Dynamics of uniform texture in ^{3}He–*B*

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Equations are derived to describe the low-frequency dynamics of the uniform texture which forms in a plane-parallel layer of helium whose thickness is smaller than the "healing" texture length when this layer is in a magnetic field directed parallel to the walls of the vessel. The motion occurs from a Leggett configuration, which arises in pulsed NMR experiments after the magnetic moment has relaxed to its equilibrium value. Solutions of the equations near equilibrium positions of the anisotropy axis of the order parameter are derived for both the case without dissipation and the case with a Leggett–Takagi relaxation.

1. INTRODUCTION

In the superfluid *B* phase of ^{3}He, the order parameter is proportional to an orthogonal 3×3 matrix. This is conveniently thought of as a matrix which performs a rotation through an angle θ around the $\hat{\mathbf{n}}$ axis: $\overleftrightarrow{R}(\hat{\mathbf{n}},\theta)$. The magnitude of the angle θ is fixed by the dipole energy: $\theta=\theta_0=\arccos(-1/4)$. In the absence of a magnetic field, and far from the walls, the orientation of $\hat{\mathbf{n}}$ is undetermined. The dipole energy gives rise to a slight anisotropic increment in the magnetic susceptibility, of such a nature that $\hat{\mathbf{n}}$ turns slightly toward a direction parallel or antiparallel to the magnetic field $\mathbf{H}$. The walls also orient[1] $\hat{\mathbf{n}}$. As a result of this competition between two factors, a vessel of each specific shape has a corresponding equilibrium distribution of $\hat{\mathbf{n}}$, a so-called equilibrium texture. This texture affects various phenomena which are observed in ^{3}He–*B*. Textures have been studied for various geometric configurations.[2]

The simplest case, and that most convenient for interpretating of experiments, is a uniform texture. A uniform texture occurs in a layer of ^{3}He–*B* sandwiched between plane-parallel plates. The distance L between the plates at which the orienting effect of the walls is the governing effect in the field H is estimated from the inequality L (cm) $\lesssim 10/H$, where L is in centimeters, and H in oersteds.[3] At the fields ordinarily used, ~ 100 Oe, this distance is fairly large: $L\lesssim 1$ mm. Under these conditions the orientation of $\hat{\mathbf{n}}$ is determined by the minimum of the magnetic surface energy[3–5]

$$F_H{}^S=-q(\mathbf{H}\overleftrightarrow{R}\hat{\mathbf{s}})^2,$$

where $\hat{\mathbf{s}}$ is the normal to the surface. If the field lies in the plane of the plates, there are four equivalent minima. In a coordinate system with $\hat{\mathbf{z}}\|-\mathbf{H}$ and $\hat{\mathbf{y}}\|\hat{\mathbf{s}}$, these directions are (the notation is that of Ref. 6)

$$\hat{\mathbf{a}}=\frac{1}{5^{1/2}}(3^{1/2},-1,-1),\quad \hat{\mathbf{b}}=\frac{1}{5^{1/2}}(3^{1/2},1,1),$$
$$\hat{\mathbf{c}}=\frac{1}{5^{1/2}}(-3^{1/2},1,-1),\quad \hat{\mathbf{d}}=\frac{1}{5^{1/2}}(-3^{1/2},-1,1).$$

If $\hat{\mathbf{n}}$ does not lie along any of these four directions, the order parameter moves, ultimately relaxing toward one of the equilibrium orientations.

Significant deviations from equilibrium texture arise in a natural way in pulsed NMR experiments with ^{3}He–*B* in the geometry described above. Experiments of this sort were first carried out with a stack of plates by Borovik-Romanov *et al.*;[7,8] subsequent experiments were carried out with one slit by Ishikawa *et al.*[6]

After the magnetization relaxation, a so-called Leggett configuration arises. This is a texture in which $\hat{\mathbf{n}}$ is parallel or antiparallel to the field $\mathbf{H}$. Experiments show that this texture relaxes to an equilibrium in a time on the order of 1 ms. Ishikawa *et al.*[6] also carried out a numerical simulation of experiments of this sort, using the complete system of Leggett equations. They found satisfactory agreement with the observations.

