Collective excitations in planar 2*D* phase of superfluid ^{3}He

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The complete spectrum of collective excitations in the planar 2*D* phase of ^{3}He is calculated for the first time, using a functional-integration technique developed for these purposed by Brusov and Popov (*Superconductivity and Collective Properties of Quantum Liquids* [in Russian], Nauka, 1988, p. 216). The spectrum contains the *A*-phase spectrum modes previously known{P. N. Brusov and V. N. Popov, Zh. Eksp. Teor. Fiz. **79**, 1871 (1980) [Sov. Phys. JETP **52**, 714 (1980)]} [Goldstone (gd), clapping (cl), pair-breaking (pb)] as well as new modes existing only in the 2*D* phase [P. N. Brusov and M. V. Lomakov, Physica (Utrecht) **B 165–166**, 635 (1990)]. Interestingly, the frequencies of the cl and pb modes, for which a linear Zeeman effect exists in the *A* phase, are independent of the magnetic field in the 2*D* phase. The collective-mode energies turn out to be complex, since the gap in the single-particle spectrum vanishes in a selected direction (as in the *A* phase).

INTRODUCTION

The collective-excitation spectrum was investigated in the superfluid *A*-phase[1] and *B*-phase[2] of ^{3}He. The superfluid phases of ^{3}He also include a 2*D*-phase,[3] known as planar and having an order parameter

$$c_{ia}^{(0)}(p)=c(\beta V)^{1/2}\delta_{p0}(\delta_{i1}\delta_{a1}+\delta_{i2}\delta_{a2}).$$

Here C is a constant, $\beta = T^{-1}$, V is the system volume, $p = (k,\omega)$ is the 4-momentum, and i and a are the vector and isotopic indices. This phase has not yet been observed in experiment, but its existence under various conditions was deduced by many researchers. In particular, Alonso and Popov[4] predicted a phase transition from the *B*- to the 2*D*-phase at $H = H_c$ and proved the stability of the 2*D*-phase to small perturbations for $H > H_c$. Fujita *et al.*,[5] by considering the *B*-phase in a semibounded space, have shown that a 2*D*-phase is realized on the boundary: in this situation it is energetically more favored than the *A*-phase. (Collective excitations for this case were investigated by Brusov and Bukshpun.[6]) One of the possible explanations of the double splitting of the sq (squashing) mode in the *B*-phase, recently observed in experiment by Ketterson's group,[7] is an assumed existence on the cell boundary of a 2*D* phase one of the collective modes of which leads to the appearance of a second peak in ultrasound absorption.

These examples suffice, in our opinion, to understand the importance of investigating the planar 2*D*-phase, and particularly the spectrum of its collective excitations. We calculate this spectrum below by the path-integration method.

1. HYDRODYNAMIC-ACTION FUNCTIONAL FOR THE PLANAR 2*D* PHASE OF ^{3}He

All the properties of the model ^{3}He system obtained by successive path integration over the "fast" and "slow" Fermi fields are determined by the functional S_h of the hydrodynamic action, given by

$$S_h = \frac{1}{g}\sum_{p,i,a} c_{ia}^{+}(p)c_{ia}(p) + \frac{1}{2}\ln\det[\hat{M}(c^{+},c)/\hat{M}(c^{+(0)},c^{(0)})]. \tag{1.1}$$

Here $c_{ia}(p\}$ is the Fourier transform of the Bose field $c_{ia}(x,\tau)$ describing the Cooper pairs of the quasifermions on the Fermi surface, the operator $\hat{M}$ is given by

$$\hat{M}=\begin{pmatrix} Z^{-1}(i\omega-\xi+\mu H\sigma_3)\delta_{p_1p_2} & \dfrac{(n_{1i}-n_{2i})}{(\beta V)^{1/2}}\sigma_a c_{ia}(p_1+p_2) \\ -\dfrac{(n_{1i}-n_{2i})}{(\beta V)^{1/2}}\sigma_a c_{ia}^{+}(p_1+p_2) & Z^{-1}(-i\omega+\xi+\mu H\sigma_3)\delta_{p_1p_2} \end{pmatrix}, \tag{1.2}$$

