The structure and elastic moduli of flux-line lattices in anisotropic superconductors

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The elastic moduli of flux-line lattices in anisotropic superconductors are investigated. In addition to the well-known bulk, shear, and tilt moduli we observe moduli that relate deformations in the basal plane of the lattice to vortex tilt. These moduli vanish when the superconductor is magnetized along the axis of anisotropy. The vortex structure continuum realized in this case has identical bulk and shear moduli and different tilt moduli. A hexagonal flux-line lattice is realized in superconductors with "easy axis" anisotropy when a weak magnetic field is applied. When the field **H** is applied in the *ab*-plane of the crystal the lattice becomes an oblique lattice with orthorhombic symmetry. This results in a sharp growth of its elastic moduli and the induction in the sample. Vortex chain structures are the only stable structures in "easy plane" superconductors. The elastic moduli characterizing the rigidity of an isolated chain are exponentially large compared to the moduli describing interchain interaction. The tilt moduli may reverse their sign when **H** is oriented near the axis of anisotropy *c* for strongly anisotropic superconductors. In this case, the vortex structure and all related elastic moduli undergo a discontinuous irreversible change.

1. INTRODUCTION

Since their discovery high-T_c superconductors have become the principal focus of investigations of the superconducting state. The most evident feature of the new materials is the strong anisotropy of the superconducting states. Its characteristic manifestations include anisotropy of the critical fields and current as well as asymmetry of the flux-line lattices (FLL).

There are also a number of features in the magnetization of anisotropic superconductors. In many respects such magnetization is analogous to the magnetization of a ferromagnet whose principal processes in an external magnetic field **H** include displacement of the domain boundaries and rotation of the spontaneous magnetization vectors. Such processes also determine the reversible magnetization curve **B**(**H**) in anisotropic superconductors. The magnetic flux in fields **H** exceeding the first critical field penetrates the superconductor as a FLL, with the vortex concentration rising with increasing H. A change in the induction B in the sample (vortex concentration) is analogous to displacement of the domain boundaries in a ferromagnet and is the only process in isotropic superconductors that determines the form of the curve **B**(**H**).

Generally the orientation of isolated vortices in anisotropic superconductors does not coincide with either the crystal axes or the direction of field **H**. Reference 1 was the first to identify the rotation of magnetic vortices in an increasing field **H**. For $H = 0$ the fluctuation vortex lies along the axis of symmetry with the lowest effective electron mass $\hat{\mu}$ (Ref. 2). In the threshold field H_{c1} such a vortex is oriented in the direction $\hat{\mu}^{-1}\mathbf{H}$ (Ref. 3). The vortices rearrange into a FLL and rotate towards **H** with increasing field magnitude. Only when **H** is oriented along one of the crystal axes do the equilibrium direction of the vortices in the superconductor coincide with **H**. In the case of an arbitrary field orientation, the vortices asymptotically rotate toward the vector **H** yet do not coincide with this direction with any finite H (Ref. 3). This conclusion is entirely consistent with the analogous result for ferromagnets.

The FLL elastic moduli represent an important characteristic of vortex structures. An array of elastic moduli can be used to describe the fluctuation part of superconductor energy attributable to FLL deviation from equilibrium and to determine the stability of an equilibrium FLL. The elastic moduli can also be employed to express the critical current and the anomalies on the I-V characteristic of the superconductors.[4] The dimensions of the short-range order domains in FLL's are also determined by the elastic moduli.[5]

The structure and elastic moduli of FLL's have been exhaustively analyzed in isotropic superconductors.[6–8] Equilibrium FLL's[9,3,10] and magnetization curves[11,12] for anisotropic crystals have only been described for layered superconductors (these also include the new high-T_c superconductors). The elastic moduli of FLL's in superconductors of arbitrary anisotropy have not yet been investigated; Ref. 13 only reports the shear moduli of dense FLL's.

This paper is devoted to a description of equilibrium vortex structures and their elastic moduli in anisotropic superconductors. Section 2 contains the equilibrium equations for the FLL parameters together with expressions for the elastic moduli and descriptions of the FLL structure in isotropic superconductors. Superconductor magnetization along one of the axes of anisotropy is considered in Sec. 3. Section 4 contains a description of sparse FLL's and their elastic moduli in uniaxial "easy axis" superconductors, while Sec. 5 provides the same information for "easy plane" superconductors. The elastic moduli of dense FLL's are discussed in the conclusion.

2. EQUILIBRIUM EQUATIONS AND ELASTIC MODULI

1. The thermodynamics of a system of rectilinear vortices uniformly distributed in space is determined by the de-

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pendence of the density of the Gibbs potential on $\mathbf{H}$ (see, for example, Refs. 1, 3)

$$G=n_L\frac{\Phi_0}{4\pi}\left[H_{c1}-\mathbf{H}\boldsymbol{\nu}+\frac{1}{2}\int d^3q\,\delta(\mathbf{q}\boldsymbol{\nu})Sh\right]. \qquad (1)$$

The Gibbs potential is written in terms of the longitudinal (parallel to the total magnetic flux $\boldsymbol{\nu}$) Fourier-component of the vortex field[2]

$$h(\mathbf{q},\boldsymbol{\nu})=\frac{\Phi_0}{(2\pi\lambda)^2}\frac{1+q^2\boldsymbol{\nu}\mu\boldsymbol{\nu}}{(1+q^2\boldsymbol{\nu}\hat{\mu}\boldsymbol{\nu})(1+[\mathbf{q}\boldsymbol{\nu}]\hat{\mu}[\mathbf{q}\boldsymbol{\nu}])-q^2(\boldsymbol{\nu}\hat{\mu}[\mathbf{q}\boldsymbol{\nu}])^2} \qquad (2)$$

and the structure factor of the FLL

$$S(\mathbf{q})=\sum_{\mathbf{x}}\exp\left(\frac{i\mathbf{q}\mathbf{x}}{\lambda}\right)-1. \qquad (3)$$

The anisotropy of the superconducting properties is expressed by the electron effective mass tensor $m\hat{\mu}$ ($\det\hat{\mu}=1$). The following notation is used in Eqs. (1)-(3): $H_{c1}(\boldsymbol{\nu})=H_{c1}^{0}(\boldsymbol{\nu}\hat{\mu}\boldsymbol{\nu})^{1/2}$ is the first critical field in an anisotropic superconductor, $H_{c1}^{0}=(\Phi_0/4\pi\lambda^2)\ln(\lambda/\xi)$ is the first critical field in an isotropic material; Φ_0 is the flux quantum; and $\delta(q)$ is the delta-function. Let us assume that the vortices are uniformly distributed in space, are parallel, and oriented in the direction of $\boldsymbol{\nu}$. This structure is described by the two-dimensional vectors $\{\mathbf{x}\}$ perpendicular to $\boldsymbol{\nu}$ and forming a regular two-dimensional lattice. It is specified by the translation vectors $\mathbf{A}_1$ and $\mathbf{A}_2$ (see Fig. 1). These vectors are used to express the vortex concentration $n_L=(\boldsymbol{\nu}[\mathbf{A}_1\mathbf{A}_2])^{-1}$ in the lattice.

Therefore a regular system of parallel vortices is represented by six independent parameters: the unit vector $\boldsymbol{\nu}$, the length of the translation vectors A_1 and A_2, and the angles Ξ and Γ determining the orientation of the translation vectors in the basal plane (Fig. 1). Their equilibrium values are determined from the minimum of the potential (1).

2. Let us write the Gibbs potential (1) as an expansion in the small deviations of the vortices from equilibrium. We represent the vortex displacement from the lattice sites by the distortion tensor $u_{ij}=\partial u_i/\partial x_j$. The indices $i,j=1,2$ label the Cartesian coordinates of the vectors in the basal plane perpendicular to $\boldsymbol{\nu}$. The deviation $\delta\boldsymbol{\nu}$ of the vortices from the equilibrium direction $\boldsymbol{\nu}$ is determined by the two-dimensional vector $\boldsymbol{\omega}=[\boldsymbol{\nu}\delta\boldsymbol{\nu}]$. We then obtain

$$G=G_0+\mathbf{K}\boldsymbol{\omega}+\sigma_{ij}u_{ij}+{}^1/_2T_{ij}\omega_i\omega_j+{}^1/_2C_{ijkl}u_{ij}u_{kl}+D_{ijk}\omega_iu_{jk}. \qquad (4)$$

Here the torque is equal to

$$\mathbf{K}=n_L\frac{\Phi_0}{4\pi}\left\{H_{c1}\frac{[\boldsymbol{\nu},\hat{\mu}\boldsymbol{\nu}]}{\boldsymbol{\nu}\hat{\mu}\boldsymbol{\nu}}+[\mathbf{H}\boldsymbol{\nu}]+\frac{1}{2}\int d^3q\,\delta(\mathbf{q}\boldsymbol{\nu})S\left[\boldsymbol{\nu}\frac{\partial}{\partial\boldsymbol{\nu}}\right]h\right\}, \qquad (5)$$

and the elastic stress tensor takes the form

$$\sigma_{ij}=n_L\frac{\Phi_0}{4\pi}\left\{\left[\mathbf{H}\boldsymbol{\nu}-H_{c1}-\frac{1}{2}\int d^3q\,\delta(\mathbf{q}\boldsymbol{\nu})Sh\right]\delta_{ij}+\frac{1}{2}\int d^3q\,\delta(\mathbf{q}\boldsymbol{\nu})hq_i\frac{\partial S}{\partial q_j}\right\}. \qquad (6)$$

Summation is implied over all repeating vector indices.

The equilibrium configuration of vortices is determined by the vanishing of the torque and the stress tensor:

$$\mathbf{K}=0,\quad \hat{\sigma}=0. \qquad (7)$$

We examine the equilibrium equations in greater detail below for various magnetic field orientations with respect to the crystal axes.