Numerical solutions, however, do not draw a simple physical picture of the motion of the order parameter, and they do not bring out important qualitative aspects of the process. We have accordingly carried out an analytic study of the motion of the order parameter and the paths by which it relaxes to equilibrium in a geometry corresponding to the experiments of Refs. 6–8. We will be talking in terms of the motion of the texture, i.e., in terms of motion which is induced by the orienting effect of the walls after the magnetization has relaxed to its equilibrium value.

2. EQUATIONS OF MOTION

Let us consider the problem of the motion of the order parameter in ^{3}He–*B* in a gap between plane-parallel plates, which are separated by a distance which is small enough that the motion can be regarded as spatially uniform. We assume that the magnetic field $\mathbf{H}$ is parallel to the plates and is strong enough ($\gtrsim 20$ Oe) that we can ignore the dipole surface energy. The motion of the spin and of the order parameter in superfluid ^{3}He are described by the Leggett equations,[9] in which we should include the moment of force set up by F_H^s. Under our assumptions regarding the value of L, we can, as in Ref. 6, "smear" this energy over the volume of the helium in the gap. In other words, we can assume that we have a bulk energy

$$V=\frac{1}{L}F_H{}^S=-\frac{a}{H^2}(\mathbf{H}\overleftrightarrow{R}\hat{\mathbf{s}})^2.$$

The units which we will be using below are such that the magnetic susceptibility of the ^{3}He–*B* and the gyromagnetic

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ratio for the ^{3}He nuclei are unity. The energy density has the dimensions of a square frequency; the Zeeman energy is on the order of the square of the Larmor frequency, ω_L^2; the dipole energy is on the order of the square of the frequency of the longitudinal vibrations; and we have $V \sim \omega_L^2 \xi / L$, where ξ is a correlation length. We see from these estimates that the energy V is substantially lower than the two other energies involved in the problem. Its ratio to the Zeeman energy, for example, is characterized by the small parameter

$$\lambda = \frac{a}{\omega_L^2} \sim \frac{\xi}{L},$$

which is on the order of 10^{-3} for the experimental conditions of Refs. 6–8. The frequencies of the motions caused by V should also be small. To describe these low-frequency motions, we will use the procedure proposed in Ref. 10, but with some modifications.

In order to use that procedure, we should express the order parameter in terms of the Euler angles α, β, γ in the following way:

$$\overleftrightarrow{R}(\hat{\mathbf{n}}, \theta) = \overleftrightarrow{R}(\alpha, \beta, \gamma) = \overleftrightarrow{R}_z(\alpha)\, \overleftrightarrow{R}_y(\beta)\, \overleftrightarrow{R}_z(\gamma), \tag{1}$$

where the matrix $\overleftrightarrow{R}_z(\alpha)$ performs a rotation through an angle α around the $\hat{\mathbf{z}}$ axis, etc. The Leggett Hamiltonian should be written in terms of the angles β, γ, $\Phi = \alpha + \gamma$ and their canonical-conjugate spin projections, S_β, $Q = S_\zeta - S_z$, and S_z, respectively, where S_z is the spin projection onto the $\hat{\mathbf{z}}$ axis, S_ζ is the projection onto the $\hat{\boldsymbol{\zeta}} = \overleftrightarrow{R}\hat{\mathbf{z}}$ axis, and S_β is the projection onto the line of nodes $[\hat{\mathbf{z}}, \hat{\boldsymbol{\zeta}}]$:

$$\mathcal{H} = \frac{1}{2\sin^2\beta}[Q + S_z(1-\cos\beta)]^2 + \frac{S_\beta^2}{2} + U_D(\beta, \Phi) + \frac{(S_z-\omega_L)^2}{2} + V(\beta, \gamma). \tag{2}$$

The dipole energy U_D does not depend on γ; the explicit expression for it will not be needed below. The energy V is given by the following expression in terms of the Euler angles:

$$V = -a\sin^2\beta\sin^2\gamma. \tag{3}$$

We break up the Hamiltonian (2) into a "main" Hamiltonian $\mathcal{H}^{(0)}$ and a perturbation W, in which we include the last two terms:

$$W = V + \frac{(S_z-\omega_L)^2}{2}. \tag{4}$$

Here we are assuming motions for which these two terms are comparable in magnitude. The energy $(S_z - \omega_L)^2/2$ was incorporated in $\mathcal{H}^{(0)}$ in Ref. 10. The grouping which we have used here is more convenient for describing the motion of the order parameter near singular points of the unperturbed solutions. Now following the procedure of Ref. 10, we write equations of motion corresponding to the Hamiltonian (2), moving to the right side all the terms which contain W and derivatives with respect to the energy, which are assumed to be small quantities of the same order of magnitude:

