Bessel beams of electromagnetic waves: self-effect and nonlinear structures

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Nonlinear structures which arise as Bessel beams of electromagnetic radiation propagate through a plasma are analyzed. Various mechanisms for focusing nonlinearities of the medium and their role in the spatial modulation of the field intensity are analyzed. For certain types of nonlinearities, which are identified here, a large-scale structure arises at the same time as the small-scale longitudinal structure. Photographs of spatially periodic plasma formations which have been observed in experiments on the propagation of Bessel beams, and which correspond to small- and large-structures accompanying a modulation of the beams, are presented. The self-modulation of Bessel beams is shown to be of a universal nature, independent of the particular mechanism responsible for the nonlinearity of the medium.

1. INTRODUCTION

Effects which arise as wave beams of intense electromagnetic radiation in the optical and microwave ranges propagate through a nonlinear medium and which are determined by self-effects obviously depend on the radial intensity profiles of the beams. For beams with monotonic intensity profiles, e.g., Gaussian or hyper-Gaussian, self-effects have been studied quite thoroughly both in media with a local nonlinearity[1] and in models with a nonlocal nonlinear response.[2] It is also worthwhile to study these processes for beams of other types: beams with a nonmonotonic but regular radial intensity profile[3] which arise when radiation is focused by conical lenses of the axicon type.[4,5]

The radial profile of the field amplitude in the beams focused by an axicon can be described near the symmetry axis by a Bessel function of order zero, $J_0(kr\sin\gamma)$, where $\gamma=(N_a-1)\alpha$ is the angle between the rays and the symmetry axis, N_a is the refractive index, α is the angle at the base of the axicon, and $k=(\omega/c)\varepsilon_0^{1/2}$ is the wave vector of the radiation. Because of the specific features of the spatial structure of such beams, they can be put in a special category of "Bessel beams" (B beams). The radius of the caustic of a B beam, bounded by the first zero of the function J_0, is constant over the entire focusing distance $L_a=R_a/\tan\gamma\approx R_a/\gamma$, where R_a is the aperture of the axicon. For this reason, these beams are sometimes called "diffraction-free."[6]

The work with B beams has resulted in the development of a new physical entity: the unbroken long laser spark,[7] whose unique properties suggest several possible applications.[8]

One characteristic property of an unbroken long laser spark is the formation of a periodic spatial structure along the beam axis while the plasma channel is being created by the pump light.[9,10] The period of this spatial structure is independent of the type of gas and of the gas pressure over the pressure range which has been studied (5–1000 kPa; Ref. 11). This result suggests that the formation of these periodic structures is of a general nonlinear-electrodynamic nature.

In this paper we present the results of a theoretical study of the self-effects of B beams in media with various types of local and nonlocal nonlinearities.[12] We analyze experimental data on the formation and structure of unbroken long laser sparks. These data provide support for models which have been developed for the propagation of a B beam in the nonlinear medium which arises when a gas breaks down.

2. BASIC EQUATIONS

To describe the steady-state distribution of the cylindrically symmetric beam beyond the axicon, we use the equation for the field strength $E(r,z)$, which varies slowly along the propagation direction (z) of a beam of complex amplitude

$$\mathbf{E}(\mathbf{r},t)=\mathrm{Re}\,\{\mathbf{e}E(r,z)\exp[-i(\omega t-kz)]\}:$$

$$2ik\frac{\partial E}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial E}{\partial r}\right)+\left(\frac{\omega}{c}\right)^2[i\varepsilon''+\varepsilon_{NL}(|E|^2)]E=0, \tag{1}$$

Here $k^2=(\omega/c)^2\varepsilon_0$ is determined by the real part ε_0 of the linear permittivity ε of the medium. We write the latter as

$$\varepsilon=\varepsilon_0+i\varepsilon''+\varepsilon_{NL}(|E|^2),$$

where ε'' is the imaginary part, which causes damping of the field. The nonlinear functional $\varepsilon_{\rm NL}$ depends on the field intensity and is determined by the nonlinear constitutive equations of the medium. The boundary condition, which corresponds to focusing of the radiation by the axicon (with an aperture R_a), which forms a conical wavefront, is

$$E(r,z=0)=E_{in}(r)\exp(-ikr\sin\gamma), \tag{2}$$

where $E_{\rm in}(r>R_a)=0$, $E_{\rm in}(r\leqslant R_a)=I^{1/2}(r)$, and $I(r)$ is the radial intensity profile of the focused beam. This profile might be, for example, hyper-Gaussian:

$$I(r)=I_0\exp[-(r/a_0)^{2M}],\quad M\geqslant1, \tag{3}$$

where a_0 is the beam radius.

