Energy exchange between light waves in media with bipolar response

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The interaction of waves with media having a diagonal bipolar nonlinearity is studied theoretically. The properties of parametric coupling which arises between waves having different frequencies when permittivity gratings are excited are discussed. The conditions of efficient energy exchange in the region of optical anisotropy of the crystal, including for a frequency-degenerate interaction, are found. In the general case weak waves are amplified in a certain range of pump intensities. Then the gain is also found to be limited. However, there exists an interaction geometry in which these restrictions are removed.

INTRODUCTION

It is well known (see, for example, Refs. 1–4) that the exchange of energy between light waves is possible only under certain conditions, which are determined by the conservation laws.

There are four known ways to avoid the conditions preventing energy exchange:[1] the response of the medium to the light waves must be nonlocal in time (1) or space (2); the waves undergo self-diffraction and high orders of diffraction appear (3); and three or more beams interact under conditions of spatial synchronism. In particular, the requirement that in isotropic media the interaction must be local in space and time means in practice that energy exchange between two oppositely propagating light beams is forbidden. This is also found to be valid for the case when the waves are elliptically polarized and induce birefringence in an isotropic medium.[5]

In Refs. 6–8 it was observed that stationary exchange of energy between two oppositely propagating waves having the same frequency becomes possible in media whose nonlinear response has certain properties (for example, in some photorefractive and liquid crystals). In Ref. 8 it was found that dissipation of the light energy can bring about such a nonlinearity. In this paper it is shown that such dissipation with a thermal nonlinearity mechanism can remove the anisotropy of the optical properties of the crystal and thereby fundamentally alter the formulation of the problem. The possibilities of efficient wave interaction (i.e., achievement of complete synchronism) in the "anisotropic" phase of a nonlinear crystal are also discussed. The main purpose of this work is to investigate in detail the so-called diagonally bipolar optical nonlinearity.

In Sec. 1 the diagonally bipolar nonlinearity is defined and the basic equations for oppositely propagating waves having different frequencies are presented. In Sec. 2 it is shown that for waves having the same frequency the energy flux is conserved. It is proved that the diagonal bipolar nonlinearity is not Lagrangian. In Secs. 3–5 different geometries of amplification of weak waves are investigated and the conditions when this is possible far from the "phase transition" point are found. In Sec. 6 the interaction for which the limitation of the pump intensity can be removed owing to excitation of permittivity gratings is discussed.

1. DIAGONALLY BIPOLAR OPTICAL NONLINEARITY

Let an ordinary and an extraordinary waves (o and e waves, respectively) propagate in opposite directions along the z-axis in an optically uniaxial medium (Fig. 1):

$$\mathbf{E}(z)=\mathbf{e}_x[E_1(z)\exp(-ik_1z)+E_3(z)\exp(ik_1z)] +\mathbf{e}_y[E_2(z)\exp(-ik_2z)+E_4(z)\exp(ik_2z)]. \qquad (1)$$

Here $\mathbf{e}_i$ are the unit vectors of a Cartesian coordinate system.

The slow z-dependence of the complex amplitudes of the electric fields $E_i(z)$ of the waves is determined by the nonlinear interaction in the medium. We assume that the optical axis $\mathbf{n}$ lies in the yz plane. Then $\mathbf{k}_2$ and $\mathbf{k}_1$ will be the wave vectors of the e and i waves respectively.

We shall define as follows the optical nonlinearity of the medium by the following relations:

$$\delta\varepsilon_{xy}=\delta\varepsilon_{yx}=0, \quad \delta\varepsilon_{xx}=C_0Q(z), \quad \delta\varepsilon_{yy}=C_eQ(z), \qquad (2)$$

where $\delta\varepsilon_{ik}$ are the permittivity perturbations produced in the medium by a change in some parameter Q of the medium; C_0 and C_e are real coupling constants:

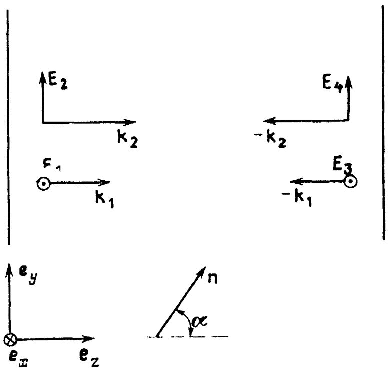

FIG. 1. The interaction geometry: The light waves propagate in opposite directions along the z-axis of a Cartesian coordinate system in an optically uniaxial medium: $\mathbf{k}_i$ are the wave vectors, $\mathbf{E}_i$ are the intensities of the electric fields of the waves ($\mathbf{E}_2$ and $\mathbf{E}_4$ are extraordinary waves and $\mathbf{E}_1$ and $\mathbf{E}_3$ are ordinary waves), $\mathbf{n}$ is the unit vector along the optic axis of the crystal. The optic axis lies in the yz-plane and makes an angle α with the z-axis.

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$$Q(z)=A_0\{|E_1(z)|^2+|E_3(z)|^2+b_0[E_1E_3^*\exp(-2ik_1z)+\text{c.c.}]\}+A_e\{|E_2(z)|^2+|E_4(z)|^2+b_e[E_2E_4^*\exp(-2ik_2z)+\text{c.c.}]\},\qquad(3)$$

where A_0, A_e, b_0, and b_e are real coefficients.

Such modulation occurs, for example, when the medium is heated, and then the difference of the coefficients A_0 and A_e is determined by the dichroism of photoabsorption and the coefficients b_0 and b_e determine how efficiently the stationary thermal gratings are recorded as compared with uniform heating ($b\leqslant 1$).

The condition (2) means that the nonlinearity is diagonal. We shall impose also the bipolarity condition

$$C_0C_e<0\,.\qquad(4)$$

As will be shown below, the most interesting phenomena occur precisely when this condition is satisfied.

Under the assumption $|q|=|k_2-k_1|\ll k_1,k_2$ the interaction of the waves will be described by truncated wave equations for the slowly varying amplitudes:

$$\begin{aligned}\frac{d}{dz}E_1&=-i\frac{\pi}{\lambda_1n_0}C_0[E_1(Q_0+b_0A_0I_3)+b_eA_eE_2E_4^*E_3e^{-2iqz}],\\ \frac{d}{dz}E_2&=-i\frac{\pi}{\lambda_2n_e}C_e[E_2(Q_0+b_eA_eI_4)+b_0A_0E_1E_3^*E_4e^{2iqz}],\\ \frac{d}{dz}E_3&=i\frac{\pi}{\lambda_1n_0}C_0[E_3(Q_0+b_0A_0I_1)+b_eA_eE_2^*E_4E_1e^{i2qz}],\\ \frac{d}{dz}E_4&=i\frac{\pi}{\lambda_2n_e}C_e[E_4(Q_0+b_eA_eI_2)+b_0A_0E_1^*E_3E_2e^{-2iqz}],\end{aligned}\qquad(5)$$

where $Q_0=A_0(I_1+I_3)+A_e(I_2+I_4)$ (here $I_i=|E_i|^2$) in the case of the thermal mechanism of nonlinearity[1)] is the change in the temperature of the medium; λ_1 and λ_2 are the wavelengths of the o and e waves, respectively; and, n_0 and n_e are the indices of refraction for the o and e waves, respectively. We thus presuppose that identically polarized waves have the same frequency.

The system of equations (5) describes the coupling resulting between two orthogonally polarized waves when two permittivity gratings are recorded in the medium as a result of the interference of identically polarized waves. Indeed, in the mechanism of nonlinearity (2) and (3) which we are studying each grating is read by waves having both polarizations. In the process the e-wave reading of an e-wave grating results in a change of phase of the oppositely propagating e wave, while e-wave reading of an o-wave grating gives rise to parametric coupling between the e and o waves.

2. CONSERVATION LAWS AND THE NONLAGRANGIAN CHARACTER OF THE NONLINEARITY

The system of equations (5) admits the conservation laws

$$\frac{d}{dz}(|E_1|^2-|E_3|^2)=0,\quad \frac{d}{dz}(|E_2|^2-|E_4|^2)=0,\qquad(6)$$

which show that photon exchange can occur only between waves having the same polarization. The expressions (6) actually express the laws of conservation of energy fluxes for both identically polarized waves and for the system as a whole.

