# Self-localized electromagnetic vortex in a dense gas

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An electromagnetic vortex self-localized in a region surrounded by a plasma with a high electron temperature is analyzed. It is assumed that the plasma in the interior of the vortex is compressed by the vortex, so its pressure is well above that in the external region. The vortex energy and the plasma pressure in the interior are directly proportional to each other in this case. The entrainment of plasma electrons by the vortex is taken into account. It leads to the generation of a magnetic field. A static magnetic field tends to reduce the energy loss due to the escape of hot particles from the plasma into the surrounding gas. The parameters of the vortex and the plasma are estimated for the case in which the vortex forms in a gas whose composition and pressure are nearly atmospheric. The mechanism for the appearance of a vortex during the propagation of an electromagnetic wave through a nonlinear medium is discussed. A hypothesis is offered regarding the mechanism for the formation of a vortex during a lightning discharge.

## 1. INTRODUCTION

An electromagnetic field may be localized in space if a self-sustaining cavity or closed waveguide is formed by the field in a nonlinear medium. In other words, an electromagnetic field may undergo a self-localization in a medium with a negative dielectric constant if the dielectric constant of the medium becomes positive in the region in which the field is strong, as a result of the interaction of the field with the medium. In a hot plasma a nonlinear interaction may be caused by the pressure of the nonuniform alternating electric field on the plasma electrons (a ponderomotive force). In this case a closed low-density region in which the electromagnetic field is concentrated may form in the plasma. Outside this region the plasma density must be high enough that the electron plasma frequency is higher than the oscillation frequency of the electromagnetic field.

The case of the localization of an electromagnetic field in a hot plasma in which the self-sustaining cavity is spherical was studied by L. V. Keldysh. (His study was reported at a session of the Division of General and Applied Physics of the Academy of Sciences of the USSR in 1964. Unfortunately, that study has not been published.)

Studies of the self-focusing of electromagnetic beams[1,2] have stimulated research on a self-localized electromagnetic wave propagating through a self-sustaining closed waveguide. The structure of a self-localized electromagnetic vortex in a hot plasma was studied in Refs. 3 and 4. In the two-dimensional case, an electromagnetic wave traveling along a circle forms an annular waveguide channel. In three dimensions, the self-localized field takes the form of a ring vortex: a tubular waveguide of toroidal configuration forms, in which the wave travels along a helix. In other words, the wave vector has components along the major and minor circumferences of the torus. The charged particles in the inner part of the torus are trapped in the vortex: Their motion is limited by the potential barrier set up by the alternating nonuniform electric field. The plasma pressure in the inner part of the torus is determined by the electromagnetic field of the vortex and by the plasma pressure in the inner region.

A self-localized high-frequency field in a dense gas was studied in Ref. 5. The structure of the hot plasma which forms near the self-localized field was analyzed. It was assumed there that the plasma was at a dynamic equilibrium with the surrounding neutral gas: neutral particles diffused into the plasma volume, where they were ionized in collisions with hot electrons. The ions and electrons which formed as a result escaped from the plasma by ambipolar diffusion. The energy which the electrons expended on maintaining this dynamic picture was replenished through dissipation of the high-frequency field energy. The energy contained in this self-localized high-frequency field and the energy loss were interrelated with the electron temperature and the plasma pressure near the boundary of the self-localized field.

In the present paper we take up the case in which the plasma in the inner region is compressed by the electromagnetic vortex in such a way that its pressure is well above the plasma pressure in the outer part of the vortex. In this case the electromagnetic field energy in the vortex and the plasma pressure in the inner region are directly proportional to each other, and the energy contained in the system is essentially independent of the pressure of the surrounding gas. Under these conditions the energy of the system may be far higher than in the cases discussed in Refs. 3 and 5.

In addition, we take account of the effect of the entrainment of the electrons by the electromagnetic wave on the structure of the vortex. This effect leads to generation of a magnetic field near the vortex. This magnetic field may strongly influence the transport of energy and particles of the plasma in the outer region. In this region the plasma is bordered on one side by the high-frequency field which is heating the electrons. On the other side, it is bordered by the surrounding neutral gas. Most of the plasma energy loss results from the escape of plasma particles into the surrounding gas. The magnetic field substantially reduces the electron thermal conductivity. As a result, the electron temperature near the self-localized field may be well above the electron temperature near the boundary of the plasma with the surrounding gas. This circumstance will promote a decrease in the energy loss from the system. With increasing energy in the system, and with decreasing energy loss, the vortex lifetime increases.

