Quantum interference in the backscattering of charged particles from a disordered medium in a magnetic field

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The interference component of the angular distribution of the backscattering of charged particles from disordered $3D$ and $2D$ systems in a uniform magnetic field is calculated under weak-localization conditions. In addition to rounding of the peak in the angular distribution, as a result of the disruption of the coherence of the forward and backward waves, there may also be an asymmetry of the spectrum. If the magnetic field is directed parallel to the surface of the medium, the peak in the angular distribution of the backscattered particles is shifted away from the "exactly backward" direction. The magnitude of this shift is determined by the average magnetic flux linking a loop formed by the particle paths in the disordered medium.

1. INTRODUCTION

During multiple scattering by an ensemble of disordered centers, waves which are propagating along the same paths but in opposite directions acquire identical phase shifts. The interference between such waves is known to be responsible for a weak localization of waves and particles in random media; this localization is in turn associated with quantum corrections to the kinetic coefficients of metals and semiconductors.[1-8] The most prominent effect is a coherent summation of waves during backscattering from disordered media. In this case the interference contribution to the intensity near the "exactly backward" direction is comparable in magnitude to the backscattering intensity itself.[9-24]

The coherent summation of waves propagating in different directions along the same paths is a consequence of the symmetry of the scattering process under time reversal. Interference effects are thus exceedingly sensitive to factors which would disrupt T invariance: motion of the scattering particles,[11,20] spin-spin interaction with impurity centers,[2,5,6,24] an external magnetic field,[2-8] and gyrotropy of the medium, which is related to the external magnetic field by the Faraday effect.[11,19]

The violation of the symmetry under time reversal does not necessarily cause complete suppression of interference effects. If the phase relaxation of the forward and backward waves over distances on the order of the mean free path l is slight, the absence of T invariance would simply lead to a limitation on the number of collisions over which the wave functions would retain their coherence. Associated with this limitation are a large number of interesting aspects of the behavior of the quantum corrections to the kinetic coefficients of solids[2-8] and in the angular distribution of the backscattering intensity.[19,20,23,24] In particular, fine structure forms in the angular distribution near the exactly backward direction in the case of spin-spin (magnetic) and spin-orbit interactions with scattering centers.[23,24]

The quantum interference accompanying multiple scattering of charged particles in disordered media is very sensitive to the application of an external magnetic field.[6,7] The anomalous behavior of the magnetoresistance of $2D$ and $3D$ systems might be thought of as one manifestation of this sensitivity.[2-8] When a magnetic field $\mathbf{H}$ is applied, the wave function of a particle acquires an additional phase shift. Since the wave function transforms in accordance with $\Psi^T(\mathbf{H}) = \Psi^*(-\mathbf{H})$ under time reversal, the additional phase shifts of the forward and backward waves have the same sign. The interference component of the backscattering intensity thus acquires a phase factor in a magnetic field ($\hbar = c = 1$):

$$AA^{\mathrm{T}}=A(\mathbf{H})A^*(-\mathbf{H})=A_0A_0^*\exp(2i\varphi_{\mathbf{H}}), \tag{1}$$

where A and A^T are the amplitudes of the forward and backward waves, $\varphi_{\mathbf{H}} = e\phi$, e is the charge of the particle, $\phi = H\Sigma_\perp$ is the magnetic flux through the loop formed by the particle paths, and $\Sigma_\perp$ is the area of the projection of this loop onto the plane perpendicular to the vector $\mathbf{H}$. The phase $\varphi_{\mathbf{H}}$ in (1) is a random quantity, which depends on the shape of the particle path. The coherence of the wave functions is disrupted over distances

$$s\sim(r_H/p_0)^{1/2} = (|e|H)^{-1/2}$$

(p_0 is the momentum of the particle, and $r_H = p_0/|e|H$ is the Larmor radius), over which the phase shift $\varphi_H \sim eHs^2$ reaches a value on the order of unity.

Under weak localization conditions ($p_0 l \gg 1$), the most interesting case of quantum interference is that in a weak magnetic field,

$$p_0l^2/r_H\ll 1, \tag{2}$$

in which case the coherence of the wave functions is disrupted after many elastic-scattering events. Quantum effects dominate the situation under condition (2). The corrections of classical origin, which stem from the curvature of the particle paths in the magnetic field, are on the order of $l/r_H \ll p_0 l^2/r_H$ and can be ignored.

The disruption of the quantum interference of charged particles in a magnetic field has been discussed previously only in connection with an analysis of the behavior of global quantities: corrections to the conductivity[2-7] and to other kinetic coefficients[8] and the dispersion of the conductance.[25,26] There has been no study of how a magnetic field affects the coherent enhancement of the backscattering of particles from disordered media.

Below we use assumption (2) to derive an analytic solution of the problem of calculating the angular distribution of the backscattering of charged particles from disordered $3D$

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and $2D$ systems in a uniform external magnetic field. We analyze in detail features in the angular distribution of the backscattered particles which stem from disruption of the interference of the wave functions in a magnetic field. We find that the angular distribution at angles other than the exactly backward direction, $\vartheta \lesssim (p_0 r_H)^{-1/2}$, depends on the orientation of the magnetic field. We find that the rounding of the peak in the backscattering spectrum, which stems from a loss of coherence of the forward and backward waves, may be accompanied by a shift of the intensity peak with respect to the exactly backward direction. In the backscattering of charged particles in a magnetic field directed parallel to the surface, the peak in the angular distribution is shifted an angle

$$\Delta\vartheta_{\max} \simeq 0.75(p_0 r_H)^{-1/2}$$

away from the backward direction. The physical reason for this shift is the nonzero average phase shift of the wave functions of the interfering particles. This shift is proportional to the average magnetic flux linking the loop formed by the particle paths. The direction of this shift (clockwise or counterclockwise around the field **H**) depends on the sign of the particle's charge.

2. GENERAL RELATIONS

Let us consider the motion of a nonrelativistic charged particle in a disordered system of small-radius scattering centers, with $p_0 a \ll 1$. We assume that the medium occupies the half-space $z > 0$ (the lower z axis runs perpendicular to the surface).

To calculate the flux density of backscattered particles we need to find the one-particle density matrix averaged over the positions of the centers:

$$\langle\rho(\mathbf{r},\mathbf{r}')\rangle = \langle\Psi(\mathbf{r})\,\Psi^*(\mathbf{r}')\rangle. \quad (3)$$

If we ignore the recoil in the collisions with the incident particle, the density matrix can be written[27]

$$\rho(\mathbf{r}_1,\mathbf{r}_2)=\rho^{(0)}(\mathbf{r}_1,\mathbf{r}_2)+\int\ldots\int d\mathbf{R}_1\,d\mathbf{R}_2\,d\mathbf{r}_1'\,d\mathbf{r}_2'$$
$$\times G(\mathbf{r}_1,\mathbf{R}_1)G^*(\mathbf{r}_2,\mathbf{R}_2)\Gamma(\mathbf{R}_1,\mathbf{r}_1';\mathbf{R}_2,\mathbf{r}_2')\rho^{(0)}(\mathbf{r}_1',\mathbf{r}_2'), \quad (4)$$

where $G(\mathbf{r},\mathbf{r}')$ is the Green's function of the scattering problem, and $\rho^{(0)}(\mathbf{r},\mathbf{r}')$ is the density matrix of the particles which have not undergone incoherent interactions in the medium (this is the density matrix of the coherent wave field). For the case in which there is a source at the point $\mathbf{R}_0$, $\rho^{(0)}(\mathbf{r},\mathbf{r}')$ is

$$\rho^{(0)}(\mathbf{r},\mathbf{r}')=G(\mathbf{r},\mathbf{R}_0)G^*(\mathbf{r}',\mathbf{R}_0). \quad (5)$$

If there is no magnetic field outside the medium (at $z < 0$; this situation is possible if the field **H** inside the medium is directed parallel to the surface), the source can be specified as a unidirectional particle flux incident on the surface of the medium. We can then write

$$\rho^{(0)}(\mathbf{r},\mathbf{r}')=\Psi(\mathbf{r})\Psi^*(\mathbf{r}'), \quad (6)$$

where $\Psi(\mathbf{r})$ is the wave function of the scattering problem. This wave function corresponds to the case in which a plane wave with a momentum $\mathbf{p}_0$ is incident on the vacuum-medium interface from $z = -\infty$:

$$\Psi_{\rm inc}(\mathbf{r})\,|_{z=-\infty}=\exp(i\mathbf{p}_0\mathbf{r}). \quad (7)$$

The function $\Psi(\mathbf{r})$ satisfies a Schrödinger equation in a magnetic field with a non-Hermitian potential. The imaginary part of this potential is related to the scattering cross section by the optical theorem.[27,28]

Under weak localization conditions, with $p_0 l \gg 1$, the function $\Gamma(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ is dominated by the series of ladder and fan (maximal-crossing) diagrams.[6,7]

The sum of the ladder-diagram series corresponding to a sequence of independent incoherent-scattering events can be written in the form

$$\mathcal{L}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')=(\pi n\sigma/m^2)\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1'-\mathbf{r}_2')$$
$$\times[\delta(\mathbf{r}_1-\mathbf{r}_1')+F_{\mathcal{L}}(\mathbf{r}_1,\mathbf{r}_1')], \quad (8)$$

where n is the number of scattering centers per unit volume, σ is the cross section for elastic scattering by a center, and m is the mass of a particle. To avoid unimportant complications, we assume that the scattering is purely elastic: $\omega = \sigma/\sigma_{\rm tot} = 1$, where $\sigma_{\rm tot}$ is the total interaction cross section. The results derived below can be generalized without difficulty to the case $\omega < 1$; this is in fact done at the end of Sec. 5.