From the system of equations (5) there follows for the number of photons propagating, for example, along the z-axis

$$\frac{c}{8\pi}\frac{d}{dz}\left(\frac{n_0|E_1|^2}{\omega_1}+\frac{n_e|E_2|^2}{\omega_2}\right)=\frac{i}{16\pi}(b_eA_eC_0-b_0A_0C_e)(E_1E_2^*E_3^*E_4e^{2iqz}-E_1^*E_2E_3E_4^*e^{-2iqz}).\qquad(7)$$

Obviously, if

$$b_eA_eC_0=b_0A_0C_e\qquad(8)$$

then the flux along the z-axis is conserved. In the case of waves having the same frequency this would correspond to prohibition of energy exchange between oppositely propagating waves. In order to understand the physical meaning of the condition (8) we shall construct the free-energy function $F(E,E^*)$, variation of which with respect to the intensity of the electric field determines the nonlinear induction:[4]

$$\delta D_i=\frac{\partial F}{\partial E_i^*}.\qquad(9)$$

For the present nonlinearity mechanism, substitution of the expression for δD_i, determined with the help of Eqs. (2) and (3), into Eq. (9) gives a system of equations for F:

$$\begin{aligned}\frac{\partial F}{\partial E_x^*}&=C_0A_0E_xE_x^*E_x+C_0A_eE_yE_y^*E_x,\\ \frac{\partial F}{\partial E_y^*}&=C_eA_0E_xE_x^*E_y+C_eA_eE_yE_y^*E_y.\end{aligned}\qquad(10)$$

It follows from Eqs. (10) that the condition for the existence of the function F

$$\frac{\partial^2F}{\partial E_x^*\partial E_y^*}=\frac{\partial^2F}{\partial E_y^*\partial E_x^*}$$

is satisfied only if the condition (8) is satisfied.

Thus the condition (8) is the condition for the existence of the free-energy function for our system, and in this case exchange of energy is forbidden by the conservation laws, as shown in Ref. 4. Under the bipolarity condition (4) the condition (8) for the free energy function F to be Lagrangian is not satisfied, and the expressions (6) and (7) show that in our case the law of conservation of momentum of the system is not satisfied.

Therefore our "non-Lagrangian" mechanism of nonlinearity can only be dissipative. In addition, the energy dissipation can be so small that it can be neglected in the wave equation.

We examine next the interaction of strong beams, which are assumed not to be depleted, with signal waves.

3. AMPLIFICATION OF AN OPPOSITELY PROPAGATING WEAK WAVE

Let the waves E_1 and E_2 be weak compared with the waves E_3 and E_4. Then Eqs. (5) can be simplified, retaining in them only the terms that are linear in E_1 and E_2:

$$\frac{d}{dz}E_1=-i\frac{\pi}{\lambda_1n_0}C_0[E_1(Q_0'+b_0A_0I_3)+b_eA_eE_2E_4^*E_3e^{-2iqz}],$$

$$\frac{d}{dz}E_2=-i\frac{\pi}{\lambda_2 n_e}C_e[E_2(Q_0'+b_eA_eI_4)+b_0A_0E_1E_3^*E_4e^{2iqz}],$$

$$\frac{d}{dz}E_3=i\frac{\pi}{\lambda_1 n_0}C_0E_3Q_0'\equiv iP_3E_3, \tag{11}$$

$$\frac{d}{dz}E_4=i\frac{\pi}{\lambda_2 n_e}C_eE_4Q_0'\equiv iP_4E_4,$$

where $Q_0' = A_0I_3 + A_eI_4$.

Thus for strong waves only a change in phase occurs:

$$E_3(z)=B_3\exp(iP_3z),\quad E_4(z)=B_4\exp(iP_4z), \tag{12}$$

where B_3 and B_4 are complex constants.