The linear solution $E^{(0)}$ of the boundary-value problem (1), (2) (with $\varepsilon''=\varepsilon_{\rm NL}=0$) takes the following form in the axial region $r<z\sin\gamma$, $kr^2<z$, of the focusing distance ($\lambda/\sin^2\gamma\ll z\ll L_a$, $\lambda=2\pi/k$):

$$E^{(0)}(r,z)=E_0\exp\left(-i\frac{k}{2}z\sin^2\gamma\right)\left[J_0(kr\sin\gamma)-i\frac{r}{2z\sin\gamma}J_1(kr\sin\gamma)\right]+E_a\exp\left[\frac{ik(R_a^2+r^2)}{2z}\right]J_0\left(\frac{kR_ar}{z}\right). \tag{4}$$

The first term in this expression is, to within a small term on the order of $r/(z\sin\gamma)$, the field of a diffraction-free beam with a wave vector $k_\perp = k\sin\gamma$ normal to the axis [correspondingly, we have $\delta k_\parallel = (1/2)k\sin^2\gamma$] and with a slowly varying amplitude

$$E_0(z)=2\pi(z/\lambda)^{1/2}\sin\gamma E_{in}(z\sin\gamma)\exp(-i\pi/4).$$

The second (last) term, with the amplitude

$$E_a=\frac{E_{in}(R_a)}{1-z/L_a}\exp(-ikR_a\sin\gamma)$$

describes diffraction at the edge of the axicon. This second term is small in comparison with the first by a ratio $(\lambda/z)^{1/2}\ll 1$. Furthermore, if the beam radius a_0 does not exceed the aperture size R_a, the amplitude E_a is exponentially small, proportional to the factor $\exp[-(R_a/a_0)^{2M}]$, according to (2) and (3). Under these conditions the radial intensity profile of the linear solution (4) is approximately a Bessel profile near the axis:

$$|E^{(0)}(r,z)|^2=|E_0(z)|^2J_0^2(k_\perp r), \qquad (5)$$

The monotonic intensity variation $|E_0(z)|^2$ along the beam axis (the z direction), which is determined by geometric-optics growth proportional to z/λ and by the radial intensity profile of the initial beam (the beam to be focused) [$I(r)$ in (3)], is small over distances Δz which are small in comparison with the focusing distance L_a.

The interaction of a B beam with a nonlinear medium and the particular features of the propagation of this beam are determined by the nonlinear part of the permittivity of this medium, $\varepsilon_{\rm NL}(|E|^2)$, and by its dependence on the field strength. Let us consider interactions in which the nonlinearity is characterized by a local power-law dependence on $|E|^2$ without and with saturation of the nonlinearity and also by a nonlocal dependence characteristic of, for example, thermal nonlinearity of the plasma.

In the partially ionized plasma which forms during the breakdown of the gas, with the change in electron temperature being determined by the balance between the energy acquired from the radiation as a result of inverse bremsstrahlung and the energy lost by the electrons, primarily in collisions with neutral particles, a quasisteady distribution of the electron density n_e can be found from the pressure balance condition. If the length scale L_E on which the electromagnetic field varies is far larger than the electron mean free path ($l_e = v_{Te}/\nu_e$), $l_e/L_E < \delta_e^{1/2}$, where δ_e is the part of the energy which is transferred to neutrals from electrons in collisions (in the case of elastic collisions would have $\delta = 2m_e/M_i$). The nonlinear part of the permittivity, $\varepsilon_{\rm NL}$, which is determined by the excursion of the electron density from the equilibrium value n_0 ($\delta n = n_e - n_0$), is given by

$$\varepsilon_{NL}=\frac{-\delta n}{n_c}=\frac{n_0}{n_c}\frac{|E|^2/E_T^2}{1+|E|^2/E_T^2}, \qquad (6)$$

Here $E_T^2 = (12\pi\delta_e n_c T)^{1/2}$ is a characteristic plasma field for the thermal nonlinearity, and $n_c = m_e\omega^2/(4\pi e^2)$ is the critical density for the radiation frequency ω.

If the electron mean free path is long, and the conditions $\delta_e^{1/2} < l_e/L_E < 1$ hold, the plasma nonlinearity becomes nonlocal and is determined by the electron thermal conductivity. The permittivity $\varepsilon_{\rm NL} = -\delta n/n_c$ is found from the solution of the following equation in the case $|\delta n/n_0| < 1$:

$$\frac{2}{3}\frac{l_e^2}{\delta_e}\Delta\left(\frac{\delta n}{n_c}\right)-\frac{\delta n}{n_c}=\frac{n_0}{n_c}\frac{|E|^2}{E_T^2}, \qquad (7)$$

where Δ is the Laplacian.