$$0 = \frac{\partial Q}{\partial t} + \frac{\partial W}{\partial \gamma}, \tag{5}$$

$$\frac{\partial \mathcal{H}^{(0)}}{\partial \beta} = -\frac{\partial S_\beta}{\partial t} - \frac{\partial W}{\partial \beta}, \tag{6}$$

$$\frac{\partial \mathcal{H}^{(0)}}{\partial \Phi} = -\frac{\partial S_z}{\partial t}, \tag{7}$$

$$\frac{\partial \mathcal{H}^{(0)}}{\partial Q} = \frac{\partial \gamma}{\partial t}, \tag{8}$$

$$\frac{\partial \mathcal{H}^{(0)}}{\partial S_z} = \frac{\partial \Phi}{\partial t} - \frac{\partial W}{\partial S_z}, \tag{9}$$

$$\frac{\partial \mathcal{H}^{(0)}}{\partial S_\beta} = \frac{\partial \beta}{\partial t}. \tag{10}$$

Assuming in a zeroth approximation that the right sides of this system of equation are zero, we find a system of equations which determines steady-state solutions of the unperturbed Hamiltonian. These solutions have been found previously.[11] The motion of the texture is described as a change in the adjustable parameters of the solution 1a of Ref. 11:

$$S_\beta = 0, \quad Q = S_z(\cos\beta - 1), \quad \cos\Phi = \frac{1/2 - \cos\beta}{1+\cos\beta}. \tag{11}$$

In contrast with Ref. 11, the quantity S_z is not fixed in this case, and the solution (11) is degenerate in terms of three parameters: γ, S_z, and one of their angles, Φ or β. Our problem is to derive equations of motion for these parameters.

As variables we introduce (for example) γ, $u = \cos\beta$, and $\sigma = (S_z - \omega_L)/\omega_L$. One of the three equations is found by substituting (11) into Eq. (5):

$$(1+\sigma)\frac{\partial u}{\partial t} - (1-u)\frac{\partial \sigma}{\partial t} = -\frac{1}{\omega_L}\frac{\partial V}{\partial \gamma}. \tag{12}$$

Two more equations are found from the conditions that the right sides of Eqs. (6)–(10) are orthogonal with respect to the tangent vectors of the degeneracy space of the solutions (11). As these vectors we adopt

$$\mathbf{l}_1 = \left(\frac{d\beta}{du}, \frac{d\Phi}{du}, S_z, 0, 0\right)$$

and

$$\mathbf{l}_2 = (0,\ 0,\ \omega_L(u-1),\ \omega_L,\ 0).$$

The order in which the variables appear corresponds to the order in which Eqs. (6)–(10) are written.[1)] All the variables which appear on the right sides of Eqs. (6)–(10) should be expressed in terms of γ, σ, and u in accordance with (11). As a result we find

$$(1+\sigma)\frac{\partial \gamma}{\partial t} - \frac{d\Phi}{du}\frac{\partial \sigma}{\partial t} = \frac{1}{\omega_L}\frac{\partial V}{\partial u}, \tag{13}$$

$$\frac{d\Phi}{du}\frac{\partial u}{\partial t} - (1-u)\frac{\partial \gamma}{\partial t} = \omega_L \sigma. \tag{14}$$

Equations (12)–(14) constitute a closed system of equations in the variables γ, u, σ. The derivative $d\Phi/du$ has a singularity at $u = -1/4$:

$$\left(\frac{d\Phi}{du}\right)^2 = \frac{3}{(1+u)^2(1+4u)}. \tag{15}$$

For orbits which do not pass through the singularity, Eqs. (12)–(14) can be simplified even further. As we see from (14), σ is a quantity of first order in the derivatives $\partial\gamma/\partial t$ and $\partial u/\partial t$, while its time derivative is a quantity of second order. When we omit from Eqs. (12)–(14) all small quanti-

ties which are of second order, we find the equations

$$\omega_L \frac{\partial \gamma}{\partial t} = \frac{\partial V}{\partial u}, \tag{16}$$

$$\omega_L \frac{\partial u}{\partial t} = -\frac{\partial V}{\partial \gamma}. \tag{17}$$

This system of equations was derived previously.[10] It is in Hamiltonian form, and it conserves the energy

$$V(\gamma, u) = -\lambda \omega_L^2 (1-u^2) \sin^2 \gamma = \text{const}.$$

Two variables are no longer enough for describing the motion near $u + 1/4 \sim \lambda$. With $u = -1/4$ (this value corresponds to a configuration $\hat{\mathbf{n}} \perp \mathbf{H}$), the frequency of the longitudinal vibrations vanishes. As a result, a change in S_z becomes possible.