Substituting Eqs. (12) into Eqs. (11) and carrying out the transformation

$$E_1(z)=B_1(z)e^{i\Delta z},\quad E_2(z)=B_2(z)e^{-i\Delta z},$$
$$\Delta=-q+\frac{1}{2}(P_3-P_4), \tag{13}$$

we obtain equations describing the parametric coupling of the weak waves E_1 and E_2:

$$\frac{d}{dz}B_1+iP_1B_1+is_{12}B_2=0,$$
$$\frac{d}{dz}B_2+iP_2B_2+is_{21}B_1=0, \tag{14}$$

where

$$P_1=\Delta+\frac{\pi}{\lambda_1 n_0}C_0(Q_0'+b_0A_0I_3),$$
$$P_2=-\Delta+\frac{\pi}{\lambda_2 n_e}C_e(Q_0'+b_eA_eI_4),$$
$$s_{12}=\frac{\pi}{\lambda_1 n_0}C_0b_eA_eB_4^*B_3,$$
$$s_{21}=\frac{\pi}{\lambda_2 n_e}C_eb_0A_0B_4B_3^*. \tag{15}$$

The condition for the existence of the nontrivial solution $B_{1,2}\sim\exp(gz)$ of the system of equations (14) gives

$$g_{1,2}=-\frac{1}{2}i(P_1+P_2)\pm\frac{1}{2}i[(P_1-P_2)^2+4s_{12}s_{21}]^{1/2}. \tag{16}$$

Thus the character of the interaction is determined by the sign of the square root: If it is positive, only phase modulation of the waves occurs. If, however,

$$\frac{1}{4}(P_1-P_2)^2+s_{12}s_{21}<0, \tag{17}$$

then g acquires a real part which gives rise to amplitude modulation also, i.e., amplification of the weak waves.

The condition (17) can be satisfied only if $s_{12}s_{21}<0$, i.e., in the case of bipolar nonlinearity, $C_0C_e<0$. In the case of thermal nonlinearity the bipolarity condition (4) corresponds to the fact that when the crystal is heated the changes in the indices of refraction for orthogonally polarized waves have different signs. Such a situation, as we have already mentioned, is realized, for example, in nematic liquid crystals,[8] where the dielectric constant increases on heating for waves of the o type and decreases for waves of the e type, and as a result we have

$$C_0C_e\propto(\partial\varepsilon_0/\partial T)\cdot(\partial\varepsilon_e/\partial T)<0.$$

It is obvious that the gain should be highest in the case of synchronous interaction ($P_1-P_2=0$). This means that the following condition for the total intensity, proportional to the quantity $I=I_3+I_4$, should also be satisfied for the parameter $\eta=I_3/(I_3+I_4)$:

$$I_s=2(\lambda_1n_e-\lambda_2n_0)n_0n_e\left\{\eta n_e\lambda_2A_0\left[C_0\left(1+\frac{1}{2}b_0\right)-C_e\frac{\lambda_1n_0}{\lambda_2n_e}\right]+(1-\eta)n_0\lambda_1A_e\left[C_0\frac{\lambda_2n_e}{\lambda_1n_0}-C_e\left(1+\frac{1}{2}b_e\right)\right]\right\}^{-1}. \tag{18}$$

For fixed wavelengths the medium must have a certain combination of linear and nonlinear optical properties in order for the condition (18) to be satisfied: If the indices of refraction of the medium are such that $\lambda_1n_e-\lambda_2n_0>0$, then the medium must also be characterized by the quantities $C_0>0$ and $C_e<0$ (and vice versa). We emphasize that for given optical properties the condition (18) can be satisfied by choosing appropriate wavelengths.

Substituting Eq. (18) into Eq. (16) and using the expressions for the parameters s_{12} and s_{21} we obtain

$$g_m=\left[\frac{\pi^2}{\lambda_1\lambda_2}C_0|C_e|b_0b_eA_0A_e\right]^{1/2}f(\eta)(n_0n_e)^{-1/2}, \tag{19}$$

where

$$f(\eta)=[\eta(1-\eta)]^{1/2}I_s.$$

The expressions (18) and (19) simplify in some particular cases. Let us assume, first, that the medium has a small anisotropy of the linear and nonlinear optical properties and grating permittivity perturbations have little effect:

$$A_0\sim A_e=A,\quad C_0\sim|C_e|=C,\quad b_0\sim b_e=b\ll1. \tag{20}$$

Then

$$I_s=\frac{2}{AC}n_0^2n_e^2\frac{q_2-q_1}{q_2n_e^2+q_1n_0^2}, \tag{21}$$

where

$$q_{1,2}=2\pi n_{0,e}/\lambda_{1,2}.$$

Thus in this case I_s does not depend on the parameter η and g_m is maximum for $\eta=0.5$, i.e., when the strong beams have the same intensity. In the general case the maximum of the function $f(\eta)$ is determined by the relations between the parameters of the medium.

