Source of a regular flux of photons

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A new type of light source is proposed. It is based on an optical resonator with variable Q and a resonant medium inside this resonator. The resonant medium is excited into an emitting state in a noise-free manner. The excitation is pulsed. The Q is switched in synchronization with the pulsed excitation. At the time of excitation, the resonator is blocked, and it does not emit light. In the time intervals between excitation events, the resonator is unblocked, and light reaches a photocathode. The conditions under which one can assume, very accurately, that each observed light pulse contains the same number of photons, equal to the number of atoms in the resonator, are discussed. The accuracy of this assumption is considerably better than that attainable in the case of a Poissonian photon flux. Since the length of the observed pulse is determined entirely by the low Q of the unblocked resonator, it becomes possible to produce a flux of photons with a regularity better than that of lasers. The reason is that the shortest time scale of the flux (the time over which the fluctuations in the number of photons are unable to cancel out) is determined exclusively by the pulse length in this case.

Increasing the sensitivity and accuracy of optical measurements requires a well-regulated flux of photons. A fairly large number of corresponding sources, of various types, have been proposed. One version, which has been examined theoretically by Sokolov and the present author[1] and studied experimentally by Yamamoto *et al.*,[2] is based on a laser with an ordered excitation of atoms to the working levels. This excitation gives rise to photon antibunching in the output at times on the order of the photon lifetime in the resonator set by the finite Q of the resonator, $t_0 \sim C^{-1}$, where C is the spectral width of the resonator. As a particular result, if one counts the photons which pass through a cross section over a time $\Delta t \gg t_0$ one finds that this number remains very accurately the same at any time t; i.e., we have $\overline{\Delta n^2_{\Delta t}} \ll \bar{n}_t$. At $\Delta t \lesssim t_0$, on the other hand, the fluctuations are large, as for an ordinary laser: $\overline{\Delta n^2_{\Delta t}} \sim \bar{n}_t$.

A light flux of this sort would thus be suitable for highly accurate measurements over long times. In spectral terms, a light flux of this sort has, for example, an elevated sensitivity to switching at frequencies far lower than C. On the other hand, this flux has no advantage in the case of brief measurement procedures or in the case of switching over a broad spectral region. If such advantages are to be realized, we need an even greater order in the photon flux. In this paper we propose one possible version of a source which, we believe, provides the necessary quantum properties of the light. While the time scale of the photon antibunching in Ref. 1 was equal to the reciprocal of the resonator width, in the case at hand this time scale can in principle be arbitrarily small. At any rate, we will see how this time can be reduced, i.e., how the degree of order of the photon flux can be increased.

As we know, parametric effects can be exploited to convert coherent light into a sub-Poissonian flux of photons over a broad spectral region.[3] However, the circumstance that the high degree of order is achieved by virtue of a quantum phase squeezing can be regarded as a disadvantage of such systems: Implementing this idea in practice requires surmounting some major difficulties in matching the wavefronts of the various optical fields, in selecting suitable nonlinear crystals, and in resolving other problems which are characteristic of parametric phenomena.

1. LIGHT SOURCES USING A VARIABLE-*Q* OPTICAL RESONATOR

We assume that the optical resonator contains N atoms in the ground (nonworking) state. We assume that at some instant t_i (these times are spaced along the time axis with a period T) all N atoms are put in the upper working level by some efficient mechanism. The intention here is to have all the atoms back in their ground state by the next time $t_{i+1} = t_i + T$. Upon the transition to the lower working level, N photons appear in the working mode (there is of course a certain accuracy level associated with this assumption; we will be discussing this accuracy below). Under the assumption that no relaxation processes occur from the upper working level, we conclude that the transition from the lower level to the nonworking ground state results in an N-photon excitation of the working mode of the resonator. We assume that the conversion of the atomic excitation into electromagnetic radiation (i.e., into N photons) in the time interval between t_i and $t_i + T_1$ occurs at a very high value of the resonator. We also try to arrange events so that the working mode is again in the vacuum state by the time the next excitation of the medium occurs. To arrange this situation, we require that the resonator reach a very low value over the time interval between $t_i + T_1$ and $t_i + T$ and that it do so exclusively as a result of the degree of transmission of the optical output mirror. We are thus assuming that light in the form of periodic pulses, each containing N photons, is incident on the photocathode. By choosing the temporal characteristics of the problem appropriately we can pack these pulses in densely along the time axis and thereby increase the order of the photons in a nominally steady-state flux. Experiments with isolated pulses with definite number of photons might also be interesting.