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A self-purification of the plasma by escape of heavy particles[6] also tends to reduce the energy loss of the system. This self-purification can be outlined as follows: If a vortex forms in a dense gas with a small light-gas impurity, the heavy and light ions will separate in the plasma. The plasma volume will contain primarily the ions of light particles, while heavy particles will play the major role in forming the double layer at the boundary of the plasma with the surrounding gas.

In the conclusion to this paper we discuss a possible mechanism for the appearance of a self-localized vortex during the propagation of an electromagnetic wave through a nonlinear medium. We offer a hypothesis regarding the mechanism for the formation of a vortex during a lightning discharge.

## 2. STRUCTURE OF THE ELECTROMAGNETIC FIELD

In the two-dimensional case, the electric field of the wave in the cylindrical coordinate system $(r,\varphi,z)$ is $E = E_z \propto \exp(iq\varphi - i\omega t)$, decaying as $r\to 0$ and also as $r\to\infty$. (All properties are uniform along the $z$ axis.) We are interested in the situation in which the high-frequency electric field is so strong that the oscillation energy of an electron which has entered the waveguide region is well above the average thermal energy of an electron. Under these conditions the potential barrier created by the spatially nonuniform high-frequency field divides the plasma into two regions which are essentially isolated from each other: an inner region and an outer one. These regions generally differ in electron temperature and in unperturbed plasma density.

We denote by $\rho$ the point at which the field amplitude $E(r)$ reaches its maximum, and we denote by $\Delta r$ a characteristic width of the waveguide region. We denote the electron temperature in the inner plasma region $(r<\rho)$ by $T_{e0}$. We denote the plasma density, unperturbed by the high-frequency field, in this inner region by $n_{e0}$. The corresponding properties in the outer region $(r>\rho)$ are $T_{e1}$ and $n_{e1}$.

In both plasma regions, the electron energy distribution is approximately Maxwellian, with a truncated tail. The electron distribution in the inner region is essentially Maxwellian, except at electron energies close to the height of the potential barrier created by the high-frequency field. In this case the distribution function falls off more rapidly than a Maxwellian function, since the time scale for the escape of the high-energy electrons through the barrier is far shorter than the time scale of the relaxation of the distribution function to a Maxwellian function. In the outer plasma region, the distribution function becomes depleted of electrons at lower energies. The reason is that the potential barrier due to the double layer at the boundary of the plasma with the neutral gas (Ref. 7, for example) is far lower than the barrier formed by the high-frequency field in the case under consideration here.

Let us consider the case in which the oscillation frequency of the electromagnetic field is considerably higher than the effective rate of electron collisions with plasma particles $(\omega\gg\nu_e)$. In the case of a Maxwellian electron distribution we have the following expression for the plasma density:[8,9]

$$n_e=n_{e\alpha}\exp(-E^2(r)/\mathscr{E}_\alpha^2), \tag{1}$$

where

$$n_{e\alpha}=\begin{cases} n_{e0} & \text{for } r<\rho,\\ n_{e1} & \text{for } r>\rho,\end{cases}$$

$$\mathscr{E}_\alpha^2=4mT_{e\alpha}\omega^2/e^2,$$

$$T_{e\alpha}=\begin{cases} T_{e0} & \text{for } r<\rho,\\ T_{e1} & \text{for } r>\rho.\end{cases}$$

Since Eq. (1) ignores the deviation of the electron distribution from Maxwellian, this equation cannot be used near the edge of the potential well, where the plasma density is exponentially small (i.e., near the point $r=\rho$). When we recall the truncated tail of the electron energy distribution, we find a function for the plasma density which is continuous at the point $r=\rho$. We will not derive these corrections to the plasma density, since they have no significant effect on the parameters of the entity under consideration here. (The spatial distribution of the plasma density in the case of a truncated electron energy distribution is discussed in Ref. 10.)