3. MOTION OF THE ORDER PARAMETER

We now use Eqs. (12)–(14) to describe the motion of the order parameter. The functional dependence $\Phi(u)$ is not single-valued. The two branches of this function differ in sign and merge at $u = -1/4$. Consequently, motions which may pass through the singularity $u = -1/4$ are described more conveniently in terms of the variables Φ, γ, σ. In terms of these variables we have

$$V = -3\lambda \omega_L^2 \sin^2 \gamma \frac{(1/4 + \cos \Phi)}{(1+\cos \Phi)^2}. \tag{18}$$

Treating Eqs. (12)–(14) as an algebraic system of equations in terms of the time derivatives, we note that this system of equations is consistent only under the condition

$$\sigma(\sigma+1) + \frac{1}{\omega_L^2} \frac{d\Phi}{du} \left[\frac{\partial V}{\partial \gamma} + (1-u) \frac{\partial V}{\partial \Phi} \right] = 0. \tag{19}$$

Equation (19), along with (15) and (18), determines a surface A in terms of the coordinates Φ, γ, σ:

$$\sigma(\sigma+1) = \frac{\lambda}{2}(1+4\cos \Phi) \left\{ \frac{\sin 2\gamma}{\sin \Phi} - \frac{(1-2\cos \Phi)}{(1+\cos \Phi)^2} \sin^2 \gamma \right\}. \tag{20}$$

A direct check shows that Eqs. (12)–(14) have an integral of motion which represents an energy:

$$E = \omega_L^2 \frac{\sigma^2}{2} + V(\gamma, \Phi). \tag{21}$$

The orbits of the motion of the image point in the coordinates Φ, γ, σ are the lines along which the surface (20) intersects the surfaces (21). Different initial deviations of the order parameter from an equilibrium orientation correspond to different energies E. Figure 1 shows the surface A and the orbits of motion for several values of the energy. The angles γ and Φ vary over the intervals $-\pi \leqslant \gamma \leqslant \pi$ and $-\theta_0 \leqslant \Phi \leqslant \theta_0$. Only that part of the surface A which corresponds to $0 \leqslant \gamma \leqslant \pi$ is drawn in this figure. Since Eqs. (20) and (21) are not changed by a change of $m\pi$ in γ, the remainder can be found by making a shift of $-\pi$ along the γ axis. The equilibrium positions correspond to the points with the coordinates $\Phi = \pm \pi/3$, $\gamma = \pm \pi/2$, $\sigma = 0$. [In the notation of Ref. 6, the signs $(-,+)$ correspond to the $\hat{\mathbf{a}}$ orientation, $(+,+)$ to the $\hat{\mathbf{b}}$ orientation, $(-,-)$ to the $\hat{\mathbf{c}}$ orientation, and $(+,-)$ to the $\hat{\mathbf{d}}$ orientation.] All equilibrium positions correspond to $E/\omega_L^2 = -\lambda$.