The first term in (8) is the distribution of singly scattered particles. The second term in (8),

$$L(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')=(\pi n\sigma/m^2)\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1'-\mathbf{r}_2')$$
$$\times F_{\mathcal{L}}(\mathbf{r}_1,\mathbf{r}_1') \quad (9)$$

corresponds to multiple scattering. The propagator $F_{\mathcal{L}}(\mathbf{r},\mathbf{r}')$ in (8), (9) describes the spatial distribution of particles which have been scattered incoherently by a point source of unit intensity. It satisfies the equation

$$F_{\mathcal{L}}(\mathbf{r},\mathbf{r}')=\frac{\pi n\sigma}{m^2}|G(\mathbf{r},\mathbf{r}')|^2+\frac{\pi n\sigma}{m^2}\int_V d\mathbf{R}\,|G(\mathbf{r},\mathbf{R})|^2F_{\mathcal{L}}(\mathbf{R},\mathbf{r}'). \quad (10)$$

The integration on the right side of (10) is carried out over the volume V occupied by the medium.

We know that the series of fan diagrams corresponds to interference of the wave functions of particles which have passed by the same scattering centers, going in opposite directions. The sum of the fan-diagram series (a "Cooperon") is

$$C(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')=(\pi n\sigma/m^2)\delta(\mathbf{r}_1-\mathbf{r}_2')\delta(\mathbf{r}_2-\mathbf{r}_1')$$
$$\times F_c(\mathbf{r}_1,\mathbf{r}_1'), \quad (11)$$

where the propagator $F_c(\mathbf{r},\mathbf{r}')$ satisfies the equation

$$F_c(\mathbf{r},\mathbf{r}')=\frac{\pi n\sigma}{m^2}G(\mathbf{r},\mathbf{r}')G^*(\mathbf{r}',\mathbf{r})$$
$$+\frac{\pi n\sigma}{m^2}\int_V d\mathbf{R}\,G(\mathbf{r},\mathbf{R})G^*(\mathbf{R},\mathbf{r})F_c(\mathbf{R},\mathbf{r}'). \quad (12)$$

Equations (8)–(12) have been written for a $3D$ system. In the case of multiple scattering of particles in a disordered $2D$ medium, the factor $\pi n\sigma/m^2$ in (8)–(12) must be replaced by $n\sigma p_0/m^2$, where σ is now the scattering cross section in the $2D$ case (it has the dimensionality of a length), and n is the number of scatterers per unit area.

The reciprocity theorem for the Green's function in an external magnetic field **H** is known to be

$$G(\mathbf{r}, \mathbf{r}', \mathbf{H}) = G(\mathbf{r}', \mathbf{r}, -\mathbf{H}). \tag{13}$$

Accordingly, $G(\mathbf{r},\mathbf{r}')$ loses its symmetry under a simple interchange of its arguments: $G(\mathbf{r},\mathbf{r}') \neq G(\mathbf{r}',\mathbf{r})$. Because of this aspect of the motion of charged particles in a magnetic field, expression (11) for the sum of the fan diagrams cannot be reduced to expression (9) through any interchange of arguments. The diffusion and the interference of particles in this case are described by quite different equations of motion. The physical reason why the symmetry which existed between $L(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ and $C(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$ in the case $\mathbf{H} = 0$ (Ref. 21) no longer exists is that the T invariance of the scattering process is violated in an external magnetic field.

In a weak field, such that the Larmor radius is far larger than the wavelength of a particle ($r_H \gg \lambda = 2\pi/p_0$), we can use the semiclassical approximation for the Green's function of the scattering problem. Under the condition $|\mathbf{r} - \mathbf{r}'| \ll r_H$, the function $G(\mathbf{r},\mathbf{r}')$ $(z,z' > 0)$ can be written in the form[29]

$$G(\mathbf{r}, \mathbf{r}') = \exp[i\varphi(\mathbf{r}, \mathbf{r}')]\, G_0(|\mathbf{r}-\mathbf{r}'|), \tag{14}$$

where $G_0(|\mathbf{r} - \mathbf{r}'|)$ is the Green's function of the scattering problem in the absence of a magnetic field.[27,28] In the 3D case, for example, we would have

$$G_0(|\mathbf{r}-\mathbf{r}'|) = -\frac{m}{2\pi}\frac{1}{|\mathbf{r}-\mathbf{r}'|}\exp\left[ip_0|\mathbf{r}-\mathbf{r}'| - \frac{n\sigma}{2}|\mathbf{r}-\mathbf{r}'|\right], \tag{15}$$

and the phase $\varphi(\mathbf{r},\mathbf{r}')$ would be given by

$$\varphi(\mathbf{r}, \mathbf{r}') = e\int_{\mathbf{r}'}^{\mathbf{r}} d\mathbf{R}\, \mathbf{A}(\mathbf{R}), \tag{16}$$

where $\mathbf{A}(\mathbf{r})$ is the vector potential, and the integral in (16) is evaluated along the straight line connecting $\mathbf{r}$ and $\mathbf{r}'$.

Under the approximation (14), Eq. (10) for the function $F_{\mathscr{L}}(\mathbf{r},\mathbf{r}')$ goes over to the corresponding equation describing diffusion of particles in the absence of an external magnetic field. All the results derived in Ref. 21 for the sum of the ladder series, $\mathscr{L}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')$, thus remain valid.

After (14) is substituted into (12), the equation for $F_c(\mathbf{r},\mathbf{r}')$ becomes

$$F_c(\mathbf{r}, \mathbf{r}') = \frac{\pi n\sigma}{m^2}|G_0(|\mathbf{r}-\mathbf{r}'|)|^2\exp[2i\varphi(\mathbf{r}, \mathbf{r}')] + \frac{\pi n\sigma}{m^2}\int_V d\mathbf{R}\,|G_0(|\mathbf{r}-\mathbf{R}|)|^2\exp[2i\varphi(\mathbf{r}, \mathbf{R})]F_c(\mathbf{R}, \mathbf{r}'). \tag{17}$$

Using inequality (2) along with (14), we can transform the integral term in Eq. (17) as follows:[30]

$$\int d\mathbf{R}\,|G_0(|\mathbf{r}-\mathbf{R}|)|^2\exp[2i\varphi(\mathbf{r}, \mathbf{R})]F_c(\mathbf{R}, \mathbf{r}') = \int d\mathbf{R}_1|G_0(R_1)|^2\exp[-i\mathbf{R}_1\hat{\mathbf{p}}]F_c(\mathbf{r}, \mathbf{r}'), \tag{18}$$

where

$$\hat{\mathbf{p}} = \frac{1}{i}\frac{\partial}{\partial\mathbf{r}} - 2e\mathbf{A}(\mathbf{r}) \qquad (\operatorname{div}\mathbf{A} = 0). \tag{19}$$

Now substituting (18) into (17) for the propagator $F_c(\mathbf{r},\mathbf{r}')$, we find the equation

$$F_c(\mathbf{r}, \mathbf{r}') = \frac{\pi n\sigma}{m^2}\int d\mathbf{R}_1|G_0(R_1)|^2\exp[-i\mathbf{R}_1\hat{\mathbf{p}}][\delta(\mathbf{r}-\mathbf{r}') + F_c(\mathbf{r}, \mathbf{r}')]. \tag{20}$$

The angular distribution of the backscattering in a magnetic field can be determined by the method of Refs. 21 and 22. However, we must allow for the circumstance that the magnetic field may be nonzero outside the medium, and in general we cannot ignore the effect of this field on the motion of the particles on the ballistic part of their paths.

