Electron tunneling in quantum wires

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The tunneling of an electron in two quantum wires separated by a potential barrier is examined theoretically. The electron moves ballistically under the influence of a longitudinal field. If the electron is localized in the first quantum wire, there is a nearly unit probability that it will go into the second wire. In a strong electric field, an electron does not make such a transition.

1. INTRODUCTION

Electron transport in real space has attracted considerable interest recently. Hess *et al.*[1] have proposed a mechanism for establishing a negative differential resistance in $GaAs/Al_xGa_{1-x}As$ heterostructures, with the electric field directed parallel to the planes in which the layers touch each other. That mechanism is based on thermionic emission of hot electrons from the GaAs layer, with a high carrier mobility, into the $Al_xGa_{1-x}As$ layer, which has a dopant and a low carrier mobility. Coleman *et al.*[2] found experimental confirmation of this mechanism. The spatial transport of electrons had a diffusive nature in their study. Many of the problems associated with the high expenditure of energy and heating can be avoided in the case of quantum ballistic transport or quasiballistic transport of electrons in real space.[3]

For a quantum system consisting of two potential wells, the wave functions of the ground states do not overlap if the energy levels are far apart. If a change in some parameter of the system causes the energy levels to move closer together, the states cease to be localized in each of the wells, and spatial tunneling of an electron becomes possible. This effect was studied experimentally by Kirchoefer *et al.*[4] in a two-level superlattice structure. A negative differential resistance was found, and it was shown that an analog of the Gunn effect in real physical space could occur. Resonant tunneling between states in two different channels can be utilized to control parallel transport and conductance in a field-effect transistor.[5] Resonant tunneling in an isolated quantum well was studied by Kastalsky *et al.*[6] Takagaki *et al.*[7] have observed that the electron transition probability depends on the geometric shape of the channel and that the transport in quantum wires is not local.

In the present paper we take a theoretical look at the tunneling of an electron in two quantum wires separated by a potential barrier. All the results below can be generalized to the case of two-dimensional heterostructures.

2. FORMULATION OF THE TIME-DEPENDENT TUNNELING PROBLEM; SOLUTION METHOD

We consider two quantum wires, 1 and 2, separated by potential barrier 3 (Fig. 1). The x axis runs along the wires; the cross section of the overall structure is a square. The y and z axes run along the sides of this square. The motion of the electron is localized inside the walls of the infinite rectangular region 1 + 3 + 2. This system could be a heterostructure made from $Al_xGa_{1-x}As$ with different aluminum concentrations in the different regions. The effective mass of the electron is different in the different wires. For convenience in the calculations, we adopt a system of units with $\hbar = 1$ and in which the side of the square in the cross section of the quantum wires is unity. The dynamics of an electron is described by the time-dependent Schrödinger equation

$$i\frac{\partial\Psi}{\partial t} = -\frac{1}{2}\operatorname{div}\frac{\operatorname{grad}\Psi}{m} + U\Psi. \tag{1}$$

It is assumed that the electron is initially localized in region 1 and that a uniform, time-dependent electric field $E(t)$ is applied along the x axis. Our purpose is to learn about the time-dependent tunneling of an electron through the potential barrier (hatched region 3).

In the system of units we have adopted, the potential energy can be written

$$U(t, x, y, z) = -xE(t) + V_y(y) + V_z(z). \tag{2}$$

Here

$$V_y(y) = \begin{cases} +\infty, & y \leqslant 0 \\ V_1, & 0 < y \leqslant b_1 \\ V_3, & b_1 < y \leqslant b_2, \\ V_2, & b_2 < y \leqslant 1 \\ +\infty, & 1 < y \end{cases} \quad V_z(z) = \begin{cases} +\infty, & z \leqslant 0 \\ 0, & 0 < z \leqslant 1. \\ +\infty, & z > 1. \end{cases} \tag{3}$$

We set $V_1 = 0$; then V_2 and V_3 are constants, which are found from the gaps in the conduction band. The effective mass $m = m(y)$ is a piecewise-constant function. Since it is independent of the coordinate z, the dimensionality of the problem can be lowered. For this purpose we seek a solution of the Schrödinger equation in the form

$$\Psi(t, x, y, z) = \psi(t, x, y)\sin \pi z, \tag{4}$$

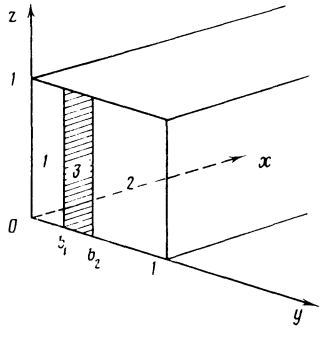

FIG. 1. Schematic diagram of two quantum wires, 1 and 2, separated by a potential barrier, 3.