It is difficult to suggest at the outset just how we should

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achieve this Q switching, since this problem might require the development of some specific resonator for which the standard methods might be unsuitable. However, until we start talking about specifics we can choose (for example) the following method as one possibility. We replace the output mirror by a Fabry-Perot resonator whose optical thickness can be varied by means of the piezoelectric effect in a ceramic layer. By applying an alternating voltage to the ceramic and synchronizing this voltage with the pulses which excite the medium, we can meet the necessary conditions here.

We choose as the working medium a system of atoms with the energy structure shown in Fig. 1, which is the same as that chosen in Ref. 1. The atoms are assumed to be immobile, and they are all assumed to be in the ground state (0) at the time t_i. By means of an intense light pulse operating on the 0–3 transition and by means of a very fast spontaneous 3–2 decay, we excite all the atoms into the upper working level, 2. These atoms then make a transition to the lower level 1, primarily as a result of induced emission. In the process, N photons are emitted into one working mode of the resonator. Here we must require physical conditions such that the induced emission becomes dominant even if there is only a single working photon in the mode. As we will see below, it is completely feasible to meet these conditions. After the transition in which N atoms are excited into an N-photon excitation of the field in the resonator, the atoms from level 1 go back to the ground state quite rapidly by virtue of spontaneous decay. The entire process then repeats itself.

2. EQUATION FOR THE DENSITY MATRIX OF THE FIELD RADIATED BY THE ATOMS

The density matrix behaves in different ways over the characteristic time intervals from t_i to $t_i + T_1$ and from $t_i + T_1$ to $t_i + T$. We can therefore write

$$\dot{\rho}=\sigma_1(\dot{\rho})_1+\sigma_2(\dot{\rho})_2, \tag{1}$$

where σ_1 and σ_2 ($\sigma_1 + \sigma_2 = 1$) are functions with the time evolution shown in Fig. 2; $\sigma_1 + \sigma_2 = 1$.

Damping is the only process which deforms the density matrix in the time intervals from $t_i + T_1$ to $t_i + T$. We can thus write

$$(\dot{\rho})_2=-\frac{C}{2}(a^+a\rho-2a\rho a^++\rho a^+a). \tag{2}$$

We will be interested below only in the diagonal matrix elements:

$$(\dot{\rho}_{nn})_2=-C(n\rho_{nn}-(n+1)\rho_{n+1,\,n+1}).$$

Only the working medium plays a role in shaping $(\rho)_1$. We can use a result from Ref. 1, since the model of the medium used in Ref. 1 is exactly the same as that used here. We then have

$$(\dot{\rho}_{nn})_1=\frac{N}{T_1}\Big[-(\rho_{nn}-\rho_{n-1,n-1})-\frac{1}{2}(\rho_{nn}-2\rho_{n-1,n-1}+\rho_{n+1,n+1})\Big]. \tag{3}$$

We use a generating-function method to find a solution. We introduce the function

$$G(z,t)=\sum_{n=0}^{\infty}z^n\rho_{nn}(t). \tag{4}$$

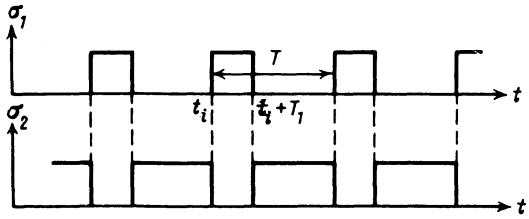

FIG. 2.

We can write all the necessary averages by making use of this function and the following equation for this function:

$$\frac{\partial G}{\partial t}=\frac{N}{2T_1}\sigma_1(1-z)(z-3)G+C\sigma_2(1-z)\frac{\partial G}{\partial z}. \tag{5}$$

This equation follows from definition (4) and from Eqs. (1)–(3).

3. ONE-PULSE SOLUTION

The complete solution of Eq. (5) can be written as a superposition of one-pulse solutions:

$$G(z,t)=\sum_i G_i(z,t). \tag{6}$$

The solutions G_i satisfy the same equation, (5), but with the functions $\sigma_{1,2}$ replaced by $\sigma_{1i,2i}$, which are defined by

$$\sigma_{1i}(t)=\begin{cases}1, & t_i<t<t_i+T_1,\\ 0, & t<t_i,\quad t>t_i+T_1,\end{cases}$$

$$\sigma_{2i}(t)=\begin{cases}1, & t_i+T_1<t<t_i+T,\\ 0, & t<t_i+T_1,\quad t>t_i+T,\end{cases} \tag{7}$$

$$\sigma_1=\sum_i\sigma_{1i},\qquad \sigma_2=\sum_i\sigma_{2i}.$$

We see that once we know the one-pulse solution we can determine the complete solution. The one-pulse solution may of course be of interest in its own right.