We assume that the source and detector of the charged particles are far from the surface of the medium, at the points $\mathbf{R}_0$ and $\mathbf{R}_1$, respectively (Fig. 1). In this case we need to substitute $\rho^{(0)}$ as in (5) into expression (4); here G is the Green's function of the scattering problem in the vacuum-medium system with a given configuration of the magnetic field. Assuming, as above, that the condition for the semiclassical approximation is satisfied, we write the product of Green's functions $G(\mathbf{r},\mathbf{R}_0)G^*(\mathbf{r}',\mathbf{R}_0)$ in (4) near the interface as

$$G(\mathbf{r}, \mathbf{R}_0)G^*(\mathbf{r}', \mathbf{R}_0) = aa^*\exp[-i\mathbf{p}_0(\mathbf{r}-\mathbf{r}') - i[\varphi(\mathbf{r}, \mathbf{R}_0) - \varphi(\mathbf{r}', \mathbf{R}_0)] - {}^1\!/_2(n\sigma/\mu_0)(z+z')], \tag{21}$$

where p_0 is the momentum of the incident particles at the vacuum-medium interface $(z = 0)$, given by $\mathbf{p}_0 = (\nabla S - e\mathbf{A})|_{z=0} = m\mathbf{v}_0$. Here $S(\mathbf{r},\mathbf{R}_0)$ is the semiclassical action; $\mathbf{v}_0$ is the velocity of the particle; $\mu_0 = \cos\theta_0 = p_{0z}/p_0$, $\varphi(\mathbf{R}_\alpha,\mathbf{R}_\beta)$ are some additional phase shifts of the wave functions of the incident particles which result from the external magnetic field, given by

$$\varphi(\mathbf{R}_\alpha, \mathbf{R}_\beta) = e\int_{\mathfrak{L}(\mathbf{R}_\alpha, \mathbf{R}_\beta)} d\mathbf{R}\mathbf{A}(\mathbf{R}), \tag{22}$$

and the integral in (22) is evaluated along an arc of the particle path $\mathfrak{L}(\mathbf{R}_\alpha,\mathbf{R}_\beta)$ (Fig. 1). An expression like (21) can be written for the product $G(\mathbf{R}_1,\mathbf{r})G^*(\mathbf{R}_1,\mathbf{r}')$.

By writing GG^* as in (21), we can find the angular distribution of the backscattered particles with a "resolution"

$$\Delta\vartheta = p_0^{-1}|\Delta\mathbf{p}| \lesssim l/r_H.$$

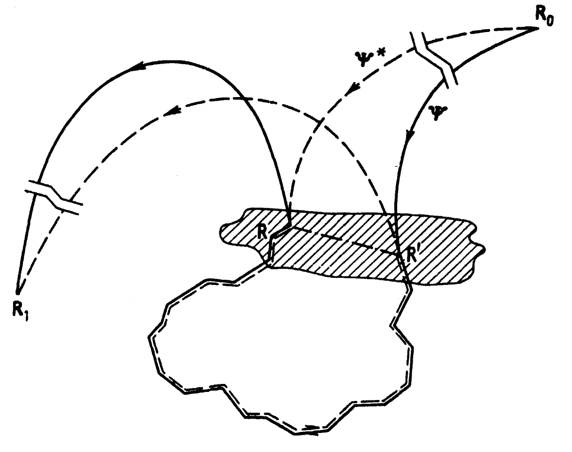

FIG. 1.

The reason for this limitation is the error in the determination of the direction in which the particles incident on the surface of the medium (and also the backscattered particles) are moving in the representation (21): An uncertainty in the momentum stems from the rotation around a magnetic field line as the particle moves between the vacuum-medium interface and the first (or last) collision with a scattering center.

At very small angles between the velocity of the particles and the field **H**, the quantum-mechanical nature of the motion in the XY plane becomes important, and we must abandon the representation (21). In this case the uncertainty regarding the direction of the momentum of the incident particles is due entirely to quantum-mechanical effects (Sec. 4).

Using relations (9) and (11) and representation (21) in (4), we find the following expression for the angular distribution of the backscattering, i.e., for the ratio of the flux density of particles moving in the **p** direction, across a plane oriented parallel to the boundary of the medium, to the flux density of incident particles:

$$J(\mathbf{p},\mathbf{p}_0)=J_{\mathscr{L}}(\mathbf{p},\mathbf{p}_0)+J_c(\mathbf{p},\mathbf{p}_0). \tag{23}$$

Here

$$J_{\mathscr{L}}(\mathbf{p},\mathbf{p}_0)=\frac{n\sigma}{4\pi\Sigma}\Big[\int d\mathbf{R}|\Psi(\mathbf{R},-\mathbf{p})|^2|\Psi(\mathbf{R},\mathbf{p}_0)|^2 + \iint d\mathbf{R}\,d\mathbf{R}'|\Psi(\mathbf{R},-\mathbf{p})|^2 F_{\mathscr{L}}(\mathbf{R},\mathbf{R}')|\Psi(\mathbf{R}',\mathbf{p}_0)|^2\Big] \tag{24}$$

is the angular distribution of incoherently scattered particles, and

$$J_c(\mathbf{p},\mathbf{p}_0)=\frac{n\sigma}{4\pi\Sigma}\iint d\mathbf{R}\,d\mathbf{R}'\Psi(\mathbf{R},-\mathbf{p})\Psi^*(\mathbf{R},\mathbf{p}_0)F_c(\mathbf{R},\mathbf{R}') \times\Psi^*(\mathbf{R}',-\mathbf{p})\Psi(\mathbf{R}',\mathbf{p}_0)\exp[i\hat{\varphi}(\mathbf{R}_0\mathbf{R}_1;\mathbf{R}\mathbf{R}')] \tag{25}$$

is the interference component of the backscattering intensity, where $\mathbf{p}_0=(p_{0\parallel},p_{0z})$ and $\mathbf{p}=(p_\parallel,p_z)$ are the momenta of the incident and backscattered particles at the vacuum-medium interface ($z=0$), and

$$p_{0z}=p_0\cos\theta_0=p_0\mu_0,\quad p_z=p_0\cos\theta=p_0\mu.$$

The function $\Psi(\mathbf{r},\mathbf{p})$ in (24), (25) is given by

$$\Psi(\mathbf{r},\mathbf{p})=\exp(i\mathbf{p}\mathbf{r}-n\sigma z/2|\mu|). \tag{26}$$

The propagators $F_{\mathscr{L}}(\mathbf{R},\mathbf{R}')$ and $F_c(\mathbf{R},\mathbf{R}')$ satisfy Eqs. (10) [with Green's function (14)] and (17) [or (20)], and Σ is the surface area of the medium.

The phase shift $\hat{\varphi}(\mathbf{R}_0\mathbf{R}_1;\mathbf{R}\mathbf{R}')$ in the argument of the exponential function in (25) is

$$\hat{\varphi}=-\varphi(\mathbf{R},\mathbf{R}_0)+\varphi(\mathbf{R}',\mathbf{R}_0)-\varphi(\mathbf{R}_1,\mathbf{R}')+\varphi(\mathbf{R}_1,\mathbf{R}), \tag{27}$$

where $\varphi(\mathbf{R}_\alpha,\mathbf{R}_\beta)$ is given by (22). The phase shift $\hat{\varphi}$ can also be written in the form

$$\hat{\varphi}=e\phi_0+e\phi_1-2\varphi(\mathbf{R},\mathbf{R}'), \tag{28}$$

where ϕ_0 and ϕ_1 are the magnetic fluxes linking the closed loops $\mathfrak{L}_0$ and $\mathfrak{L}_1$, formed by the ballistic parts of the paths of the particles and by the line segment connecting the points of entry into the medium and of exit from it (more precisely, connecting the points of the first and last collisions), **R** and **R**′:

$$\phi_{0,1}=\oint_{\mathfrak{L}_{0,1}}\mathbf{A}(\mathbf{R})\,d\mathbf{R},$$
$$\mathfrak{L}_0=\mathfrak{L}(\mathbf{R}_0,\mathbf{R}',\mathbf{R}),\quad \mathfrak{L}_1=\mathfrak{L}(\mathbf{R}_1,\mathbf{R}',\mathbf{R}). \tag{29}$$

The quantity $\varphi(\mathbf{R},\mathbf{R}')$ is given by the integral in (16).

Under conditions (2), the incoherent-scattering intensity (24) is given by the same expression as in the case $\mathbf{H}=0$ (Refs. 21, 31, and 32). In contrast with the situation in the $\mathbf{H}=0$ case, examined in Ref. 21, the interference component in (25) is not the same as the multiple-scattering component in (24) in the case $\mathbf{p}=-\mathbf{p}_0$ (the exactly backward direction). The reason is the violation of T invariance of the scattering of the charged particles in a magnetic field.

The expression for $J_c(\mathbf{p},\mathbf{p}_0)$ in (25), which depends on the magnetic field, is gauge-invariant. This invariance can be verified directly, by comparing (25) with (17) [or (20)].

In order to experimentally observe features of the backscattering angular distribution which are directly related to the quantum interference of particles in the disordered medium, it would be necessary to cancel the phase shift of the wave functions due to the motion of the particles in the magnetic field on the ballistic parts of their paths. Determining the best experimental arrangement for canceling this ballistic phase shift is a separate problem, which we will not discuss further here. We would simply like to point out that the simplest approach would seem to be to bring the boundary of the spatial volume occupied by the field **H** into coincidence with the surface of the disordered medium. Such a configuration is possible only if the field lines of **H** run parallel to the vacuum-medium interface $\mathbf{H}\mathbf{n}=0$, where **n** is the inward normal to the surface). In this situation we would have $\mathbf{H}=0$ outside the medium, and there would be absolutely no magnetic flux linking the loops $\mathfrak{L}_0$ and $\mathfrak{L}_1$ (Fig. 1) formed by the ballistic parts of the paths.