In deriving (6) and (7) we ignored the disruption of the ionization balance of the plasma. The corresponding ionization rate ν_i and the corresponding recombination rate ν_r should therefore be small in comparison with the reciprocal of the relaxation time of the electron temperature ($\delta_e\nu_e$) and that of the pressure [v_s/L_E, where $v_s = (T_e/M_i)^{1/2}$ is the velocity of sound]. In a plasma with a low degree of ionization, in which the rate of ion-neutral collisions is high, $\nu_{\rm in} > v_s/L_E$, we should use the ambipolar diffusion time

$$(D_a/L_E^2)^{-1}=[v_s^2/(L_E^2\nu_{in})]^{-1}$$

as the relaxation time for perturbations of the electron density. The disruption of the ionization balance can be ignored under the condition

$$\nu_i\approx\nu_r<v_s^2/L_E^2\nu_{in},\quad \delta_e\nu_e.$$

Equations (6) and (7) correspond to a small deviation of the thermodiffusion ratio from unity.[13]

If the plasma is sufficiently hot, and the electron mean free path is greater than the length scale of the field variations ($l_e > L_E$), the perturbation of the plasma density is determined by the ponderomotive force. As in (6), the plasma nonlinearity is local in this case, and for $|\delta n/n_0| < 1$ the nonlinear permittivity is a quadratic function of the field:

$$\varepsilon_{NL}=\frac{n_0}{n_c}\frac{|E|^2}{E_s^2}, \qquad (8)$$

where $E_s = (16\pi n_c T)^{1/2}$ is the characteristic field of the ponderomotive nonlinearity of the plasma.

A local power-law nonlinearity prevails not only in plasmas but also in other media, e.g., those exhibiting the Kerr effect.[1] In a gas with excited atoms, nonlinear polarizability may be associated with multiphoton transitions from a metastable state. In this case the corresponding contribution to the nonlinear permittivity is determined by powers of the field higher than the second.[14] We will accordingly also discuss a permittivity which is a quadratic function of the intensity:

$$\varepsilon_{NL}=n_4|E|^4/E_p^4. \qquad (9)$$

Equation (1) takes the following form for the dimensionless field amplitude $\mathscr{E} = E/E_*$, normalized to the characteristic field $E_* = E_{T,s,p}$ corresponding to the type of nonlinearity under consideration [see (6)–(9)]:

$$i\frac{\partial\mathscr{E}}{\partial z}+\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\mathscr{E}}{\partial\rho}\right)+(i\Gamma+\beta Y)\mathscr{E}=0, \qquad (10)$$

where $\Gamma = \varepsilon''/(\varepsilon_0\sin^2\gamma)$ is the dimensionless field absorption coefficient, $\rho = kr\sin\gamma$, and $z' = (kz/2)\sin^2\gamma$ (the prime on the dimensionless coordinate z has been omitted from this equation). In a plasma we would have $\beta = (n_0/(n_c - n_0))\sin^{-2}\gamma$, and the relative density perturbation $Y = -\delta n/n_0$ for local nonlinearity (6) becomes

$$Y=|\mathscr{E}|^2/(1+|\mathscr{E}|^2), \qquad (11)$$

Expression (8) corresponds to expression (11) with $|\mathscr{E}|^2\ll 1$.

For the case of a nonlocal plasma nonlinearity, we use $\sin^2\gamma\ll 1$ and ignore the small spatial derivatives with respect to z. From Eq. (7) we find

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial Y}{\partial\rho}\right)=\alpha(-|\mathscr{E}|^2+Y), \quad (12)$$

where $\alpha = (3\delta_e/2)(kl_e \sin\gamma)^{-2}$ and $\alpha \ll 1$. Finally, the results

$$Y=|\mathscr{E}|^4$$
$$\beta=n_4/(\varepsilon_0\sin^2\gamma). \quad (13)$$

and $\beta = n_4/(\varepsilon_0 \sin^2\gamma)$ correspond to expression (9).