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where the function ψ satisfies the equation

$$2im\frac{\partial\psi}{\partial t}+\frac{\partial^2\psi}{\partial x^2}+m\frac{\partial}{\partial y}\left(\frac{1}{m}\frac{\partial\psi}{\partial y}\right)+\{2m[xE(t)-V_y]-\pi^2\}\psi=0. \tag{5}$$

We assume that at the time $t=0$ there is a wave packet which is bounded along the x direction; i.e., $|\psi(0,x,y)|\to 0$ as $|x|\to\infty$. To determine the time evolution of the function ψ, we use a method similar to that which has been used[8] to study the tunneling between subbands in a uniform electric field. We write the wave function as a series in the set $\{F_n(y)\}$, which is complete on the interval [0, 1], and as a Fourier integral in the variable x:

$$\psi(t,x,y)=\int_{-\infty}^{\infty}dk_0\sum_n C_n(t,k_0)F_n(y)\times\exp\left[i\int_0^t E(t)x\,dt+ik_0x-i\int_0^t\varepsilon_n(t,k_0)dt\right]. \tag{6}$$

Here k_0 is the wave number at the time $t=0$. Substituting (6) into (5), we find

$$\int_{-\infty}^{\infty}dk_0\sum_n\left\{2im\left(C_n\frac{\partial F_n}{\partial t}+F_n\frac{\partial C_n}{\partial t}\right)+C_n\left[m\frac{\partial}{\partial y}\left(\frac{1}{m}\frac{\partial F_n}{\partial y}\right)+(2m(\varepsilon_n-V_y)-\pi^2-k^2)F_n\right]\right\}\exp\left\{i\int_0^t(Ex-\varepsilon_n)dt+ik_0x\right\}=0. \tag{7}$$

We require that the expression in square brackets vanish:

$$m\frac{\partial}{\partial y}\left(\frac{1}{m}\frac{\partial F_n}{\partial y}\right)+[2m(\varepsilon_n-V_y)-\pi^2-k^2]F_n=0, \tag{8}$$

$$k=k_0+\int_0^t E(t)dt. \tag{9}$$

At a fixed value of k, Eq. (8) is a steady-state one-dimensional Schrödinger equation for an electron in a quantum well with a potential

$$V(y)=V_y+\frac{k^2+\pi^2}{2m(y)}$$

and the boundary conditions

$$F_n(0)=F_n(1)=0. \tag{10}$$

Consequently, $\{F_n\}$ is a complete system of eigenfunctions corresponding to the discrete spectrum of the real eigenvalues $\{\varepsilon_n\}$. The time dependence enters F_n and ε_n through the wave number $k(t)$ [see (9)].

Using (8), and requiring that expression (7) vanish, we find, for each k_0,

$$\sum_n\left(C_n\frac{\partial F_n}{\partial t}+F_n\frac{\partial C_n}{\partial t}\right)\exp\left(-i\int_0^t\varepsilon_n\,dt\right)=0. \tag{11}$$

Multiplying this equation by

$$\bar{F}_l\exp\left(i\int_0^t\varepsilon_l\,dt\right)$$

(the superior bar means the complex conjugate), integrating the resulting expression over y, and making use of the orthogonality condition

$$\int_0^1 F_n(y)\bar{F}_l(y)dy=\delta_{nl}, \tag{12}$$

we find a system of ordinary differential equations which determines the time evolution $C_n(t)$ at a fixed k_0:

$$\frac{dC_n}{dt}=\sum_l W_{nl}C_l\exp\left[i\int_0^t(\varepsilon_n-\varepsilon_l)dt\right], \tag{13}$$

$$W_{nl}=-\int_0^1\frac{\partial F_l}{\partial t}\bar{F}_n\,dy, \tag{14}$$

$$C_n(0)=C_{0,n}(k_0). \tag{15}$$

The initial values $c_{0,n}(k_0)$ depend on the value of the wave function at $t=0$. From (6) we find

$$C_{0,n}(k_0)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}dx\int_0^1 dy\,\bar{F}_n(y,k_0)\exp(-ik_0x)\psi(0,x,y). \tag{16}$$

3. RESULTS OF A NUMERICAL ANALYSIS

A study was made of a heterostructure consisting of two quantum wires, 1 and 2, separated by a potential barrier 3 (Fig. 1). Region 1 is pure GaAs; region 3 is $Al_{0.3}Ga_{0.7}As$; and region 2 is $Al_{0.02}Ga_{0.98}As$. The energy at the bottom of the Γ valley of the conduction band and the effective mass for the alloy $Al_xGa_{1-x}As$ were found from

$$E_g^{\Gamma}(x)=1{,}425+1{,}155x+0{,}37x^2\ \text{[eV]},$$
$$m/m_e=0{,}063+0{,}087x.$$

The overall dimension of the cross section of the quantum wires (the unit used in converting lengths to dimensionless form) is 40 nm. The dimensionless coordinates of the layers are $b_1=0.306$ and $b_2=0.403$. The longitudinal electric field of 100 V/cm is $E=0.5284$ in dimensionless units. The other dimensionless parameters for the various regions are shown in Table I.