In reality there exists a region of intensities I for which exponential amplification of weak waves is possible. This region is determined from the condition (17) and has the form

$$\frac{1}{1+g_m/2q}<I/I_s<\frac{1}{1-g_m/2q}. \tag{22}$$

The parametric amplification phenomenon which we are studying can also occur in the case when, for example, the waves E_2 and E_3 are weak and the waves E_1 and E_4 are strong, i.e., opposite to each strong wave a weak wave propagates having the same polarization.

4. CONDITIONS FOR EXCHANGE OF ENERGY IN THE CRYSTALLINE (ANISOTROPIC) PHASE OF THE MEDIUM

If the waves have the same wavelength, i.e., $\lambda_1 = \lambda_2 = \lambda$, then the intensity I_s of in-phase interaction as well as the gain are proportional to the anisotropy of the index of refraction $n_a = n_e(\lambda) - n_0(\lambda)$. On the other hand, in the case of a thermal perturbation the heating of the medium by the light field $Q \approx AI$ changes the anisotropy by the amount

$$\delta n_a \approx \frac{1}{2} ACI \frac{n_e + n_0}{n_e n_0}. \tag{23}$$

Thus it follows from Eqs. (22) and (23) that for intensity of the order of $I_s(\lambda_1 = \lambda_2 = \lambda)$ the optical anisotropy of the medium can vanish as a result of heating, if n_e and n_0 correspond to the principal values of the permittivity tensor $n_e = \varepsilon_\parallel^{1/2}$, and $n_0 = \varepsilon_\perp^{1/2}$. For liquid crystals this corresponds to a transition into the isotropic state.

Here heating of the medium up to the point at which the optical anisotropy vanishes can be avoided by the following methods:

1. Rotation of the optic axis. If the optic axis makes an angle α with the z-axis (see Fig. 1), then the intensity I_s, determined by $n_e(\alpha)$, can be reduced by choosing an appropriate angle α to values when $\delta n_a < n_a$:

$$\cos\alpha > \frac{n_\perp}{n_\parallel + n_\perp}\left(1 + 2\frac{n_\parallel}{n_\parallel + n_\perp}\right)^{1/2}. \tag{24}$$

Figure 2 shows the region of these angles in a section of the surface of wave normals. This limits the maximum possible value of g_m:

$$g_m' = \frac{\pi b}{2\lambda}\frac{n_\perp(n_\parallel^2 - n_\perp^2)}{n_\parallel + 3n_\perp}. \tag{25}$$

2. Waves with different frequency. In order to show that in-phase interaction of waves is in principle possible in the region of optical anisotropy of the medium we shall study a nondispersive medium whose optic axis is oriented along the y-axis (i.e., $\alpha = \pi/2$). Then it follows from Eqs. (21) and (23)

$$1 + \frac{n_\parallel^2 - n_\perp^2}{n_\parallel^2 + 2n_\perp^2 + n_\parallel n_\perp} < \lambda_2/\lambda_1 < 1 + \frac{n_a}{n_\perp}. \tag{26}$$

As one can see from the inequalities (26), the longer one of the wavelengths the wider the range of possible frequencies of the other wave is.

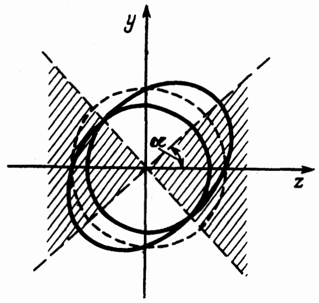

FIG. 2. A section of the surface of wave normals to the plane yz for positive uniaxial crystals. The dashed circle has a radius of $0.5(n_\parallel + n_\perp)$; the hatched region is the region where interaction is realized in the optically anisotropic phase.