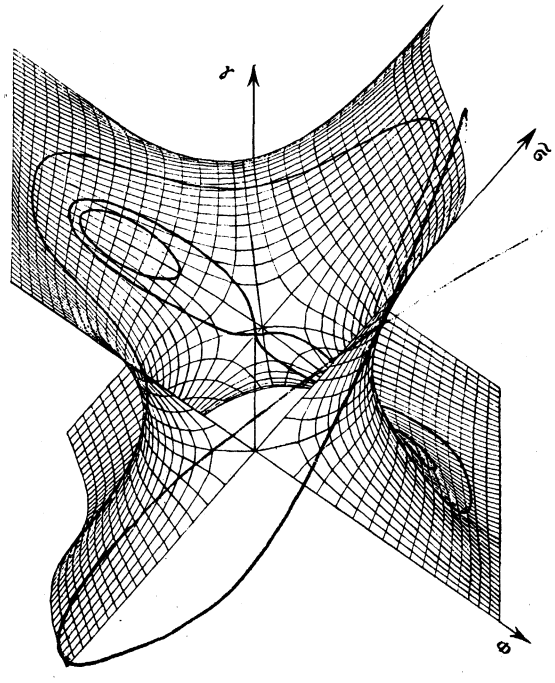

FIG. 1. Orbits of the image point in the coordinates Φ, γ, $\bar{\sigma} = \sigma(5/\lambda)^{1/2}$ for the three values $J^2 = \frac{5}{8}, \frac{15}{16}, \frac{31}{32}$. The inner curves correspond to larger values of J^2.

Since λ is small, the deviation of the surface A from the plane $\sigma = 0$ becomes important only in the region $\Phi \sim \lambda^{1/2}$. Near each equilibrium position there is a region of initial values of Φ, γ, σ for which the subsequent motion occurs along orbits which enclose only the given equilibrium position and which do not venture into the "dangerous" region $\Phi \sim \lambda^{1/2}$. The equations of these orbits are found by substituting (18) into (21) and setting $\sigma = 0$. The motion along such orbits is described by Eqs. (16) and (17). This motion can be interpreted by going over to the variables J and ψ:

$$J = \sin \beta \sin \gamma, \quad \operatorname{tg} \psi = -\cos \gamma \operatorname{tg} \beta. \tag{22}$$

In terms of these variables, Eqs. (16) and (17) become

$$\frac{dJ}{dt} = 0, \tag{23}$$

$$\frac{d\psi}{dt} = 2\lambda \omega_L J. \tag{24}$$

In other words, J is an integral of motion, and ψ is the corresponding "angular variable," which varies linearly with the time. The motion described by (23) and (24) can be interpreted graphically in the coordinate system $(\hat{\xi}, \hat{\eta}, \hat{\zeta})$, which moves with the order parameter. In the basis $(\hat{\xi}, \hat{\eta}, \hat{\zeta})$, the unit vector $\hat{\mathbf{z}}$ has the coordinates $(-\sin \beta \cos \gamma, \sin \beta \sin \gamma, \cos \beta)$; i.e., J is the projection of $\hat{\mathbf{z}}$ onto $\hat{\eta}$, and ψ is the angle between the projection of $\hat{\mathbf{z}}$ onto the $\hat{\xi}\hat{\zeta}$ plane and the $\hat{\zeta}$ axis. The order parameter thus moves in such a way that the unit vector $\hat{\mathbf{z}}$ in the related basis $(\hat{\xi}, \hat{\eta}, \hat{\zeta})$ undergoes a uniform revolution along a cone with axis along $\hat{\eta}$. The projection of $\hat{\mathbf{z}}$ onto $\hat{\eta}$ is equal to J, and the angular velocity of the revolution is $2\lambda J \omega_L$.

For $J \approx \pm (15/16)^{1/2}$, the phase orbits enter the dangerous region, where the difference between σ and zero becomes important. For $\Phi = 0$ and for $\gamma = 0, \pi/2, \pi$, the surface A has saddle points. For $J^2 < 15/16$, the orbits

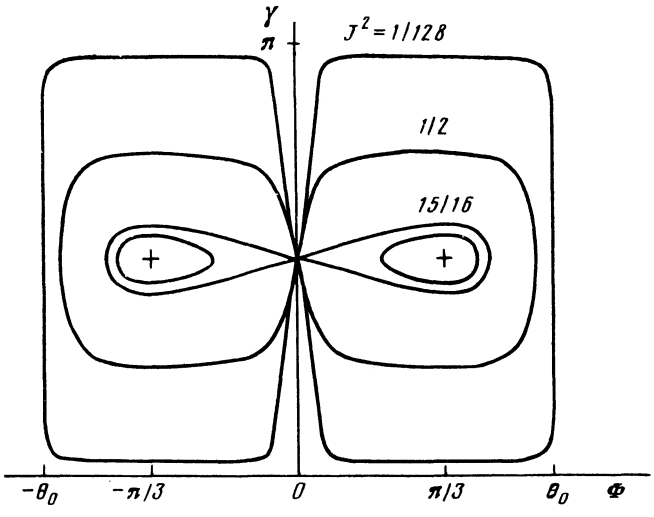

FIG. 2. Phase orbits of the motion of the order parameter in the Φ, γ plane for various values of the integral J. The two inner curves correspond to the value $J^2 = \frac{31}{32}$.

transform into figure-eights, which pass near the saddle point $\Phi = 0$, $\gamma = \pi/2$ and which run around two equilibrium points, e.g., $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$, shown in Fig. 2. To describe the time evolution of Φ, γ, and σ as we move along these figure-eights, we need to integrate Eqs. (12)–(14). Although we have not been able to find an analytic solution of this system of equations, the corresponding behavior for specific initial conditions can be found easily through numerical integration with the help of the integrals of motion which are present here.