In the two-dimensional case, the electric field of the self-localized wave is described by[3]

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dE}{dr}\right)-\frac{q^2}{r^2}E=-\frac{\omega^2}{c^2}\left[1-\frac{\omega_{L\alpha}^2}{\omega^2}\exp\left(-\frac{E^2}{\mathscr{E}_\alpha^2}\right)\right]E, \tag{2}$$

where $\omega_{L\alpha}=4\pi n_{e\alpha}e^2/m$.

The spatial distribution of the electric field has essentially no effect on the plasma density distribution in the waveguide region, in which the condition $E^2\gg\varepsilon_\alpha^2$ holds.

Multiplying the right and left sides of Eq. (2) by $dE/dr$, and integrating over $r$ from 0 to $\infty$, we find

$$n_{e1}T_{e1}-n_{e0}T_{e0}=\frac{1}{8\pi}\frac{c^2}{\omega^2}\int_0^\infty\frac{1}{r}\left[\frac{q^2}{r^2}E^2-\left(\frac{dE}{dr}\right)^2\right]dr. \tag{3}$$

The pressure drop between the inner and outer regions is balanced by the pressure and strength of the electromagnetic field. We are interested in the case in which the plasma in the inner region is compressed by the vortex, so the pressure in this region is well above the plasma pressure in the outer region: $n_{e0}T_{e0}\gg n_{e1}T_{e1}$. We assume that the following condition holds here:

$$\int_0^\infty\frac{1}{r}\left(\frac{dE}{dr}\right)^2dr\gg\int_0^\infty\frac{1}{r}\frac{q^2}{r^2}E^2\,dr. \tag{4}$$

This condition means that the component of the wave magnetic field along the wave propagation direction is considerably larger than the transverse component. In this case we find from (3)

$$n_{e0}T_{e0}=\frac{1}{8\pi}\frac{c^2}{\omega^2}\int_0^\infty\frac{1}{r}\left(\frac{dE}{dr}\right)^2dr. \tag{5}$$

Under the assumption that the characteristic width of the waveguide region is considerably smaller than the radius of curvature of the waveguide, $\Delta r\ll\rho$, we can use (5) to find the following expression for the energy of the electromagnetic field per unit length (along the $z$ axis) of the self-localized vortex:

$$W \approx 2 \cdot 10^{-18} \rho^2 n_{e0} T_{e0} \quad (\text{J}), \tag{6}$$

($W$ is expressed in joules). In estimating values here and below, we express distances in centimeters, plasma densities in particles per cubic centimeter, and temperatures in electron volts.

We wish to call attention to the following point. In analyzing the structure of the electromagnetic field we assumed that the electric field amplitude of the wave has a unique maximum at the point $r=\rho$. However, there can also be localized solutions such that there are several maxima,[3] i.e., such that the spatial distribution of the electric field amplitude in a $z=$ const cross section has the form of concentric rings. We denote the number of such rings by $s$. We then have the following expression for the width of the waveguide region:

$$\Delta r = sc/\omega. \tag{7}$$

The energy of the electromagnetic vortex given by (6) does not depend on the width of the waveguide region (or on the wave mode) for a given plasma pressure in the inner part of the vortex and for a given radius of curvature of the waveguide if the condition $\Delta r \ll \rho$ holds.

The field falls off with distance from the waveguide region in proportion to $\exp[(-\omega_{La}/c)|r-\rho|]$. The width of the plasma layer in the outer part of the vortex must be considerably greater than the effective depth to which the field penetrates into the plasma, so that we can ignore the energy leakage due to decay of the self-localized field. The integration in Eqs. (3)–(5) is intended to be carried out up to those values of $r$ in the outer plasma region at which the effect of the electromagnetic field on the plasma becomes inconsequential ($E^2 \ll \mathscr{E}_1^2$) while the interaction of the plasma with the surrounding neutral gas still has only a slight effect on the spatial distribution of the plasma properties.

In the three-dimensional case, in which an annular vortex forms, the total energy of the vortex is $2\pi R W$ if the major radius of the toroidal waveguide is significantly greater than the minor radius, $R \gg \rho$. Under these conditions the strength and pressure of the electromagnetic field can be balanced if the wave propagates along not only the minor circumference but also the major circumference of the torus.[4]

The case $k \ll \omega/c$, where $k$ is the component of the wave vector along the major radius of the torus, was discussed in Ref. 5. In this case the plasma formation is oval or spherical. The electromagnetic wave propagates around the oval and then travels along the axis of the formation, creating a cylindrical waveguide.

