. (4.1) with a constant diffusion coefficient satisfies the initial condition $\Omega = 0$ at $t = 0$:

$$\Omega(x,t)=\frac{1}{2a\pi^{1/2}}\int\limits_0^t dt'\int\limits_{-\infty}^{\infty}dx'\,\frac{G(x',t')}{(t-t')^{1/2}}\exp\Big[-\frac{(x-x')^2}{4a^2(t-t')}\Big]. \tag{4.3}$$

Here we must assume $\nu(x,t) = \nu_m N(x)S(t)S_0^{-1}$. Expression (4.3) describes a competition among three processes: excitation of the magnetic field by the source, diffusion of the magnetic field out of the regions in which it is excited most efficiently, and dissipation of the anisotropy energy, which results in the disappearance of the magnetic field.

In the case of a weakly ionized plasma we have $\nu(x) = \nu_m N(x)$ and $a^2 = \nu_m c^2/\omega_m^2 S(t)$. We introduce a new time variable

$$\tau=\int\limits_0^t\frac{dt'}{S(t')}\,,$$

and we reduce (4.1) to an equation with a constant diffusion coefficient. Solving this equation, we find (4.3) in which $t-t'$ is replaced by $\int_{t'}^{t}dt''S^{-1}(t'')$. The solutions found thus determine the space-time distribution of the magnetic field as a function of the spatial distribution of the density N and the shape of the ionizing pulse (S and Q).

A case of practical interest is the excitation of quasistatic electromagnetic fields by an intense, short ionizing pulse. Specifically, the pulse length t_0 is short in comparison with the time scale of the magnetic-field evolution. For both a fully ionized plasma and a weakly ionized plasma we can use the following approximation for the source which generates the magnetic field [cf. (3.1)]:

$$G(x,t)\approx v_T{}^2\frac{\partial}{\partial x}\frac{1}{N}\frac{\partial}{\partial x}Ne^{-\nu_m N t}. \tag{4.4}$$

Using this expression in (4.3), we find the explicit space-time distribution of the magnetic field. Under the conditions for the applicability of Eqs. (4.1) and (4.4), we can replace (4.3) by a simpler and more convenient form of the solution $\Omega(x,t)$. Here we are making use of the circumstance that in the case under consideration here ($L^2 \gg c^2/\omega_p^2$) essentially no magnetic-field diffusion can occur over the time scale ($\sim\nu_m^{-1}$) on which the source (4.4) exists, because of the relation $a^2/L^2 \ll \nu_m$. We can thus ignore the second term in (4.1) and write

$$\Omega(x,t)\approx\int\limits_0^t dt'\,G(x,t'), \tag{4.5}$$

where the source $G(x,t)$ is given by (4.4). We find

$$\Omega(x,t)=v_T{}^2te^{-\nu_m N t}\Big[\frac{d^2\ln N}{dx^2}-\nu_m N t\Big(\frac{d\ln N}{dx}\Big)^2\Big]. \tag{4.6}$$

In the common case

$$\beta(x)\equiv(d^2\ln N/dx^2)(d\ln N/dx)^{-2}<0,$$

the magnetic field described by (4.6) evolves in the following way. In the initial stage of the evolution ($t \ll \nu_m^{-1}$) the magnetic field increases linearly with the time (Sec. 3). It then goes through a maximum and falls off to zero. The instant at which the magnetic field reaches its maximum value at a given spatial point is given by

$$\nu_m t=\frac{1}{2N(x)}\{\beta(x)+2+[\beta^2(x)+4]^{1/2}\}. \tag{4.7}$$

This dynamic picture is illustrated by Fig. 6 for the particular density distribution

$$N(x)=N_0+(1-N_0)e^{-k^2x^2},\quad N_0=0{,}1.$$

For density profiles (or parts thereof) characterized by

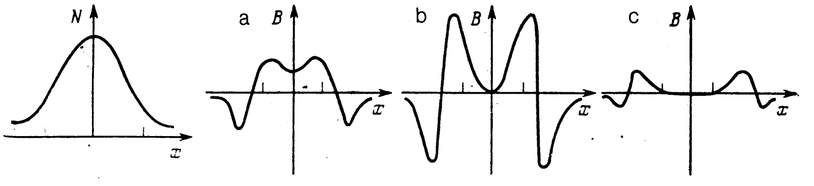

FIG. 6. Space-time evolution of the magnetic field in a plasma slab ($k^2c^2/\omega_m^2 \ll 1$) a—$\nu_m t = 1$; b—5; c—30.

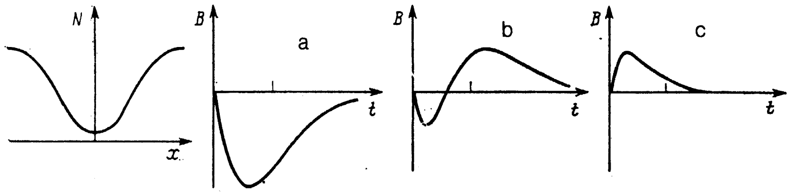

FIG. 7. Time evolution of the magnetic field for a density well ($k^2c^2/\omega_m^2 \ll 1$). a—$kx = 0.2$; b—0.35; c—1.