We provide the second derivatives of the potential G in Eq. (4) in equilibrium. The tensor components of the tilt moduli take the form

$$T_{ij}=n_L\frac{\Phi_0H_{c1}}{4\pi}\frac{\mu_{ij}^{-1}}{(\boldsymbol{\nu}\hat{\mu}\boldsymbol{\nu})^2}+n_L\frac{\Phi_0}{4\pi}\int d^3q\,\delta(\mathbf{q}\boldsymbol{\nu})S\left\{\delta_{ij}\left(2h-\boldsymbol{\nu}\frac{\partial h}{\partial\boldsymbol{\nu}}\right)+\left[\boldsymbol{\nu}\frac{\partial}{\partial\boldsymbol{\nu}}\right]_i\left[\boldsymbol{\nu}\frac{\partial}{\partial\boldsymbol{\nu}}\right]_jh-\left(\boldsymbol{\nu}\frac{\partial}{\partial\mathbf{q}}\right)\left([\boldsymbol{\nu}\mathbf{q}]_i\left[\boldsymbol{\nu}\frac{\partial}{\partial\boldsymbol{\nu}}\right]_jh+[\boldsymbol{\nu}\mathbf{q}]_j\left[\boldsymbol{\nu}\frac{\partial}{\partial\boldsymbol{\nu}}\right]_ih\right)+[\boldsymbol{\nu}\mathbf{q}]_i[\boldsymbol{\nu}\mathbf{q}]_j\left(\boldsymbol{\nu}\frac{\partial}{\partial\mathbf{q}}\right)^2h\right\}. \qquad (8)$$

The elastic shear and bulk moduli are equal to

$$C_{ijkl}=n_L\frac{\Phi_0}{4\pi}\int d^3q\,\delta(\mathbf{q}\boldsymbol{\nu})h\left[q_iq_k\frac{\partial^2S}{\partial q_j\partial q_l}+(\delta_{il}\delta_{jk}-\delta_{ij}\delta_{kl})q_i\frac{\partial S}{\partial q_i}\right]. \qquad (9)$$

The tilt-compression and tilt-shear moduli are cast as

$$D_{ijk}=n_L\frac{\Phi_0}{4\pi}\int d^3q\,\delta(\mathbf{q}\boldsymbol{\nu})q_j\frac{\partial S}{\partial q_k}\left[\boldsymbol{\nu}\frac{\partial}{\partial\boldsymbol{\nu}}\right]_ih. \qquad (10)$$

Labush[7] was the first to expand the FLL elastic energy for an isotropic crystal for the case of spatially homogeneous strains and Brandt[8] was the first to carry out this procedure in the spatially inhomogeneous case. Note that the last three terms in potential (4) can be recast equivalently as

$$T_{ij}\omega_i\omega_j+C_{ijkl}u_{ij}u_{kl}+2D_{ijk}\omega_iu_{jk}\rightarrow C_{i\alpha j\beta}u_{i\alpha}u_{j\beta}.$$

In this case the Greek subscripts α and β run over values of 1, 2, 3 so we have $u_{23}=\omega_1$, while $u_{13}=\omega_2$.

3. One anomaly of an anisotropic crystal is the appearance of "tilt-compression (shear)" terms in the expansion of Gibbs potential (4). These terms are proportional to the tensor of rank three $\hat{D}$. The FLL elastic strains in the basal plane u_{ij} are related to vortex tilt $\boldsymbol{\omega}$ by the dependence of the

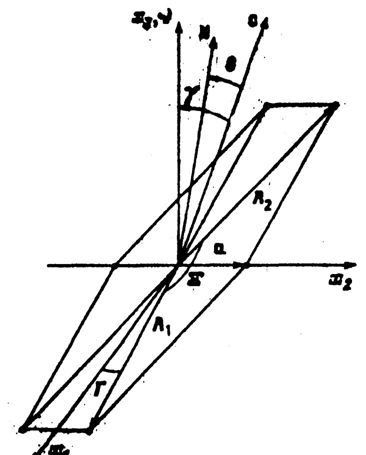

FIG. 1. Vortex lattice cell in a superconductor in external field $\mathbf{H}$: $\mathbf{A}_1$ and $\mathbf{A}_2$ are the translation unit vectors in the basal plane x_1, x_2 perpendicular to the vortex direction ν; $\mathbf{c}$ is the axis of anisotropy.

magnetic field configuration of an individual vortex on its orientation $\mathbf{v}$. The elastic moduli $\hat{D}\equiv 0$ in the isotropic case. In an anisotropic crystal the torque

$$\frac{\partial G}{\partial \omega_i}=T_{ij}\omega_j+D_{ijk}u_{jk}$$

is manifested not only upon vortex tilting but also upon strains in the basal plane. Elastic stresses in the FLL

$$\frac{\partial G}{\partial u_{kl}}=C_{ijkl}u_{ij}+D_{ikl}\omega_i$$

likewise arise not only from FLL strain (as in the isotropic case) but also due to vortex tilt.

The last three terms in relation (4) represent the fluctuation energy of the Gibbs potential caused by FLL deviation from equilibrium. Arbitrary fluctuations are conveniently described by the six-dimensional vector $\zeta=\{u_{11},u_{12},u_{21},u_{22},\omega_1,\omega_2\}$. The fluctuation part of the energy is then cast as $\delta G=\frac{1}{2}\zeta\hat{\chi}\zeta$, where the 6×6 matrix is given in block form:

$$\hat{\chi}=\left(\begin{array}{c|c} C & D \\ \hline D^+ & T\end{array}\right)=\begin{pmatrix} C_{1111} & C_{1112} & C_{1121} & C_{1122} & D_{111} & D_{211} \\ C_{1211} & C_{1212} & C_{1221} & C_{1222} & D_{112} & D_{212} \\ C_{2111} & C_{2112} & C_{2121} & C_{2122} & D_{121} & D_{221} \\ C_{2211} & C_{2212} & C_{2221} & C_{2222} & D_{122} & D_{222} \\ D_{111} & D_{112} & D_{121} & D_{122} & T_{11} & T_{12} \\ D_{211} & D_{212} & D_{221} & D_{222} & T_{21} & T_{22} \end{pmatrix} \tag{11}$$

4. The tensor $\hat{T}$ is a symmetrical tensor in the general case of a biaxial superconductor and has three independent components; the tensor $\hat{C}$ is symmetrical under permutation of the first and last pair of indices and has ten independent elements; all eight components of the tensor $\hat{D}$ are independent.

The orthorhombic symmetry of the FLL will reduce the number of nonzero components of the matrix $\hat{\chi}$ in a uniaxial superconductor. The presence of this symmetry can be directly determined from the general equations $\sigma_{12}=\sigma_{21}=0$ in Eq. (7). Indeed, for uniaxial superconductors it follows from $K_2=0$ in Eq. (7) that the anisotropy axis $\mathbf{v}$ and the magnetic field $\mathbf{H}$ always lie in the same plane. By placing the axis of the Cartesian coordinate system in this plane we see that the Fourier component of the vortex field

$$h(\mathbf{q})=\frac{\Phi_0}{(2\pi\lambda)^2}\frac{1+q^2\mu_{33}}{(1+q^2\mu_a)(1+q_1^2\mu_c+q_2^2\mu_{33})} \tag{12}$$

is an even function of q_1 and q_2 (here $\mu_c=\mu_a^{-2}$ and μ_a are the principal values of the tensor $\hat{\mu}$ parallel and perpendicular to the axis of anisotropy $\mathbf{c}$).

The structure factor $S(\mathbf{q})$ from Eq. (3) must be even with respect to q_1 and q_2 for the nondiagonal components of the tensor $\hat{\sigma}$ in Eq. (7) to vanish. An FLL symmetrical under the substitution $x_1\to -x_1$, $x_2\to -x_2$ will have such a structure factor. This means that the unit cell is a rhombus whose diagonals lie along the x_1 and x_2 axes, perpendicular and parallel to the projection of the axis of symmetry $\mathbf{c}$ in the basal plane (see Fig. 1).

This additional symmetry element causes the nondiagonal components of $\hat{T}$ and the components of $\hat{C}$ in which the indices 1 and 2 appear an odd number of times to vanish identically. This leaves six linearly independent components of $\hat{C}$.

The components of tensor $\hat{D}$ do not follow the symmetry of the field h given by (12), but rather the symmetry of its derivatives: $\partial h/\partial v_2\propto q_1^2$, while $\partial h/\partial v_1\propto q_1q_2$. The corresponding evenness of the derivatives $\partial h/\partial\mathbf{v}$ guarantees that only four of the eight moduli of $\hat{D}$ are nonzero.

The fluctuation part of the FLL energy in a uniaxial crystal can therefore be given as $\delta G=\frac{1}{2}\zeta'\hat{\chi}'\zeta'$, where the matrix $\hat{\chi}$ from Eq. (11) after permutation of the rows and columns becomes block diagonal

$$\hat{\chi}'=\begin{pmatrix} C_{1111} & C_{1122} & D_{111} & & & \\ C_{2211} & C_{2222} & D_{122} & & 0 & \\ D_{111} & D_{122} & T_{11} & & & \\ & & & C_{1212} & C_{1221} & D_{212} \\ & 0 & & C_{2112} & C_{2121} & D_{221} \\ & & & D_{212} & D_{221} & T_{22} \end{pmatrix} \tag{13}$$

the coupled fluctuations $\zeta'=\{u_{11},u_{22},\omega_1,u_{12},u_{21},\omega_2\}$ represent noninteracting combinations of FLL compression dilation and vortex tilt in the plane containing the axis of anisotropy $\mathbf{c}$ and $\mathbf{H}$ as well as shear strains and vortex deviations from this plane.

5. The elastic modulus matrix (13) becomes substantially simpler in the case of an isotropic crystal. In such a crystal the vortex direction $\mathbf{v}$ always coincides with $\mathbf{H}$. This follows directly from the equilibrium equations (7) with $\mathbf{K}$ from Eq. (5). Indeed, in the isotropic case $\hat{\mu}\equiv 1$ holds, while the vector $\partial h/\partial\mathbf{v}$ in integrals with $(\mathbf{qv})=0$ is parallel to $\mathbf{v}$. Hence the only identically nonzero term in Eq. (5) for $\mathbf{K}$ is the term proportional to $[\mathbf{Hv}]$.