These calculations require knowledge of the eigenfunctions F_n and eigenvalues ε_n of boundary-value problem (8), (10). Since Eq. (8) is an ordinary second-order differential equation with piecewise-constant coefficients, we seek a solution in the following form:

$$F=B_1\exp(i\lambda_1 y)+B_2\exp(-i\lambda_1 y),\quad \lambda_1=[2m_1(\varepsilon-V_1)-\pi^2-k^2]^{1/2}, \tag{17}$$

for region 1,

$$F=B_3\exp(\lambda_3 y)+B_4\exp(-\lambda_3 y),\quad \lambda_3=[2m_3(V_3-\varepsilon)+\pi^2+k^2]^{1/2}, \tag{18}$$

for region 3, and

$$F=B_5\exp(i\lambda_2 y)+B_6\exp(-i\lambda_2 y),\quad \lambda_2=[2m_2(\varepsilon-V_2)-\pi^2-k^2]^{1/2} \tag{19}$$

for region 2.

TABLE I.

Region	Width, nm	Al concentration	V_i	Effective mass
1	12,24	0	0	1
2	23,88	0,02	30,71	1,0276
3	3,88	0,3	501,76	1,4143

The requirement that the differentiation in Eq. (8) be correct imposes conditions at the boundaries between regions:

$$F(b-0)=F(b+0), \quad \frac{1}{m}\frac{dF}{dy}\Big|_{y=b-0}=\frac{1}{m}\frac{dF}{dy}\Big|_{y=b+0}. \quad (20)$$

Conditions (10) and (20) lead to a homogeneous system of linear algebraic equations in the constants B_1–B_6. If this system of equations is to have a solution, the following dispersion relation must be satisfied:

$$\left[\sin(\lambda_1 b_1)+\frac{\lambda_1 m_3}{\lambda_3 m_1}\cos(\lambda_1 b_1)\right] \times\left\{\sin[\lambda_2(1-b_2)]+\frac{\lambda_2 m_3}{\lambda_3 m_2}\cos[\lambda_2(1-b_2)]\right\} =\left[\sin(\lambda_1 b_1)-\frac{\lambda_1 m_3}{\lambda_3 m_1}\cos(\lambda_1 b_1)\right]\left\{\sin[\lambda_2(1-b_2)] -\frac{\lambda_2 m_3}{\lambda_3 m_2}\cos[\lambda_2(1-b_2)]\right\}\exp[-2\lambda_3(b_2-b_1)]. \quad (21)$$

The vector of constants B_i is determined within a normalization factor:

$$B_1=1, \quad (22)$$

$$B_2=-1, \quad (23)$$

$$B_3=i\left[\sin(\lambda_1 b_1)+\frac{\lambda_1 m_3}{\lambda_3 m_1}\cos(\lambda_1 b_1)\right]\exp(-\lambda_3 b_1), \quad (24)$$

$$B_4=i\left[\sin(\lambda_1 b_1)-\frac{\lambda_1 m_3}{\lambda_3 m_1}\cos(\lambda_1 b_1)\right]\exp(\lambda_3 b_1), \quad (25)$$

$$B_5=-\frac{\sin(\lambda_1 b_1)-\cos(\lambda_1 b_1)\lambda_1 m_3/(\lambda_3 m_1)}{\sin[\lambda_2(1-b_2)]+\cos[\lambda_3(1-b_2)]\lambda_2 m_3/(\lambda_3 m_2)} \times\exp[-\lambda_3(b_2-b_1)-i\lambda_2], \quad (26)$$

$$B_6=\frac{\sin(\lambda_1 b_1)-\cos(\lambda_1 b_1)\lambda_1 m_3/(\lambda_3 m_1)}{\sin[\lambda_2(1-b_2)]+\cos[\lambda_3(1-b_2)]\lambda_2 m_3/(\lambda_3 m_2)} \times\exp[-\lambda_3(b_2-b_1)+i\lambda_2]. \quad (27)$$

After normalization (12), we evaluate the matrix coefficients W. For this purpose we eliminate the differentiation with respect to the time from (14). We consider two different eigenvalues ε_n and ε_l, and we rewrite (8) in operator form:

$$\hat{H}F_n=\varepsilon_n F_n, \quad \hat{H}F_l=\varepsilon_l F_l,$$

where

$$\hat{H}=-\frac{1}{2}\frac{\partial}{\partial y}\frac{1}{m}\frac{\partial}{\partial y}+V_y+\frac{k^2+\pi^2}{2m}.$$

Differentiating the first equation with respect to k, multiplying the result by $\bar{F}_l$, integrating over y, and using (12) and the relation $\partial/\partial t = E\partial/\partial k$, we find the expression[8]

$$W_{nl}=\frac{kE}{\varepsilon_l-\varepsilon_n}\int_0^1\frac{F_n\bar{F}_l}{m}dy. \quad (28)$$