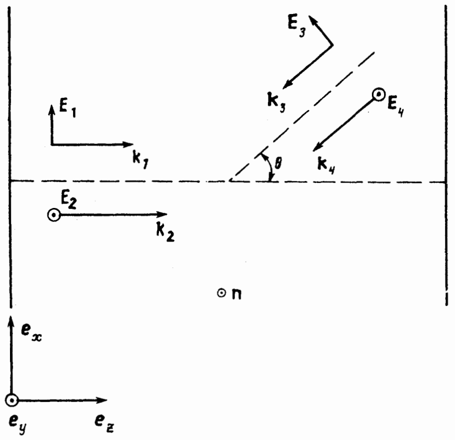

FIG. 3. The geometry of noncollinear interaction. The axis n of the crystal is perpendicular to the plane of incidence xz. $\mathbf{E}_{1,3}$ and $\mathbf{E}_{2,4}$ are waves of the ordinary and extraordinary types, respectively; $k_{1,3} = 2\pi n_\perp/\lambda$ and $k_{2,4} = 2\pi n_\parallel/\lambda$ are the corresponding wave vectors, λ is the wavelength of light in vacuum, and θ is the angle of refraction of the strong wave in the crystal.

5. INTERACTION OF NONCOLLINEAR WAVES

By studying the interaction of waves which are not strictly collinear it is easy to show that wave synchronism can also be realized in an anisotropic phase. This is also interesting from the viewpoint of dynamic holography, where the signal wave consists of a collection of plane waves which make different angles with the pump wave and the relative efficiency of the interaction between these components and the pump wave is important.

Of many possible geometries, for simplicity we shall study the geometry in which the period of the interference gratings is determined only by the angle of convergence of the waves. Namely, let the optic axis be oriented along the y-axis and let the wave vectors of all interacting waves lie in the xz-plane (Fig. 3). In this case the indices of refraction for the characteristic modes do not depend on the angle of refraction θ.

Then the complex vector of the total intensity of the electric field has the form

$$\mathbf{E} = \mathbf{e}_x[E_1\exp(-ik_\perp z) + E_3\cos\theta\cdot\exp(ik_\perp z\cos\theta + ik_\perp x\sin\theta)] + \mathbf{e}_y[E_2\exp(-ik_\parallel z) + E_4\exp(ik_\parallel z\cos\theta + ik_\parallel x\sin\theta)] + \mathbf{e}_z E_3\sin\theta\cdot\exp(ik_\perp z\cos\theta + ik_\perp x\sin\theta). \tag{27}$$

The expression (3) for the photoinduced change in the parameter Q of the medium is modified as follows:

$$Q = A_0[|E_1|^2 + |E_3|^2 + b_0(E_1E_3^*\cos\theta\cdot\exp\{-ik_\perp z(1+\cos\theta) - ik_\perp x\sin\theta\} + \text{c.c.})] + A_e[|E_2|^2 + |E_4|^2 + b_e(E_2E_4^*\exp\{-ik_\parallel z(1+\cos\theta) - ik_\parallel x\sin\theta\} + \text{c.c.})]. \tag{28}$$

Assuming that the waves E_3 and E_4 are strong and not depleted and neglecting the appearance of new waves (the volume-hologram approximation), the parametric-coupling equations can be derived from Eq. (14) taking into account, as follows from Eqs. (17) and (28), that 1) A_e is replaced by $A_\parallel$, 2) E_3 transforms into $E_3\cos\theta$ but the intensity $|E_3|^2$ in the expression for Q_0' does not change, 3) $q = (k_\parallel - k_\perp)\cos^2(\theta/2)$, and 4) P_3 and P_4 increase by the amount $1/\cos\theta$ owing to the increase in the effective thickness.

The expression for the pump intensity giving in-phase interaction

$$I_s=2n_a n_\| n_\perp(\cos\theta+\cos^2\theta)\{A_\perp\eta[C_o n_\|(1+\cos\theta+b_\perp\cos^3\theta)-C_e n_\perp(1+\cos\theta)]+A_\|(1-\eta)[C_o n_\|(1+\cos\theta)-C_e n_\perp(1+\cos\theta+b_\|\cos\theta)]\}^{-1} \tag{29}$$

depends in a very complicated manner on the angle θ. However this dependence simplifies significantly in the approximation determined by the conditions (20):

$$I_s=\frac{2}{AC}\frac{n_a n_\perp n_\|}{n_\|+n_\perp}\cos\theta. \tag{30}$$

The decrease in I_s as the angle θ increases results from the decrease in the wave vectors of the interference gratings.