## 3. PLASMA IN THE INNER REGION OF THE VORTEX

We assume that the heating of the plasma by the high-frequency field occurs primarily through collisions of electrons with plasma particles. We have the following expression for the energy dissipated per unit time by the electromagnetic wave:[11]

$$Q_E = \frac{e^2}{m\omega^2} \int dV \, \nu_e n_e E^2. \tag{8}$$

We assume that the plasma in the inner region contains mostly hydrogen ions. The effective collision rate of the electrons is then given by

$$\nu_e \approx 5{,}9 \cdot 10^{-5} \, T_e^{-3/2} n_e. \tag{9}$$

Using (1) and (9), we find from (8) the power which the plasma in the inner region acquires over a unit length of the plasma cylinder:

$$Q_{E0} \approx 5{,}4 \cdot 10^{-17} \rho T_{e0}^{-1/2} \, n_{e0}^{3/2} \quad (\text{W}). \tag{10}$$

The electron energy loss in this region stems from an escape of electrons through the high-frequency barrier and from bremsstrahlung. The energy loss due to collisions of electrons with ions is unimportant under these conditions and can be ignored in the energy balance equation.

Let us find the electron flux coming out of the inner region. Since the electrons have a fairly high temperature, and the mean free path of the electrons with respect to electron-electron collisions is much longer than the spatial dimensions of the plasma, essentially all the electrons which acquire an energy equal to the height of the potential barrier escape from the inner region of the plasma. Under the assumption that the electron diffusion in velocity space stems primarily from electron-electron collisions, we have

$$j_{e0} = \pi\rho^2 \, 4\pi v^{*2} D_v(v^*) \frac{\partial f_{e0}}{\partial v}\bigg|_{v=v^*}, \tag{11}$$

where

$$v^* = (2\varepsilon^*/m)^{1/2}, \qquad \varepsilon^* = e^2 E_{max}^2 / 4m\omega^2,$$
$$D_v(v^*) = 4\pi e^4 n_{e0} T_{e0} \ln \Lambda / m^3 v^{*3},$$

where $\ln\Lambda$ is the Coulomb logarithm.

Using (11), we find the following expression for the energy flux of electrons out of the inner region:

$$Q_{e0} = j_{e0}\varepsilon^* \approx 3 \cdot 10^{-24} \frac{\rho^3 n_{e0}^3}{s\omega T_{e0}^{1/2}} \exp\left(-\frac{\rho n_{e0}}{10 s\omega}\right) \quad (\text{W}). \tag{12}$$

The electron energy loss to bremsstrahlung is given by (Ref. 12, for example)

$$Q_{\text{rad}} \approx 5 \cdot 10^{-32} \rho^2 n_{e0}^2 T_{e0}^{1/2} \quad (\text{W}). \tag{13}$$

Using (10), (12), and (13), we can write an energy balance equation for the plasma electrons in the inner region of the vortex, $Q_{E0} = Q_{e0} + Q_B$. We find

$$5 \cdot 10^{-8} \frac{\rho^2 n_{e0}^{3/2}}{s\omega} \exp\left(-\frac{\rho n_{e0}}{10 s\omega}\right) + 10^{-15} \rho n_{e0}^{1/2} T_{e0} = 1. \tag{14}$$

The particle-number balance equation for the plasma in the inner region of the vortex can be written

$$j_{e0} = j_0, \tag{15}$$

where $j_0$ is the flux of neutral particles going into the inner region.

We denote by $r_1$ the boundary between the waveguide region and the outer plasma, and we denote by $r_2$ the boundary between the plasma and the neutral gas. We recall that the width of the transition layer between the waveguide region and the plasma and also the thickness of the double layer at the boundary of the plasma with the surrounding gas are significantly smaller than the width of the plasma in the outer region, $\delta r = r_2 - r_1$. We consider the case $\delta r \ll \rho$. For the density of neutrals at the point $r = r_1$ we have[7]

$$n_1=n_2 \exp\left(-\frac{1}{v_r}\int_{r_1}^{r_2} v_e\sigma_i(T_e)\,n_e\,dr\right), \quad (16)$$

where $v_r$ is the velocity component of a neutral particle toward the axis of the plasma cylinder, and $\sigma_i(T_e)$ is the cross section for electron-impact ionization. Expression (16) was derived for the case in which the ionization cross section is large in comparison with the cross section for the collision of a neutral particle with an ion. In the opposite case, the neutral particles penetrate into the central plasma region by diffusion.[5]