4. DISSIPATION

If we wish to describe the relaxation of the order parameter to one of the equilibrium positions, we should add some dissipative terms to Eqs. (5)–(10). For temperatures which are not too low, it is sufficient to consider the bulk dissipation which arises by the Leggett–Takagi mechanism.[12] Dissipative terms arise from the dipole energy and also from the interaction with the walls. In order to write out these dissipative terms, we need to add a torque $\mathbf{R}_S$—due to the magnetic surface energy—to the Leggett–Takagi equations for the rate of change of the spin density, $\dot{\mathbf{S}}$, and the angular rotation velocity of the order parameter, $\boldsymbol{\nu}$, as was done in Ref. 6. We then need to project these equations onto the unit vectors $\hat{\mathbf{z}}$ and $\hat{\boldsymbol{\zeta}}$ and onto the line of nodes $[\hat{\mathbf{z}},\hat{\boldsymbol{\zeta}}]$. Here we need to allow for the rotation of the unit vector $\hat{\boldsymbol{\zeta}}$ and of the line of nodes. In the hydrodynamic limit we find the following corrections in Eqs. (5)–(10):

$$0 = \frac{\partial Q}{\partial t} + \frac{\partial W}{\partial \gamma} - \frac{\varkappa}{2}\frac{\partial U}{\partial(\cos\theta)} \times\{(1+\cos\Phi)(Q\cos\beta - S_z(1-\cos\beta)) + S_\beta \sin\beta\sin\Phi\} + 2\varkappa a\{(S_z(1-\cos\beta) - Q\cos\beta)\cos\beta\sin^2\gamma + S_\beta\sin\beta\cos\beta\sin\gamma\cos\gamma\}, \quad (25)$$

$$\frac{\partial \mathcal{H}^{(0)}}{\partial\beta} = -\frac{\partial S_\beta}{\partial t} - \frac{\partial W}{\partial\beta} - \frac{\varkappa}{2}\frac{\partial U}{\partial(\cos\theta)}\frac{\sin\Phi}{\sin\beta} \times(Q+S_z(1-\cos\beta)) + 2\varkappa a(Q+S_z(1-\cos\beta))\operatorname{ctg}\beta\sin\gamma\cos\gamma, \quad (26)$$

$$\frac{\partial \mathcal{H}^{(0)}}{\partial\Phi} = -\frac{\partial S_z}{\partial t}, \quad (27)$$

$$\frac{\partial \mathcal{H}^{(0)}}{\partial Q} = \frac{\partial\gamma}{\partial t} - \frac{\varkappa}{2}\frac{\partial U}{\partial(\cos\theta)}\sin\Phi - 2\varkappa a\sin\gamma\cos\gamma, \quad (28)$$

$$\frac{\partial \mathcal{H}^{(0)}}{\partial S_z} = \frac{\partial\Phi}{\partial t} - \frac{\partial W}{\partial S_z} - \varkappa\frac{\partial U}{\partial(\cos\theta)}\sin\Phi - 2\varkappa a(1-\cos\beta)\sin\gamma\cos\gamma, \quad (29)$$

$$\frac{\partial \mathcal{H}^{(0)}}{\partial S_\beta} = \frac{\partial\beta}{\partial t} - \frac{\varkappa}{2}\frac{\partial U}{\partial(\cos\theta)}\sin\beta(1+\cos\Phi) - 2\varkappa a\sin\beta\cos\beta\sin^2\gamma. \quad (30)$$

The effective time $\varkappa$ was introduced previously.[11]

The dissipative terms due to the dipole energy are vastly larger in order of magnitude than the "wall" terms, and they can be moved to the left-hand side. In the solution *Ia* of Ref. 11, all the dipole dissipative terms, being proportional to the derivative $\partial U/\partial(\cos\theta)$ vanish and thus do not alter the solution itself. They do, on the other hand, change the eigenvectors onto which the right-side is to be projected. As a result, increments $\sim\varkappa a$ arise in the equations of motion, and increments of the same order of magnitude arise directly from the wall dissipative terms. The three equations which result, however, do not have an integral of motion, and they do not lend themselves to a qualitative analysis like that in Sec. 3.