The partial differential equation (5) is uncomplicated. It can be solved easily by (for example) switching from the variables t, z to the new variables t and

$$z_i=1+(z-1)f_i(t,t_0), \tag{8}$$

$$f_i(t,t_0)=\exp\Big[-C\int_{t_0}^{t}dt'\sigma_{2i}(t')\Big].$$

We are then left with only a time derivative in this equation, and solving the problem becomes an exceedingly simple matter. The solution is finally written in the form

$$G_i(z,t)=G_i(z_i,t_0)$$
$$\times\exp\Big\{\frac{N}{2T_1}(1-z)\int_{t_0}^{t}dt'\sigma_{1i}(t')f_i(t,t')[(z-1)f_i(t,t')-2]\Big\} \tag{9}$$

for $t_i<t_0<t<t_i+T$.

To write a solution which is physically meaningful, we assume that not only does the atomic system revert to its original ground state by the time t_i (or t_i+mT) but also that the electromagnetic field becomes the vacuum field. The field is "cleaned out of the resonator" by satisfying the condition $T - T_1 \gg C^{-1}$. We must therefore require $G_i(z, t_i) = 1$. The physical one-pulse solution then takes the form

$$G_i^\phi(z,t) = \exp\left\{\frac{N}{2T_1}(1-z)\int_{t_i}^{t} dt'\sigma_{1i}(t')f_i(t,t') \times [(z-1)f_i(t,t')-2]\right\}. \quad (10)$$

We can make use of this quantity to construct physical averages of the type $\overline{n_i(t)}$, the average number of photons in pulse i in the optical resonator at a time $t_i < t < t_i + T$, or the mean square fluctuation of this number of photons, $\overline{\Delta n_i^2(t)} = \bar{n}_i(t)(1 + \xi_i(t))$ (the statistical parameter ξ_i is sometimes called the "Mandel parameter"). It is easy to derive

$$\bar{n} = \left.\frac{\partial G(z,t)}{\partial z}\right|_{z=1}, \quad \overline{n^2} - \bar{n} = \left.\frac{\partial^2 G(z,t)}{\partial z^2}\right|_{z=1}. \quad (11)$$

Hence

$$\bar{n}_i(t) = \frac{N}{T_1}\int_{t_i}^{t} dt'\sigma_{1i}(t')f_i(t,t') = \begin{cases} N\dfrac{t-t_i}{T_1}, & t_i<t<t_i+T_1, \\ N\exp[-C(t-t_i-T_1), & t>t_i+T_1, \end{cases} \quad (12)$$

and we find the following result for the Mandel parameter:

$$\xi_i(t) = -\int_{t_i}^{t} dt'\sigma_{1i}(t')f_i^2(t,t')\left[\int_{t_i}^{t} dt'\sigma_{1i}(t')f_i(t,t')\right]^{-1} = \begin{cases} -1, & t_i<t<t_i+T_1, \\ -\exp[-C(t-t_i-T_1)], & t>t_i+T_1. \end{cases} \quad (13)$$

4. ROLE OF SPONTANEOUS EMISSION FROM THE UPPER WORKING LEVEL

In writing Eq. (1) and thus Eq. (5), we ignored the fact that once an atom reaches the upper working level it might emit not only the photon of the working mode in which we are interested but also a photon of any other mode, through spontaneous decay. The latter emission can substantially distort the statistical picture. In general, the uncertainty in the number of photons in each pulse can be large. To prevent this distortion from occurring, we require that even one photon of the working mode confer an advantage on induced processes over spontaneous processes. The uncertainty in the number of photons in a pulse will then be determined exclusively by the uncertainty in the time at which the first photon goes into the working mode. That uncertainty is apparently given by $\Omega/(4\pi)$, where Ω is the solid angle subtended by the mode spot on the cavity mirror at the atom. This uncertainty determines the mean square fluctuation of the number of photons: $\overline{\Delta n^2} = (4\pi/\Omega)^2$. We would like to require that this uncertainty be much smaller than the Poissonian level: $\overline{\Delta n^2} \ll \bar{n}$. We see that this condition is met if the number of atoms in the resonator is sufficiently large:

$$N \gg (4\pi/\Omega)^2.$$

What are the consequences of requiring that the transition be saturated by a single photon? If the induced emission is to dominate, we must require that the Rabi frequency $d_{21}E$ be greater than the rate of spontaneous decay, γ_2:

$$d_{21}E \gg \gamma_2, \quad \gamma_2 = {}^3/_2\omega^3 d_{21}{}^2.$$

Since this condition is correct if the amplitude E remains constant over time, we should also require that E remain constant over the time of the transition from the upper level to the lower one:

$$d_{21}E \gg C.$$

These two conditions can be rewritten as

$$V \ll v, \quad V \ll v(\gamma_2/C)^2,$$

where we have used $E = (\omega\bar{n}/2V)^{1/2}$ and where we have assumed that the number of photons, $\bar{n}$, is unity. We see that the actual volume of the resonator, V, must be small in comparison with the wave volume $v = \lambda^2/\gamma_2$.