Since nearly all the particles which enter the inner region are ionized in collisions with electrons, the flux of neutral particles going into the inner region is

$$j_0=2\pi\rho v_r n_1. \quad (17)$$

We turn now to the generation of a static magnetic field by the electromagnetic vortex. As a result of the interaction of the electromagnetic wave with the plasma, the plasma electrons obviously acquire momentum as well as energy. The entrainment of the electrons by the electromagnetic wave is the reason for the generation of a static magnetic field.[13,14] Under the assumption that the electromagnetic wave is traveling along a helix in a self-sustaining tubular waveguide, we find the electric current directed along the axis of the plasma cylinder in the inner plasma region. For the momentum acquired by electrons from the wave in the inner region we find

$$F_{Ez}=\gamma Q_{E0}/c, \quad (18)$$

where $\gamma = k\rho/(q^2 + k^2\rho^2)^{1/2}$, and $k$ is the component of the wave vector along the $z$ axis.

The electrons lose momentum primarily as a result of friction with ions. We write the friction force as

$$F_{fr}=mI\nu_e, \quad (19)$$

where $I = \pi\rho^2 n_{e0} u$ is the current in the inner region of the vortex, and $u$ is the electron flux velocity along the $z$ axis.

Working from the momentum balance equation $F_{Ez} = F_{mp}$, along with (9) and (10), we find the following expression for the magnitude of the electric current:

$$I\approx 3\cdot 10^{-8}\,\gamma\rho n_{e0}^{1/2} T_{e0} \ (\text{A}). \quad (20)$$

The static magnetic field $H = H_\varphi$ near the point $r = \rho$ is given by

$$H=I/2\pi\rho\approx 5\cdot 10^{-9}\,\gamma n_{e0}^{1/2} T_{e0} \ (\text{Oe}). \quad (21)$$

We are interested in the case in which the static magnetic field acts primarily on the plasma structure in the outer region. We assume that the effect of the magnetic field on the magnitude of the ponderomotive forces is negligible, since the oscillation frequency of the wave field is much higher than the electron Larmor frequency: $\omega \gg \omega_{He}$. We ignore the effect of the magnetic field on the motion of electrons in the inner region, under the assumption that the electron temperature in this region is so high that the electron Larmor radius is larger than the characteristic dimension of the plasma.

## 4. PLASMA STRUCTURE IN THE OUTER REGION OF THE VORTEX

A plasma formation in the steady state, at dynamic equilibrium with the surrounding gas, was studied in Refs. 15 and 7 for the case without a static magnetic field. The problem was studied there for the case in which the ions escape from the plasma under conditions of ambipolar diffusion[15] and under "transit" conditions, such that the ion mean free path with respect to collisions with neutral particles is large in comparison with the length scale of the plasma.[7] In each case, the plasma density distribution in the interior of the formation is almost spatially uniform. The plasma density decays in a narrow boundary layer. The density of neutral particles increases with distance from the central part of the plasma in such a way that the sum of the partial pressures of the plasma components remains constant, equal to the gas pressure at infinity. Under transit conditions, the characteristic thickness of the boundary region in which the plasma density is very nonuniform is given by[7]

$$l\approx\frac{v}{v_e\sigma_i n_e}, \quad (22)$$

where $v_e$, $\sigma_i$, and $n_e$ are respectively the electron velocity, the cross section for the ionization of a neutral particle, and the plasma density near the boundary layer. The quantity $v$ is the velocity of a neutral particle. We will refer to this boundary region of the plasma as the "$l$ layer." In the $l$ layer, the plasma density falls off progressively more steeply with increasing distance from the center of the plasma formation. In the boundary region of the $l$ layer, where the length scale of the plasma variations becomes comparable to the Debye length, the quasineutrality of the plasma is disrupted. This region is the "double layer" in Langmuir's terminology.[16] The electric potential drop occurs primarily across this double layer. The height of the potential barrier for electrons is several times the electron temperature.