In the case of complete cancellation of the ballistic phase shift[1)] ($\phi_1+\phi_0=0$), we can write

$$\hat{\varphi}=-2\varphi(\mathbf{R},\mathbf{R}') \tag{30}$$

and from (25) we find the expression

$$J_c(\mathbf{p},\mathbf{p}_0)=\frac{n\sigma}{4\pi\Sigma}\iint d\mathbf{R}\,d\mathbf{R}'\exp\Big[-i(\mathbf{p}_0+\mathbf{p})(\mathbf{R}-\mathbf{R}') -\frac{1}{2}n\sigma\Big(\frac{1}{\mu_0}+\frac{1}{|\mu|}\Big)(z+z')-2i\varphi(\mathbf{R},\mathbf{R}')\Big]F_c(\mathbf{R},\mathbf{R}'). \tag{31}$$

This expression gives us the quantum-interference contribution to the backscattering angular distribution formed directly as a result of multiple collisions of particles with randomly positioned, small-radius centers. It is not distorted by the effect of the magnetic field on the motion of the particles outside the medium.

Expression (31) is written for the $3D$ case. For the reflection of particles from a disordered $2D$ system (in the YZ plane), the expression for the angular distribution $J_c(\mathbf{p},\mathbf{p}_0)$ differs from (31) only by a common factor: $(4\pi\Sigma)^{-1}$ must be replaced by $(2\pi L)^{-1}$, where L is the length of the vacuum-medium interface. The only interesting case of $2D$ motion is that in which the magnetic field is oriented perpendicular to the scattering plane. If the vector **H** lies in the *yz* plane, the magnetic field has no effect on multiple scattering in the medium.

3. ANGULAR DISTRIBUTION OF THE BACKSCATTERING OF PARTICLES IN A MAGNETIC FIELD ORIENTED PARALLEL TO THE SURFACE OF THE MEDIUM

Let us consider the reflection of a flux of charged particles which are incident on a disordered system of isotropically scattering centers. We assume that a uniform and constant magnetic field is directed along the X axis $[\mathbf{H} = (H,0,0)]$ and is zero except inside the medium. We choose the vector potential

$$\mathbf{A}=(0,\ -Hz,\ 0). \tag{32}$$

According to expression (25), the problem of calculating the interference contribution to the flux density of the backscattered particles reduces to one of finding the propagator $F_c(\mathbf{r},\mathbf{r}')$.

In the case under consideration here, with the magnetic field oriented parallel to the surface of the medium ($\mathbf{H}\cdot\mathbf{n} = 0$), Eq. (20) for $F_c(\mathbf{r},\mathbf{r}')$ cannot be solved analytically in its general form. However, in order to calculate the backscattering spectrum at angles from the exactly backward direction at which the magnetic field is influential we do not need to know the behavior of $F_c(\mathbf{r},\mathbf{r}')$ over the entire range of the variables $\mathbf{r}$, $\mathbf{r}'$. Since the magnetic field affects the interference of the wave functions of the particles only after many collisions $[N\sim(r_H/p_0l^2)\gg1]$ with scattering centers, it is sufficient to calculate the asymptotic behavior of the function $F_c(\mathbf{r},\mathbf{r}')$ at large $|\mathbf{r}-\mathbf{r}'|\gg l$. For this purpose we can use the diffusion approximation in Eq. (20) (Ref. 6). In this approach, the equation for the asymptotic behavior of $\widetilde{F}_c(\mathbf{r},\mathbf{r}') = F_c(\mathbf{r},\mathbf{r}')$ at $|\mathbf{r}-\mathbf{r}'|\gg l$ is

$$\frac{l^2}{d}\left[\frac{1}{i}\frac{\partial}{\partial\mathbf{r}}-2e\mathbf{A}(\mathbf{r})\right]^2\widetilde{F}_c(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}'), \tag{33}$$

where d is the dimensionality of the system.

Equation (33) must be supplemented with a boundary condition. The usual approach is to use a condition of the type (Refs. 18 and 28, for example)

$$(l_0\mathbf{n}\partial/\partial\mathbf{r}-1)\widetilde{F}_c(\mathbf{r},\mathbf{r}')|_{z=0}=0, \tag{34}$$

where $\mathbf{n}$ is the inward normal to the surface, and l_0 is the so-called extrapolated length. In the $3D$ case this length is $l_0\approx0.7104l$.

We could take a different approach to calculate the angular distribution of the backscattering, $J_c(\mathbf{p},\mathbf{p}_0)$, in a weak magnetic field [inequality (2)]. Specifically, we could require, as an additional condition on Eq. (33), that the spectrum found in the diffusion approximation for angular deviations $(p_0r_H)^{-1/2}\ll\vartheta\ll(p_0l)^{-1}$ from the exactly backward direction agree with the exact result found for $\mathbf{H}=0$ in Ref. 21. In this angular region, we can use the diffusive asymptotic behavior of the exact solution $[\vartheta\ll(p_0l)^{-1}]$, and at the same time the magnetic field does not affect the interference between the wave functions of the particles $[\vartheta\gg(p_0r_H)^{-1/2}]$.

The solution of Eq. (33) is

$$\widetilde{F}_c(\mathbf{r},\mathbf{r}') =\frac{d}{8\pi^2l^2}\left(\frac{r_H}{p_0}\right)^{1/2}\iint dp_x\,dp_y\exp[ip_x(x-x')+ip_y(y-y')] \times\widetilde{F}_c\left(2\left(\frac{p_0}{r_H}\right)^{1/2}\left(z+\frac{p_yr_H}{2p_0}\right),\ 2\left(\frac{p_0}{r_H}\right)^{1/2}\left(z'+\frac{p_yr_H}{2p_0}\right),\ \nu\right), \tag{35}$$

where

$$\widetilde{F}_c(\tau,\tau',\nu)=\frac{1}{W}\begin{cases}[\alpha D_\nu(\tau)+D_\nu(-\tau)]D_\nu(\tau'), & \tau<\tau'\\ [\alpha D_\nu(\tau')+D_\nu(-\tau')]D_\nu(\tau), & \tau'<\tau\end{cases}, \tag{36}$$

$$\nu=-{}^1/_2-{}^1/_4k_x^2,\quad k_x=p_x(r_H/p_0)^{1/2}. \tag{37}$$

Here $D_\nu(\tau)$ is a parabolic cylinder function;[33] the Wronskian $W=W[D_\nu(\tau),D_\nu(-\tau)]$ is given by

$$W=(2\pi)^{1/2}/\Gamma(-\nu), \tag{38}$$

where $\Gamma(x)$ is the gamma function;[33] and d is the dimensionality of the space. In the $d=2$ case, the function $\widetilde{F}_c(\mathbf{r},\mathbf{r}')$ does not depend on the variables x, x', so we can set $p_x=0$ and $\nu=-1/2$ in (37).

The unknown constant of the diffusive solution, α, which appears in (36) is determined by the condition at the boundary of the medium. For boundary condition (34) we would have

$$\alpha=\frac{D_\nu(-k_y)+\tau_0D_\nu'(-k_y)}{-D_\nu(k_y)+\tau_0D_\nu'(k_y)}, \tag{39}$$

where

$$k_y=p_y(r_H/p_0)^{1/2},\quad \tau_0=2l_0(p_0/r_H)^{1/2}.$$

Substituting solution (35) and (36) into (31), we find the following result for the interference part of the backscattering angular distribution:

$$J_c(\mathbf{p},\mathbf{p}_0)=\frac{1}{4\pi(d-1)}\left\{A+2l\left(\frac{p_0}{r_H}\right)^{1/2}B\Phi\left(\left(\frac{r_H}{p_0}\right)^{1/2}(\mathbf{p}_0+\mathbf{p})_\parallel\right)\right\}, \tag{40}$$

where the universal function $\Phi(\mathbf{q})$ is defined by

$$\Phi(\mathbf{q})=\Phi(q_x,q_y)=\frac{D'_{\nu(q_x)}(q_y)}{D_{\nu(q_x)}(q_y)}. \tag{41}$$

Since (40) was derived with the help of the solution for $F_c(\mathbf{r},\mathbf{r}')$ in the diffusion approximation, the range of applicability of (40) is limited by the inequality $|(\mathbf{p}_0+\mathbf{p})_\parallel|l\ll1$.

The constants A and B in (40) do not depend on the direction of the magnetic field. They are determined only by the additional condition on Eq. (33).

For the case of a condition of the type (34) at the surface, the coefficients A and B are precisely the same as the known values of the universal constants which appear in the diffusion solution in the case $\mathbf{H}=0$ (Ref. 18). For normal incidence ($\mu_0=1$) in the $3D$ case, for example, we would have

$$A=3(1+2l_0/l)\approx7{,}26,\qquad B=6(1+l_0/l)^2\approx17{,}55. \tag{42}$$

Another way to calculate the coefficients A and B is to compare (40) with the diffusion limit of the exact expression for the backscattering angular distribution in the absence of a magnetic field which was found in Ref. 21 for $2D$ and $3D$ disordered systems. That approach makes it possible to find the exact values of the coefficients A and B.