The gain, in this case, is equal to

$$g_m=\frac{2\pi}{\lambda}bn_a[\eta(1-\eta)n_\| n_\perp]^{1/2}\cos^2\theta. \tag{31}$$

Thus the decrease in I_s as θ increases results, as expected, in a decrease of g_m. As we have already mentioned above, such a decrease in I_s can be employed to achieve in-phase interaction while preserving the optical anisotropy of the crystal (for liquid crystals—without achieving the isotropic phase). In the simplified case studied above, this minimum angle is determined by the expression

$$\cos\theta_{min}\simeq(n_\perp+n_\|)/(2n_\| n_\perp). \tag{32}$$

6. ENERGY EXCHANGE UNDER CONDITIONS OF LINEAR SYNCHRONISM

In the wave-interaction geometry studied above permittivity gratings are excited with wave vectors that differ as a result of the optical anisotropy. Because of this, energy exchange, which acquires a threshold character, occurs in a certain range of pump intensities, and this limits the maximum achievable gains.

It turns out that the interaction geometries for which the wave vectors of both gratings are equal in magnitude can be realized even with the help of two waves incident on the crystals. Namely, consider a crystal slab whose optic axis is oriented along **n** and let two elliptically polarized waves be incident in the *yz*-plane on the crystal slab symmetrically with respect to the optic axis (Fig. 4). Each wave initiates refracted waves of the *e* and *o* type, propagating at some angle δ with respect to one another. The *z*-axis is chosen in the direction of refraction of the weak *e* wave. In this case the condition of linear synchronism is satisfied ($\mathbf{k}_i$ are the wave vectors, neglecting nonlinear corrections)

$$\mathbf{k}_1-\mathbf{k}_2=\mathbf{k}_3-\mathbf{k}_4, \tag{33}$$

and the truncated equations in the inexhaustible-pump approximation assume the form

$$\frac{d}{dz}E_1=-i\frac{\pi}{\lambda n_e}C_e[E_1(Q_0'+b_eA_eI_3\cos^2\theta)+A_0b_0E_2E_4^*E_3\cos\theta],$$

$$\frac{d}{dz}E_2=-i\frac{\pi}{\lambda n_0\cos\delta}C_0[E_2(Q_0'+b_0A_0I_4)+A_eb_eE_1E_3^*E_4\cos\theta],$$

$$\frac{d}{dz}E_3=i\frac{\pi}{\lambda n_e\cos\theta}C_eQ_0'E_3, \tag{34}$$

$$\frac{d}{dz}E_4=i\frac{\pi}{\lambda n_0\cos(\theta-\delta)}C_0Q_0'E_4,$$

where θ is expressed in terms of the angle of refraction α_{ref} of the *e* waves by the relation $\theta=\pi+2\alpha_{\text{ref}}$.

The following transformations reduce the system of equations (34) to Eqs. (14):

$$P_3=\frac{\pi C_eQ_0'}{\lambda n_e\cos\theta},\quad P_4=\frac{\pi C_0Q_0'}{\lambda n_0\cos(\theta-\delta)},$$

$$P_1=+\Delta+\frac{\pi C_e}{\lambda n_e}(Q_0'+A_eb_eI_3\cos^2\theta),$$

$$P_2=-\Delta+\frac{\pi C_0}{\lambda n_0\cos\delta}(Q_0'+A_0b_0I_4), \tag{35}$$

$$\Delta=\frac{1}{2}(P_3-P_4),$$

$$s_{12}=\frac{\pi}{\lambda n_e}C_eA_0b_0B_4^*B_3\cos\theta,$$

$$s_{21}=\frac{\pi}{\lambda n_0\cos\delta}C_0A_eb_eB_3^*B_4\cos\theta.$$

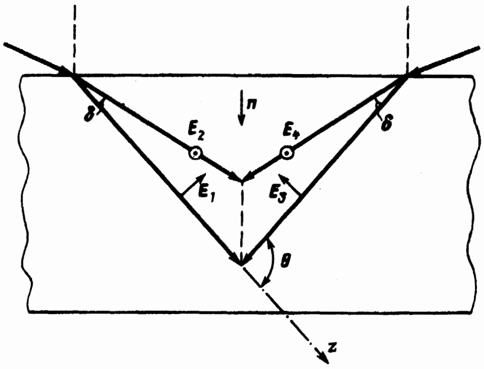

FIG. 4. Energy exchange under conditions of linear synchronism. Two elliptically polarized waves are incident on the crystal symmetrically relative to the optic axis (which is normal to the surface of the crystal). $\mathbf{E}_{1,3}$ are extraordinary waves and $\mathbf{E}_{2,4}$ are ordinary waves; the angle δ takes into account birefringence; θ is the angle of convergence of the waves; and, the *z*-axis is oriented along the direction of refraction of the weak *e*-type wave.