3. NONLINEAR STRUCTURES IN BESSEL BEAMS

The problem (10)–(13), with the boundary conditions (2) ($\mathscr{E}(\rho,z=0) = \mathscr{E}_{in}(\rho)\exp(-i\rho)$), has been solved numerically for the case of the focusing by an axicon of a hyper-Gaussian beam of the form (3) with $M = 8$ and radius $a_0 < R_a$. The field strength was assumed to vanish $\rho = \rho_{max} \simeq 10^3 > a_0 k \sin\gamma = 0.8\rho_{max}$ at the boundary of the computation region, ($\mathscr{E}(\rho_{max},z) = 0$). In addition, the radial shape of the beam to be focused, $\mathscr{E}_{in}(\rho)$, was assigned a profile such that the amplitude of the linear solution, (5), was constant over the greater part of the focal distance L_a and corresponded to the experimental conditions of Refs. 7–12. Those experiments used beams with $a_0 < R_a \simeq 1$ cm and $L_a \gtrsim 10$ cm, and on the scales of the nonlinear structures studied, $\Delta z \approx \lambda/\sin^2\gamma \sim 10^2$–$10^3\ \mu$m $\ll L_a$, the geometric-optics change in the amplitude in (5), which is proportional to $(z/\lambda)^{1/2}$ was negligible. The accuracy of the solution was monitored on the basis of the conservation of the energy flux, Eq. (10).

The first result which emerges from these calculations is that when the absorption is linear [$\Gamma \neq \Gamma(\mathscr{E}^2)$], and the optical thickness τ of the medium is small, i.e.,

$$\tau=kl\varepsilon''<1, \quad (14)$$

the field structure of a B beam is essentially the same as its structure in a dissipationless medium. Since the beam path length is $l \simeq R/\gamma$, where R is the characteristic radius of the absorbing region, condition (14), under which we can ignore dissipation (we assume below that this condition holds), leads to the restriction

$$\frac{R}{\lambda}<\frac{\gamma}{2\pi}\frac{1}{\varepsilon''}. \quad (15)$$

For example, when the beam from a neodymium laser ($\lambda = 1.06\,\mu$m) interacts with a plasma with $n_0 \approx 10^{19}$ cm^{-3}, $\nu_e \lesssim 10^{13}$ s, and $\gamma = 0.1$, the inequality (15) reduces to the limitation $R < 10^2\ \mu$m. Consequently, the effect of dissipation on the field distribution can be ignored, at least in the initial stage of the formation of an unbroken long laser spark, with $R < 100\ \mu$m.

Calculations carried out for a medium with a cubic local nonlinearity [i.e., without saturation, so we have $|\mathscr{E}|^2 \ll 1$ and $Y = |\mathscr{E}|^2$ in expression (11)] revealed a periodic spatial structure in the intensity distribution of the nonlinear solution of Eq. (10), with a length scale l_1 parallel to the beam axis given by

$$l_1=2\pi/\delta k=2\lambda/\sin^2\gamma\approx kr_1^2 \quad (16)$$

This length scale is close to the diffraction size of an "axial caustic," limited by the first zero of the Bessel function, $r_1 = 2.4/(k \sin\gamma)$ (Fig. 1a). The self-modulation of a B beam which is observed[3] is manifested when the radiation power in the axial caustic and thus in each annular zone of radial profile (5) is close to the critical value for self-focusing, $\mathscr{E}^2_{cr} = 1/\beta$, i.e., lies in the following interval of subthreshold beam intensities:

$$0{,}2\lesssim\beta|\mathscr{E}^{(0)}|^2\lesssim 1, \quad (17)$$

Here $\mathscr{E}^{(0)}$ is the dimensionless amplitude of the linear solution (4) at the axis ($\rho = 0$). With increasing intensity of the beam to be focused ($|\mathscr{E}^{(0)}|^2$), the amplitude of the spatial oscillations of the nonlinear solution increases significantly along the focusing section in the interval (17), and the length scale of the oscillations decreases slightly (Fig. 1b). At $\beta|\mathscr{E}^{(0)}|^2 \gtrsim 1$, a structural feature appears at the beam