Let us consider the case in which the electron Larmor radius is much larger than the depth to which the high-frequency field penetrates into the plasma and also much larger than the thickness of the $l$ layer at the boundary of the plasma with the neutral gas. The length scale of the plasma in the outer region is then larger than the electron Larmor radius; i.e., we have

$$c/\omega_{Li}, \ l\ll r_{He}\ll\delta r. \quad (23)$$

Under these conditions the static magnetic field affects only transport processes in the interior of the plasma.

The $l$ layer thus has the same structure in this case as in the case without a magnetic field. Under the assumption of transit conditions, we can draw the following picture of the physical phenomena at the boundary of the plasma with the neutral gas. The electric field which results from charge separation accelerates the ions, tending to push them out of the plasma into the surrounding gas. The electrons, in contrast, are retarded by this field: Only the high-energy electrons which have surmounted the potential barrier break out of the plasma. The electron partial pressure transforms into a convective ion pressure, so outside the double layer we have an ion flux pressure which is roughly equal to the gas pressure at infinity.

The plasma self-purification by escape of heavy particles from the gas under conditions such that a vortex has formed in a mixture of a heavy gas and a light gas occurs primarily in the $l$ layer, where the electric field due to charge separation is strong. The heavy particles, whose ionization potential and thermal velocity are both lower than those of the light neutral particles, are ionized more rapidly. In other words, the mean free path of the heavy neutral particles in the plasma is shorter than that of the light particles. Consequently, the heavy particles, being ionized in the region with a strong electric field, are ejected from the plasma.[6] Outside the double layer the plasma flux is slowed down by friction between ions and neutral particles. The ion pressure falls off, and the neutral gas pressure rises. Those electrons which have moved outside the double layer lose most of their kinetic energy in the process of surmounting the potential barrier. They enter a dense neutral gas, where a cold recombining plasma forms. We will not go into the structure of this cold plasma here. In our analysis, the point $r = r_2$ is the boundary of the plasma containing the hot electrons.

The pressure of the hot electrons near the $l$ layer is equal to the gas pressure at infinity. We can write

$$n_{e2}T_{e2}\approx 6\cdot 10^{17}\,p, \tag{24}$$

where $p$ is the gas pressure at infinity, in atmospheres.

The energy loss due to the escape of plasma particles into the surrounding gas is given by[7]

$$Q_2\approx 8\cdot 10^5 \rho p (T_{e2}/M)^{1/2} \quad (\mathrm{W}), \tag{25}$$

where $M$ is the mass of a neutral particle, in atomic units. Under conditions such that the vortex has formed in a heavy gas with a small admixture of a light gas, we should use the mass of the heavy particle in (25) if the flux of light ions at the plasma-gas interface is small in comparison with the flux of heavy ions. Under the steady-state conditions which we are assuming here, the oppositely directly fluxes of ions and neutral particles are identical in composition and equal in magnitude at each point in the system. Consequently, the composition of the neutral gas in this case must be such that the flux of heavy neutral particles out of the gas into the hot plasma is greater than the flux of light particles.

As we mentioned earlier, the neutral gas pressure at the boundary with the hot plasma is lower than the gas pressure at infinity, because the ion flux pushes the neutral gas away from the plasma boundary. We have the following expression for the neutral gas density at the point $r = r_2$ (Ref. 7):

$$n_2\approx 0{,}3(T/T_{e2})^{1/2} n_\infty, \tag{26}$$

where $T$ is the temperature of the neutral gas. Expression (26) was derived for the case of a homogeneous gas, but it can also be used under conditions such that the gas composition is not homogeneous, provided that the impurity content is low enough that the convective pressure of impurity ions can be ignored. In this case we can assume that the density of the neutral impurity particles at the boundary of the hot plasma is equal to the density of these particles at infinity, provided that the cross section for the collisions of neutral particles with ions of the impurity gas is much lower than that for collisions with ions of the same gas (a resonant charge exchange).

