Electronic inelastic scattering of light in an absorbing medium

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It is shown that the well-known result of L. D. Landau and E. M. Lifshitz (*Electrodynamics of Continuous Media*, Pergamon, Oxford, 1960) describing the scattering of light is altered when the distribution of the field in the medium is taken into account. Scattering with excitation of electron-hole pairs and plasmons is analyzed.

1. INTRODUCTION

In the general theory of the scattering of light by a continuous medium the electromagnetic field is found by superposing the fields produced at a large distance by separate scattering centers.[1] In this case the amplitude of the scattered light can be obtained by calculating the response of the system to the corresponding external force.[2,3] The scattering cross section is proportional to the imaginary part of the inverse dielectric function.

Prior to leaving the conductor, however, the electromagnetic radiation appearing as a result of scattering interacts with the charge carriers. Just like the excitation radiation, the scattered light penetrates into the conductor only to a distance of the order of the thickness of the skin layer, and the scattered radiation observed outside the conductor is the result of collective interaction with charge carriers inside the conductor. In essence, it is also necessary to solve the boundary-value problem of the propagation of the scattered radiation observed outside the conductor is the result of collective interaction with charge carriers inside the conductor. In essence, it is also necessary to solve the boundary-value problem of the propagation of the scattered radiation from the conductor into the vacuum, where the scattering is observed.

The inelastic scattering of light in this formulation was first studied in Ref. 4, which is devoted to scattering in a superconductor, with the help of the principle of detailed balance, which makes it possible to relate scattering with absorption in a prescribed field. The result obtained in Ref. 4 is qualitatively valid only for media in which the absorption coefficient is greater than the refraction coefficient and for the case when the transferred frequency is small. The same remark also pertains to Ref. 5, where electronic scattering in a normal metal was studied with the help of the kinetic equation.

Scattering of light in superconductors was also studied in Refs. 6 and 7 and their results agree qualitatively with Ref. 4, but they are qualitatively incorrect because the fields were not accurately matched at the metal-vacuum boundary.

The scattering of light in superconductors and normal metals has recently been arousing special interest as one of the most direct methods for determining the superconducting gap as well as for checking the results of recent theories.[8,9] The experimental data are interpreted with the help of the results of Refs. 1–7, which are not entirely correct. The purpose of this paper is to fill this gap.

2. RELATION BETWEEN THE SCATTERING CROSS SECTION AND THE DENSITY CORRELATION FUNCTION

We write Maxwell's equation

$$\mathrm{rot\,rot}\,\mathbf{E}+\frac{1}{c^2}\frac{\partial^2\mathbf{D}}{\partial t^2}=-\frac{4\pi}{c^2}\mathbf{j} \quad (1)$$

for the Fourier components of the scattered field

$$\mathbf{E}(t,\mathbf{r})=\int\frac{d\omega\,d^2k}{(2\pi)^3}\mathbf{E}(\omega,\mathbf{k},z)\exp(-i\omega t+i\mathbf{k}\boldsymbol{\sigma}). \quad (2)$$

The Fourier expansion is made with respect to the time and the two-dimensional coordinate $\boldsymbol{\sigma}$ parallel to the surface, the z-axis is oriented normal to the metal-vacuum interface, and the metal occupies the half-space $z>0$.

We distinguish the incident field from the scattered field by assigning an index to the frequency (and the vector $\mathbf{k}$): ω_i for incident light and ω_s for the scattered light. We call the transferred frequency $\omega=\omega_i-\omega_s$. We confine our attention to optical frequencies ω_i and ω_s, where the spatial dispersion of the permittivity ε is insignificant.

We transferred the part of the current $\mathbf{j}$ that is proportional to the scattered field to the left-hand side of Eq. (1), incorporating this part of the current into $\mathbf{D}$. The part of the current remaining on the right-hand side is proportional to the incident field $\mathbf{E}(\omega_i,\mathbf{k}_i,z)$. An expression for this part can be obtained by introducing an effective Hamiltonian, describing the scattering in second-order perturbation theory:

$$H=\frac{e^2}{c^2}\gamma A_iA_s\psi^+\psi, \quad (3)$$

where A is the vector potential of the electromagnetic field, we designate by $\mathbf{e}_i$ and $\mathbf{e}_s$ the polarization vectors of the incident and scattered fields, $\gamma=e_i^{(\alpha)}m_{\alpha\beta}^{-1}e_s^{(\beta)}$, and m^{-1} is the generalized inverse effective mass tensor:

$$m_{\alpha\beta}^{-1}=m_0^{-1}\delta_{\alpha\beta}+\frac{1}{m_0^2}\sum_v\left[\frac{p_{cv}^{(\beta)}p_{vc}^{(\alpha)}}{E_c(\mathbf{p})-E_v(\mathbf{p})-\omega_s}+\frac{p_{cv}^{(\alpha)}p_{vc}^{(\beta)}}{E_c(\mathbf{p})-E_v(\mathbf{p})+\omega_i}\right]. \quad (4)$$