At angles $(r_H p_0)^{-1/2} \ll \vartheta \ll (p_0 l)^{-1}$ from the exactly backward direction, the magnetic field does not affect the shape of the backscattering angular distribution, so expression (40) should become the corresponding expression derived in Ref. 21. Again in this angular region, using the Darwin expansion[33] for the function $D_\nu(\tau)$, we can put Eq. (40) in the form

$$J_c(\mathbf{p}, \mathbf{p}_0) = \frac{1}{4\pi(d-1)}[A - Bl|(\mathbf{p}+\mathbf{p}_0)_\parallel|]. \tag{43}$$

Comparing (43) with expression (50) from Ref. 21, we find the following expressions for the coefficients A and B:

$$A = \mu_0[Z^2(\mu_0, 1) - 1], \quad B = 2\mu_0^2 Z^2(\mu_0, 1), \tag{44}$$

where the function $Z(\mu, 1)$ in the $3D$ case is the same as the Chandrasekhar function $H(\mu, 1)$ (Refs. 31 and 32). In the $2D$ case, it is the same as its $2D$ analog $h(\mu, 1)$ (Ref. 21). According to (44), the values of A and B for normal incidence ($\mu_0 = 1$) are

$$\begin{aligned} &A = 7{,}45\ldots, \quad B = 16{,}91\ldots \ (d=3), \\ &A = 5{,}42\ldots, \quad B = 12{,}84\ldots \ (d=2). \end{aligned} \tag{45}$$

Expression (41) for the universal function $\Phi(\mathbf{q}_\parallel)$ can be simplified if one of the components of the vector $\mathbf{q}_\parallel$ vanishes. In the case of backscattering of particles in the plane parallel to the magnetic field lines ($q_y = 0$), we would have

$$\Phi(q_x, q_y=0) = -2^{1/2}\frac{\Gamma(3/4 + 1/8 q_x^2)}{\Gamma(1/4 + 1/8 q_x^2)}. \tag{46}$$

If the momentum transfer $(\mathbf{p} + \mathbf{p}_0)_\parallel$ is perpendicular to $\mathbf{H}$, the angular distribution of the reflected particles can be expressed in terms of the function $\Phi(q_x = 0, q_y)$, given by

$$\Phi(q_x=0, q_y) = -\frac{|q_y|}{2}\frac{I_{-3/4}(q_y^2/4) - (q_y/|q_y|)I_{3/4}(q_y^2/4)}{I_{-1/4}(q_y^2/4) - (q_y/|q_y|)I_{1/4}(q_y^2/4)}, \tag{47}$$

where $I_\alpha(x)$ is the modified Bessel function.[33]

The linear decrease in $J_c(\mathbf{p}, \mathbf{p}_0)$ with increasing momentum transfer $|(\mathbf{p} + \mathbf{p}_0)_\parallel|$ which prevails in the absence of a magnetic field (Refs. 9, 18, and 21, for example) is therefore replaced in the case at hand by (40), which is determined by the universal function $\Phi(\mathbf{q})$.

4. ANGULAR DISTRIBUTION OF THE BACKSCATTERING OF PARTICLES IN A MAGNETIC FIELD DIRECTED PERPENDICULAR TO THE SURFACE OF THE MEDIUM

In contrast with the case discussed above, in this section of the paper we are interested in the reflection of particles from only a $3D$ disordered medium. When the field $\mathbf{H}$ is directed perpendicular to the surface, it has no effect on the $2D$ motion.

There is one distinctive feature in the formulation of the problem of calculating the backscattering angular distribution in a magnetic field oriented perpendicular to the surface of the medium. The uniform field $\mathbf{H}$ can no longer be assumed to be zero except inside the medium. By virtue of the continuity of the field lines, there is also a magnetic field in the vacuum, and it affects the motion of the particles on the ballistic parts of their paths.

In this case, the uncertainty regarding the projection of the momentum of the incident particles (and of the backscattered particles) onto the $z = 0$ plane results from the "classical" curvature of the paths in the magnetic field, which we mentioned above (Sec. 2) and also from quantum-mechanical effects associated with the motion of the particles at small angles from the direction of $\mathbf{H} = (0,0,H)$.

We assume that the charged-particle flux is incident on the surface of the disordered medium at an angle θ_0 (θ_0 is the angle between the velocity of the particles and the z axis).

If the angle θ_0 is small $[\theta_0 \lesssim (p_0 r_H)^{-1/2}]$, the quantum-mechanical nature of the motion in the external field leads to an uncertainty on the order of $|\Delta(\mathbf{p}_0)_\parallel| \sim (p_0 r_H)^{1/2}$ in the projection of the momentum of the incident particles onto the xy plane. It thus becomes impossible to observe those changes in the backscattering angular distribution which are due to the disruption of interference.

The effect of the magnetic field on the angular distribution can be observed only at angles of incidence $\theta_0 \gg (p_0 r_H)^{-1/2}$. In this case the motion of the particles in the external field outside the medium is semiclassical, and the uncertainty in the projection of the initial momentum $|\Delta(\mathbf{p}_0)_\parallel| \sim (r_H\theta_0)^{-1}$ is much lower than the scale value of the momentum transfer, $|(\mathbf{p} + \mathbf{p}_0)_\parallel| \sim (p_0/r_H)^{1/2} \gg |\Delta(\mathbf{p}_0)_\parallel|$, which characterizes the shape of the backscattering angular distribution $J_c(\mathbf{p},\mathbf{p}_0)$ near the backward direction.

For angles of incidence $\theta_0 \gg (p_0 r_H)^{-1/2}$, with complete cancellation of the ballistic phase shift of the wave functions, we can use (31) for the interference part of the angular distribution $J_c(\mathbf{p},\mathbf{p}_0)$, and we can reduce the problem to one of calculating the propagator $F_c(\mathbf{r},\mathbf{r}')$.

If the magnetic field is oriented perpendicular to the surface, we can find an exact analytic solution of Eq. (20). The solution method is based on the utilization of the following circumstance. Since the medium is unbounded in the plane perpendicular to H, the kernel of the integral equation (20) is an even function of the variable $(\mathbf{R}_1)_\parallel$. The external field thus enters the kernel only in the following combination with derivatives with respect to $\boldsymbol{\rho} = \mathbf{r}_\parallel$:

$$\hat{\mathbf{p}}_\parallel^2 = \left(\frac{1}{i}\frac{\partial}{\partial\boldsymbol{\rho}} - 2e\mathbf{A}\right)^2.$$

The $\hat{\mathbf{p}}_\parallel^2$ dependence of the kernel of the equation allows us to seek a solution as an expansion of $F_c(\mathbf{r},\mathbf{r}')$ in eigenfunctions of the operator $\hat{\mathbf{p}}_\parallel^2$. This approach to the solution of an equation like (20) was first proposed in Ref. 30, for the case of an infinite medium (one not bounded by a surface). This approach has subsequently been taken in several theoretical papers on impure superconductors.[34,35]

We choose the vector potential in the form $\mathbf{A} = (-Hy,0,0)$. Expanding the propagator $F_c(\mathbf{r},\mathbf{r}')$ in eigenfunctions of the operator $\hat{\mathbf{p}}_\parallel^2$, we find

$$F_c(\mathbf{r},\mathbf{r}')=\sum_{n,p_x}F_c(z,z',n)\exp[ip_x(x-x')]\chi_{np_x}(y)\chi_{np_x}(y'),$$

(48)

where $\chi_{np_x}(y)\exp(ip_x x)$ are the eigenfunctions of the operator $\hat{\mathbf{p}}_{\parallel}^2$:

$$\hat{\mathbf{p}}_{\parallel}^2\chi_{np_x}(y)\exp(ip_x x)=\varepsilon_n\chi_{np_x}(y)\exp(ip_x x), \quad (49)$$

$$\chi_{np_x}(y)=\left(\frac{2}{\pi}\right)^{1/4}\frac{1}{(2^n n!)^{1/2}}\left(\frac{p_0}{r_H}\right)^{1/4}$$

$$\times\exp\left\{-\frac{p_0}{r_H}\left(y+\frac{p_x r_H}{2p_0}\right)^2\right\}H_n\left(\left[\frac{2p_0}{r_H}\right]^{1/2}\left[y+\frac{p_x r_H}{2p_0}\right]\right). \quad (50)$$

Here $H_n(x)$ is the Hermite polynomial of index n (Ref. 33). The eigenvalue ε_n in (49) is given by

$$\varepsilon_n=(\hat{\mathbf{p}}_{\parallel}^2)_n=4(p_0/r_H)(n+{}^1/_2). \quad (51)$$

Substituting expansion (48) into our original equation, (20), we find an integral equation with a difference kernel for $F_c(z,z',n)$:

$$F_c(z,z',n)=K_n(|z-z'|)+\int_0^\infty dz''K_n(|z-z''|)F_c(z'',z',n). \quad (52)$$

For the kernel $K_n(|z|)$ we can use the representation

$$K_n(|z|)=\frac{\pi n\sigma}{m^2}\int_0^\infty d^2\rho|G_0([\rho^2+z^2]^{1/2})|^2\exp\left(-\frac{1}{2}\frac{p_0}{r_H}\rho^2\right)L_n\left(\frac{p_0}{r_H}\rho^2\right), \quad (53)$$

where $L_n(x)$ is the Laguerre polynomial.[33] In going from Eq. (20) to Eq. (52) with kernel (53), we need to allow for the circumstance that the operators $\hat{p}_x$ and $\hat{p}_y$ do not commute: $[\hat{p}_x\hat{p}_y]=2ieH$ (Refs. 30, 34, and 35).