An important feature of this interaction geometry, resulting in the satisfaction of the condition of linear synchronism (33), is that synchronous energy exchange becomes impossible: P_1-P_2 cannot vanish owing to the condition of bipolarity of the nonlinearity. This is also obvious from physical considerations: Since the condition of synchronism is satisfied in the zeroth-order approximation in the intensities of the waves, the change induced in indices of refraction can only cause this condition to be violated.

At the same time, the condition of energy exchange—the condition that the square root in Eq. (16), together with Eq. (35), is negative—can be satisfied for a certain relation between the angle θ, the parameter η, and the material parameters of the medium ($\cos\delta\to1$):

$$\frac{\pi I}{\lambda n_0n_e\cos\theta}\{\eta A_e[C_en_0(1+\cos\theta+b_e\cos^3\theta)-C_0n_e(1+\cos\theta)]+(1-\eta)A_0[C_en_0(1+\cos\theta)-C_0n_e(1+\cos\theta+b_0\cos\theta)]\}$$

$$-2\frac{\pi}{\lambda}\left|\frac{\eta(1-\eta)}{n_0n_e}A_0A_eb_0b_eC_0C_eI^2\cos^2\theta\right|^{1/2}<0, \quad (36)$$

$$\frac{\pi I}{\lambda n_0n_e\cos\theta}\{\eta A_e[C_en_0(1+\cos\theta+b_e\cos^3\theta)-C_0n_e(1+\cos\theta)]$$
$$+(1-\eta)A_0[C_en_0(1+\cos\theta)-C_0n_e(1+\cos\theta+b_0\cos\theta)]\}$$
$$+2\frac{\pi}{\lambda}\left|\frac{\eta(1-\eta)}{n_0n_e}A_0A_eb_0b_eC_0C_eI^2\cos^2\theta\right|^{1/2}>0.$$

Under some simplifying assumptions it is easy to check that the system of inequalities (36) can be satisfied. However we did not perform any specific calculations because of their difficulty. It is important to note that no restrictions are imposed on the wave intensities and the gain can be increased, since it is proportional to the pump intensity.

CONCLUSIONS

Thus in this paper we investigated theoretically the interaction of light waves in media with diagonally bipolar nonlinearity. The characteristics of the interaction as a function of the geometric and material parameters were determined.

Among the possible processes, the parametric coupling of waves having different frequencies via the recording of static gratings is especially important. The advantage of recording static gratings is made evident in situations when the times over which the nonlinearity is established are longer than the characteristic beam time and for this reason traveling gratings are not recorded efficiently.

The most interesting feature is energy exchange occurring in the single-frequency case between two beams incident on a crystal, including both when the waves propagate in strictly opposite directions and in the stationary regime, thanks to the bipolarity of the medium. It is true that this is achieved with the interaction of four waves in the crystal, but there is an important difference from the traditional degenerate four-wave interaction. First of all, the amplification of the waves is exponential, including in the case of codirectional propagation. We also note the additional possibilities of achieving synchronism in our case by selection of the proper pump intensity and polarization and the arrangement of the anisotropy axis of the medium or the angle of convergence of the weak and strong beams.

In the general case the energy exchange which we discussed occurs in a certain range of pump intensities. However the situation when both the upper and lower limits on the intensity are removed can also be realized.

The interaction mechanism studied in this paper could find different applications in problems of opto-optical modulation, in particular, phase conjugation schemes (including in the traditional four-wave mixing of waves) for amplification of weak plane waves with the help of a strong beam with a distorted front, etc.

[1)] For example, the same liquid-crystalline materials have a quite high thermal nonlinearity owing to the large values of $\partial n/\partial T \approx (3\text{–}10)\cdot 10^{-4}$ deg^{-1}.

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