An expression like Eqs. (3) and (4) is encountered in the theory of scattering of light by atomic electrons.[10] Such an expression for electrons in a solid is obtained in Ref. 11, if the

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correction in second order in the electric field for a particle in a periodic potential is written in the Luttinger-Kohn representation, analogously to the manner in which this is done in the derivation of an expression for the effective mass. The index c refers to the band which the particle occupies before and after scattering and to which the ψ operators in Eq. (3) correspond; the summation index v enumerates all possible intermediate states which an electron can occupy after emitting or absorbing a photon.

In contrast to the standard effective mass, characterizing the electronic spectrum at the bottom of the band, the generalized mass Eq. (4) describes the effect of resonance enhancement in the case when one of the frequencies ω_i or ω_s is close to an interband transition frequency. Strictly speaking, the value of E must be taken for $\mathbf{p} \pm \mathbf{k}$, where $\mathbf{k}$ is the wave vector of the absorbed or scattered photon. In all cases of practical interest, however, k is small compared with the Fermi momentum p.

With the help of Eq. (3) we obtain an expression for the current on the right-hand side of Eq. (1):

$$j=-c\frac{\delta H}{\delta A}=-\frac{e^2}{c}\gamma A_i\psi^+\psi. \tag{5}$$

We note that since Eq. (1) was written for the Fourier components corresponding to the frequency of the scattered field, the current Eq. (5) vanishes when the electronic operators are averaged.

For simplicity we shall confine our attention to the case when the scattered light, like the incident light, propagates along the normal to the surface of the sample and this direction and the direction of polarization of the light, are oriented along the principal axes of the permittivity tensor. Then Eq. (1) assumes the form

$$-\frac{d^2E(\omega_s,z)}{dz^2}-\frac{\omega_s^2}{c^2}\varepsilon_sE(\omega_s,z)=\frac{4\pi i\omega_s}{c^2}j(\omega_s,z), \tag{6}$$

where ε_s is the principal value of the permittivity tensor, corresponding to the polarization and frequency of the scattered light. Among the $E(\omega_s, z)$ arguments, we omit $k=0$.

In the region $z>0$ Eq. (6) has the following solution, which decreases as $z\to\infty$:

$$E(\omega_s,z)=-\frac{2\pi}{c\varepsilon_s^{1/2}}\int_0^\infty dz'j(\omega_s,z')\exp[ik_z(\omega_s)|z-z'|]$$

$$+E_1(\omega_s)\exp[ik_z(\omega_s)z], \tag{7}$$

where the second term is the solution of the homogeneous equation $k_z(\omega_s)=\omega_s/c\varepsilon_s^{1/2}\equiv\omega_s/c(n_s+i\varkappa_s)$, corresponding to Eq. (7).

In the vacuum ($z<0$) the scattered wave has the form

$$E(\omega_s,z)=\tilde{E}(\omega_s)\exp\left(-i\frac{\omega_s}{c}z\right). \tag{8}$$

The constants E_1 and $\tilde{E}$ are found from the conditions that the electric and magnetic fields are continuous at $z=0$:

$$\begin{gathered}E_0(\omega_s)+E_1(\omega_s)=\tilde{E}(\omega_s),\\ \varepsilon_s^{1/2}[-E_0(\omega_s)+E_1(\omega_s)]=-\tilde{E}(\omega_s),\end{gathered} \tag{9}$$

where we have introduced the notation

$$E_0(\omega_s)=-\frac{2\pi}{c\varepsilon_s^{1/2}}\int_0^\infty dz'j(\omega_s,z')\exp[ik_z(\omega_s)z']. \tag{10}$$

The conditions (9) make it possible to find the amplitude of the field in the vacuum:

$$\tilde{E}(\omega_s)=\frac{2\varepsilon_s^{1/2}}{1+\varepsilon_s^{1/2}}E_0(\omega_s). \tag{11}$$