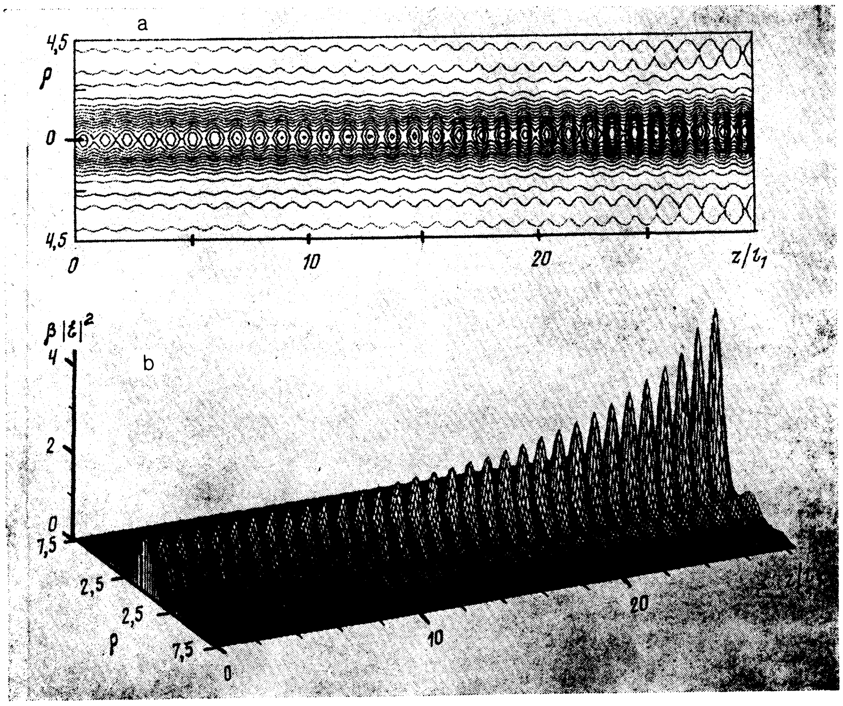

FIG. 1. Spatial distribution of the field intensity in the case of a cubic nonlinearity [see expression (8)] for two intensities of the incident beam. a—Contour lines $|\mathscr{E}(\rho,z)|^2 = $ const, $\beta|\mathscr{E}^{(0)}|^2 = 0.47$; the isometry $|\mathscr{E}(\rho,z)|^2$, $\beta|\mathscr{E}^{(0)}|^2 = 0.6$.

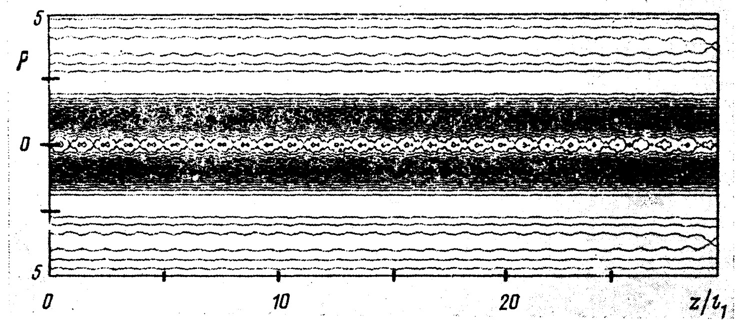

FIG. 2. Spatial distribution of the field intensity (contour lines $|\mathscr{E}(\rho,z)|^2 = \text{const}$) in the case of a power-law nonlinearity [see (13)], with $\beta|\mathscr{E}^{(0)}|^4 = 0.27$.

axis in a medium without a saturation of the nonlinearity.

For a local power-law nonlinearity of higher order, as in (13), self-modulation of the beam again occurs, with a periodic structure having a longitudinal length scale $\sim l_1$ [see (16) and Fig. 2]. As in the case of a cubic nonlinearity, the threshold intensity for the onset of oscillations (and the range in which the self-modulation effect is seen) is determined in order of magnitude by the condition for cancellation of the diffraction effects by the nonlinear change in the refractive index:

$$0{,}2 \leqslant \beta|\mathscr{E}^{(0)}|^4 \leqslant 0{,}5. \qquad \text{(f)}$$

A similar type of nonlinear periodic field structure occurs in a medium with a nonlocal nonlinear response as in (12). Figure 3 shows results calculated for $\alpha = 0.1$ and $\alpha\beta|\mathscr{E}^{(0)}|^2 = 0.3$. It follows from this figure that the period of the intensity oscillations near the threshold for the appearance of the self-modulation effect ($\alpha\beta|\mathscr{E}^{(0)}_{\text{thr}}|^2 \simeq 0.2$) is determined by a length scale on the order of l_1 [see (16)], as in the case of a local power-law nonlinearity.

With increasing beam intensity in a medium with a nonlocal nonlinearity as in (12), a more complex field structure arises, as in the case of a saturating nonlinearity [see (11)]. In addition to the small-scale modulation with period $\sim l_1$ [see (16)] described above, oscillations with a length scale $l_2 \lesssim 10.l_1$ appear in the intensity distribution along the beam (Fig. 4). In the case of a local nonlinearity with saturation as in (11), these large-scale oscillations are most obvious at $\beta \approx 2$ and $|\mathscr{E}^{(0)}| \gtrsim 1$. For a nonlocal nonlinearity as in (12), they are most obvious at $\alpha\beta|\mathscr{E}^{(0)}|^2 \gtrsim 1$. The calculated results shown in Fig. 4 indicate that the large-scale modulation is manifested not only at the beam axis but also in the side maxima of the radial distribution of the field intensity.