Here we discuss only the final stage of the relaxation, in which the motion of the order parameter occurs about one of the equilibrium positions. In this case the motion is described by two equations. In terms of the variables J and ψ, the equations with the dissipative terms are

$$\frac{\partial J}{\partial t} = 2\varkappa\lambda\omega_L^2 J(1-J^2), \quad (31)$$

$$\frac{\partial\psi}{\partial t} = 2\lambda\omega_L J. \quad (32)$$

These equations can be integrated. As a result we find

$$\frac{J^2}{J_0^2} = \{J_0^2 + (1-J_0^2)\exp[4\varkappa\lambda\omega_L^2(t_0-t)]\}^{-1}, \quad (33)$$

$$\psi - \psi_0 = \frac{1}{2\varkappa\omega_L}\ln\frac{(1+J)(1-J_0)}{(1-J)(1+J_0)}. \quad (34)$$

In other words, the relaxation occurs over a time scale $\tau \sim 1/\varkappa\lambda\omega_L^2$. The variables u and γ are expressed in terms of J and ψ in the following way:

$$u = (1-J^2)^{1/2}\cos\psi, \qquad \operatorname{ctg}\gamma = -\frac{(1-J^2)^{1/2}}{J}\sin\psi. \quad (35)$$

The initial conditions for the motion of the texture which arise under the experimental conditions of Refs. 6–8, after an equilibrium magnetization ($\hat{\mathbf{n}}\|\mathbf{H}$ or $-\hat{\mathbf{n}}\|\mathbf{H}$) is reached, correspond to values $\Phi = \pm\theta_0$, i.e., $E \approx 0$. The meaning here is that the motion initially occurs along a figure-eight which encloses either the pair of equilibrium states $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ or the pair $\hat{\mathbf{c}}$ and $\hat{\mathbf{d}}$, depending on the initial value of γ.

As the relaxation progresses, the figure-eight shrinks and splits up into two loops. The particular loop along which the order parameter moves depends strongly on the initial conditions, so we would expect the relaxation to proceed to different equilibrium values in different regions of the planar layer of helium. In other words, domains would arise, separated by walls (solitons), which would make the transition from the orientation $\hat{\mathbf{a}}$ to $\hat{\mathbf{b}}$ or from $\hat{\mathbf{c}}$ to $\hat{\mathbf{d}}$. It is also clear from this analysis that the nature of the relaxation should change upon the transition from motion along a figure-eight to motion along a loop, but at the present level of experimental precision it would not be possible to observe that transition.

As we mentioned earlier, the conditions for the relaxation of the order parameter from the Leggett configuration to the equilibrium position determined by the walls which were set up in the experiments of Refs. 6–8 are similar to those which we have been discussing here. Unfortunately, however, no data were reported in Refs. 7 and 8 with which we could make a comparison of the results of the present study. We can make a comparison with data from Ref. 6. In that previous study, the relaxation time of the uniform texture, τ_2, was found as a function of T/T_c by a two-pulse NMR method involving a change in the frequency of the free-induction signal. A comparison of τ_2 with the relaxation time found in the present paper, $\tau \sim 1/\varkappa\lambda\omega_L^2$ (as in Ref. 6, we take ξ/L to be $3\cdot 10^{-4}$) shows that τ agrees in order of magnitude with the experimental value. The shape of the curves generated by Ishikawa *et al.* in the course of their numerical integration of the complete system of Leggett equations also supports the conclusions of the present study. The oscillations of the angle θ and of the component n_z of the vector $\hat{\mathbf{n}}$ over the texture relaxation time indicate penetration into the dangerous region and a related change in S_z. We can thus say that the system of equations which we have identified here gives an adequately accurate description of the behavior of the uniform texture in a thin plane-parallel layer of helium.

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[1)] The order of the variables was mixed up in Eq. (44) of Ref. 10, but the order was correct in the actual derivation, and the equations turned out to be correct.

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