The energy flux in the scattered wave is determined by the square of the field:

$$|\tilde{E}(\omega_s)|^2=\frac{(4\pi/c)^2}{|1+\varepsilon_s^{1/2}|^2}\int_0^\infty dz'\,dz\langle j^+(\omega_s,z)j(\omega_s,z')\rangle$$

$$\times\exp[ik_z(\omega_s)z'-ik_z^*(\omega_s)z], \tag{12}$$

where the brackets designate both quantum-mechanical averaging of the electronic operators and statistical averaging (for nonzero temperature).

In Eq. (12) the integration over the half-space $z>0$ can be extended to all space, if it is assumed that the electrons are mirror-reflected from the inner surface of the sample. In order to do this, we note that the average of the product of four ψ operators reduces to two Green's functions:

$$\langle\psi^+(z)\psi(z)\psi^+(z')\psi(z')\rangle=G(z,z')G(z',z).$$

Under conditions of mirror reflection the Green's function vanishes when one of the coordinates z or z' lies on the surface. This condition can be satisfied by setting

$$G(z,z')=G_\infty(z-z')-G_\infty(z+z'), \tag{13}$$

where G_∞ is the Green's function of an unbounded metal. For $z, z'>0$ the function Eq. (13) satisfies the same equation as $G_\infty(z-z')$ and vanishes at the boundary, since G_∞ depends only on the magnitude of its argument (we assume that the surface of the sample is also a symmetry plane).

The integral (12) also contains exponentials corresponding to the scattered field (with frequency ω_s) and the incident field [see Eq. (5)]. The incident field has the form

$$A_i(z)=\frac{2A_i^0}{1+\varepsilon_i^{1/2}}\exp[ik_z(\omega_i)z-i\omega_it], \tag{14}$$

where $k_z(\omega_i)=\omega_i/c)\varepsilon_i^{1/2}$, where ε_i is the corresponding principal value of the permittivity tensor at the frequency ω_i. We normalize the amplitude of the vector potential in a vacuum A_i^0 so that the field would be equal to unity: $A_i^0=c/\omega_i$. Thus, for example, in the integral over z' the exponential factor has the form

$$f(z')=\exp\{iz'[k_z(\omega_i)+k_z(\omega_s)]\}. \tag{15}$$

It is convenient to continue it into the region $z'<0$ in an even manner:

$$f(z')=f(-z'). \tag{16}$$

Using the formulas (13) and (16) we can see that if we neglect terms of the form $G_\infty(z'-z)G_\infty(z'+z)$ the two-particle electron correlation function of an unbounded metal can be integrated over infinite limits. The error made in so doing is of order $1/p_F\delta$, where $1/\delta = \mathrm{Im}(k_z)$ is the inverse penetration depth of light in the metal and p_F is the characteristic electronic momentum. The final result acquires only a factor of 1/2.

Returning now from the Fourier components to the space-time variables and keeping in mind Eqs. (5) and (14), we write

$$\int dz'\, dz \langle j^+(z,\omega_s) j(z',\omega_s)\rangle$$

$$\sim \int dx'\, dx \rho_{\gamma^2}(x-x') f^+(z) f(z') \exp[i\omega(t-t') - i\mathbf{q}(\boldsymbol{\sigma}-\boldsymbol{\sigma}')], \tag{17}$$

where the transferred frequency is $\omega = \omega_i - \omega_s$, and the dependence on the transferred wave vector $\mathbf{q} = \mathbf{k}_i - \mathbf{k}_s$ parallel to the surface is presented for an arbitrary direction of propagation of the light; we denote by x the four-dimensional coordinate $(t,\boldsymbol{\sigma},z)$. The correlation function $\rho_{\gamma^2}(x-x')$, appearing in Eq. (17), depends on the difference $x - x'$:

$$\rho_{\gamma^2}(x-x') = \langle \gamma^2 \psi^+(x)\psi(x)\psi^+(x')\psi(x')\rangle. \tag{18}$$