where $\xi = c_F(k-k_F)$, $n_i = k_i/k_F$, H is the magnetic field and μ is the magnetic moment of the quasiparticle, σ_a ($a = 1,2,3$) are two-by-two Pauli matrices, and $\omega = (2n+1)\pi T$ are the Fermi frequencies. The negative constant g in (1.1) is proportional to the scattering amplitude of two quasifermions near the Fermi sphere under the assumption that the amplitude is equal to $g(\mathbf{k}_1-\mathbf{k}_2, \mathbf{k}_3-\mathbf{k}_4)$, where $\mathbf{k}_1$ and $\mathbf{k}_2$ are the momenta of the incident fermions, and $\mathbf{k}_3$ and $\mathbf{k}_4$ are those of the outgoing ones.

Expanding the functional (1.1) in the Ginzburg–Landau region $T_c - T \ll T_c$ in powers of the fields c and c^+ we obtain[1]

$$S_h = -\frac{20k_F^2(\Delta T)^2\beta V}{21\xi(3)c_F}\Pi,$$

where

$$\Pi=-\mathrm{tr}\,AA^{+}+\nu\,\mathrm{tr}\,A^{+}AP+(\mathrm{tr}\,A^{+}A)^{2}+\mathrm{tr}\,AA^{+}AA^{+} + \mathrm{tr}\,AA^{+}A^{*}A^{T}-\mathrm{tr}\,AA^{T}A^{*}A^{+}-\frac{1}{2}\mathrm{tr}\,AA^{T}\,\mathrm{tr}\,A^{+}A^{*}.$$

Here

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$$\nu=7\xi(3)\mu^2H^2/4\pi^2T_c\Delta T,$$

P is the projector on the third axis along which the field is directed. Minimizing Π, we obtain the matrix A that determines the condensate density. The equation $\delta\pi = 0$ or

$$-A+\nu AP+2(\operatorname{tr} A^+A)A+2AA^+A+2A^*A^TA$$
$$-2AA^TA^*-A^*\operatorname{tr} AA^T=0$$

has several nontrivial solutions corresponding to the superfluid phases. One of them has an order parameter

$$\frac{1}{2}\begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix}.$$

This is in fact the planar $2D$-phase. Calculation of the second variation yields

$$\delta^2\Pi=(\nu-{}^1/_2)u_{33}{}^2+(\nu+{}^1/_2)v_{33}{}^2+\nu(u_{13}{}^2+u_{23}{}^2)$$
$$+(\nu+2)(v_{13}{}^2+v_{23}{}^2)+{}^1/_2[3u_{11}{}^2+3u_{22}{}^2+2u_{11}u_{22}+(u_{12}+u_{21})^2]$$
$$+{}^1/_2[3v_{12}{}^2+3v_{21}{}^2-2v_{12}v_{21}+(v_{11}-v_{22})^2].$$

Here

$$u_{ia}=\operatorname{Re} A_{ia},\quad v_{ia}=\operatorname{Im} A_{ia}.$$

For $\nu < 1/2$ the $\delta^2\Pi$ variation is of alternating sign, while for $\nu > 1/2$ it is non-negative. This means that the $2D$-phase is stable in a magnetic field $H > H_c = [\pi\mu 2T_c\Delta T/7\xi(3)]^{1/2}$. As indicated in the introduction, Alonso and Popov[4] have shown that at $H = H_c$ a phase transition takes place from the B- to the $2D$-phase.

As $T\to 0$, the functional (1.1) must be expanded in terms of the fluctuations of the fields $c_{ia}(p)$ above their condensate values $c_{ia}{}^{(0)}(p)$. In this temperature region the Dose spectrum of the system is determined in first-order approximation by the quadratic part of the functional S_h (1.1), a part obtained via the shift $c_{ia}(p)\to c_{ia}(p) + c_{ia}^{(0)}(p)$, where $c_{ia}^{(0)}(p) = c(\beta V)^{1/2}\delta_{p0}(\delta_{i1}\delta_{a1} + \delta_{i2}\delta_{a2})$ and is obtained from the equation $\det Q = 0$, where Q is the matrix of a quadratic form. The quadratic part of S_h is given by