We conclude that $W_{nl}=\bar{W}_{ln}$. In addition, it is simple to show that $W_{ll}=0$. Another consequence of this fact is the existence of a dynamic invariant for Eqs. (13):

$$\sum_l |C_l|^2=\text{const}. \quad (29)$$

Knowing the initial data in (16), we can solve the Cauchy problem (13), (15). Then evaluating the sum and the integral in (6), we find the time-dependent wave function $\psi(t, x, y)$. This procedure has been carried out numerically. Equations (13) were solved by the Runge-Kutta method of fourth-order accuracy. The steps in the integration over k_0 and over time and the number of terms in the sum (6) were chosen so that the final result varied only slightly upon variations in these parameters. For a sufficiently high potential barrier, the coefficients W_{nl} are essentially zero, except near the synchronization point t_*, where $\varepsilon_n \approx \varepsilon_l$. If, at $t=0$, we have $C_1(k_0)\neq 0$ and $C_i(k_0)=0$ for $i\geqslant 2$, then the number of modes which are to be taken into account depends on the number of synchronization points over the time integration interval. In this case, there is one synchronization point, and an increase in the number of modes from 2 to 20 does not result in any significant change in the function ψ. The results of the calculation are shown in Fig. 2, as a plot of the distribution of $|\psi|^2$ at four successive times.

Figure 2a shows the distribution of $|\psi|^2$ at the initial time. The state of the electron is specified in such a way that the electron is localized predominantly in the first quantum wire. Along the coordinate x, the packet has a Gaussian shape, is centered at $x_c=0$, and has a zero average velocity. At this time, a uniform electric field is turned on. The field gives rise to an average velocity and to intermode exchange. Figure 2, b–d, shows the evolution of the initial state; the time is given in picoseconds, and the x coordinate of the center of the packet is given in microns. At $t=17.4$ ps, part of the wave packet goes into region 2 (Fig. 2b). The two humps, of approximately the same height, are separated by a

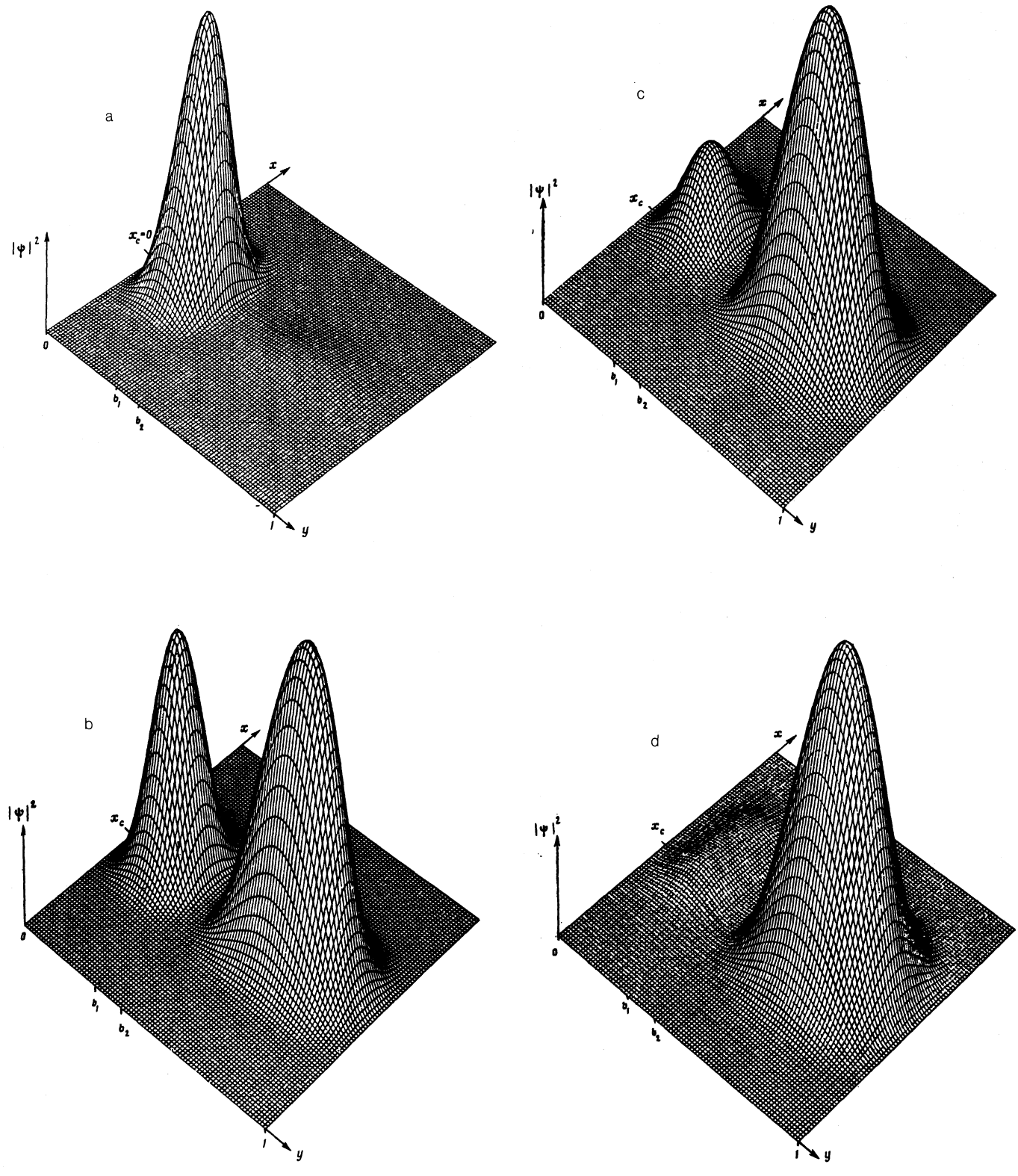

FIG. 2. Distribution of the probability density at various times. a — $t = 0$, $x_c = 0$; b — $t = 17.4$ ps, $x_c = 4.16\,\mu$m; c—$t = 21.8$ ps, $x_c = 6.44\,\mu$m; d—$t = 26.1$ ps, $x_c = 9.28\,\mu$m.