$\beta > 0$ the dynamic picture of the relaxation of the magnetic field is more complicated. After an initial linear increase ($t \ll \nu_m^{-1}$), the magnetic field reaches a maximum. It then decreases, changes direction, increases in magnitude, goes through a second extremum at the time

$$\nu_m t = \frac{1}{2N(x)}\{\beta(x)+2-[\beta^2(x)+4]^{1/2}\}, \tag{4.8}$$

and then drops to zero. This case is illustrated in Fig. 7 by the time evolution of the magnetic field at a certain point x for the density well

$$N=N_0\exp\left(\frac{k^2x^2}{1+k^2x^2}\ln\frac{1}{N_0}\right),\quad N_0=0,1$$

(Fig. 7b). Shown for comparison are curves of the magnetic-field relaxation for two other points on the density profile.

In the course of the relaxation, the magnetic field reaches the value determined by

$$\frac{B^2}{4\pi N_m T_{xz}} \sim \frac{v_T^2c^2}{\nu_m^2\omega_m^2L^4}. \tag{4.9}$$

It follows that if the collision rate is sufficiently high, specifically, $\nu_m > \omega_m (v_T/c)(c/\omega_m L)^2$, the energy of the magnetic field which is excited is small in comparison with the electron thermal energy. One might suggest that in a situation of this sort it would be legitimate to use the linear approximation to describe the excitation of quasistatic electromagnetic fields.

We turn now to the relaxation of the magnetic field in the case of small-scale variations: $L^2 \ll c^2/\omega_p^2$. In this case we can ignore the term $\partial\Omega/\partial t$ in Eq. (2.16) and rewrite this equation as

$$-c^2\frac{\partial}{\partial x}\frac{1}{\omega_p^2}\left(\frac{\partial^2\Omega}{\partial x\,\partial t}+\nu\frac{\partial\Omega}{\partial x}\right) = G(x,t). \tag{4.10}$$

As above, we restrict the discussion to the case in which a short ionizing pulse is applied to the medium, and the excitation of the magnetic field caused by this source can be described by Eq. (4.4). From Eq. (4.10) we find

$$\frac{\partial^2\Omega}{\partial x\,\partial t}+\nu\frac{\partial\Omega}{\partial x} = -\frac{v_T^2}{c^2}\omega_m^2 S(t)\frac{\partial}{\partial x}Ne^{-\nu_m N t}. \tag{4.11}$$

Assuming that we have $\Omega = 0$ at $N = 0$, we find the following solution from (4.11):

$$\Omega(x,t) = -\frac{v_T^2}{c^2}\omega_m^2\int_0^t\frac{S(t')dt'}{\rho(t,t')}\left[\frac{1}{\nu_m}(1-e^{-\nu_m\rho N}) - \frac{t'}{\nu_m\rho}(1-e^{-\nu_m\rho N})+t'Ne^{-\nu_m\rho N}\right],$$

where $\rho(t,t') = t$ holds in the case of a weakly ionized plasma ($\nu = \nu_m N$) or

$$\rho(t,t') = t' - S_0^{-1}\int_t^{t'} S(\tau)\,d\tau$$

for the case of a fully ionized plasma ($\nu = \nu_m NSS_0^{-1}$). At times $t \gg t_0$ we then find

$$\Omega(x,t) = -S_0\frac{\omega_m^2 v_T^2}{2c^2}\left[\frac{1}{\nu_m}(1-e^{-\nu_m N(x)t})+tN(x)e^{-\nu_m N(x)t}\right]. \tag{4.12}$$

Expression (4.12) describes an increase of the magnetic field to its maximum value over a time $t \approx 2/\nu_m N(x)$, followed by a decrease and an approach to a steady-state value ($\nu_m N t \gg 1$), determined by

$$\frac{B^2}{4\pi N_m T_{xz}} = \frac{1}{4}\frac{\omega_m^2}{\nu_m^2}\frac{v_T^2}{c^2}S_0^2. \tag{4.13}$$

The maximum value which the magnetic field reaches in the course of its relaxation is only 13% higher than the steady-state level. It is physically obvious, however, that the magnetic field could not go into a steady state with a finite magnitude in a dissipative system. Consequently, expression (4.12) is not valid after a long time. The reason is that in the stage in which the magnetic field decreases ($t > 2/\nu_m N$) the length scale of this field is increased by diffusion, and the condition under which (4.12) is applicable, $L^2 \ll c^2/\omega_p^2$, is violated. The subsequent relaxation of the magnetic field occurs in a regime of large-scale variations, as discussed above. Figure 8 shows spatial distributions of the magnetic field according to (4.12) in a plasma slab with a density distribution described by (3.3). We see from these figures that as time elapses there is a transition to larger-scale variations in the magnetic field.