$$\sum_{p,i,a} c_{ia}{}^+(p)c_{jb}(p)\Big[-\delta_{ij}\delta_{ab}g^{-1}$$
$$+\frac{2Z^2}{\beta V}\sum_{p_1+p_2=p} n_{1i}n_{1j}\operatorname{tr}(A_1-B_1\sigma_3)\sigma_a(A_3+B_2\sigma_3)\sigma_b$$
$$+\sum_p c_{ia}{}^+(p)c_{jb}{}^+(-p)\frac{Z^2}{\beta V}$$
$$\times\sum_{p_1+p_2=p} n_{1i}n_{1j}\operatorname{tr}(C_1\sigma_1-D_1\sigma_2)\sigma_a(C_2\sigma_1+D_2\sigma_2)\sigma_b$$
$$+\sum_p c_{ia}(p)c_{jb}(-p)\frac{Z^2}{\beta V}$$
$$\times\sum_{p_1+p_2=p} n_{1i}n_{1j}\operatorname{tr}(\bar C_1\sigma_1-\bar D_1\sigma_2)\sigma_a(\bar C_1\sigma_2+\bar D_2\sigma_2)\sigma_b, \qquad (1.2)$$

where the coefficients are given by

$$\begin{aligned}A&=A_1=M^{-1}[-(i\omega-\xi)(\omega^2+\xi^2+\mu^2H^2+\Delta^2)+2\xi\mu^2H^2],\\ B&=-B_1=M^{-1}\mu H[(\omega^2+\xi^2+\mu^2H^2+\Delta^2)-2\xi(i\omega+\xi)],\\ C&=\bar C_1=M^{-1}\Delta_0[n_1(\omega^2+\xi^2+\mu^2H^2+\Delta^2)-2i\xi\mu Hn_2],\\ D&=\bar D_1=M^{-1}\Delta_0{}^2[n_2(\omega^2+\xi^2+\mu^2H^2+\Delta^2)+2i\xi\mu Hn_1],\\ M&=(\omega^2+\xi^2+\mu^2H^2+\Delta^2)^2-4\xi^2\mu^2H^2,\\ \Delta&=\Delta_0\sin\theta.\end{aligned} \qquad (1.4)$$

The equation for the gap in the $2D$-phase is

$$g^{-1}=\frac{2Z^2}{\beta V}\sum_p\frac{\sin^2\theta}{\omega^2+\xi^2+\Delta^2}. \qquad (1.5)$$

2. SPECTRUM OF COLLECTIVE MODES

After calculating the quadratic-form coefficients (1.2) by taking the trace and replacing g with the aid of Eq. (1.4), we obtain from the equation $\det Q = 0$ the following equations for the collective-mode spectrum

$$\int_0^1 dx(1-x^2)(1+4c)J(c)=0;\quad u_{11}-u_{22}\pm(v_{12}-v_{21}), \qquad (2.1)$$

$$\int_0^1 dx(1-x^2)(1+2c)J(c)=0;\quad u_{11}+u_{22}\pm(v_{12}+v_{21}),$$
$$v_{11}+v_{22}\pm(u_{12}+u_{21}), \qquad (2.2)$$

$$\int_0^1 dx(1-x^2)J(c)=0;\quad v_{11}-v_{22}\pm(u_{12}-u_{21}), \qquad (2.3)$$

$$\int_0^1 dx\, x^2[(1+2c)J(c)-1]=0;\quad u_{31}\pm v_{32},\quad v_{31}\pm u_{32}, \qquad (2.4)$$

$$\int_0^1 dx\, x^2[(1+4c_+)J(c_+)+(1+4c_-)J(c_-)-2]=0;\quad u_{33}, \qquad (2.5)$$

$$\int_0^1 dx\, x^2[J(c_+)+J(c_-)-2]=0;\quad v_{33}, \qquad (2.6)$$

$$\int_0^1 dx(1-x^2)[(1+4c_+)J(c_+)+(1+4c_-)J(c_-)]=0,\quad u_{13},u_{23}, \qquad (2.7)$$