4. RESULTS OF EXPERIMENTS WITH B BEAMS

For an experimental test of the ideas presented above, we can compare the results of the calculations presented here with data on the formation of an unbroken long laser spark in gases.[9-12] This comparison can be based on some simple considerations. If the intensity distribution of the radiation in the interior of the gas is nonuniform, breakdown occurs first in the zone with the highest field intensity. For this reason, as the output power of the laser is raised, and as the breakdown field of the gas is reached, the localization of the breakdown zones in a B beam should correspond to the distribution of intensity maxima which is formed because of the self-effect of the B beam.

The experimental data presently available on this question span a fairly wide range of conditions. The gas breakdown and the formation of the plasma channel of an unbroken long laser spark have been studied in air, argon, helium, neon, xenon, and carbon dioxide at pressures from 5 to 1000 kPa. Electromagnetic radiation at a wavelength of 1.06 μm, with a power from 1 to 30 GW, was generated in a pulse ranging in length from 1 to 50 ns by four neodymium lasers, which differed in design and parameters. One of them[15] was a single-frequency laser, while the others were single-mode lasers with a divergence close to the diffraction limit.[7,11] The distribution in the initial laser beams corresponded to a hyper-Gaussian distribution with a parameter value $M = 5$–8. The hyper-Gaussian beam was transformed into a B beam by an axicon. A set of axicons made it possible to vary the focusing angle γ from 2.5° to 10°.

Among the experimental results, we should first note the data of Ref. 16 on the light intensity distribution in a B beam found under conditions such that the field of the electromagnetic wave had not yet reached the threshold for gas breakdown, and no self-effect of the beam was seen. Figure 5a shows a representative intensity distribution in a cross section. At a divergence level of 10^{-4}–10^{-3} rad of the original beam, clearly defined equidistant annular maxima and minima of the intensity were detected. When the quality of the initial beam was deliberately degraded by means of a phase plate, the diameters of the rings remained the same.

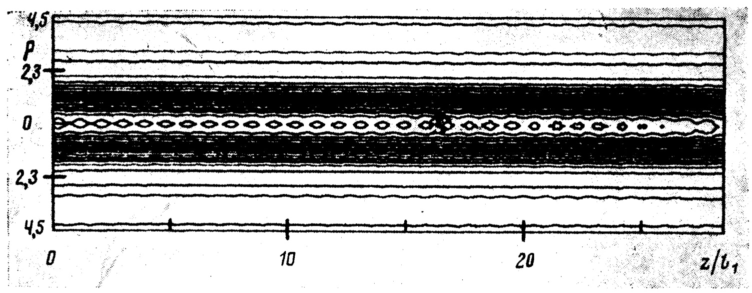

FIG. 3. Contour lines of the field intensity in the case of a nonlocal nonlinearity [see (12)]. $\alpha = 0.1, \beta = 1,\ \alpha\beta|\mathscr{E}^{(0)}|^2 = 0.3$.

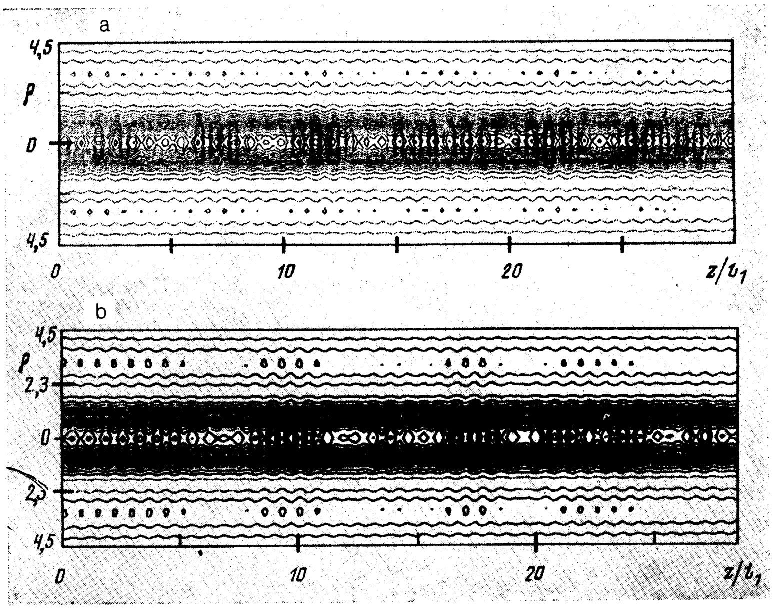

FIG. 4. Spatial distribution of the field ($|\mathscr{E}(\rho,z)|^2 = \mathrm{const}$) as the intensity of the pump beam is raised. a—Nonlinearity with saturation [see (11)], $\beta = 2.0$, $|\mathscr{E}^{(0)}|^2 = 3.12$; b—nonlocal nonlinearity [see (12)], $\alpha = 0.1$, $\alpha\beta|\mathscr{E}^{(0)}|^2 = 1.4$.