For this reason, for example, one of the integrals over t or t' gives the observation time T while the other integral reduces to the Fourier component with respect to the difference coordinate. The integral over the coordinate parallel to the surface is calculated in a like manner. The observation time and the surface area were assumed to be equal to unity when normalizing the incident radiation. The remaining integration over the coordinates z and z' leads to the Fourier component of the function $f(z)$:

$$f(q_z) = \int_0^\infty dz f(z)[\exp(iq_z z) + \exp(-iq_z z)]$$

$$= \frac{2i[k_z(\omega_i)+k_z(\omega_s)]}{[k_z(\omega_i)+k_z(\omega_s)]^2 - q_z^2}. \tag{19}$$

The spectral density of the radiation, as is evident from the expansion Eq. (2), is calculated for the interval

$$\frac{d\omega_s\, d^2k_s}{(2\pi)^3} = \frac{k_z\omega_s}{c}\frac{d\omega_s\, do_s}{(2\pi)^3}, \tag{20}$$

where do_s is the solid angle near the direction of scattering.

The final expression for the effective scattering cross section, i.e., for the relative number of photons reflected with a change of frequency $\omega = \omega_i - \omega_s$ into the interval of angles do_s and frequencies $d\omega_s$, can be written in the form

$$d\sigma = \left(\frac{2\pi e^2\omega_s}{c^2\omega_i}\right)^2 \frac{16}{[(1+n_s)^2+\varkappa_s^2][(1+n_i)^2+\varkappa_i^2]}$$

$$\times \frac{d\omega_s\, do_s}{(2\pi)^3}\int_0^\infty \frac{dq_z}{2\pi}\rho_{\gamma^2}(\omega,q)|f(q_z)|^2, \tag{21}$$

where $\rho_{\gamma^2}(\omega,q)$ is the Fourier component of the correlation function Eq. (18).

We present $|f(q_z)|^2$ for the simplest case when the incident and scattered light have the same polarization and their frequencies ω_i and ω_s differ by a small amount ($n_s = n_i$, $\varkappa_s = \varkappa_i$):

$$|f(q_z)|^2 = \left(4\frac{\omega_i}{c}\right)^2 (n_i^2+\varkappa_i^2) \Big/ \left\{\left[\left(2\frac{\omega_i}{c}\right)^2 (n_i^2-\varkappa_i^2) - q_z^2\right]^2 + 2^6\frac{\omega_i^4}{c^4} n_i^2\varkappa_i^2\right\}. \tag{22}$$

Hence it is obvious that if the frequency of the incident light lies below the threshold of transmission, when n_i is small compared with $\varkappa_i$, then the expression (22) can be put into the form

$$|f(q_z)|^2 = 2^4/\varkappa_i\delta^2(q_z^2+4/\delta^2)^2, \tag{23}$$

where $\delta = c/\omega_i\varkappa_i$. At higher frequencies, where the real part of ε becomes positive and n is large compared with $\varkappa$, $|f(q_z)|^2$ has a pole, which can be represented by a δ-function:

$$|f(q_z)|^2 = \frac{1}{\left(q_z - 2\frac{\omega_i}{c}n_i\right)^2 + \left(2\frac{\omega_i}{c}\varkappa_i\right)^2} = \delta\frac{\pi}{2}\delta\left(q_z - 2\frac{\omega_i}{c}n_i\right). \tag{24}$$

The meaning of this formula is quite transparent: when the light is scattered backward twice the wave vector of the light is transferred to the electronic system and the scattering cross section, under otherwise equal conditions, is proportional to the penetration depth δ in the metal.

3. SCATTERING BY SINGLE-PARTICLE EXCITATIONS AND PLASMONS

The correlation function (18), shown in Fig. 1 by the first diagram, differs from the standard density correlation function by the presence of the vertex factors γ. It can be expressed in terms of the corresponding retarded Green's function $K^R_{\gamma^2}(\omega,q)$ in the standard manner:[12]

$$\rho_{\gamma^2}(\omega,q) = -\frac{1}{\pi}\frac{\mathrm{Im}\, K_{\gamma^2}^R(\omega,q)}{1-\exp(-\omega/T)}. \tag{25}$$

For a noninteracting Fermi gas

$$K_{\gamma^2}^R(\omega,q) = \frac{2}{(2\pi)^3}\int d^3p\, \gamma^2\left(-\frac{dn}{d\varepsilon}\right)\frac{\mathbf{vq}}{-\mathbf{vq}+\omega+i\delta}. \tag{26}$$

The Coulomb interaction of electrons leads to the series of diagrams shown in Fig. 1. The result of summing this series reduces to replacing one of the vertices in the loop $\gamma(p) \to \Gamma(p,q)$. The following equation is obtained for the vertex function:

$$\Gamma(p,q) = \gamma(p) + V(q)T\sum_{\omega_l}\int \frac{d^3p_1}{(2\pi)^3} G(p_1)G(p_1+q)\Gamma(p_1,q), \tag{27}$$

where the potential $V(q) = 4\pi e^2/\varepsilon^{(0)}_{\alpha\beta}q_\alpha q_\beta$ describes the Coulomb interaction taking into account the permittivity $\varepsilon^0_{\alpha\beta}$ of the ionic core.