$$\int_0^1 dx(1-x^2)[J(c_+)+J(c_-)]=0,\quad v_{13},v_{23}. \qquad (2.8)$$

Here

$$J(c)=(1+4c)^{-1/2}\ln\frac{(1+4c)^{1/2}+1}{(1+4c)^{1/2}-1},$$
$$c_\pm=\frac{\Delta_0{}^2(1-x^2)}{\omega^2+[c_F(nk)\pm 2\mu H]^2},\qquad c=\frac{\Delta_0{}^2(1-x^2)}{\omega^2+c_F{}^2(n,k)^2},$$
$$u_{ia}=\operatorname{Re}(c_{ia}),\quad v_{ia}=\operatorname{Im}(c_{ia}).$$

Let us examine Eqs. (2.1)–(2.8) at zero momenta ($k = 0$) of the collective excitations. In this case Eqs. (2.1)–(2.3) coincide with those obtained earlier by Brusov and Popov for the A phase without a magnetic field,[2] while Eqs. (2.6)–(2.8) go over into the aforementioned Brusov-Popov equation for an A-phase without a magnetic field following the substitution $\omega^2 + 4\mu^2H^2\to\omega^2$. These equations can thus be solved by using the results of Ref. 2. Finding also the roots of Eqs. (2.4) and (2.5), we obtain the following result of the spectrum of the collective modes at $k = 0$, as listed in Table I.

Thus, the spectrum of a planar $2D$ phase in a magnetic field contains modes similar to those in the A phase without a magnetic field, as well as a number of new modes. The former consist of six gd modes, four cl modes, and two pb modes. Two quasigoldstone (qgd) modes and two quasi-

TABLE I.

No.	Type	Frequency	Variables
6	*gd*	$E=0$	$v_{11}+v_{22}\pm(u_{12}+u_{21}),\ u_{31}\pm v_{32},\ u_{32}\pm v_{31}$
4	*cl*	$E=(1,17-0,13i)\,\Delta_0$	$u_{11}+u_{22}\pm(v_{12}+v_{21}),$ $u_{11}-v_{22}\pm(u_{12}-u_{21})$
2	*pb*	$E=(1,96-0,31i)\,\Delta_0$	$u_{11}-u_{22}\pm(v_{12}-v_{21})$
2	*qgd*	$E=2\mu H$	$u_{13},\ u_{23}$
2	*qpb*	$E^2=(1,96-0,31i)^2\Delta_0{}^2+4\mu^2H^2$	$v_{13},\ v_{23}$
1	*	$E^2=(0,518)^2\Delta_0{}^2+4\mu^2H^2$	u_{33}
1	*	$E^2=(0,495)^2\Delta_0{}^2+4\mu^2H^2$	v_{33}

Notation: *N*—number of modes of a given type; the types as designated as follows: gd—goldstone, cl—clapping, pb—pairbreaking, qgd—quasigoldstone, qpb—quasipairbreaking. Asterisks mark modes whose frequencies depend on the magnetic field.

pairbreaking (qpb) modes are obtained from the gd and pb modes respectively by substituting $E^2\to E^2-4\mu^2H^2$. The gap in the qgd-mode spectrum is $\sim 2\,\mu H$. Finally, we obtained two new modes having no analogs in the *A*-phase. They correspond to the variables u_{33} and v_{33}, are not degenerate, and the difference between their frequencies is small. Interestingly, whereas for the cl and pb modes there exists in the *A* phase a linear Zeeman effect[9] (threefold splitting in a magnetic field), the frequencies of these modes in the 2*D* phase are independent of the magnetic field, while the energies of the qpb modes and of the two "new" modes are quadratic in the field. Note also that the energies of all the nonphonon modes, except the two "new" ones, have imaginary parts due, just as in the *A*-phase, to the vanishing of a Fermi-spectrum gap in a special direction (that of the magnetic field). The frequencies of all the nonphonon modes of the spectrum turn out to be complex, in view of the possible decay of the collective excitations into the initial fermion (owing to the vanishing of the Fermi-spectrum gap along the field direction). Just as in the *A* and *B* phases, collective modes can be excited in the 2*D* phase in ultrasound and NMR experiments.