We now consider the plasma in the region between the waveguide and the $l$ layer, i.e., in the region $r_1 < r < r_2$. In the absence of a magnetic field, the electron temperature and the plasma density in this region are essentially uniform over space.[7] The reason for this situation lies in the high thermal conductivity of the electrons and the circumstance that only a negligible number of neutral particles enter this region. There is essentially no plasma flux, since ionization is negligible. We can write

$$j_e = \frac{d}{dr}(D_a n_e) = 0\,. \tag{27}$$

When there is a transverse magnetic field, the plasma diffusion coefficient is given by (Ref. 17, for example)

$$D_a = \nu_e T_e / m_e \omega_{He}^2,$$

where $\omega_{\mathrm{He}}$ is the electron Larmor frequency. Assuming $\nu_e \sim n_e T_e^{-3/2}$, and working from Eq. (27) for the region $r_1 < r < r_2$ under consideration, we find

$$n_e^4/T_e = \mathrm{const}. \tag{28}$$

The electron energy loss in the interior of the plasma plays an unimportant role in the energy balance equation. Most of the energy loss is due to the escape of electrons from the plasma into the surrounding gas and is described by (25). Using (8), we find the power acquired by the plasma from the high-frequency field in the outer region of the vortex:

$$Q_{E1}\approx 5{,}4\cdot 10^{-17}\,\rho T_{e1}^{-1/2}\, n_{e1}^{3/2} \quad (\mathrm{W}). \tag{29}$$

The energy balance equation for the electrons in this region ($Q_{E1} = Q_2$) can be written

$$T_{e1}^{1/2}\, T_{e2}^{1/2} p \approx 7\cdot 10^{-23} n_{e1}^{3/2} M^{1/2}. \tag{30}$$

Energy transport between the plasma boundaries, from $r_1$ to $r_2$, results from the thermal conductivity of the plasma. Ignoring the energy loss in the interior of the plasma, we write

$$Q_{E1} = -\varkappa_e \frac{d}{dr} T_e, \tag{31}$$

where $\varkappa_e = 4.66 n_e T_e \nu_e / m\omega_{\mathrm{He}}^2$ is the electron thermal conductivity in a transverse magnetic field.[17]

Using (29) and (31), we find the following approximate relation:

$$\rho\delta r H^2 \approx 2\cdot 10^{-6} n_{e1}^{1/2} T_{e1}. \tag{32}$$

## 5. ESTIMATE OF THE PROPERTIES OF THE VORTEX AND THE PLASMA

We can work from the relations derived above to determine the properties of the self-localized high-frequency field in the plasma as a function of the length scale of the vortex, the wave frequency, the wave mode, and the plasma pressure in the inner region, provided that the pressure, temperature, and composition of the gas far from the vortex are all given. We assume that the neutral gas consists of nitrogen molecules with a small hydrogen admixture. We assume that the gas pressure at infinity is close to atmospheric. Working from Eqs. (14), (15), (21), (24), (28), (30), and (32), we find the following results. For the outer plasma region we find

$$n_{e1} \approx 2\cdot 10^{15}\left(\frac{\rho\gamma p_{e0} T^{1/4}}{s^{1/2}}\right)^{24/43}, \tag{33}$$

$$T_{e1} \approx 10^{-47}\, n_{e1}^{19/6}, \tag{34}$$

$$n_{e2} \approx 10^{13}\, n_{e1}^{1/6}, \tag{35}$$

$$T_{e2} \approx 7\cdot 10^{4}\, n_{e1}^{-1/6}. \tag{36}$$

For the plasma density in the inner region we have the relation

$$n_{e0} \ln^{-1}(5\cdot 10^{-13}\rho^2 n_{e0}^{3/2}/s n_{e1}^{1/2}) = 5\cdot 10^{5}\, s n_{e1}^{1/2}/\rho. \tag{37}$$

The strength of the static magnetic field near the waveguide region ($r \approx \rho$) is

$$H \approx 3\cdot 10^{9}\, \gamma n_{e0}^{-1/2} p_{e0}. \tag{38}$$

The length scale of the outer region of the plasma can be estimated from

$$\delta r \approx 2\cdot 10^{-72}\, n_{e1}^{11/3}\, n_{e0}/\rho\gamma^2 p_{e0}^2. \tag{39}$$