It is a straightforward matter to solve Eq. (52) by the Wiener–Hopf method. Using results from Ref. (21), we find the following expression for the Laplace transform of the function $F_c(z,z',n)$:

$$F_c(p,p',n)=(p+p')^{-1}[U_n(p)U_n(p')-1], \quad (54)$$

where

$$U_n(p)=\exp\left[-\frac{p}{\pi}\int_0^\infty\ln\Lambda_n(q_z)\frac{dq_z}{q_z^2+p^2}\right], \quad (55)$$

$$\Lambda_n(q_z)=1-K_n(q_z), \quad (56)$$

$$K_n(q_z)=\int_{-\infty}^\infty dzK_n(|z|)\exp(-iq_z z). \quad (57)$$

The explicit expression for the function $\Lambda_n(q_z)$ is

$$\Lambda_n(q_z)=1-(-1)^n\int_0^\infty\xi\,d\xi\,L_n(\xi^2)\exp\left(-\frac{\xi^2}{2}\right)\frac{\operatorname{arctg}[l(q_z^2+p_0\xi^2/r_H)^{1/2}]}{l(q_z^2+p_0\xi^2/r_H)^{1/2}}$$

$$=\frac{1}{3}l^2\left[q_z^2+4\frac{p_0}{r_H}\left(n+\frac{1}{2}\right)\right]$$

$$-\frac{1}{5}l^4\left[q_z^2+4\frac{p_0}{r_H}\left(n+\frac{1}{2}\right)\right]^2-\frac{4}{5}l^4\frac{p_0^2}{r_H^2}+\ldots \quad (58)$$

We can use Eq. (54) to find the final expression for the backscattering angular distribution. Substituting expansion (48) into (43), and noting that the additional phase shift $\hat{\varphi}$ in (43) is

$$\hat{\varphi}=-2\varphi(\mathbf{R},\mathbf{R}')=\frac{p_0}{r_H}(y+y')(x-x'),$$

we find

$$J_c(\mathbf{p},\mathbf{p}_0)=\frac{1}{2\pi}\frac{|\mu|\mu_0}{|\mu|+\mu_0}\sum_{n=0}^\infty(-1)^n\left[\left|U_n\left(\frac{1}{l\tilde{\mu}}\right)\right|^2-1\right]$$

$$\times\exp\left\{-\frac{1}{2}\frac{r_H}{p_0}[(\mathbf{p}+\mathbf{p}_0)_\parallel]^2\right\}L_n\left(\frac{r_H}{p_0}(\mathbf{p}+\mathbf{p}_0)_\parallel^2\right), \quad (59)$$

where

$$\tilde{\mu}^{-1}={}^1/_2(|\mu|^{-1}+\mu_0^{-1})+ip_0l(|\mu|-\mu_0).$$

Let us analyze this result. In the situation under consideration here, in which the inequality (2) holds, a large number of terms, $1\ll n_{\rm eff}\lesssim r_H/p_0l^2$, contribute to the sum in (48). At large values of n we can use the following asymptotic expression[33] for $L_n(x^2)$:

$$\exp(-{}^1/_2x^2)L_n(x^2)\approx J_0(2x[n+{}^1/_2]^{1/2}). \quad (60)$$

We can write the kernel of Eq. (52) as

$$K_n(|z|)\approx\frac{\pi n\sigma}{m^2}\int d^2\rho|G_0([z^2+\rho^2]^{1/2})|^2J_0\left(2\rho\left[\frac{p_0}{r_H}\left(n+\frac{1}{2}\right)\right]^{1/2}\right), \quad (61)$$

There is a slightly different way to derive (61). In this approach we note that at large values of n the circumstance that the operators $\hat{p}_x$ and $\hat{p}_y$ do not commute is irrelevant, and we can deal with them as if they were ordinary numbers.

Integrating over the angle between $\boldsymbol{\rho}$ and $\hat{\mathbf{p}}_\parallel$, we then find

$$\left[\int_0^{2\pi}d\varphi\exp(-i\boldsymbol{\rho}\hat{\mathbf{p}}_\parallel)\right]\chi_{np_x}(y)\exp(ip_x x)$$

$$\approx2\pi J_0(\rho[\hat{p}_\parallel^2]^{1/2})\chi_{np_x}(y)\exp(ip_x x)$$

$$=2\pi J_0(\rho\varepsilon_n^{1/2})\chi_{np_x}(y)\exp(ip_x x), \quad (62)$$

where ε_n is given by (51). Approximation (62) leads immediately to (61).

Evaluating the integral in (61), we find the following approximation for the function $\Lambda_n(q_z)$:

$$\Lambda_n(q_z)\approx 1-\frac{\operatorname{arctg}\{[l(q_z^2+4p_0(n+{}^1/_2)/r_H]^{1/2}\}}{l[q_z^2+4p_0(n+{}^1/_2)/r_H]^{1/2}}$$

$$=\frac{1}{3}l^2\left[q_z^2+4\frac{p_0}{r_H}\left(n+\frac{1}{2}\right)\right]$$

$$-\frac{1}{5}l^2\left[q_z^2+4\frac{p_0}{r_H}(n+{}^1/_2)\right]^2+\ldots \qquad (63)$$

A comparison of the approximation (63) for the function $\Lambda_n(q_z)$ with the exact expression (58) reveals that the results in (63) and (58) differ by a small quantity on the order of $(p_0l^2/r_H)^2\ll 1$. Ignoring this difference, we can express the function $U_n(1/l\tilde{\mu})$, which appears in the expression for the angular distribution, in terms of a "generalized" Chandrasekhar function $H(\mu,1|\nu)$:

$$U_n(1/l\tilde{\mu})=H(\mu,\ 1|\nu_n), \qquad (64)$$

where

$$\nu_n=\left[4\frac{p_0l^2}{r_H}\left(n+\frac{1}{2}\right)\right]^{1/2}. \qquad (65)$$

For $H(\mu,\omega|\nu)$ we can use the integral representation[21]

$$H(\mu,\ \omega|\nu)=\exp\left[-\frac{\mu}{\pi}\int_0^\infty \ln\left[1-\omega\frac{\operatorname{arctg}(\xi^2+\nu^2)^{1/2}}{(\xi^2+\nu^2)^{1/2}}\right]\frac{d\xi}{1+\mu^2\xi^2}\right]. \qquad (66)$$

Substitution of (64) into (59) makes it possible to write an expression for the backscattering angular distribution:

$$J_c(\mathbf{p},\mathbf{p}_0)=\frac{1}{2\pi}\frac{|\mu|\mu_0}{|\mu|+\mu_0}\sum_{n=0}^\infty(-1)^n[|H(\tilde{\mu},1|\nu_n)|^2-1]$$

$$\times\exp\left[-\frac{1}{2}\frac{r_H}{p_0}(\mathbf{p}+\mathbf{p}_0)_\parallel^2\right]L_n\left(\frac{r_H}{p_0}(\mathbf{p}+\mathbf{p}_0)_\parallel^2\right). \qquad (67)$$

For angular deviations $\vartheta\ll(p_0l)^{-1}$ from the backward direction, the quantity $J_c(\mathbf{p},\mathbf{p}_0)$ is dominated by the first $n\lesssim r_H/p_0l^2$ terms. We can therefore use an approximation of $H(\mu,\ 1|\nu)$ in (67) under the condition $\nu\ll 1$:

$$H(\mu,1|\nu)=\frac{H(\mu,1|0)}{1+\mu|\nu|}, \qquad (68)$$

where $H(\mu,\ 1)=H(\mu,\ 1|0)$ the ordinary Chandrasekhar function.[31,32] For $J_c(\mathbf{p},\mathbf{p}_0)$ under the condition $|(\mathbf{p}+\mathbf{p}_0)_\parallel|\ll l^{-1}$ we find the expression

$$J_c(\mathbf{p},\mathbf{p}_0)\approx\frac{\mu_0}{4\pi}\left\{-\frac{1}{2}+\sum_{n=0}^\infty\frac{(-1)^nH^2(\mu_0,1)}{\{1+\mu_0[4(p_0l^2/r_H)(n+{}^1/_2)]^{1/2}\}^2}\right.$$

$$\times\exp\left[-\frac{1}{2}\frac{r_H}{p_0}(\mathbf{p}+\mathbf{p}_0)_\parallel^2\right]L_n\left(\frac{r_H}{p_0}(\mathbf{p}+\mathbf{p}_0)_\parallel^2\right). \qquad (69)$$

Analysis of the results in (67) and (69) shows that the effect of the magnetic field on the interference part of the backscattering angular distribution is manifested only at angular deviations

$$\vartheta<(p_0r_H)^{-1/2}\quad(|(\mathbf{p}+\mathbf{p}_0)_\parallel|<(p_0/r_H)^{1/2})$$

from the backward direction. The rounding of the backscattering peak observed in this angular region stems from a disruption of the coherence of the wave functions of the particles which have traveled a distance $s>(r_H/p_0)^{1/2}$ through the disordered medium.