potential barrier (region 3), in which the solution is exponentially small. The probability for finding an electron is approximately the same in the first and second quantum wires. In Fig. 2c, the probability for finding the electron in region 1 is considerably lower, while at the time corresponding to Fig. 2d the electron is essentially localized in region 2. Over a time of 26 ps, tunneling from one quantum wire to the other thus occurs under the influence of the longitudinal electric field.

4. HIGH POTENTIAL BARRIER

To reach a better understanding of the effects described above, we carry out an asymptotic analysis for the case of a high potential barrier, with $V_3 \to +\infty$. Actually, it is more

convenient to use the parameter $\lambda_3 \gg 1$ in the calculations; this parameter is related to V_3 by expression (18).

Within small terms of higher order, we rewrite the dispersion relation (21) as

$$\sin(\lambda_1 b_1)\sin[\lambda_2(1-b_2)] = A, \tag{30}$$

where

$$A = 4\frac{\lambda_1\lambda_2 m_3^2}{\lambda_3^2 m_1 m_2}\cos(\lambda_1 b_1)\cos[\lambda_2(1-b_2)]\exp[-2\lambda_3(b_2-b_1)]. \tag{31}$$

In general, the constant A in expression (30) should be replaced by zero. At the so-called synchronization points k_*, however, at which we have, simultaneously,

$$\sin(\lambda_1 b_1) = \sin[\lambda_2(1-b_2)] = 0, \tag{32}$$

in the leading approximation, this exponentially small constant is important. The reason is that the exchange between modes is most intense near the synchronization points.

We first consider the dispersion relation (30) under the assumption $A = 0$. In this case we have two noninteracting quantum wires, for which

$$\lambda_1 b_1 = \pi l_1, \quad \lambda_2(1-b_2) = \pi l_2, \quad l_1, l_2 = 1, 2 \ldots,$$

where l_1 and l_2 are the indices of the discrete subbands in the first and second quantum wires. Substituting expressions (17) and (19) here, we find the dependence $\varepsilon(k)$:

$$\varepsilon_1(k) = V_1 + \frac{1}{2m_1}\left(\pi^2 + k^2 + \frac{\pi^2 l_1^2}{b_1^2}\right), \tag{33}$$

$$\varepsilon_2(k) = V_2 + \frac{1}{2m_2}\left[\pi^2 + k^2 + \frac{\pi^2 l_2^2}{(1-b_2)^2}\right], \tag{34}$$

The plot of these dispersion relations consists of two sets of parallel straight lines in the (ε, k^2) plane. If $m_2 > m_1$, they acquire intersection points:

$$\varepsilon_1(k_*) = \varepsilon_2(k_*) = \varepsilon_*,$$
$$k_*^2 = -\pi^2 + \frac{1}{m_2 - m_1}\left[\frac{m_1\pi^2 l_2^2}{(1-b_2)^2} - \frac{m_2\pi^2 l_1^2}{b_1^2} + 2m_1 m_2(V_2 - V_1)\right].$$

We need to recall that at these points A is not equal to zero, and $\varepsilon(k)$ is as shown in Fig. 3. With increasing k^2 [k is related to the time by (9)], a transition occurs from the set (33) to the set (34), and vice versa, at k_*. This transition is the primary reason for the tunneling (Fig. 2), which occurs near the synchronization point k_*. Let us examine the behavior $\varepsilon(k)$ in this region (Fig. 4).

As $k - k_* \to 0$, relations (30) and (31) can be rewritten as

$$[p_1(\varepsilon-\varepsilon_*) - q_1(k-k_*)][p_2(\varepsilon-\varepsilon_*) - q_2(k-k_*)] = A, \tag{35}$$

where

$$p_1 = b_1 m_1/\lambda_1, \quad p_2 = (1-b_2)m_2/\lambda_2,$$
$$q_1 = b_1 k_*/\lambda_1, \quad q_2 = (1-b_2)k_*/\lambda_2,$$
$$A = 4\frac{\lambda_1\lambda_2 m_3^2}{\lambda_3^2 m_1 m_2}\exp[-2\lambda_3(b_2-b_1)].$$

We are assuming here that all the λ_i are calculated from the parameters k_* and ε_*. Solving the quadratic equation (35) for ε, we find two branches of the dispersion relation:

$$\varepsilon_{2,1} = \frac{(p_1q_2 + p_2q_1)(k-k_*) \pm [(p_1q_2 - p_2q_1)^2(k-k_*)^2 + 4p_1p_2A]^{1/2}}{2p_1p_2}. \tag{36}$$