Results (33)–(39) were derived under the assumption that the wave frequency is close to the plasma frequency of the plasma at the point $r = r_1$:

$$\omega \approx 5{,}6\cdot 10^{4}\, n_{e1}^{1/2}. \tag{40}$$

If we set the plasma pressure in the inner region equal to $3\cdot 10^3$ atm, take the spatial size of the vortex to be $\rho \approx 10$ cm, take the width of the waveguide region to be $\Delta r \approx 2$ cm (in which case we have $q \approx \Delta r\omega/c$), and set $T \approx 0.1$ eV and $\gamma \approx 10^{-2}$, we find the following results from (33)–(40):

$$n_{e1} \approx 2\cdot 10^{16},\quad T_{e1} \approx 3\cdot 10^{4},\quad n_{e2} \approx 6\cdot 10^{15},$$

$$T_{e2} \approx 10^{2},\quad n_{e0} \approx 5\cdot 10^{16},\quad T_{e0} \approx 10^{5},\quad H \approx 3\cdot 10^{3},\quad \delta r \approx 1.$$

The vortex energy found from (6) under these conditions is $W \approx 10^6$ J. The vortex lifetime can be found by dividing the vortex energy by the energy lost by the system per unit time. We find $\tau \approx 0.2$ s.

As we mentioned earlier [see (6)], the energy in the vortex is proportional to the plasma pressure in the inner region. In the case of higher plasma pressures in this inner region, the vortex lifetime will obviously be relatively long.

## 6. MECHANISM FOR THE OCCURRENCE OF A VORTEX

Talanov[2] studied the self-focusing of a plane beam with an electric field

$$E = E_z(x)\exp(iky - i\omega t) \tag{41}$$

He found a solution for the function $E_z(x)$ which falls off in each direction away from the waveguide axis, $x = 0$.

If the transverse dimensions of the beam are smaller than the wavelength, a rectilinear beam trajectory is unstable: The field of the wave may deform the self-sustaining waveguide and curve the beam trajectory.

An instability occurs if forces which tend to increase this bending arise at a certain curvature of the waveguide channel. Let us assume that the beam trajectory is curved at some point. Under the assumption that the radius of curvature $\rho$ is much larger than the effective width of the waveguide, we find the following expression for the time-averaged force acting along $\rho$ due to the wave field:

$$F_\rho \propto \frac{1}{\rho}\int dx\left[k^2 E_z^2 - \left(\frac{dE_z}{dx}\right)^2\right].$$

We see that the beam is unstable with respect to curvature if the wavelength is larger than the effective width of the waveguide. If the beam intersects itself, and the waveguide channel forms a loop, an electromagnetic vortex will arise in the plasma. The loop will then contract, raising the plasma pressure in the inner region of the vortex. This process will proceed until the plasma pressure and the field balance each other [see (3)].

Volkov[8] examined the case in which a one-dimensional cavity forms as the result of the effect of the high-frequency electromagnetic field on the plasma. The electric field in this case is given by (41) with $k = 0$. In other words, it is assumed that the wavelength is infinite. It follows that a one-dimensional, self-sustaining resonator of this sort will be unstable with respect to bending.

Abakarov *et al.*[18] took up the self-focusing of a three-dimensional cylindrical beam in which the electric field lines form concentric circles around the axis of a self-sustaining tubular waveguide. In a cylindrical coordinate system, the field of this beam is

$$E = E_\varphi(r)\exp(ikz - i\omega t),$$

where $E_\varphi(r) \to 0$ as $r \to \infty$.

As in the two-dimensional beam, the force due to the field may deform the waveguide and give rise to a vortex if the wavelength is large in comparison with the transverse dimensions of the beam. An annular vortex forms in this case.

We see that conditions which favor the appearance of a vortex may arise during the self-contraction of a beam or when a beam breaks up into filaments in a nonlinear medium.

We conclude with a few words about the possible formation of an electromagnetic vortex during a lightning discharge. We know that the plasma channel which arises in a lightning discharge (we are thinking of ordinary streak lightning) may be subject to compression because of the pinch effect. The necks formed in the course of this pinch effect may give rise to a spatial structure in the plasma. In particular, when bead lightning, with a periodic structure, arises, the necks accompanying the pinch effect play a leading role. If the flux of high-energy electrons present in the discharge generates an electromagnetic wave, the self-contraction of this wave or the compression of the waveguide channel as the result of the pinch effect may give rise to an electromagnetic vortex in the atmosphere.


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