Aside from the two relations used so far equilibrium equations (7) with $\hat{\sigma}$ from Eq. (6) also contain two relations for the diagonal components of $\hat{\sigma}$: $\sigma_{11}-\sigma_{22}=0$, $\sigma_{11}+\sigma_{22}=0$. Consider the first relation:

$$\sigma_{11}-\sigma_{22}=n_L\frac{\Phi_0}{4\pi}\int d^3q\,\delta(\mathbf{qv})h\left(q_1\frac{\partial S}{\partial q_1}-q_2\frac{\partial S}{\partial q_2}\right)=0. \tag{14}$$

We show that square and hexagonal FLLs satisfy this condition. For this purpose let us introduce the polar coordinate systems $x_1+ix_2=\rho e^{i\varphi}$, $q_1+iq_2=Qe^{i\psi}$. Using this notation, we rewrite Eq. (14) as

$$\begin{aligned}\sigma_{11}-\sigma_{22}&=n_L\frac{\Phi_0}{4\pi}\int d\psi\,dQ\,Qh(Q)\left(\cos 2\varphi\cdot Q\frac{\partial S}{\partial Q}-\sin 2\varphi\frac{\partial S}{\partial\psi}\right)\\ &=in_L\frac{\Phi_0}{4\pi}\int d\psi\,dQQ^2h(Q)\sum_\rho\exp(iQ\rho\cos\psi)\\ &\quad\times\sum_\varphi\cos(\psi+2\varphi)=0.\end{aligned} \tag{14a}$$

We have taken account of the fact that the field h is only dependent on $Q=(q_1^2+q_2^2)^{1/2}$ in the isotropic case, while the sum over the FLL sites in the structure factor (3) has been decomposed into a sum over the coordination spheres of radius ρ and a sum over sites lying along the circle of given radius ρ and having the azimuthal angle φ. It is easily determined that the sum $\Sigma_\varphi\cos(\psi+2\varphi)$ is equal to zero in Eq. (14a) with an arbitrary ψ for only two FLL's: a square FLL and a hexagonal FLL.

The last equation $\sigma_{11}+\sigma_{22}=0$ in (7) in the form

$$H=H_{c1}+\frac{1}{2}\int d^3q\,\delta(\mathbf{qv})h\left(S-q_1\frac{\partial S}{\partial q_1}\right) \tag{15}$$

determines the magnetic field dependence of the unit cell size of the FLL.

Let us now consider the elastic moduli. In the isotropic case $\hat{D}\equiv 0$ since the vectors $\mathbf{v}$ and $\partial h/\delta\mathbf{v}$ are parallel with $(\mathbf{q}\mathbf{v})=0$ in the integrals.

The rotational axis of symmetry in the fourth or sixth order FLL will cause the following elastic moduli to coincide: $C^0_{1111}=C^0_{2222}$, $C^0_{1212}=C^0_{2121}$. The "zero" superscript here denotes the values in an isotropic superconductor. Moreover, the fact that vectors $\mathbf{v}$ and $\mathbf{H}$ are parallel results in an additional symmetry element: the entire crystal can be rotated by an arbitrary angle $\mathbf{\Omega}\|\mathbf{H}$ about the direction of $\mathbf{H}$. Such a rotation corresponds to a deformation of the flux-line lattice

$$u_{ij}=\frac{\partial}{\partial x_j}[\mathbf{x}\mathbf{\Omega}]_i$$

as well as a shift by $\delta G=(C^0_{1212}-C^0_{1221})\Omega$ in the FLL energy. The relation

$$C^0_{1212}=C^0_{1221}$$

follows from the constancy of the Gibbs potential.

The FLL rigidity in an isotropic crystal is therefore determined by four elastic moduli.

An important feature of an isotropic crystal is the isomorphic transformation of the FLL under a varying magnetic field: the lattice period $\mathbf{A}$ changes, while the angle Ξ remains constant. This property of the FLL can be attributed to the fact that the elastic stresses $\sigma_{11}-\sigma_{22}$ in relation (14a) turned out to be zero for the vortices of each coordination shell. By virtue of the isomorphism of the FLL the derivative of the structure factor S with respect to $\mathbf{q}$ in the relations for $\hat{T}$ in Eq. (8) and $\hat{C}$ in Eq. (9) can be expressed through the derivatives with respect to $B=n_L\Phi_0$:

$$\mathbf{q}\frac{\partial S}{\partial\mathbf{q}}=-2B\frac{\partial S}{\partial B}.$$

This relation allowed Labush[7] to express the elastic moduli of a hexagonal FLL in terms of the characteristics of the equilibrium magnetization curve $B=B_\Delta(H)$:

$$T_{11}{}^0=T_{22}{}^0=\frac{BH}{4\pi},$$

$$C^0_{1111}(B)=\frac{B^2}{4\pi}\frac{\partial H}{\partial B}+\frac{1}{8\pi}\int_0^B B^2\frac{\partial^2 H}{\partial B^2}dB,$$

$$C^0_{1122}(B)=\frac{B^2}{4\pi}\frac{\partial H}{\partial B}-\frac{1}{8\pi}\int_0^B B^2\frac{\partial^2 H}{\partial B^2}dB,$$

$$C^0_{1212}(B)=\frac{1}{8\pi}\int_0^B B^2\frac{\partial^2 H}{\partial B^2}dB. \qquad (16)$$

Note that the Cauchy relation

$$C^0_{1111}-C^0_{1122}=2C^0_{1212}$$

holds for a hexagonal FLL. With low-level induction $(n_L\lambda^2\ll 1)$ the elastic moduli $\hat{C}$ are similar in magnitude, $C^0_{1122}\approx C^0_{1212}\approx\frac{1}{3}C^0_{1111}$ and are exponentially small,

$$C^0_{1122}(B)\approx\frac{B^2}{8\pi}\frac{\partial H}{\partial B}\approx\left(\frac{\Phi_0}{4\pi\lambda^2}\right)^2\left(\frac{B\lambda^2}{\Phi_0}\right)^{1/4}\exp\left[-\left(\frac{2\Phi_0}{3^{1/2}B\lambda^2}\right)^{1/2}\right],$$

$$C^0_{1122}(H)\approx\frac{\Phi_0}{4\pi\lambda^2}(H-H_{c1}{}^0)\ln\left[\left(\frac{H}{H_{c1}{}^0}-1\right)^{-1}\right]. \qquad (17)$$

The tilt modulus considerably exceeds the moduli $\hat{C}$

$$T_{11}{}^0\approx BH_{c1}{}^0/4\pi. \qquad (18)$$

The compression moduli C^0_{1111} and C^0_{1122} are comparable in magnitude to the tilt modulus $T^0_{11}=H^2/4\pi$ with increasing induction $(n_L\lambda^2)\gg 1)$ and substantially greater than the shear modulus[8]

$$C^0_{1212}=\Phi_0 H/64\pi^2\lambda^2.$$

The positive definiteness of the block diagonal matrix (13) determines the stability of the hexagonal FLL; in this matrix all diagonal elements and the minor of rank two are positive

$$\Delta_c=C_{1111}C_{2222}-C^2_{1122}. \qquad (19a)$$

The minor of rank 2

$$\Delta_s=C_{1212}C_{2121}-C^2_{1221} \qquad (19b)$$

is identically equal to zero due to the degeneracy of the FLL energy under rotation in the basal plane.

A square FLL is unstable.[6] It is unstable against compression-dilation strains. The minor Δ_c in Eq. (19a) is negative across the entire field range. With low-level induction $(n_L\lambda^2\ll 1)$ it is exponentially small

$$\Delta_c\approx-\left(\frac{\Phi_0}{2\pi\lambda^2}\right)^3\frac{B}{2}\exp\left[-2\left(\frac{\Phi_0}{B\lambda^2}\right)^{1/2}\right],$$

while with high a level induction $(n_L\lambda^2\gg 1)$ it is equal to

$$\Delta_c\approx-\frac{\Phi_0}{2\pi\lambda^2}\left(\frac{B}{2\pi}\right)^3.$$

In both of these cases the principal terms in the functions $\hat{C}(B)$ cancel and the sign of the minor Δ_c (19a) is already determined by small corrections to the fundamental relation. This statement once again confirms the well known fact that the energies of hexagonal and square FLLs differ by a negligible amount.[6,14]

6. In the general case of an anisotropic superconductor all 21 linearly-independent elastic moduli $\hat{C}$, $\hat{T}$, and $\hat{D}$ are nonzero. The minor of rank two Δ_s in Eq. (19b) is nonzero and describes the rigidity of the FLL with respect to unit cell rotation.

3. MAGNETIZATION IN THE SYMMETRICAL DIRECTION; DEGENERATE VORTEX STRUCTURES

1. We first determine the equilibrium orientation of a rectilinear isolated vortex. We can determine from Eqs. (7) with $\mathbf{K}$ from Eq. (5) that for $\mathbf{H}=0$ the vortex lies along the axis of symmetry with the lowest value of the tensor $\hat{\mu}$ (henceforth we label this the a axis). In this case, the self-energy of the vortex $\Phi_0 H_{c1}/4\pi$ is minimized.

For magnetization along the $\mathbf{c}$ axis (the "hard" axis in accordance with the value of μ) $\mathbf{v}$ rotates in the ac plane as

$$\mathbf{v}\mathbf{c}=\left[1+\frac{\mu_c}{\mu_a}\left(\frac{\tilde{H}^2}{H^2}-1\right)\right]^{-1/2},\quad H\leqslant\tilde{H}. \qquad (20)$$

In this case $\mathbf{v}$ rotates from $\mathbf{a}$ to $\mathbf{c}$ in the field range from zero through

$$\tilde{H}=H_{c1}\left(1-\frac{\mu_a}{\mu_c}\right). \qquad (21)$$

The vortex is stationary in fields exceeding $\tilde{H}$ and oriented along **H**.