This expression determines two hyperbolas (Fig. 4). If the second quantum well is "heavier" than the first (i.e., if $m_2 > m_1$), then we have $q_1/p_1 = k_*/m_1 > q_2/p_2$, and as $|k - k_*| \to \infty$ the asymptotic behavior of hyperbolas (36) is

$$\varepsilon_1 - \varepsilon_* \to \frac{q_1}{p_1}(k-k_*), \quad \varepsilon_2 - \varepsilon_* \to \frac{q_2}{p_2}(k-k_*) \quad \text{as} \quad k - k_* \to -\infty;$$
$$\varepsilon_1 - \varepsilon_* \to \frac{q_2}{p_2}(k-k_*), \quad \varepsilon_2 - \varepsilon_* \to \frac{q_1}{p_1}(k-k_*) \quad \text{as} \quad k - k_* \to +\infty.$$

This asymptotic behavior shows that, to the left of the synchronization point k_*, the quantity ε_1 characterizes the motion of the electron in the first quantum wire, and ε_2 characterizes the motion in the second wire. To the right of k_*, on the other hand, ε_1 corresponds to the second wire, and ε_2 to the first.

Knowing the asymptotic dispersion relation, we can find the asymptotic behavior of the matrix element W. For

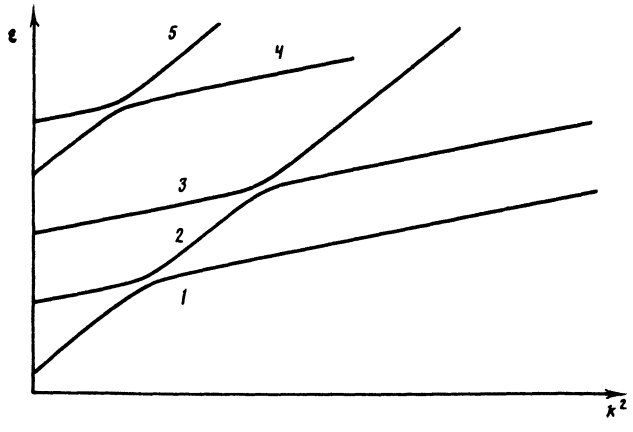

FIG. 3. Positions of the quantum-size subbands for two quantum wires.

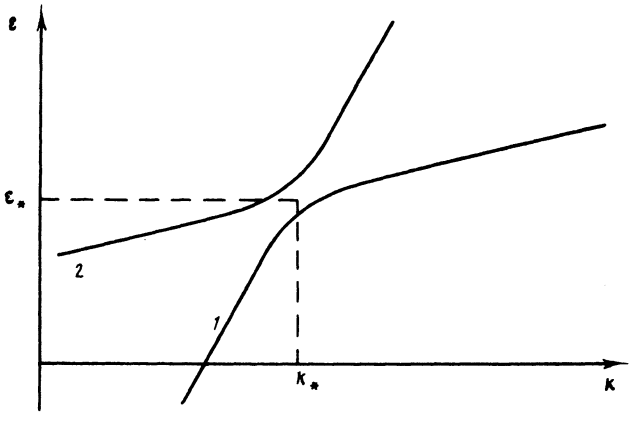

FIG. 4. Dispersion relation near a synchronization point.

this purpose we need to normalize the eigenfunctions which are determined by the sets of constants in (22)–(27) corresponding to the values ε_1 and ε_2.

The constants B_3 and B_4 can be set equal to zero in these calculations, since they make an exponentially small contribution. Substituting the eigenfunctions in (28), and carrying out the tedious but straightforward calculations, we find, in the leading approximation,

$$W_{1,2} = -\frac{Eg}{2(1+g^2(k-k_*)^2)}, \tag{37}$$

where g^{-1} determines the region along the k scale in which the two modes interact:

$$g = \frac{k_*(m_2-m_1)\lambda_3}{4\lambda_1\lambda_2 m_3}[b_1(1-b_2)]^{1/2}\exp[\lambda_3(b_2-b_1)].$$

Since we have $\lambda_3 \gg 1$, the interaction between modes takes place in an exponentially small neighborhood of the synchronization point. Furthermore, at this point itself the value of W_{12} is exponentially large and is proportional to the strength of the longitudinal electric field. For a weak field $E \ll g^{-1}$, because of the small value of W_{12}, the exchange between modes is weak, and the electron should make an essentially complete transition from the first quantum wire to the second. Let us examine this case in more detail.