The only changes observed were distortions in the contrast of the rings. Above a certain permissible level of the wavefront distortion (e.g., when the beam reached a divergence angle of $3\cdot 10^{-2}$ rad), adjacent maxima overlapped, and a B beam simply did not form. In addition, the distortions of the conical shape of the wavefront resulted in the appearance of a diffusive component, which also reduced the overall contrast in the radial distribution of the radiation. It was for this reason that the intensity of the minima did not reach zero, as can be seen from the densitometer trace in Fig. 5b, which represents the intensity distribution shown in the photograph in Fig. 5a. Note that the diameters of both the maxima and the minima agree precisely with the theoretical predictions and are described by a Bessel function (the dashed curves), in agreement with (5).

We can draw conclusions about the intensity distribution along the axis of the B beam by observing the time of gas breakdown. As an example, Fig. 6 shows photographs of the breakdown of argon at pressures of 10, 20, and 100 kPa. We see from these frames that the gas breaks down at discrete centers, rather than continuously along the axis. These centers form a regular small-scale spatial structure with a period $l_1 = 2\lambda/\gamma^2$ over the entire angular range $\gamma = 2.5$–$10°$. The period of this structure does not depend on the type of gas or its pressure; it is also independent of the length of the laser pulse and of the mode composition of the light, at least over the ranges specified. It is equal to its theoretical value (16). At a pressure of 100 kPa, there are additional features in the breakdown: A large-scale structure with a period $l_2 \sim 10.l_1$, which also spans the part of the B beam away from the axis, appears along with the small-scale structure.

It is interesting to trace the evolution of the pattern. Plasma formations appear in the breakdown zones. These formations remain at rest at the axis, but as they expand toward each other at a velocity $\sim 5\cdot 10^6$ cm/s and higher, they merge into an unbroken plasma channel in 2–3 ns. The plasma formations distant from the symmetry axis participate in a more complex interaction. As a result, the large-scale structure is preserved as the breakdown centers merge,

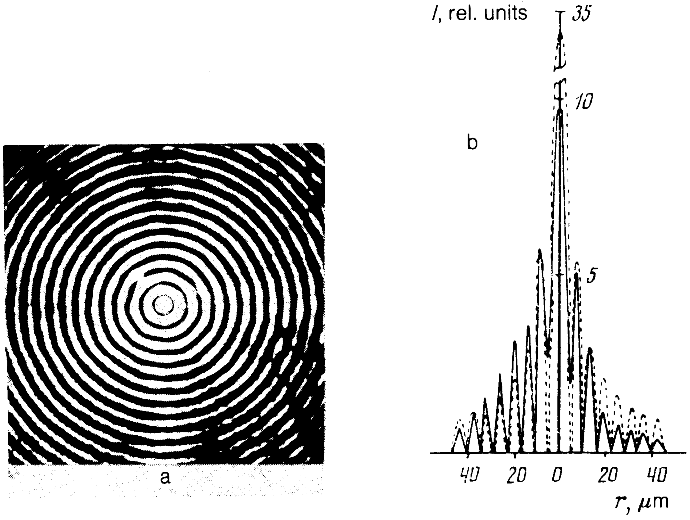

FIG. 5. Radial intensity distribution in a B beam. a—Photograph; b—densitometer trace and Bessel function (dashed line).