The moduli $\hat{C}$ and $\hat{D}$ are identically equal to zero for an isolated vortex. The tilt moduli behave as follows:

$$T_{11}=\begin{cases}\frac{B}{4\pi}H_{c1}\frac{\mu_b}{\mu_a}\left[1-\frac{H^2}{\tilde{H}^2}\left(1-\frac{\mu_a}{\mu_c}\right)\right], & H\leqslant\tilde{H},\\ \frac{B}{4\pi}H_{c1}\frac{\mu_b}{\mu_c}, & H>\tilde{H},\end{cases}$$

$$T_{22}=\begin{cases}\frac{B}{4\pi}H_{c1}\frac{\mu_c}{\mu_a}\left[1-\frac{H^2}{\tilde{H}^2}\left(1-\frac{\mu_a}{\mu_c}\right)\right]^2, & H\leqslant\tilde{H},\\ \frac{B}{4\pi}H_{c1}\frac{\mu_a}{\mu_c}, & H>\tilde{H}.\end{cases} \quad (22)$$

Here the x_1 axis perpendicular to **v** rotates in the *ac* plane for $H<\tilde{H}$, while the axis is directed along **c** for $H\geqslant\tilde{H}$. The x_2 axis is parallel to the **b** axis of the crystal. Magnetization of the crystal in the direction of the "intermediate" **b** axis is also described by Eqs. (20)–(22) in which the indices *b* and *c* are transposed.

If the crystal is magnetized along the **a** axis, the vortex is always oriented parallel to **H** and we have

$$T_{11}=T_{bb}=(B/4\pi)H_{c1}\mu_c/\mu_a,\quad T_{22}=T_{cc}=(B/4\pi)H_{c1}\mu_b/\mu_a.$$

2. Let us now consider FLL formation in a superconductor magnetized parallel to the **c** axis by an external magnetic field of magnitude exceeding $\tilde{H}$. To describe a crystal in the intermediate direction **b** or the "easy" plane **a** it is necessary to carry out the cyclic substitution of indices $a\to b\to c\to a$ in all preceding equations.

First of all, note that taking account of vortex interactions has no effect on the equilibrium orientation of the vortices for $H>H_{c1}$. This orientation remains unchanged from the case of an isolated vortex: $\nu\|\mathbf{H}\|\mathbf{c}$. This follows from the zero value of the integral in Eq. (5) since we have $\partial h/\partial\mathbf{v}\|\mathbf{v}$ for $\mathbf{v}\|\mathbf{c}$.

The results obtained for an isotropic superconductor can be employed to describe the FLL structure in fields $H>H_{c1}$. Indeed, for $\nu\|\mathbf{c}$ the expression for h in Eq. (2) is simplified significantly:

$$h=\frac{\Phi_0}{(2\pi\lambda)^2}\frac{1}{1+q_1^2\mu_b+q_2^2\mu_a}. \quad (23)$$

The basal plane anisotropy in the integrals with respect to **q** in Eqs. (6) and (9) can be accounted for by means of the scale transformation:

$$q_1'=q_1\mu_b^{1/2},\quad q_2'=q_2\mu_a^{1/2}. \quad (24)$$

The structure factor $S(\mathbf{q})$ in Eq. (3) remains unchanged under this transformation if a scaling transformation of the coordinates in the basal plane is carried out in conjunction with the transformation (24):

$$x_1'=x_1\mu_b^{-1/2},\quad x_2'=x_2\mu_a^{-1/2}. \quad (25)$$

The expression for the Gibbs energy (1) in the primed coordinate system retains virtually the same form as in an isotropic crystal. The only difference lies in the additional multiplier $\mu_c^{1/2}$ in front of the integral with respect to **q** in Eq. (1), the expression for the lower critical field $H_{c1}=H_{c1}^0\mu_c^{1/2}$ and the expression for the vortex concentration:

$$n_L=\mu_c^{1/2}(\mathbf{v}[\mathbf{A}_1\mathbf{A}_2])^{-1}.$$

A stable FLL in the primed coordinate system is a hexagonal structure with $A_1'=A_2'=A'$ and $\Xi'=\pi/3$. The unit cell parameters are independent of its orientation in the basal plane. The superconductor induction B and the field H are related by

$$B(H/H_{c1}-1)=\mu_c^{1/2}B_\Delta(H/H_{c1}-1) \quad (26)$$

in terms of the reversible magnetization curve of an isotropic superconductor $B_\Delta(H/H_{c1}^0-1)$.

3. FLL degeneracy is one of the characteristics of vortex structures in an isotropic crystal. The energy G and the unit cell remain unchanged as the cell rotates about **v**. Basal plane anisotropy exists in a biaxial crystal and $\mu_a\neq\mu_b$ holds. Nonetheless, when a biaxial superconductor is magnetized in the symmetrical direction FLL degeneration is also present. However, the unit cell shape depends on its orientation with respect to the crystal axes. To confirm this we use the inverse coordinate transform associated with (25). The parameters A_1, A_2, and Ξ of the flux-line lattice are expressed in terms of the parameters A' and $\Xi'=\pi/3$ of a hexagonal FLL in the primed coordinate system by

$$A_1=\left[\frac{\mu_b}{1+(\eta^2-1)\sin^2\Gamma}\right]^{1/2}A'(H),$$
$$A_2=\left[\frac{\mu_b}{1+(\eta^2-1)\sin^2(\Xi+\Gamma)}\right]^{1/2}A'(H),$$
$$\operatorname{tg}\Xi=\frac{3^{1/2}(1+\eta^2\operatorname{tg}^2\Gamma)}{\eta(1+\operatorname{tg}^2\Gamma)+3^{1/2}(1-\eta^2)\operatorname{tg}\Gamma}. \quad (27)$$

Here the relation $A'(H)$ is described by relations that are standard for a hexagonal FLL:

$$A'(H)=\begin{cases}\lambda\ln\left[\frac{3\Phi_0}{8\pi^{1/2}\lambda^2(H-H_{c1})}\right], & n_L\lambda^2\ll1,\\ \left[\frac{2\Phi_0}{3^{1/2}(H-H_{c1})}\right]^{1/2}, & n_L\lambda^2\gg1.\end{cases}$$

Relations (27) permit an unambiguous determination of FLL structure in an anisotropic crystal. In these relations $\eta=(\mu_b/\mu_a)^{1/2}$ characterizes the basal plane anisotropy, while the angle Γ gives the unit cell orientation relative to the crystal axes (see Fig. 1).

The FLL energy in equilibrium is independent of the angle Γ. Indeed, $n_L=B/\Phi_0$ is a function only of magnetic field (26). As follows from the transformation (24) and (25), the structure factor does not depend on the angle Γ. This means that the continuum of FLL's that continuously transform into one another with varying Γ have the same energy. This conclusion was initially derived in Ref. 9 for uniaxial superconductors.

4. The FLL unit cell can easily be plotted in graph form (see Fig. 2). An ellipse with semiaxes $\mu_b^{1/2}$ and $\mu_a^{1/2}$ is drawn in the basal plane *ab*. The vector $\mathbf{A}_1$ is determined by the intersection with the ellipse of a ray running from the center of the ellipse at an angle Γ relative to the x_1 axis. As we see from Fig. 2, the second vector $\mathbf{A}_2$ is found by plotting the chord *pp'* parallel and equal in length to the vector $\mathbf{A}_1$. The vector $\mathbf{A}_2$ connects one end of the chord to the center of the ellipse.

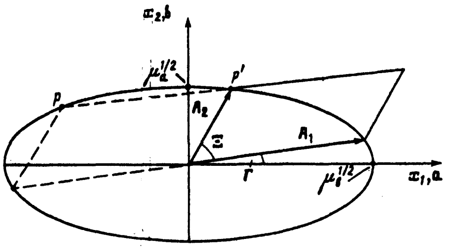

FIG. 2. Method of plotting the unit cell of a flux-line lattice in a superconductor magnetized along the **c** axis for a given angle Γ. The chord pp' is parallel and equal in length to the vector $\mathbf{A}_1$.

As Γ goes from 0 to 2π the FLL's transform continuously so that each structure repeats six times. All topologically nonequivalent structures can be obtained by varying the angle Γ on the interval $[0, \tan^{-1}(3^{1/2}/\eta)]$.

Figure 3 provides a convenient illustration of the parameters of an anisotropic FLL. This figure shows how the ratio A_1/A_2 depends on the angle Ξ between them for different values of the anisotropy parameter η. A hexagonal lattice (point $A_1/A_2 = 1$, $\Xi = \pi/3$ in Fig. 3) corresponds to the isotropic case $\eta = 1$. The closed curve in Fig. 3 corresponds to the FLL continuum for a specific value of η. It describes the family of structures that are continuously transformed into one another. Each point on the closed curve corresponds to a specific value of Γ. We can easily observe motion along the closed curve as Γ varies. Curve I in Fig. 3 is the locus of points with $\Gamma = 0$ and different values of η. Line II is $\Gamma = \tan^{-1}(3^{1/2}/\eta)$. In this orientation the unit cell is a rhombus ($A_1 = A_2$) for any values of η. Curve III corresponds to $\Gamma = \pi - \tan^{-1}(3^{1/2}/\eta)$. Therefore, counter-clockwise rotation of $\mathbf{A}_1$ in Fig. 2 corresponds to clockwise motion of a point along the closed curve in Fig. 3.

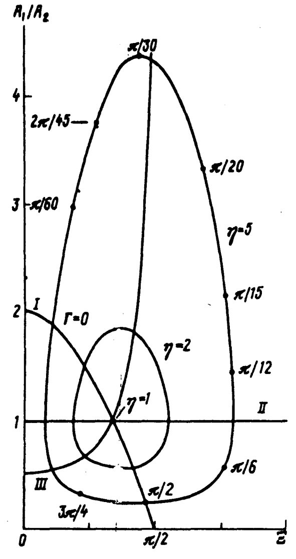

FIG. 3. The length ratio of the translation vectors A_1/A_2 versus the angle Ξ between them for different anisotropy parameters $\eta = (\mu_b/\mu_a)^{1/2}$. The corresponding values of the angle Γ are noted on the closed curve for $\eta = 5$.

A 180° rotation of the vector $\mathbf{A}_1$ corresponds to tracing the curve in Fig. 3.