The solution of Eq. (27) has the form

$$\Gamma(p,q) = \gamma(p) + \frac{V(q)K_\gamma^R(\omega,q)}{1-V(q)K^R(\omega,q)}, \tag{28}$$

where the function K^R differs from $K^R_{\gamma^2}$ (Ref. 26) by the fact that the factors γ are not present in the integrand and K^R_γ contains only one factor γ.

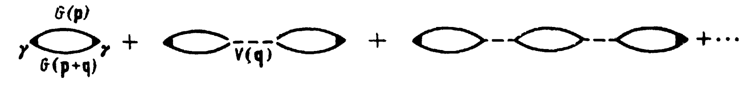

FIG. 1.

We discuss two limiting cases.

a) The transferred frequency is small compared with the plasma frequency of the carriers. As will be seen from what follows, here light is scattered by electron-hole excitations and it is necessary to know the asymptotic behavior of $K^R(\omega,q)$ in the region $\omega \ll vq$. For temperatures that are low compared with the Fermi energy the integration in Eq. (26) over the energy is performed with the help of a relation of the form $-dn/d\varepsilon = \delta(\varepsilon - \varepsilon_F)$:

$$K^R(\omega, q)=\frac{2}{(2\pi)^3}\int \frac{dS}{v}\left[-1-i\pi\frac{\omega}{qv}\delta(\mu)\right], \qquad (29)$$

where μ is the cosine of the angle between the velocity $\mathbf{v}$ at the Fermi surface and the vector $\mathbf{q}$.

In the denominator of Eq. (28) a quantity of order

$$\frac{4\pi e^2}{\varepsilon^0 q^2}\frac{p_F m}{\pi^2}\sim \varkappa_D{}^2/q^2,$$

where $\varkappa_D^2 = (4\pi e^2/\varepsilon^0)dn/d\varepsilon_F$ is the inverse Debye radius, is added to unity. Since q is of the order of the inverse depth of the skin layer and the Debye radius is usually on the order of the interatomic distance, in the case at hand we have $(\varkappa_D/q)^2 \gg 1$. For this reason unity in the denominator of the fraction in Eq. (28) should be neglected, and we obtain for the vertex function

$$\Gamma(p, q)=\gamma-\bar{\gamma}, \qquad (30)$$

where the overbar denotes averaging over the Fermi surface:

$$\bar{\gamma}=\int\frac{dS}{v}\gamma\Big/\int\frac{dS}{v}.$$

Substituting Eq. (30) into the expression for the electron loop and keeping in mind Eq. (25) we find

$$\rho_{\gamma^2}(\omega, q)=\frac{\omega/q}{1-\exp(-\omega/T)}\frac{2}{(2\pi)^3}\int\frac{dS}{v^2}\delta(\mu)\gamma(\gamma-\bar{\gamma}). \qquad (31)$$

The integration in Eq. (31) extends over the strip on the Fermi surface $\mu = 0$ in which the projection of $\mathbf{v}$ on the vector $\mathbf{q}$ vanishes.

The final integration over q_z in Eq. (21) is elementary. The integral actually extends from ω/v up to a quantity of order $|n_i + i\varkappa_i|\omega_i/c$, since for large q_z the field factor Eq. (22) drops off rapidly. This results in the fact that the scattering cross section increases in the region of small transferred frequencies up to values on the order of $\omega \sim (v/c)\omega_i|n_i + i\varkappa_i|$, and then drops off rapidly. The scattering fails to vanish in this region only due to the momentum dependence of γ, i.e., the mass. The angle-independent part, as is evident from Eq. (31), drops out of the result. This is a natural consequence of the Debye screening of the charge-density fluctuations at low frequencies.[13] A specific example of a calculation performed with the help of the formulas (21) and (31) is presented in the brief paper Ref. 14.