Let us look at a numerical estimate. With $\lambda \sim 5 \cdot 10^{-4}$ cm, $\gamma_2 \sim 10^4$ s^{-1}, and $C \sim 10^5$ s^{-1}, we find $V < 10^{-2}$ cm^3. This figure is completely realistic.

5. SPECTRAL STRUCTURE OF THE PHOTOCURRENT

It is clear simply from quite general considerations that the light flux incident on the photodetector is more ordered. The time C^{-1} is a characteristic parameter of the order. This time is very small in comparison with the lifetime of a photon in an optical resonator in the case of steady-state lasing. Below we will calculate the photocurrent spectrum, which should apparently have a dip of significant width at the level of shot noise.

The photocurrent spectrum is the sum of two terms,

$$i_\omega^{(2)} = (i_\omega^{(2)})_{\text{shot}} + (i_\omega^{(2)})_{\text{exc}}. \quad (14)$$

The first term,

$$(i_\omega^{(2)})_{\text{shot}} = Cq\int_{-T_0/2}^{T_0/2}\frac{dt}{T_0}\sigma_2(t)\bar{n}(t) \quad (15)$$

is the shot noise of the photodetection. The second term,

$$(i_\omega^{(2)})_{\text{exc}} = q^2C^2 2\,\text{Re}\int_{-T_0/2}^{T_0/2}\frac{dt_1\sigma_2(t_1)}{T_0} \times \int_{t_1}^{T_0/2} dt_2\sigma_2(t_2)g(t_1,t_2)\exp[i\omega(t_1-t_2)] \quad (16)$$

is the excess noise, whose frequency spectrum depends on an average of the type

$$g(t_1, t_2) = \langle a^+(t_1)a^+(t_2)a(t_2)a(t_1)\rangle, \quad (17)$$

which must be calculated (q is the quantum efficiency of the photocathode). A procedure is available for doing this by using a generating function $G(z, t)$. On the basis of very general considerations, we can show, quite easily, that

$$g(t_1, t_2) = \left. \frac{\partial G(z, t_2 | t_1)}{\partial z} \right|_{z=1}$$

Here $G(z,t_2|t_1)$ is a physically meaningless formal solution of (5) with the formal initial condition

$$G(z, t_1 | t_1) = \frac{\partial G^\phi(z, t_1)}{\partial z},$$

where G^ϕ is the physically meaningful solution of Eq. (5), which is the same as (10) in our case. Working from that assertion, and carrying out all the necessary differentiations and integrations, we find

$$i_\omega^{(2)} = q \frac{N}{T} \left(1 - q \frac{C^2}{C^2 + \omega^2} \right). \quad (18)$$

In writing this expression we have assumed that many light pulses reached the photocathode over the measurement time T_0 ($T \ll T_0$). In addition, expression (18) should actually also have a series of δ-function pulses at frequencies $2\pi n/T$ ($n = 0, 1, 2, 3, ...$), associated with the periodic structure of the light flux (Fig. 3).

If we ignore the δ-function pulses in the calculation, the photocurrent spectrum in (18) has the same form as in the case discussed in Ref. 1. Both in that previous study and in the present study, the shot noise drops to zero at zero frequency (at $q = 1$). The width of the dip is equal to the width of the resonator. In the case of steady-state lasing,[1] however, the quantity C is that spectral width of the high-Q optical resonator which leads to the excitation threshold. In our own case, C is the resonator width of a "poor" resonator, which we can, if we wish, make large, since no conditions are imposed on it. In this case, the shot noise of the photodetection is thus suppressed over a broad spectral region, and the conditions for optical measurements are correspondingly improved.

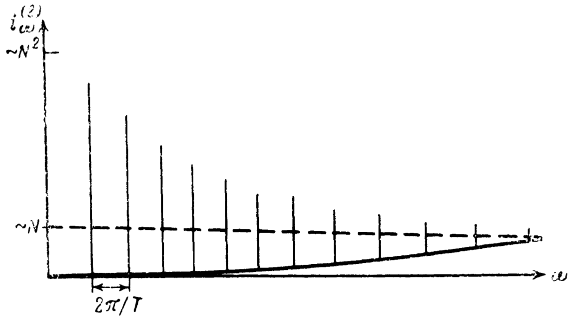

FIG. 3.

The idea of this study arose during a reading of a paper by Krause *et al.*[4]

Figure 1 was not available on the Russian original at the time when this translation was produced.

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[4] J. Krause, M. O. Scully, T. Walther, and H. Walther, Phys. Rev. A **39**, 1915 (1989).

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