With increasing angular deviation, $\vartheta>(p_0r_H)^{-1/2}$, the quantity $J_c(\mathbf{p},\mathbf{p}_0)$ starts to be dominated by the shorter paths $s<(r_H/p_0)^{1/2}$, the dependence of the angular distribution on the magnetic field fades away, and the distribution (67) becomes the exact solution,[21] valid for charged particles in the case $\mathbf{H}=0$. These assertions are easily verified by using the well-known representation of Laguerre polynomials with $n\gg 1$ in terms of Airy functions[33] and by replacing the summation over n in (67) by an integration and then evaluating the resulting integral by the stationary-phase method.

5. DISCUSSION OF RESULTS

The expressions derived above for the interference contribution to the flux density of backscattered particles, (40) and (67), make possible a detailed study of how the disruption of the coherence of the particle wave functions in a magnetic field affects the shape of the angular distribution of the backscattering.

Analysis of the results above shows that a magnetic field "deforms" the peak in the angular distribution at angular deviations $\vartheta<(p_0r_H)^{-1/2}$ from the backward direction.

When $\mathbf{H}$ is directed parallel to the surface of the medium, a structural feature appears in the angular distribution $J_c(\mathbf{p},\mathbf{p}_0)$ which has no analog in the backscattering of scalar waves, of light, or of particles with spin $s=1/2$ (Refs. 9–24). The backscattering peak is shifted with respect to the exactly backward direction and is observed at

$$\mathbf{p}_{\parallel\,max}=\left(-p_{0x},\ -p_{0y}-0{,}75\frac{e}{|e|}\left(\frac{p_0}{r_H}\right)^{1/2}\right). \qquad (70)$$

According to (70), in the reflection of electrons ($e<0$) the maximum moves clockwise away from the direction $\mathbf{p}=-\mathbf{p}_0$ around the direction of $\mathbf{H}$, through an angle $\Delta\vartheta_{\max}\approx 0.75(p_0r_H)^{-1/2}$.

The reason for this shift of the peak is quantum interference. This shift is totally unrelated to the curvature of the particle paths in a magnetic field.

When an external magnetic field is applied, the amplitude of the reflected wave acquires an additional phase factor

$$A_H(\boldsymbol{\rho},\ \boldsymbol{\rho}')=A_{H=0}(\boldsymbol{\rho},\ \boldsymbol{\rho}')\exp[ie\phi(\boldsymbol{\rho},\ \boldsymbol{\rho}')], \qquad (71)$$

where $\phi(\boldsymbol{\rho},\boldsymbol{\rho}')$ is the magnetic flux linking the loop formed by the particle path in the medium and the straight line segment which connects the points at which the particle intersects the boundary plane ($z=0$) as it leaves ($\boldsymbol{\rho}$) and as it enters ($\boldsymbol{\rho}'$). The interference contribution to the backscattering intensity is, according to (71),

$$A_H(\boldsymbol{\rho},\ \boldsymbol{\rho}')\exp[i\mathbf{p}_0\boldsymbol{\rho}'-i\mathbf{p}\boldsymbol{\rho}]A_H^*(\boldsymbol{\rho},\ \boldsymbol{\rho}')\exp[-i\mathbf{p}_0\boldsymbol{\rho}+i\mathbf{p}\boldsymbol{\rho}']$$

$$+\text{c.c.}=|A_{H=0}(\boldsymbol{\rho},\ \boldsymbol{\rho}')|^2\exp[2ie\phi(\boldsymbol{\rho},\ \boldsymbol{\rho}')-i(\mathbf{p}_0+\mathbf{p})(\boldsymbol{\rho}-\boldsymbol{\rho}')]+\text{c.c.}, \qquad (72)$$

where

$$|A_{H=0}(\boldsymbol{\rho},\boldsymbol{\rho}')|^2=\prod_{i,j}|G_0(|\mathbf{r}_i-\mathbf{r}_j|)|^2$$

is the probability density for the particle transition $\rho \to \rho'$ in the absence in a magnetic field. Taking an average of (72) over all possible particle paths, we find the backscattering intensity to be

$$J_c(\mathbf{p}, \mathbf{p}_0) = \iint d\rho\, d\rho'\, F_{H=0}(\rho, \rho') \exp[-i(\mathbf{p}+\mathbf{p}_0)(\rho-\rho') + 2ie\langle\phi\rangle_{\rho\rho'} - 2e^2\langle(\phi - \langle\phi\rangle_{\rho\rho'})^2\rangle_{\rho\rho'}$$

$$= \iint d\rho\, d\rho'\, F_{H=0}(\rho, \rho') \times \exp[-2e^2\langle(\phi - \langle\phi\rangle_{\rho\rho'})^2\rangle_{\rho\rho'}] \times \cos[(\mathbf{p}+\mathbf{p}_0)(\rho-\rho') - 2e\langle\phi\rangle_{\rho\rho'}]. \quad (73)$$

Here $F_{H=0}(\rho,\rho')$ is the probability density for the transition of a particle between ρ and ρ' in the absence of a field, and $\langle\phi\rangle_{\rho\rho'}$ and $\langle(\phi - \langle\phi\rangle_{\rho\rho'})^2\rangle_{\rho\rho'}$ are the average value and dispersion of the magnetic flux linking the loop formed by the surface of the medium and the path of a particle moving from the point ρ' to the point ρ. Expression (73) is valid under the condition that the motion of the particles is in the way of a diffusion, and the phase ϕ can be treated as a random quantity with a normal distribution.

By writing the interference contribution as in (73) we easily see the reason for the shift of the backscattering peak. It follows from (73) that the shift of the $J_c(\mathbf{p},\mathbf{p}_0)$ peak stems from a phase shift of the wave functions of the interfering particles, which is proportional to the average value of the magnetic flux, $\langle\phi\rangle_{\rho\rho'}$. Since the value of $\langle\phi\rangle_{\rho\rho'}$ is determined by the average area of the loop formed by the boundary and the particle path connecting the points ρ' and ρ, we can write the following in the gauge (32):

$$\langle\phi\rangle_{\rho\rho'} = -H(y-y')\langle z\rangle, \quad (74)$$

where $\langle z\rangle$ is the average depth to which the particles penetrate into the medium. Substituting (74) into (73), we find that the backscattering peak should be observed in the direction

$$\mathbf{p}_{\parallel\, max} = \left(-p_{0x}, -p_{0y} - 2\frac{e}{|e|} p_0 \frac{\langle z\rangle}{r_H}\right). \quad (75)$$

Comparison of (75) with (70) yields $\langle z\rangle \approx 0.38\,(r_H/p_0)^{1/2}$.

As can be seen from the expression for the distribution in (73), the rounding of the backscattering peak stems from the dispersion of the magnetic flux linking the loop formed by the particle paths. The increase in this dispersion with increasing length of the paths establishes a limiting path length over which the coherence of the particle wave functions will be preserved. The fact that the linear paths no longer contribute to $J_c(\mathbf{p},\mathbf{p}_0)$ leads to a rounding of the peak in the backscattering angular distribution, in accordance with the general understanding.[15,16,18]

It follows from expression (40) that the backscattering intensity at the peak is given by

$$J_c^{max} = \frac{1}{4\pi(d-1)}\left[A - 0{,}75l\left(\frac{p_0}{r_H}\right)^{1/2} B\right], \quad (76)$$

where expressions (44) can be used for the coefficients A and B.

In the case in which the momentum transfer is perpendicular to the magnetic field, $(\mathbf{p}+\mathbf{p}_0)_\parallel \cdot H = 0$, the angular distribution of the backscattered particles for a $3D$ medium is as shown in Fig. 2. The backscattering angular distribution has a similar shape for scattering from a $2D$ disordered system. Near the peak, the distribution $J_c(\mathbf{p},\mathbf{p}_0)$ can be written

$$J_c(p_y, p_{0y}, p_x = -p_{0x}) = J_c^{max} - \frac{0{,}75B}{8\pi(d-1)} l\left(\frac{r_H}{p_0}\right)^{1/2} (p_y - p_y^{max})^2. \quad (77)$$

With increasing angular deviation from the exactly backward direction $[\vartheta \gg (p_0 r_H)^{-1/2}]$, the effect of the magnetic field falls off, and the angular distribution approaches that in the case $\mathbf{H} = 0$:

$$J_c(p_y, p_{0y}, p_x = -p_{0x}) = \frac{1}{4\pi(d-1)}\left\{A - Bl\left[|p_y + p_{0y}| + \frac{e}{|e|}\frac{p_y + p_{0y}}{(p_y+p_{0y})^2}\frac{p_0}{r_H} + \ldots\right]\right\}. \quad (78)$$

The angular distribution of the backscattering intensity in the case $(\mathbf{p}+\mathbf{p}_0)_\parallel \cdot \mathbf{H} = 0$ is thus characterized not only by a shift of the peak but also an asymmetry, which is also a consequence of the effect of $\mathbf{H}$ on the interference of the particle wave functions.