Note that notwithstanding some similarity between the spectra of the *A* and 2*D* phases, they also have substantial differences that can possibly help identify the 2*D* phase. Just as in the latter, there exist some nonphonon modes absent from the *A* phase (and also from the *B* phase), and the behavior of the spectrum (and even of the analog modes) in the 2*D* phase and in the *A* phase is quite different: In the *A* phase we have a linear splitting of the pb and cl modes, while in the 2*D* phase one part of the spectrum is independent of the field, whereas the other part has a quadratic field dependence.

After completing the spectrum calculation the authors have learned that the excitation spectrum in the 2*D*-phase was studied by Hirashima *et al.*,[8] who, however, considered a 2*D* phase without a magnetic field. Since the 2*D* phase is stable only for $H>H_c$, the meaning of their calculations is not clear. Obviously, they could not obtain six collective modes with frequencies dependent on the magnetic field. Comparing nonetheless our results with those of Ref. 9 we note the following:

1. The main conclusions of both studies, that the 2*D*-phase spectrum coincides in part with the *A*-phase spectrum, but modes typical of the 2*D* phase are present and are close to one another.

2. The correspondence between that fraction of the modes which is the same in both phases as in the *A*-phase spectrum investigations by the kinetic-equation[10] and path-integration[11] methods. A frequency $\omega_{\rm cl}=1.23\Delta_0(T)$ was thus obtained in Ref. 9 for the cl-mode, as against $\omega_{\rm cl}=1.17\Delta_0(T)$ in the present paper, in much better agreement with the experiments in the *A*-phase (see Ref. 12 and the citations therein). The reason is that we have taken into account the collective-mode damping due to decay of Cooper pairs in view of the vanishing of the Fermi-spectrum gap (see Ref. 12 for details).

3. In Ref. 9 one mode was obtained, typical only of the 2*D* phase and having an energy somewhat lower than that of the super-flapping (sfl) mode at all temperatures. This new mode is due to spin waves with a coupling coefficient $O(k^2)$. It is neither resonant nor diffuse. Note that in our *A*-phase model[11] we obtain not sfl modes but additional Goldstone modes whose appearance is due to the presence of latent symmetry. As noted above, we have obtained in the present paper, in a magnetic field, two modes that are indicative only of the 2*D* phase. Their frequencies are close to one another and depend on the field.

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[1] P. N. Brusov and V. N. Popov, *Superfluidity and Collective Properties of 9 Quantum Liquids* [in Russian], Nauka, Moscow (1988).
[2] P. N. Brusov and V. N. Popov, Zh. Eksp. Teor. Fiz. **79**, 1871 (1980) [Sov. Phys. JETP **52**, 714 (1980)].
[3] P. N. Brusov and M. V. Lomakov, Physica (Utrecht) **B 165–166**, 635 (1990).
[4] V. Alonso and V. N. Popov, Zh. Eksp. Teor. Fiz. **73**, 1445 (1977) [Sov. Phys. JETP **46**, 760 (1977)].
[5] T. Fujita, M. Makahara, T. Ohmi *et al.*, Progr. Theor. Fiz. **64**, 396 (1980).
[6] P. N. Brusov and M. M. Bukshpun, Physica (Utrecht) B, **165–166**, 685 (1990).
[7] Z. Zhao, S. Adenwalla, M. S. Shil, B. B. Katterson, and B. K. Sarma, Physica B (Utrecht) **166–167**, (1990).
[8] P. N. Brusov, M. O. Nasten'ka, T. V. Filatova-Novoselova, M. V. Lomakov and V. N. Popov, Zh. Eksp. Teor. Fiz. **99**, 1495 (1991) [Sov. Phys. JETP **72**, 835 (1990)].
[9] D. S. Hirashima and H. Namizawa, Progr. Theor. Phys. **74**, 400 (1985).
[10] P. Wolfle, Phys. Rev. Lett. **37**, 1279 (1976).
[11] P. N. Brusov and V. N. Popov, Zh. Eksp. Teor. Fiz. **79**, 1871 (1980) [Sov. Phys. JETP **52**, 714 (1980)].
[12] P. N. Brusov, J. Low Temp. Phys. **82**, 31 (1991).

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