5. The elastic moduli $\hat{C}$ in Eq. (9) can be expressed in terms of the moduli $\hat{C}^0(B)$ from Eq. (16) for an isotropic lattice in the case $\mathbf{H}\|\mathbf{c}$ by means of the scale transformation (24) and (25):

$$C_{1111}(B) = \mu_c \overset{0}{C}_{1111}(B/\mu_c^{1/2}),$$

$$C_{2222}(B) = \mu_c \overset{0}{C}_{1111}(B/\mu_c^{1/2}),$$

$$C_{1122}(B) = C_{2211}(B) = \mu_c \overset{0}{C}_{1122}(B/\mu_c^{1/2}),$$

$$C_{1212}(B) = \mu_b^{-2} \overset{0}{C}_{1212}(B/\mu_c^{1/2}),$$

$$C_{2121}(B) = \mu_a^{-2} \overset{0}{C}_{1212}(B/\mu_c^{1/2}),$$

$$C_{1221}(B) = C_{2112}(B) = \mu_c \overset{0}{C}_{1212}(B/\mu_c^{1/2}). \qquad (28)$$

Therefore the elastic moduli in an anisotropic crystal magnetized in the symmetrical direction only depends on the induction B. The equilibrium relation of B and H is determined by expression (26).

As in the case of an isotropic crystal we can easily see from Eq. (28) that the minor (19b) will always be zero. The tilt-compression (shear) moduli are zero. The tilt moduli $\hat{T}$ cannot be expressed in terms of $\hat{T}^0$ in Eq. (16) by the scale transformation (24) and (25) for an isotropic superconductor. This is because the moduli $\hat{T}$ do not reflect the symmetry of the field h itself, but rather the symmetry of its derivatives with respect to $\mathbf{v}$. The expression for the principal components of the tilt moduli can be represented in quadrature form

$$T_i = \frac{B}{4\pi}\left(H - H_{c1} + H_{c1}\frac{\mu_i^{-1}}{\mu_c^2}\right) + \frac{B\Phi_0}{16\pi^3\lambda^2\mu_c}\int dq_1\,dq_2\,S\,\frac{(\mu_i^{-1}-\mu_c^2)\,q_i^2\,(1+q^2/\mu_i\mu_c)}{(1+q^2\mu_c)\,(1+q_1^2\mu_b+q_2^2\mu_a)^2}.$$

Here the integral term describes the dependence of the tilt moduli on the FLL unit cell orientation in the basal plane (the angle Γ). The moduli T_i reach their extremal values for values of Γ where the unit cell is a rhombus. In this case, one of the tilt moduli is a minimum while the other is a maximum. The moduli T_1 and T_2 vary in antiphase as the angle Γ changes: an increase in one modulus is accompanied by a decrease in the other. In both limiting cases (high and low induction levels) we can neglect the last integral term.

6. A continuum of FLL's is therefore realized when the crystal is magnetized along the axis of symmetry. Such FLL's have identical energies, moduli $\hat{C}$, and moduli $\hat{D} \equiv 0$, and different moduli T_1 and T_2. Such FLL degeneracy is eliminated as $\mathbf{H}$ deviates from the axis of symmetry. The resulting FLL transformations differ in the case of easy axis and easy plane crystals. We consider these cases separately based on sparse FLL's in uniaxial superconductors.

4. OBLIQUE MAGNETIC FIELD: HEXAGONAL FLUX-LINE LATTICE IN EASY-AXIS CRYSTALS ($\mu_\alpha > 1$)

1. Let us consider the magnetization of a uniaxial superconductor[1)] ($\mu_a = \mu_b > \mu_c = \mu_a^{-2}$) by an external field $\mathbf{H}$ at an angle θ to the axis of anisotropy $\mathbf{c}$ (see Fig. 1).

In the second section we obtained the most general solutions of the equilibrium equations (7) in uniaxial superconductors. That is, the vectors **c**, **H**, and **ν** that always lie in the same plane while the FLL unit cell is a rhombus:

$$A_1=A_2, \quad \Gamma=\pi/2-\Xi/2. \tag{29}$$

In this section we obtain the remaining solutions of the equilibrium equations for easy-axis crystals, write out the elastic moduli, and investigate the stability of the resulting solutions. We start by solving Eq. (14), $\sigma_{11}-\sigma_{22}=0$, which determines the relation $\Xi(\mathbf{B})$, and then determine from $\sigma_{11}+\sigma_{22}=0$ and $K_1=0$ the equilibrium magnetization curve $\mathbf{B}(\mathbf{H})$.

The structure factor $S(\mathbf{q})$ is a rapidly oscillating function with a period $q\approx\lambda/A_1\ll 1$ for a sparse FLL ($n_L\lambda^2\ll 1$). In this case, the Fourier component h (12) can be used to obtain the asymptotic forms of the vortex field $h(\mathbf{x})$ at large distances. It contains two terms with different damping rates.[1,3] The first term which is proportional to $\exp(-x/\mu_a^{1/2})$ is determined by the pole of Eq. (12) for $q^2=-1/\mu_a$ and dominates for virtually all vortex directions $\gamma\neq\pi/2$, where γ is the angle between the vortex **ν** and the axis of anisotropy **c**. The asymptotic forms of the magnetic field at large distances therefore correspond to the Fourier component of the form

$$h=\frac{\Phi_0}{(2\pi\lambda)^2}\frac{1}{1+q^2\mu_a}. \tag{30}$$

When the vortex is located in the ab plane ($\gamma\to\pi/2$) the tensor component $\mu_{33}=\mu_a\sin^2\gamma+\mu_a^{-2}\cos^2\gamma$ is comparable to μ_a. It is demonstrated below that $\gamma\to\pi/2$ holds only when the external field **H** is applied in the ab-plane (for $\theta\to\pi/2$). In this case the pole corresponding to the vanishing of the second factor in the denominator of Eq. (12) makes the primary contribution to the asymptotic form of the vortex field at large distances. This means that the asymptotic limits of the field are determined by the Fourier component

$$h=\frac{\Phi_0}{(2\pi\lambda)^2}\frac{1}{1+q_1^2\mu_a^{-2}+q_2^2\mu_a}. \tag{31}$$

The damping rate is anisotropic and the vortex field is proportional to $\exp[-\mu_a(x_1^2+\mu_a^{-3}x_2^2)^{1/2}]$.

The constant-field lines $h(\mathbf{x})$ are circles in the case of Eq. (30) and ellipses in the case of Eq. (31) and hence all equilibrium equations $\hat{\sigma}=0$ can be reduced to isotropic form by transforming the coordinates **q** and **x**. The field $h(\mathbf{q})$ in Eq. (30) differs from the isotropic case solely in the multiplier μ_a for q^2. This means that the FLL differs from the isotropic case by compression of the coordinates by a factor of $\mu_a^{1/2}$ in the basal plane. As in an isotropic crystal the solutions of the equation $\sigma_{11}-\sigma_{22}=0$ with σ_{ij} from Eq. (6) are the hexagonal and square FLL's:

$$\Xi=2\pi/3, \tag{32a}$$

$$\Xi=\pi/3, \tag{32b}$$

$$\Xi=\pi/2. \tag{32c}$$

The equilibrium vortex structures for the field distribution (31) are described in Sec. 3. The orthorhombic symmetry of Eq. (29) selects from the FLL continuum those lattices with unit cell angles

$$\Xi=2\,\mathrm{arctg}\,(3^{1/2}\mu_a^{-3/2}), \tag{33a}$$

$$\Xi=2\,\mathrm{arctg}\,(3^{-1/2}\mu_a^{-3/2}), \tag{33b}$$

$$\Xi=2\,\mathrm{arctg}\,(\mu_a^{-3/2}). \tag{33c}$$

FLL rearrangement and the transition from solutions (32) to (33) occur near the angle $\theta=\pi/2$. The size of this interval is dependent on the induction:

$$\Delta\theta=\frac{\pi}{2}-\theta\approx\mu_a^{-3/4}\left(\frac{\Phi_0}{B\lambda^2}\right)^{1/4}\exp\left[-\frac{\mu_a^{3/2}-1}{2\mu_a^{1/2}}\left(\frac{2\Phi_0}{3^{1/2}B\lambda^2}\right)^{1/2}\right]$$

$$\propto\frac{H}{H_{c1}(\pi/2)}-1. \tag{34}$$

Figure 4 shows the results from a numerical calculation of the dependence of the angle Ξ on the vortex tilt angle γ for fixed induction values.

Simultaneous solution of the equations $K_1=0$ and $\sigma_{11}+\sigma_{22}=0$ yields the equilibrium vortex orientation in the lattice (the angle γ) as well as the equilibrium vortex concentration $\mathbf{B}(\mathbf{H})$. For a hexagonal FLL the angle γ has the following dependence on the external parameters H and θ:

$$\gamma=\mathrm{arctg}\,(\mu_a^{-3}\,\mathrm{tg}\,\theta)$$

$$+\frac{\mu_a^6(\mu_a^3-1)(1+\mu_a^3\,\mathrm{tg}^2\,\theta)\,\mathrm{tg}\,\theta}{(\mu_a^6+\mathrm{tg}^2\,\theta)^2}\left[\frac{H}{H_{c1}(\theta)}-1\right],$$

$$H_{c1}(\theta)=H_{c1}^0(\mu_a^2\cos^2\theta+\mu_a^{-1}\sin^2\theta)^{-1/2}. \tag{35}$$

The first term in Eqs. (35) determines the orientation of an isolated vortex.[1] It is clear that the vortex lies in the ab plane ($\gamma\to\pi/2$) only in the limit $\theta\to\pi/2$. The second term accounts for the additional vortex rotation induced by the nearest neighbor field.