For reflection of the particles in the plane parallel to the magnetic field vector,[2)] i.e., $p_y = -p_{0y}$, the angular distribution is symmetric with respect to the backward direction (Fig. 2). Near the peak we have

$$J_c(p_x, p_{0x}, p_y = -p_{0y}) = J_c(-\mathbf{p}_0, \mathbf{p}_0) - Cl\left(\frac{r_H}{p_0}\right)^{1/2} |p_x + p_{0x}|^2, \quad (79)$$

where $J_c(-\mathbf{p}_0,\mathbf{p}_0)$ is the scattering intensity in the backward direction, given by

$$J_c(-\mathbf{p}_0, \mathbf{p}_0) = \frac{1}{8\pi}\left[A - 2^{1/2}Bl\left(\frac{p_0}{r_H}\right)^{1/2}\frac{\Gamma(3/4)}{\Gamma(1/4)}\right]. \quad (80)$$

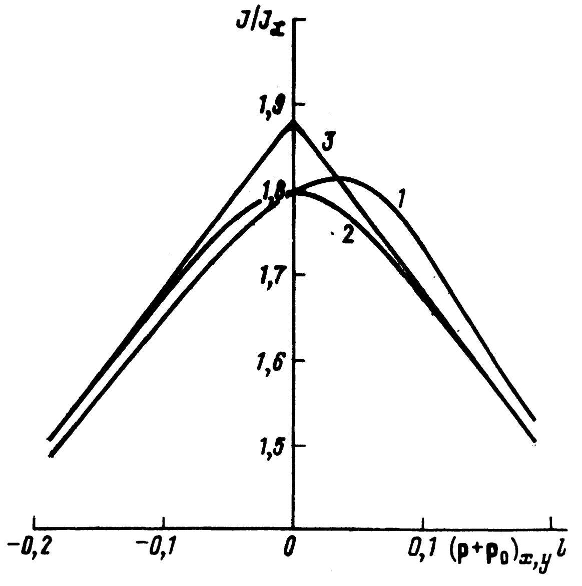

FIG. 2. Angular distribution of the electron backscattering intensity near the backward direction in a magnetic field directed parallel to the surface of the medium ($\mu_0 = 1$). *1*—$(\mathbf{p}_0 + \mathbf{p})_x = 0$; *2*—$(\mathbf{p}_0 + \mathbf{p})_y = 0$; *3*—$r_H = 500 p_0 l^2$; 3—$r_H \to \infty$ ($H = 0$).

The coefficient C in (79) is given by

$$C=\frac{1}{8\pi}\frac{B}{2^{3/4}}\frac{\Gamma(3/4)}{\Gamma(1/4)}\left[\psi\left(\frac{3}{4}\right)-\psi\left(\frac{1}{4}\right)\right] \tag{81}$$

where $\psi(x) = \Gamma'(x)/\Gamma(x)$ is the logarithmic derivative of the gamma function.[33]

With increasing deviation from the backward direction, the dependence of J_c on $|p_x + p_{0x}|$ approaches linearity; i.e., it becomes the result which has been found elsewhere in the absence of a magnetic field.[10,15-19,21]

The behavior of the angular distribution $J_c(\mathbf{p},\mathbf{p}_0)$ during backscattering from a disordered medium in a magnetic field directed parallel to the surface is qualitatively reminiscent of the dependence of J_c on $|p_x + p_{0x}|$ discussed above (Fig. 3). No shift of the $J_c(\mathbf{p},\mathbf{p}_0)$ peak is observed. The reason is that the motion of the particles in the plane perpendicular to $\mathbf{H}$ is not bounded by the surface of the medium in this case, so symmetry gives us $\langle\phi\rangle_{\mathbf{pp}'} = 0$. The rounding of the backscattering peak occurs, according to the interpretation of (73), because of fluctuations of the magnetic flux linking the projection of the loop formed by the particle paths onto the surface of the medium. The scattering intensity in the backward direction is given by

$$J_c(-\mathbf{p}_0,\mathbf{p}_0)=\frac{1}{8\pi}\left[[A-0.78]Bl\left(\frac{p_0}{r_H}\right)^{1/2}\right], \tag{82}$$

where the coefficients A and B are given by (44) for a 3D medium. In the case at hand the quantity $J_c(-\mathbf{p}_0,\mathbf{p}_0)$ is simultaneously the maximum value of the backscattering intensity.

In all the cases discussed above, the difference

$$J_c^{max}(\mathbf{H}=0)-J_c^{max}(\mathbf{H})$$

thus increases with increasing $\mathbf{H}$ in proportion to $(l^2p_0/r_H)^{1/2}\propto\sqrt{H}^{1/2}$.

We would simply like to call attention to a slight difference between the numerical coefficients of the second terms in (76) and (82). This difference stems from the scattering geometry: In the former case the value $J_c^{max}(\mathbf{H}=0) - J_c^{max}(\mathbf{H})$ is determined by fluctuations in the area of the projection of the loop formed by the paths onto the plane perpendicular to the surface, while in the latter case it is the projection onto the surface of the medium itself.

We note in conclusion that the results derived above are valid in the case in which the mean free path with respect to inelastic interactions which would disrupt the interference of the particle wave functions is large:

$$l_{in}\gg r_H/p_0l, \tag{83}$$

where $l_{in} = n\sigma_{in}$, and σ_{in} is the cross section for an inelastic interaction. If inequality (83) does not hold, the expression found above must be generalized to take the inelastic interactions into account. In the case of most interest here, that in which there is a low probability for inelastic interactions, $\sigma_{in}\ll\sigma$, the generalization procedure reduces to one of making the following changes in the results derived above.

When a magnetic field is directed parallel to the surface of the medium, the only changes are in the index ν of the parabolic cylinder function $D_\nu(\tau)$ [see (37)]. We must now replace (37) by

$$\nu=-\frac{1}{2}-\frac{1}{4}\left[k_x^2+d(1-\omega)\frac{r_H}{p_0l^2}\right], \tag{84}$$

where d is the dimensionality of the system, $\omega = \sigma/\sigma_{tot}$, and $\sigma_{tot} = \sigma + \sigma_{in}$.

For reflection of particles in a magnetic field oriented perpendicular to the surface of the medium, the inelastic interactions affect the expression for the function Λ_n in (56). In the case $\omega = 1$ we have $\Lambda_n = 1 - \omega K_n$. Correspondingly, the function $H(\mu,1|\nu)$ in (67) must be replaced by $H(\mu,\omega|\nu)$, and in making the transition from (67) to (69) we replace the approximation (68) by the more general expression[21]

$$H(\mu,\omega|\nu)=H(\mu,1)\{1+\mu[3(1-\omega)+\nu^2]^{1/2}\}^{-1}.$$

If the inequality (83) is reversed, the effect of the magnetic field on the quantum interference of the particles is suppressed. In particular, the effect of the magnetic field on the backscattering intensity leads to only small corrections which are linear in H and which are on the order of p_0ll_{in}/r_H.

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Note added in proof (2 August 1991). After this paper had been sent to press, we became aware of recent work [R. Berkovits, D. Eliyahn, and M. Kavel, Phys. Rev. C. **41**, 407 (1990)] which also treated the problem of quantum interference when electrons are reflected from a magnetic field. The definition of the backscattering angular spectrum used there is not gauge invariant and yields an erroneous result for the interference contribution J_c.

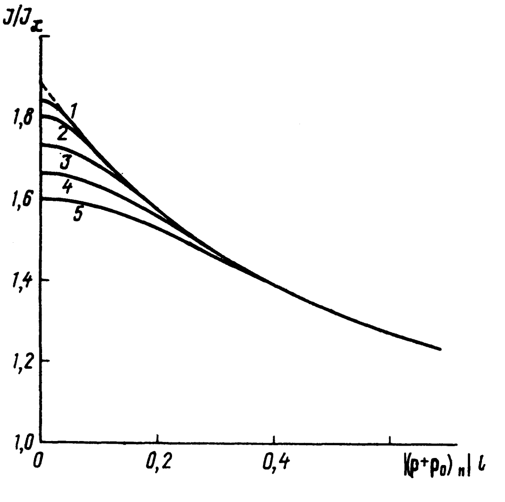

FIG. 3. Angular distribution of the backscattering intensity in a magnetic field directed perpendicular to the surface for various values of r_H: 1—$l(p_0/r_H)^{1/2} = 0.025$; 2—0.05; 3—0.1; 4—0.15; 5—0.2. Here it is assumed that the inequalities $(p_0r_H)^{-1}\ll 1-\mu_0\ll 1$ hold.

[1)] In general, perfect cancellation would not be possible because of the spread in the values of $\mathbf{R}$ and $\mathbf{R}'$ due to the uncertainty in the positions of the first and last collisions of the particle with scattering centers in coordinate space. However, this is not a point of great importance, since the phase shift caused by the spread in $\mathbf{R}$ and $\mathbf{R}'$, i.e., $e(\phi_0+\phi_1)_{min}\lesssim p_0l|\mathbf{R}-\mathbf{R}'|/r_H$ leads to an uncertainty $|\Delta(\mathbf{p}+\mathbf{p}_0)_\parallel|\lesssim p_0l/r_H$ which is on the same order of magnitude as the "classical" curvature of the particle paths in a magnetic field.

[2)] This situation does not occur in the case of backscattering from a 2D system.

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