b) The transferred frequency is greater than the plasma frequency. Here the scattering of light is associated with the excitation of a plasmon. In this case the small values of the transferred wave vector q are significant. Expanding the fraction in the integrand of Eq. (26) in powers of vq/ω and noting that owing to the symmetry of the Fermi surface under the inversion $\mathbf{v}\to -\mathbf{v}$, the odd powers of this ratio vanish on integration, we obtain

$$K^R(\omega, q_z)=\frac{2}{(2\pi)^3}\int\frac{dS}{v}\left(\frac{q_z v_z}{\omega}\right)^2\left[1+\left(\frac{q_z v_z}{\omega}\right)^2\right], \qquad (32)$$

where an infinitesimal imaginary art must be added to the frequency ω.

The pole of the second term in Eq. (28) contributes an imaginary part to the electronic loop. Its denominator, after multiplying by ω^2, can be written in the form $\omega^2 - \omega_p^2(q_z)$, where

$$\omega_p{}^2(q_z)=\omega_p{}^2(0)+q_z{}^2\overline{v_z{}^4}/\overline{v_z{}^2}, \qquad (33)$$

and

$$\omega_p{}^2(0)=\frac{8\pi e^2}{\varepsilon_{zz}{}^0(2\pi)^3}\int\frac{dS}{v}v_z{}^2. \qquad (34)$$

We note that in the isotropic case $\overline{v_z^4}/\overline{v_z^2} = 3/5v^2$ and the formula (33) gives the well-known dispersion relation for plasmons for small values of q.

Separating the imaginary part we obtain

$$\rho_{\gamma^2}(\omega, q_z)=\frac{\operatorname{sign}\omega}{1-\exp(-\omega/T)}\times q_z{}^2\delta[\omega^2-\omega_p{}^2(q_z)]\frac{(\overline{\gamma v_z{}^2})^2}{\overline{v_z{}^2}}\frac{2}{(2\pi)^3}\int\frac{dS}{v}. \qquad (35)$$

The scattering cross section is obtained by integrating (21) the correlation function (35) with the field factor (19). We note that the plasmon contribution (35) appears where the permittivity as a function of the transferred frequency is positive; in the notation employed here, it is equal to $\varepsilon_{zz}^0[1-\omega_p^2(q_z)/\omega^2]$. It is obvious that it must also be positive when the frequency of the incident light $\omega_i = \omega_s + \omega$ is large. The function $\varkappa_i$ decays slightly here as a result of interband absorption.

At the frequency of the scattered light ω_s and with high transferred frequency ω the permittivity ε_s can be positive or negative. For this reason the real and imaginary parts of the quantity $k_z(\omega_i) + k_z(\omega_s)$ gives in (19) can be comparable. The form of the plasmon peak in the scattering cross section depends on their ratio. The cross section starts to increase at $\omega = \omega_p(0)$, and near this frequency the cross section is proportional to $q_z \propto [\omega_p(0)(\omega - \omega_p(0)]^{1/2}$. The cross section decreases as q_z^{-3} for $q_z \gg |k_z(\omega_i) + k_z(\omega_s)|$, i.e., for $\omega - \omega_p(0) \gg v^2\omega_i^2(\varkappa_i^2 + \varkappa_s^2)/c^2\omega_p(0)$ if $\varkappa_s \gg n_s$. If $\mathrm{Im}[k_z(\omega_i) + k_z(\omega_s)] \ll \mathrm{Re}[k_z(\omega_i) + k_z(\omega_s)]$ the peak can be even narrower and it can be higher.

4. CONCLUSIONS

Comparing the main result Eq. (21) with the well-known formula describing scattering of light in a solid (see, for example Ref. 22), we can see that here an integration with the factor $|f(q_z)|^2$, taking into account the distributions of the incident and scattered fields in the metal, is performed over the normal component of the transferred wave vector. In the limiting case when the decay of the field is small compared with the index of refraction the integration is not performed and the transferred wave vector is merely the change in the wave vector of the light in the medium. Even in this limit, however, there appear additional factors which take into account the attenuation of the field in the metal as compared with the field in the vacuum.

The quantity which is customarily called the structure factor differs from the Fourier component of the density correlation function by factors which depend on the electronic momentum and the frequency of the light. This dependence makes possible resonance enhancement of scattering and it also makes it possible to observe the scattering at a transferred frequency less than the frequency of plasma oscillations.

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