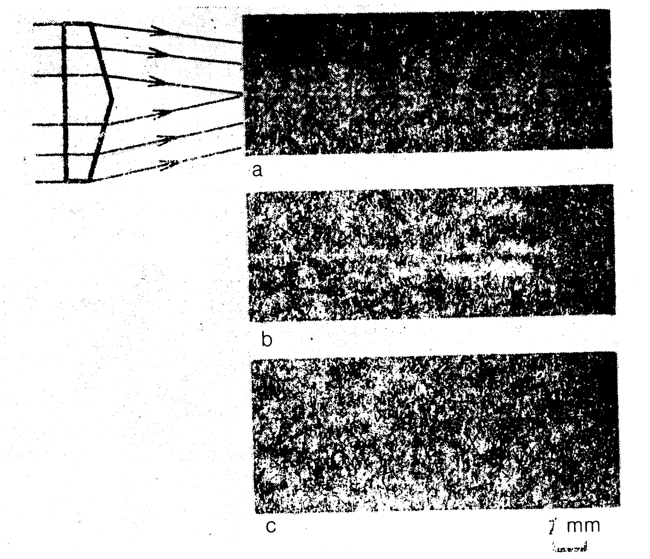

FIG. 6. Structure of the channel of a B beam at the time of breakdown in argon. a—Pressure of 100 kPa; b—20 kPa; c—10 kPa.

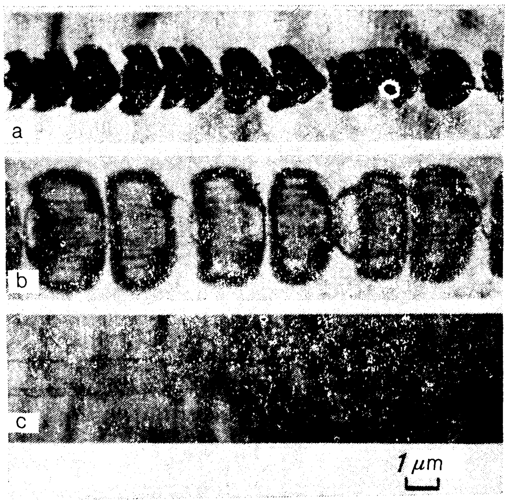

FIG. 7. Structure of the channel 20 ns after the breakdown in argon. a—Pressure of 100 kPa; b—20 kPa; c—10 kPa.

and the plasma channel acquires a structure with the same spatial period.

Figure 7 shows Schlieren photographs of the channel of the B beam found under the same conditions as in Fig. 6, but slightly later (20 ns after the breakdown). We see from these photographs that the surface of the channel is smooth in argon at 10 kPa. At a pressure of 100 kPa, the surface has a clearly defined cellular shape, reminiscent in many ways of the large-scale structure of the breakdown. The 20-kPa case is an intermediate one, with a tendency toward the formation of a smooth, unbroken channel. These experimental results provide qualitative support for the theoretical conclusion that the small-scale structure accompanies breakdown in B beams under arbitrary conditions, while the conditions under which the large-scale structure appears are restricted. In the experiments, this large-scale structure arises in a narrower pressure interval. In argon, for example, this interval is 20–300 kPa, and in air it is 20–50 kPa.

5. CONCLUSION

This analysis shows that axicon focusing of electromagnetic radiation produces beams with a nonmonotonic but regular radial intensity profile which can be described by a Bessel function: B beams. In B beams we see a self-modulation effect: near the threshold, a nonlinear periodic field structure arises with a longitudinal length scale $\sim l_1$ [see (16)]. This length scale is determined by the diffraction size of the axial caustic. The self-modulation occurs for various types of local power-law nonlinearity and also in media with a nonlocal nonlinear response.

The period of the nonlinear modulation of the field intensity depends on the wavelength of the radiation and on the radiation focusing angle γ, while it is unrelated to the particular mechanism or particular properties of the focusing nonlinearity of the medium. The manifestation of self-modulation of B beams is thus universal in nature.

In the initial stage of the avalanche development, the nonlinearity of the medium may result from an anomalously high polarizability of metastable atoms which are produced by electron impact.[17] Estimates in Ref. 14 show that a sufficient condition for the occurrence of self-modulation of a B beam with a power density $\sim 10^{11}$ W/cm^2 at its axis in an inert gas is that a fraction $\sim 10^{-3}$ of the total number of atoms be in metastable excited states. In later stages of the avalanche development, with $n_e \sim 10^{19}$ cm^{-3} and $T_e \sim 10$ eV (with $\nu_{ei} \sim \nu_{en} \sim 10^{13}$ s^{-1}), the nonlocal thermal nonlinearity of a plasma with $\gamma \sim 10^{-1}$ leads to a self-modulation at power densities no higher than 10^{11} W/cm^2. Each of these nonlinearities, acting individually or jointly, could thus give rise to the structure observed experimentally for the unbroken long laser spark.

These results indicate that a large-scale structure, with a longitudinal period $l_2 \sim 10 l_1$, will arise in the beam at the same time as the small-scale oscillations when the radiation intensity is well above the threshold for the occurrence of this effect, either in a medium with a nonlinearity saturation or in a medium with a nonlocal nonlinearity.

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