The equilibrium induction of the hexagonal structure (32a,b)

$$B(\mathrm{H})=\frac{1}{\mu_a}B_\Delta\left[\mu_a\left(\frac{\mathbf{H}\boldsymbol{\nu}}{H_{c1}(\theta)}-1\right)\right]$$

$$=\frac{1}{\mu_a}B_\Delta\left[\mu_a\left(\frac{H}{H_{c1}^0}\frac{\cos^2\theta(\mu_a^3+\mathrm{tg}^2\,\theta)^{3/2}}{\mu_a^{1/2}(\mu_a^6+\mathrm{tg}^2\,\theta)^{1/2}}-1\right)\right] \tag{36}$$

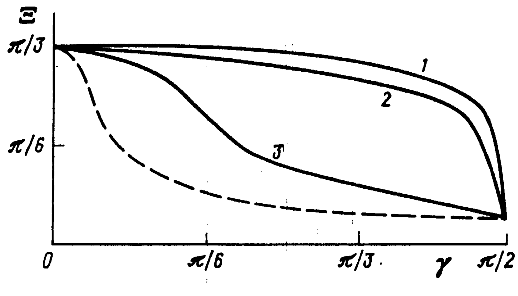

FIG. 4. The angle Ξ of the unit cell of the flux-line lattice plotted as a function of the tilt angle γ of the vortices to the axis of anisotropy of an easy axis superconductor. The curves are plotted for different inductions B: 1—$B\lambda^2/\Phi_0=0.001$; 2—0.01; 3—0.1. The primes represent relation (52) for a dense lattice, $B\lambda^2/\Phi_0\gg 1$.

is expressed through the reciprocal magnetization curve of an isotropic superconductor. When **H** is oriented in the *ab* plane the induction of the oblique structure (32a,b) is

$$B(H) = \mu_a^{1/2} B_\Delta \left(\frac{H}{H_{c1}(\pi/2)} - 1 \right). \tag{37}$$

It is clear from Eqs. (36) and (37) that the induction jumps sharply by a factor of $\mu_a^{3/2}$ in the immediate vicinity $\Delta\theta$ given by (34) of angles θ near $\pi/2$ as a hexagonal FLL [Eq. (32)] becomes an oblique FLL [Eq. (33)].

2. The strongest of the elastic moduli of a sparse FLL [see the matrix (13)] are the tilt moduli $\hat{T}$:

$$T_{ii} = \frac{BH_{c1}}{4\pi} \frac{\mu_{ii}}{\mu_{33}}; \tag{38}$$

here T_{ii} denotes the diagonal element of the matrix $\hat{T}$.

The moduli $\hat{D}$ are exponentially small in the function B and hence are due to interaction between the vortices:

$$D_{122} = \frac{3}{2} \frac{1-\mu_a^3}{\mu_a^{7/4}} \frac{\sin\theta\cos\theta}{\sin^2\theta + \mu_a^6 \cos^2\theta} \frac{B\Phi_0}{(2\pi\lambda)^2} \left(\frac{2\Phi_0}{3^{1/2} B\lambda^2} \right)^{3/4}$$
$$\times \exp\left[-\left(\frac{2\Phi_0}{3^{1/2}\mu_a B\lambda^2} \right)^{1/2} \right]$$
$$\propto (\mu_a^3 - 1)\sin\theta\cos\theta \frac{\Phi_0^2}{\lambda^4} \left(\frac{\mathbf{H}\boldsymbol{\nu}}{H_{c1}} - 1 \right) \ln\left(\frac{\mathbf{H}\boldsymbol{\nu}}{H_{c1}} - 1 \right). \tag{39}$$

The explicit dependence of the difference $(\mathbf{H}\nu)/H_{c1}(\theta) - 1$ on the external parameters H and θ is contained in Eq. (36). A property of the moduli $\hat{D}$ is that they vanish in the isotropic limit $\mu_a \to 1$ and when **H** is tilted with respect to the symmetrical directions ($\theta \to 0, \pi/2$).

The elastic moduli $\hat{C}$ have the same dependence on **H** as do the moduli $\hat{D}$

$$C \propto \frac{\Phi_0^2}{\lambda^4} \left(\frac{\mathbf{H}\boldsymbol{\nu}}{H_{c1}} - 1 \right) \ln\left[\left(\frac{\mathbf{H}\boldsymbol{\nu}}{H_{c1}} - 1 \right)^{-1} \right], \tag{40}$$

although they never vanish.

The induction jumps sharply when **H** is applied in the *ab* plane in the interval $\Delta\theta$ given by (34), while the moduli $\hat{T}$ grows simultaneously by a factor of $\mu_a^{3/2}$. The moduli $\hat{C}$ in this interval grow by a factor of $(\Delta\theta)^2 \propto [H/H_{c1}(\pi/2) - 1]^{-2}$. The moduli $\hat{D}$ vanish as $\cos\theta$ as $\theta \to \pi/2$.

3. The positive definiteness of the matrix (13) determines the stability of the solutions (32). Since the conditions $C_{1111}, C_{1212} > 0$ and $T \gg D$ hold for all the solutions (32), the stability of the solutions (32) will only depend on the sign of the minors of rank 2, Δ_c and Δ_s given by Eqs. (19).

The minor Δ_c of (19a) is negative for a square FLL [see (30c)]

$$\Delta_c = -\frac{\pi\mu_a^{21/2}}{(2\mu_a^6 + \mathrm{tg}^2\theta)^2} \frac{B\Phi_0^3}{(2\pi)^4\lambda^6} \exp\left[-\left(\frac{4\Phi_0}{\mu_a B\lambda^2} \right)^{1/2} \right]$$
$$\propto -\frac{\Phi_0^4}{\lambda^8} \left(\frac{\mathbf{H}\boldsymbol{\nu}}{H_{c1}} - 1 \right)^2 \tag{41}$$

and is positive for both hexagonal structures (32a,b). The minor Δ_s of (19b) has a different sign for the hexagonal structures of (32):

$$\Delta_s = -\frac{3^{11/4}\mu_a^5}{2^{9/2}\pi^3} \frac{\mathrm{tg}^6\theta}{(4\mu_a^6 + 3\,\mathrm{tg}^2\theta^4} \frac{B^{3/2}\Phi_0^{5/2}}{\lambda^5}$$
$$\times \exp\left[-\left(\frac{8\Phi_0}{3^{1/2}\mu_a B\lambda^2} \right)^{1/2} \right]$$
$$\propto -\sin^6\theta\cos^2\theta \frac{\Phi_0^4}{\lambda^8} \left(\frac{\mathbf{H}\boldsymbol{\nu}}{H_{c1}} - 1 \right)^2 \tag{42}$$

for the solution (32a) and

$$\Delta_s = \frac{3^{11/4}\mu_a^{11}}{2^{9/2}\pi^3} \frac{\mathrm{tg}^6\theta}{(4\mu_a^6 + \mathrm{tg}^2\theta)_4 (\mu_a^6 + \mathrm{tg}^2\theta)} \frac{B^{3/2}\Phi_0^{5/2}}{\lambda^5}$$
$$\times \exp\left[-\left(\frac{8\Phi_0}{3^{1/2}\mu_a B\lambda^2} \right)^{1/2} \right]$$
$$\propto \sin^6\theta\cos^4\theta \frac{\Phi_0^4}{\lambda^8} \left(\frac{\mathbf{H}\boldsymbol{\nu}}{H_{c1}} - 1 \right)^2 \tag{43}$$

for the solution (32b).

4. Therefore the only stable structure of the sparse vortices in "easy axis" superconductors is hexagonal FLL (32b). Unlike the case of an isotropic superconductor, the FLL unit cell has a fixed orientation in the basal plane. The long diagonal of the rhombus lies in the plane formed by the vectors **c**, **H**, and **ν**. When **H** is applied along the *ab* plane the hexagonal FLL is abruptly transformed into an oblique lattice with angle Ξ from (32b). This process occurs within the narrow range of angles (34) $\Delta\theta \propto [H/H_{c1}(\pi/2) - 1]$. The induction and the tilt moduli jump sharply (by a factor of $\mu_a^{3/2}$) when Ξ changes. The moduli $\hat{C}$ grow by a factor of $[H/H_{c1}(\pi/2) - 1]^{-2}$. When **H** is oriented along the symmetrical direction $\|$**c** or $\perp$**c** the moduli for the coupled "tilt-compression (shear)" fluctuations are moderated, the minor Δ_s (43) vanishes, and the hexagonal (oblique) FLL generates the degenerate continuum of vortex structures described in Sec. 3.

5. VORTEX CHAINS IN "EASY PLANE" CRYSTALS ($\mu_a \ll 1$)

1. Let us consider magnetization of an "easy-axis" uniaxial superconductor:[2)] $\mu_a = \mu_b < \mu_c = \mu_a^{-2}$.

In the preceding sections in solving $\sigma_{11} - \sigma_{22} = 0$ [Eq. (14)] which determines the equilibrium angle Ξ we reduced the problem to the isotropic case by a scale coordinate transformation. This method is based on the fact that in an isotropic superconductor the constant field lines $h(\mathbf{x})$ are circles, while in a biaxial crystal in the case **H**$\|$**c** and in a uniaxial crystal with $\mu_a > 1$ such lines are ellipses that become circles under a scale coordinate transformation. The distribution $h(\mathbf{x})$ is qualitatively different from the isotropic case in "easy-plane" superconductors ($\mu_a < 1$). In this case, as demonstrated by analytical and numerical calculations, the vortex field can have both positive and negative values.[3] This inversion of the longitudinal vortex field $h(\mathbf{x})$ will produce a special type of structure, vortex chains.

The nucleation threshold for isolated vortex chains H_{ch} lies below the nucleation threshold of individual vortices, $H_{ch}(\theta) < H_{c1}(\theta)$. A numerical calculation of this field as well as the vortex chain parameters $a_{ch}(\theta)$ (the distances between vortices) and $\gamma_{ch}(\theta)$ (the tilt angles of the vortices) is carried out in Ref. 3, with the results from this paper shown in Fig. 5. The curves in Fig. 5 demonstrate the existence of magnetization hysteresis in the range of small tilt angles θ of field **H**. Here, the functions $H_{ch}(\theta)$, $\gamma_{ch}(\theta)$, and $a_{ch}(\theta)$ are ambiguous and change suddenly and irreversibly as θ varies.