We restrict the discussion to two modes. We assume that the initial state (at the time $t = 0$) is such that we have $C_1 = 1$ and $C_2 = 0$, and the point $k_0 = 0$ lies to the left of k_*, outside its g^{-1} neighborhood. The meaning here is that at $t = 0$ the function $\psi(0,x,y)$ determines the state in which the electron is localized (within exponentially small terms as $\lambda_3 \to +\infty$) inside the first quantum wire. We ignore the variation of $C_1(t)$, and we take up the problem of determining $C_2(t)$. The quantity $|C_2(t)|^2$ is the probability for a transition of an electron into region 2 under the condition $k(t) < k_*$ and also the probability for finding this electron in region 1 in the case $k(t) > k_*$. Equation (13) reduces to the integral

$$C_2(t) = -\int_0^t W_{12}\exp\left(i\int_0^{t'}(\varepsilon_2-\varepsilon_1)\,dt''\right)dt'. \tag{38}$$

Since W_{12} is of a local nature, the quantity $C_2(t)$ varies only in a g^{-1} neighborhood of the synchronization point. Under the assumption that the field E is independent of the time, we go over to the new variable $\xi = g[k(t) - k_*]$ in (38). Substituting in expressions (36) and (37), and replacing the integration limits by $\pm\infty$, we find, at $k(t) > k_*$,

$$C_2(t) = \frac{J}{2}\exp\left(\frac{i}{E}\int_0^{k_*}(\varepsilon_2-\varepsilon_1)\,dk\right),$$

where

$$J = \int_{-\infty}^{\infty}\frac{d\xi}{1+\xi^2}\exp\left[i\alpha\int_0^{\xi}(1+\eta^2)^{1/2}\,d\eta\right], \tag{39}$$

$$\alpha = \frac{16m_3^2\lambda_1^2\lambda_2^2\exp[-2\lambda_3(b_2-b_1)]}{Ek_*(m_2-m_1)m_1m_2\lambda_3^2b_1(1-b_2)}.$$

Let us find the asymptotic form of the integral J as $\alpha \to \infty$. The integrand has branch points $\pm i$ in the complex ξ plane. We make cuts along the imaginary axis as shown in Fig. 5. We replace the integration path along the real axis by semicircular arc Γ_R of large radius Γ_1 by two paths (Γ_1 and Γ_2) along the cut, and by circle Γ_r, of radius r, centered at point i. In taking the limits $R \to \infty$ and $r \to 0$, we note that the integral along the contour Γ_R vanishes, while the integrals along paths Γ_1 and Γ_2 are determined by the neighborhood of the point i. The integral Γ_r is determined by the residue at this point. As a result we have the asymptotic expression

$$J = {}^2/_3\pi\,\exp(-\alpha\pi/4),\quad \alpha \to +\infty.$$

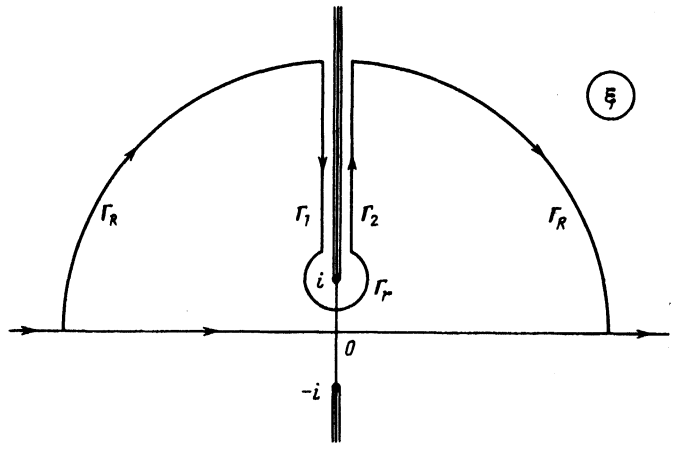

FIG. 5. Equivalent circuit for calculating the asymptotic behavior of integral (39).

For $C_2(t)$ we finally find

$$C_2(t) = \begin{cases} 0, & k(t) < k_* \\ \dfrac{\pi}{3}\exp\left[\dfrac{i}{E}\displaystyle\int_0^{k_*}(\varepsilon_2-\varepsilon_1)\,dk - \alpha\frac{\pi}{4}\right], & k(t) > k_* \end{cases}; \tag{40}$$

Inside the g^{-1} neighborhood of k_*, the quantity $C_2(t)$ changes from zero to the limiting value determined by expression (40), which is valid in the limits $\lambda_3 \to \infty$ and $d \to \infty$. The latter condition holds as $E \to 0$.

We turn now to another limiting case, that of a strong longitudinal electric field: $E \to \infty$. We assume that the potential barrier in region 3 is so high that the matrix coefficients W_{ij} are zero except in small neighborhoods of the synchronization point. We assume initial values $C_1(0) = 1$, and $C_2(0) = 0$. We rewrite the system of differential equations in (13) in the form

$$\frac{dC_1}{dt} = W_{12}C_2\exp\left[-i\int_0^t(\varepsilon_2-\varepsilon_1)\,dt\right],$$
$$\frac{dC_2}{dt} = -W_{12}C_1\exp\left[i\int_0^t(\varepsilon_2-\varepsilon_1)\,dt\right]. \tag{41}$$