If the source of anisotropic ionization is sufficiently strong ($S_0 \approx 1$), according to (4.13), the magnetic-field energy will be small in comparison with the electron thermal energy if $\nu_m > \omega_m v_T/c$. This inequality determines the con-

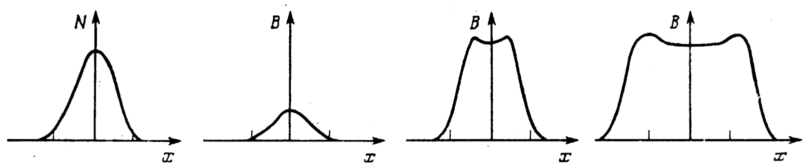

FIG. 8. Magnetic field distribution in a highly inhomogeneous ($k^2c^2/\omega_m^2 \gg 1$) plasma slab.

dition for the applicability of the linear theory for the excitation of small-scale ($L \lesssim c/\omega_p$) electromagnetic fields.

5. CONCLUSION

In this paper we have proposed a hydrodynamic model of an inhomogeneous and anisotropic plasma for the purpose of describing the excitation and dynamics of low-frequency ($\omega \ll \omega_p$) electromagnetic fields. An important distinction between this paper and preceding ones is that we have incorporated a plasma density variation, electron collisions, and a time variation of the anisotropic ionization source. On the basis of this model, we have predicted a new effect: an increase in the rate of excitation of electromagnetic fields under conditions such that two factors operate jointly. One of these factors is the variation of the plasma density, and the other is the anisotropy of the electron distribution, if the density-gradient vector and the anisotropy vector are not collinear. In contrast with the ordinary instability, which occurs only if there are initial seed perturbations, which subsequently grow exponentially, in the case at hand there is a power-law increase in the field amplitudes, starting at a zero initial level.

The excitation of low-frequency electromagnetic fields is most efficient in the case of small-scale density variations ($\lesssim c/\omega_p$). In the collisionless case, the plasma relaxation goes into a nonlinear regime over a time scale $\sim \omega_p^{-1}(c/v_T)$. A study of this nonlinear regime will require numerical simulation. In a dissipative plasma with a collision rate $\nu > \omega_p v_T/c$, the linear theory derived here draws a fairly comprehensive picture of the relaxation of low-frequency electromagnetic fields if plasma recombination is ignored or, equivalently, if the collision time is assumed to be small in comparison with the recombination time. The results derived here on the plasma relaxation describe excitation of electromagnetic fields with a spectrum bounded by a maximum frequency $\omega_{\max} \sim Q \ll \omega_p$ and a minimum frequency $\omega_{\min} \sim \nu$.

Estimates of the maximum magnetic field, $B \lesssim (4\pi n T)^{1/2}$, and the maximum electric field, $E \lesssim (\omega/\omega_p) B$, in the plasma indicate that the electron anisotropy energy could be converted into electromagnetic radiation more efficiently than in the case of the Weibel instability.[7,8]

We conclude with a discussion of the conditions under which the effect predicted here might be observed experimentally. A stimulated excitation of low-frequency electromagnetic fields in an inhomogeneous medium could be caused by ionizing x radiation. In this case the anisotropy of the electron energy distribution would result from the photoelectric effect, so the average electron energy in the direction transverse with respect to the propagation direction of the ionizing pulse would be roughly twice the average longitudinal energy. The x-ray source must generate an energy flux high enough to ionize the plasma over a time much shorter than the time scale (ν^{-1}) over which the photoelectrons would become isotropic:

$$Q \approx \sigma_{ph} q / \varepsilon_\gamma \gg \nu, \tag{5.1}$$

where σ_{ph} is the cross section for the photoelectric effect, q is the energy flux density of the x radiation, and ε_γ is the x-ray energy. The length of the radiation pulse must be shorter than the time scale ν^{-1} over which the electron distribution would become isotropic. If the pulse is instead long ($\gg \nu^{-1}$), it would become necessary to satisfy the inequality (5.1) at times $\lesssim \nu^{-1}$. In this case we would thus need a pulse with a steep leading edge. These requirements are right at the limit of present experimental capabilities. There is the possibility that the effect which we have been discussing here—the excitation of electromagnetic fields—may be realized in the laboratory in the not-too-distant future.

An excitation of low-frequency electromagnetic fields can also occur when a pulse of γ radiation passes through an inhomogeneous medium, in which case the atoms would be ionized by the Compton effect, and the energy of the electrons along the direction of the γ rays would be higher than their transverse energy. The efficiency with which the energy of the ionizing radiation is converted into an electromagnetic field would be lower in this case than in the case of x radiation, since the cross section for the Compton effect is smaller than the photoionization cross section.

We would also like to call attention to a possible manifestation of this effect under astrophysical conditions. We know that many astrophysical objects (pulsars, supernovae, etc.) are characterized by intense x-ray and γ-ray bursts. If the hard radiation of these bursts interacts with (for example) irregularities of gaseous nebulas with a length scale $\lesssim 1$ km, and if the leading edges of the pulses are steep enough to satisfy the inequality (5.1) at times $\sim \nu^{-1} \approx 0.01$–$0.1$ s, then long-wavelength electromagnetic radiation in the rf range ($\omega < 10^6\ \mathrm{s}^{-1}$) may result.

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[1)] See Ref. 6 regarding the conditions for the applicability of this model.

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