2. The only nonzero moduli for an isolated vortex chain are the tilt moduli T_{11} and T_{22}, the compression C_{2222} and shear C_{1212} moduli,[3)] and the tilt-compression D_{122} and shear-compression D_{212} moduli.

The tilt moduli are expressed through the equilibrium parameters $\gamma_{ch}(\theta)$ and $H_{ch}(\theta)$ of an isolated chain shown in Fig. 5:

$$T_{11}=\frac{BH_{ch}}{4\pi}\frac{1}{\cos(\gamma_{ch}-\theta)}\left(\frac{d\gamma_{ch}}{d\theta}\right)^{-1},$$
$$T_{22}=\frac{BH_{ch}}{4\pi}\frac{\sin\theta}{\sin\gamma_{ch}}. \qquad (44)$$

It follows from these expressions that only solutions with a positive slope of the relation $\gamma_{ch}(\theta)$ and negative signs of γ_{ch} and θ (**v** and **H** lie on the same side of the axis of anisotropy) are stable against deviations of **v** from equilibrium. This is a significant fact for strongly anisotropic superconductors such as Bi–Sr–Ca–Cu–O superconductors in which the ambiguity domain of the parameters γ_{ch} also encompasses the negative angles θ (Ref. 3).[4)]

The dependence of the elastic moduli C and D on the orientation θ of the field **H** has been obtained numerically and is shown in Fig. 6. The moduli $C_{2222}(\theta)$ and $C_{1212}(\theta)$ are always positive and determine the chain stability relative to its compression and rotation in the basal plane. Moreover, on the entire interval of angles θ the minor of rank two

$$T_{22}C_{1212}-D_{212}^2>0. \qquad (45)$$

is positive. The other minor

$$T_{11}C_{2222}-D_{122}^2, \qquad (46)$$

can reverse sign in the magnetization hysteresis domain due to the modulus T_{11}, which changes sign. The modulus T_{11} is exponentially greater than the moduli $C_{2222}\sim D_{122}$ for small values of the induction B. Hence the minor (46) becomes negative in the immediate vicinity of the points where $d\gamma_{ch}/d\theta\to\infty$. Hence the angles θ where the minor (46) vanishes are nearly identical to the boundaries of the hysteresis

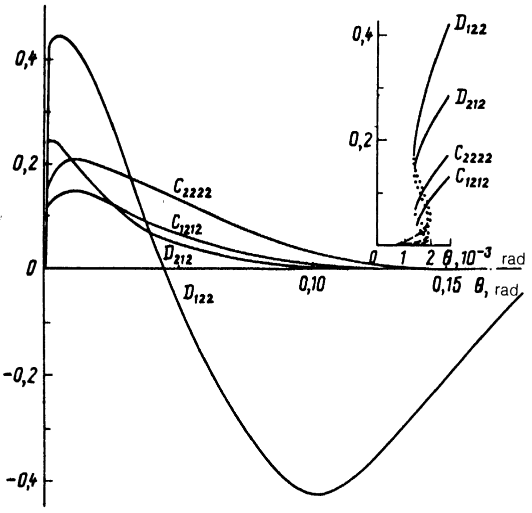

FIG. 6. Angular dependence of the elastic moduli $\hat{C}$ and $\hat{D}$ of an isolated vortex chain in a superconductor with $\mu_a=0.25$. The moduli are listed in units of $B\Phi_0(2\pi\lambda)^{-2}$. The solid line, primes, and dashed line correspond to the same states as in Fig. 5.

domain cited in Ref. 3. Within this domain two equilibrium vortex chains with different structures and nucleation field magnitudes $H_{ch}(\theta)$ correspond to each value of angle θ. The chain with the lower value of H_{ch} has the lower energy.

3. As the field **H** rises slightly above the threshold value H_{ch} in the superconductor it gives rise to a two-dimensional structure of converging vortex chains: vortex rows. The weak interaction between chains produces an exponential change in the structure of each chain. The parameters $a(\mathbf{H})$ and $\gamma(\mathbf{H})$ have only small corrections distinguishing them from the parameters of an isolated chain:[3]

$$a(\mathbf{H})=a_{ch}(\theta)-a_{ch}(\theta)\frac{4B[H-H_{ch}(\theta)]}{\pi C_{2222}(\theta)}\cos(\gamma_{ch}-\theta),$$

$$\gamma(\mathbf{H})=\gamma_{ch}(\theta)-\frac{B[H-H_{ch}(\theta)]}{4\pi T_{11}(\theta)}\sin(\gamma_{ch}-\theta).$$

Clearly the intervortex distance in the chain decreases little with increasing H, while the angle $\gamma(\mathbf{H})$ changes only at the boundaries of the hysteresis domain where $T_{11}\to 0$.

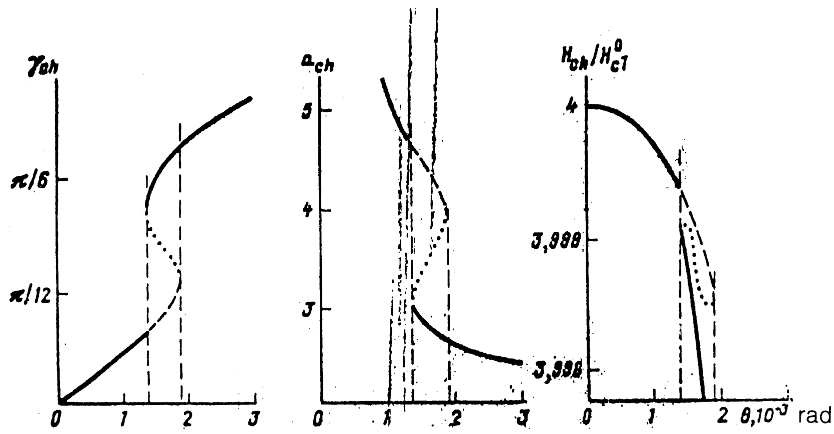

FIG. 5. The parameters a_{ch}, γ_{ch}, H_{ch} versus the angle θ for a vortex chain in a superconductor with $\mu_a=0.25$. The curve regions labeled by the dashed line correspond to unstable solutions. The states represented by the solid line result for $\theta=$ const and $H=H_{ch}(\theta)$, while those labeled by the dashed line result when θ and $H=H_{ch}(\theta)$ change simultaneously.

The length of the translation vectors $A(\mathbf{H})$ is the vortex structure parameter that changes most rapidly with the field $\mathbf{H}$ (see Fig. 1)

$$A(\mathbf{H}) = \frac{\lambda}{\mu_a} \ln\left[\frac{H_{c1}^0}{H - H_{ch}(\theta)}\right],$$

thereby producing a sharp increase in sample induction:

$$B(\mathbf{H}) = \frac{\mu_a \Phi_0}{\lambda a_{ch}(\theta) \ln\{H_{c1}^0/[H - H_{ch}(\theta)]\}}. \quad (47)$$

4. The elastic moduli of the chain rows retain all the properties of the moduli of an isolated chain. This is due to the fact that the chain structure changes little for $H \gtrsim H_{\rm ch}$. The principal terms of the moduli T_{11}, T_{22}, C_{2222}, C_{1212}, D_{212}, and D_{122} of the chain rows coincide to the analogous moduli of an isolated chain. The remaining moduli are positive and exponentially small due to mutual chain repulsion:

$$C_{1111}, C_{1122}, C_{1221}, C_{2121}, D_{111}, D_{221} \propto \exp\left(-\frac{\mu_a \Phi_0}{\lambda B a_{ch}}\right) \propto H - H_{ch}. \quad (48)$$

The exponentially small moduli $\hat{C}$ and $\hat{D}$ have no effect on the change of sign of the elastic modulus matrix (13). Therefore, rows of chains are as stable as an isolated chain.

The elastic properties of chain rows have a number of significant anomalous properties compared to FLL's in isotropic superconductors and in "easy axis" superconductors. The elastic moduli $\hat{C}$ and $\hat{D}$ in the latter are of the same order of magnitude $\propto \exp[-(2\Phi_0/3^{1/2}\mu_a B\lambda^2)^{1/2}]$. This similarity of the moduli guarantees virtually identical rigidity of the FLL's with respect to compression and shear in any direction. Strains can be divided into two types in chain rows. Certain strains break up the vortex position in the chain. In this case, the rigidity of the chain rows is maximized and proportional to the moduli of an isolated chain. In the other case, strains alter the relative configuration of chains whose structure remains unchanged. The exponentially small interaction between chains $\propto \exp(-\mu_a \Phi_0/B\lambda a_{\rm ch})$ makes the structure very flexible relative to shear strains along the chains and compressive strains in the perpendicular direction. Overall the lattices in "easy plane" crystals will be less rigid than in "easy axis" crystals or isotropic superconductors.

The hysteresis associated with magnetization of "easy-plane" superconductors causes all elastic moduli to undergo a sudden change. The sudden changes in moduli are attributable to both changes in the internal chain structure (the insert to Fig. 6 demonstrates the change in the moduli of an isolated chain) and to changes in the sample induction [see Eq. (48)].