In these equations we switch to the new variable $\xi = g[k(t) - k_*]$. Using

$$\int_0^t[\varepsilon_2(t)-\varepsilon_1(t)]\,dt = \frac{1}{gE}\int_0^k[\varepsilon_2(k)-\varepsilon_1(k)]\,dk \to 0 \quad \text{as} \quad E \to \infty,$$

we find a second-order differential equation for C_1 from (41):

$$\frac{d^2C_1}{d\xi^2}-\frac{d\tilde{W}}{d\xi}\frac{1}{\tilde{W}}\frac{dC_1}{d\xi}+\tilde{W}^2C_1=0, \tag{42}$$

where $\tilde{W}(\xi)=1/2(1+\xi^2)$. After we switch to the new variable

$$\eta=\int_0^{\xi}\tilde{W}(\xi')d\xi'$$

Eq. (42) becomes

$$\frac{d^2C_1}{d\eta^2}+C_1=0.$$

The general solution of (42) thus depends on the two constants D_1 and D_2:

$$C_1(\xi)=D_1\exp\left(i\int_0^{\xi}\tilde{W}\,d\xi'\right)+D_2\exp\left(-i\int_0^{\xi}\tilde{W}\,d\xi'\right).$$

Substituting the initial conditions, which are at $-\infty$ for the variable ξ, we find a solution of the system (41):

$$C_1(\xi)=\cos\left[\int_{-\infty}^{\xi}\tilde{W}(\xi')d\xi'\right], \tag{43}$$

$$C_2(\xi)=\sin\left[\int_{-\infty}^{\xi}\tilde{W}(\xi')d\xi'\right]. \tag{44}$$

After the neighborhood of the synchronization point is crossed, the solution $C_i(t)$ can be found from (43) and (44) by setting $\xi=\infty$. Since

$$\int_{-\infty}^{\infty}\tilde{W}(\xi')d\xi'=\pi/2, \tag{45}$$

for $k(t)>k_*$ we have $C_1(t)=0$ and $C_2(t)=-1$. These results mean that in strong longitudinal electric fields the electron does not tunnel into the second quantum wire; it instead "slips by" the synchronization point, remaining in the first wire.

This behavior of the electron should be typical for any two quantum wires separated by a potential barrier. The reasoning here is that the size of the g^{-1} neighborhood of the synchronization point does not depend on the strength of the longitudinal field, E, and the time taken to traverse this neighborhood is proportional to E^{-1}. In strong fields, the electron thus "does not have time" to overcome the potential barrier. Relation (45) and the form of the matrix coefficient in (37) should thus be valid in the vicinity of the synchronization point for any two quantum wires.

This conclusion makes it possible to generalize (40) and to find a universal expression for the electron tunneling probability in a weak longitudinal electric field. By virtue of (28) and (45), all the physical properties of the specific wires enter (38) only through the difference $\delta\varepsilon=\varepsilon_2-\varepsilon_1$. In the case of a simple intersection of the size-quantization subbands, this difference is given in the vicinity of the synchronization point by

$$\delta\varepsilon=\delta\varepsilon_*[1+g^2(k-k_*)^2]^{1/2},$$

where $\delta\varepsilon_*$ is the energy gap between the subbands at point k_*. Using these relations, we can find the probability P that an electron having passed through the neighborhood of k_* does not go into the other quantum wire:

$$P=\frac{\pi^2}{9}\exp\left\{-\frac{\pi}{2}\frac{(\delta\varepsilon_*)^{3/2}}{eE[\partial^2(\delta\varepsilon)/\partial k^2]^{1/2}}\right\}. \tag{46}$$

Here e is the charge of an electron, and the other quantities are written in dimensional form. This second derivative in (46) should be evaluated at the synchronization point. If there is such a point, then one could evidently always choose a field E which would be weak enough that P would be exponentially small. The probability for a transition into the other wire is found as $1-P$. In the direct numerical calculations, a comparison was made with (46); the comparison revealed that the probabilities for various fields E agreed within 5–10%.

Despite the differences in physical properties, the probability for a transition into the neighboring wire does not depend on the particular wire in which the electron finds itself. This conclusion follows from the symmetry of Eqs. (41).

5. DISCUSSION OF RESULTS

The motion of an electron along quantum wires is described in terms of discrete modes, from which wave packets can be formed. If two quantum wires are separated by a potential barrier, each mode will be dominant in one of the wires over an interval of the wave number k with a width on the order of unity. When the effective masses of the electron are different on the two sides of the potential barrier, there exist synchronization points, exponentially small neighborhoods of which are responsible for the exchange between modes. A wave packet formed from some mode goes entirely from one quantum wire into the other. This effect occurs when a longitudinal electric field is imposed. In the realm of practical applications, this effect might be utilized to develop quantum-wire switching devices. Devices of this type were proposed in Refs. 9 and 10.

Let us assume that electrons corresponding to a first mode with a wave number $k<k_*$ go into the first wire. If no electric field is imposed, the electrons continue to move along the first wire. The same comments apply to the second quantum wire.

After a field of 100 V/cm is imposed for a time of about 26 ps, the electrons in the first quantum wire go over into the second wire. The electrons which were in the second quantum wire go over into the first. The probability for a tunneling through the potential separating the wires is close to unity; the error probability is exponentially small and is estimated in (46).

After a packet passes through the neighborhood of the synchronization point, its average wave number k is greater than k_*. If necessary, k could be reduced to its original value by imposing a brief pulse of a strong longitudinal electric field in the opposite direction. This pulse would not cause a transition into the other wire.

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