5. The equilibrium equations (7) formally permit solutions describing sparse "two-dimensional" FLL's for "easy-plane" superconductors. The constant field lines $h(\mathbf{x})$ within narrow ranges of directions $\mathbf{x}$ near the axis x_1 are identical to the lines of ellipses, which is responsible for the existence of these solutions. The vortices located in these regions (and not along x_2, as in the chains) make the primary contribution to the interaction in a sparse FLL. In this case we can carry out a scale coordinate transformation in the equilibrium equation (7), and the resulting solutions will be similar to the results of the preceding section. The difference $\gamma = \tan^{-1}(\mu_a^{-3}\tan\theta)$ between the vortex orientation $\mathbf{v}$ and the orientation of an isolated filament is exponentially weak in sparse FLL's. FLL's can only exist in fields of intensity greater than $H_{c1}(\theta)$, while the angle Ξ is independent of H:

$$\Xi = 2\,\mathrm{arctg}\,[(\cos^2\gamma + \mu_a^3 \sin^2\gamma)^{-1/2}], \quad (49a)$$

$$\Xi = 2\,\mathrm{arctg}\,[3^{-1/2}(\cos^2\gamma + \mu_a^3 \sin^2\gamma)^{-1/2}] \quad (49b)$$

However, both of these solutions are unstable. An FLL with Ξ taken from (49a) collapses under compressional deformation:

$$\Delta_c \approx -\frac{B\Phi_0^3}{4\pi\lambda^6} \exp\left[-\frac{2\mu_a^{1/4}(\mu_a^6 + \mathrm{tg}^2\theta)^{1/4}}{(\mu_a^3 + \mathrm{tg}^2\theta)^{1/4}}\left(\frac{\Phi_0}{\lambda^2 B}\right)^{1/2}\right]. \quad (50)$$

Solution (49b) is unstable against shear deformation

$$\Delta_s \approx -\mu_a^2 \sin^4\theta \cos^2\theta \frac{B^{1/2}\Phi_0^{5/2}}{(4\pi)^4\lambda^5}$$

$$\times \exp\left[-\frac{2\mu_a^{1/4}(\mu_a^6 + \mathrm{tg}^2\theta)^{1/4}}{(\mu_a^3 + \mathrm{tg}^2\theta)^{1/4}}\left(\frac{2\Phi_0}{3^{1/2}B\lambda^2}\right)^{1/2}\right]. \quad (51)$$

6. The only stable solution of equilibrium equations (7) in "easy plane" superconductors is a vortex chain structure. Isolated vortex chains appear in a field $H_{\rm ch}(\theta) < H_{c1}(\theta)$. The chains merge as the field increases, and transform into a two-dimensional lattice. When the field $\mathbf{H}$ is tilted relative to the symmetrical direction ($\|\mathbf{c}$ or $\perp\mathbf{c}$) or as μ_a tends toward unity, the field inversion domain goes to infinity from the vortex core, the chains become increasingly sparse, $H_{\rm ch}(\theta) \to H_{c1}(\theta)$, and we obtain in the limit the vortex structure described in Sec. 2 (for $\mu_a = 1$) or Sec. 3 ($\theta = 0, \pi/2$).

6. CONCLUSION

1. The FLL structure in anisotropic superconductors is essentially different from the hexagonal structure in the isotropic case. The vortex orientation $\mathbf{v}$ does not coincide with the direction of magnetic field $\mathbf{H}$, but rather the anisotropy of the vortex field $h(\mathbf{x})$ in the basal plane produces oblique FLL's and vortex chains. A regular FLL is given by six parameters in equilibrium: two translation vectors and two angles determining vortex orientation. The stability of the vortex structures is reflected by the positive definiteness of the elastic modulus matrix. Generally an FLL is characterized by 21 independent elastic moduli (there are three such moduli in an isotropic superconductor). In addition to the known tilt $\hat{T}$ and compression and shear $\hat{C}$ moduli there also exist mixed moduli $\hat{D}$. Such moduli describe the "tilt-shear" and "tilt-compression" coupled fluctuations. Moduli $\hat{D}$ vanish when $\mathbf{H}$ is oriented along the axes of symmetry of the crystal.

2. Vortex structure degradation occurs when the superconductor is magnetized along the crystal axis. The continuum of FLL's that continuously transform into one another as the unit cell rotates relative to the axes of symmetry is stable. Such FLL's have identical moduli $\hat{C}$, moduli $\hat{D} \equiv 0$ and different moduli $\hat{T}$. The moduli $\hat{T}$ change in antiphase as the FLL's transform into one another.

3. FLL degeneration is eliminated when $\mathbf{H}$ deviates from the crystal axis. Vortex structures with orthorhombic symmetry are realized in uniaxial superconductors. The axis of anisotropy $\mathbf{c}$, the vortex axis ν, and the external field $\mathbf{H}$ lie in the same plane. This is the mirror symmetry plane for the FLL. Only 12 of the elastic moduli are independent and nonzero. They form a matrix with two nonintersecting blocks. The moduli in one block describe the compressive strains and tilt of the vortices to the **c** axis, which conserve the orthorhombic symmetry of the FLL's. The moduli of the other block describe the coupled "tilt-shear" strains that break orthorhombic symmetry. The FLL structure and the values of the elastic moduli are significantly different in "easy-axis" and "easy-plane" uniaxial superconductors.

4. Sparse hexagonal FLL's begin to form in "easy axis" superconductors from individual vortices in a field $H = H_{c1}(\theta)$. The moduli $\hat{C}$ and $\hat{D}$ of such FLL's are small compared to the tilt moduli $T_{ii} = (BH_{c1}/4\pi)(\mu_{ii}/\mu_{33})$ and are comparable in magnitude. A hexagonal FLL experiences severe angular distortions when field **H** is "applied" along the *ab*-plane. The angle between the translation vectors jumps sharply from $\pi/3$ to $2\tan^{-1}[(3\mu_a^3)^{-1/2}]$. Such changes occur within close proximity to the tilt angles θ of field **H** to the *ab* plane, $\Delta\theta \propto [H/H_{c1}(\pi/2) - 1]$, and are accompanied by a sudden increase by a factor of $\mu_a^{3/2}$ in the induction and tilt moduli. The moduli $\hat{C}$ rise by a factor of $[H/H_{c1}(\pi/2) - 1]^{-2}$. A hexagonal FLL transforms into an oblique FLL with growing magnetic field. The long diagonal of the orthorhombic unit cell is always oriented along the projection of the axis of symmetry. The limit of the angle between the translation vectors in a strong field $H \gg H_{c1}$ is equal to

$$\Xi = 2\,\mathrm{arctg}\,[3^{-1/2}(\cos^2\theta + \mu_a^3\sin^2\theta)^{-1/2}] \qquad (52)$$

(see the dashed line in Fig. 4).

5. Vortex chains are stable structures in "easy-plane" superconductors. Isolated chains appear in a field $H_{ch}(\theta)$ of lower intensity compared to the nucleation field of individual vortices $H_{c1}(\theta)$. The intervortex distance in the chain is on the order of λ, and hence the moduli T_{11}, T_{22}, C_{2222}, C_{1212}, D_{122}, and D_{212} are finite. The chain is rigid with respect to structural strains. The distance between vortex chains drops off rapidly with increasing magnetic field. Such chains form a regular FLL. Its rhombic cell is oriented so that the short diagonal of the rhombus lies along the projection of the axis of symmetry. In a strong magnetic field $H \gg H_{c1}$ the limit value of the angle Ξ in the unit cell is equal to[9]

$$\Xi = 2\,\mathrm{arctg}\,[3^{1/2}(\cos^2\theta + \mu_a^3\sin^2\theta)^{-1/2}]. \qquad (53)$$

Elastic shear moduli $\hat{C}$ and mixed tilt-shear moduli are manifested in a vortex chain structure. New moduli appear due to the weak interchain interaction and grow with the field as $H/H_{ch}(\theta) - 1$. When **H** is oriented near **c**, the superconductor is irreversibly magnetized. There is a range of angles θ between **H** and **c** where two vortex structures are stable. When **H** is applied at an angle, there is a sudden, irreversible rearrangement of the vortex configuration. The hysteresis domain shrinks with increasing induction or growth of the parameter μ_a.

6. The structure and properties of dense FLL's ($n_L\lambda^2 \gg 1$) can be analyzed using the same method employed by V. Kogan,[9,17,13] for a uniaxial crystal with $\mu_a < 1$. Only the main results will be enumerated here.

The angle between the translation vectors in dense FLL's is independent of the induction and determined by relation (52) for $\mu_a > 1$ and Eq. (53) for $\mu_a < 1$. As in sparse lattices, the short (long) diagonal of the unit cell is oriented parallel to the axis of symmetry of the crystal with $\mu_a < 1 (\mu_a > 1)$. In the primary approximation in the parameter $1/n_L\lambda^2 \ll 1$, $\mathbf{B} = \mathbf{H}$, and the vortices are oriented along the magnetic field.

Unlike sparse vortex structures the expressions for the elastic moduli of dense FLL's are identical for superconductors with $\mu_a > 1$ and $\mu_a < 1$. As in an isotropic crystal, the tilt $\hat{T}$ and compression C_{1111}, C_{2222}, and C_{1122} moduli are equal,

$$T = C = H^2/4\pi. \qquad (54)$$

The shear moduli can be rewritten by a scale transformation through the moduli of an isotropic crystal[13]

$$C_{1221} = (\mu_a^{-2}\cos^2\theta + \mu_a\sin^2\theta)^{1/2}\frac{H\Phi_0}{64\pi^2\lambda^2},$$
$$C_{1212} = (\cos^2\theta + \mu_a^3\sin^2\theta)C_{1221},$$
$$C_{2121} = (\cos^2\theta + \mu_a^3\sin^2\theta)^{-1}C_{1221}. \qquad (55)$$

The moduli $\hat{D}$ can be written analogously as

$$D_{122} = \frac{(1-\mu_a^3)\sin\theta\cos\theta}{\mu_a(\cos^2\theta + \mu_a^3\sin^2\theta)^{1/2}}\,\frac{H\Phi_0}{32\pi^2\lambda^2},$$
$$D_{111} = 3D_{122}, \quad D_{221} = D_{212} = D_{122}. \qquad (56)$$

The dependence of the elastic moduli (54)–(56) on the angle θ is smooth and reversible.

The most evident effects of the sudden change in the vortex structure and its elastic moduli is observed in threshold magnetic fields $H_{ch}(\theta)$ for crystals with $\mu_a < 1$ and $H_{c1}(\theta)$ for crystals with $\mu_a > 1$.

1) The easy axis crystals include the superconductor sodium polysulfide $(SN)_x$ (Refs. 15,16) consisting of molecular chains.

2) "Easy plane" superconductors (also called layered superconductors) include all existing metal oxide superconducting compounds.

3) For an isolated chain the modulus C_{1212} describes rigidity relative to chain deviations from the projection of the axis of symmetry in the basal plane.

4 The modulus T_{22} was not calculated in Ref. 3 and hence it was incorrectly concluded that chains with vortices tilted relative to the axis of symmetry in a direction opposite that of the magnetic field are stable.

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