Photoproduction of Wannier–Mott excitons and resonance deflection of electromagnetic waves by a magnetic field in a semiconductor crystal

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With the help of the dispersion relation for electromagnetic waves in a semiconductor crystal in a strong magnetic field, taking into account the production of Wannier–Mott excitons, we show that under certain conditions photon trapping and scattering by a curvilinear magnetic field are possible.

1. Under certain conditions the optical properties of a medium are strongly affected by the singularities of the permittivity tensor of the medium. In Refs. 1 it was shown that in vacuum the singularities arising in the polarization operator owing to photoproduction of electron–positron pairs in a magnetic field result, in particular, in the fact that the energy of the photon (wave packet) is channeled along the lines of force of the magnetic field.

In the case of curvilinear lines of force this effect can result in "capture" of a photon (motion along lines of force of the field) and in the process the photon is converted adiabatically into a free electron–positron pair (Ref. 2) or into a positronium atom (Fig. 3). On the basis of quantum electrodynamics, such effects become important in the study of pulsar magnetospheres and there is indirect experimental evidence for them.[3,4]

Similar optical effects were found previously in Ref. 5 in a study of a semiconductor crystal in a strong magnetic field. In this case these effects appear owing to the formation of a square-root singularity in the permittivity, calculated taking into account the photoproduction of free electron–hole pairs in a magnetic field. In this paper we calculate the permittivity of such a crystal by treating the possibility of production of Wannier–Mott excitations (Coulombically bound electron–hole pairs); in this case, the singularities of the permittivity are poles. We shall discuss the deflection of the trajectory of a photon (more precisely, a polariton—the mixed state of a photon and an exciton) by a magnetic field in a crystal and the optical properties of the medium which follow hence. We specially study the possibility of photon capture by a constant curvilinear magnetic field and photon scattering by a nonuniform magnetic field.

The paper is organized as follows. In Sec. 2 the permittivity of a semiconductor crystal in a uniform external magnetic field is calculated taking into account the possibility of photoproduction of Wannier–Mott excitons. For this, we first calculate the wave function of a Wannier–Mott exciton with arbitrary momentum in a constant magnetic field. In Sec. 3 we study the dispersion equation of electromagnetic waves in an isotropic cubic semiconductor crystal in a uniform magnetic field; in this case, the dispersion equation contains poles associated with the production of Wannier–Mott excitons. We show that the poles affect the way electromagnetic waves propagate and we present a qualitative picture of the propagation of a wave packet in a nonuniform field with a special configuration. In Sec. 4 we describe in detail photon capture by a circular magnetic field and in Sec. 5 we describe photon scattering by a nonuniform field. We calculate the capture radius and the maximum penetration depth into the central region of the field. In Sec. 6 we discuss the limits on the magnitude of the magnetic field.

2. In order to determine the permittivity we first need the wave function and the self-energy of an exciton in an external magnetic field. Although the problem of calculating the wave function of an exciton (or, which is actually the same thing, the wave functions of a hydrogen atom) in an external magnetic field has been quite well studied, here we need to discuss some details, since for our purposes we cannot use the standard quantum numbers and we need the wave function of an exciton with arbitrary momentum.

In a constant and uniform magnetic field, described by the vector potential $\mathbf{A}_{ext}$, in the effective-mass approximation the exciton wave function Ψ is determined by the following Schroedinger equation:

$$\left\{-\frac{\hbar^2}{2m_1}\frac{\partial^2}{\partial \mathbf{r}_1^2}-\frac{\hbar^2}{2m_2}\frac{\partial^2}{\partial \mathbf{r}_2^2}-i\frac{e\hbar}{m_1c}\mathbf{A}_{ext}(\mathbf{r}_1)\frac{\partial}{\partial \mathbf{r}_1}+i\frac{e\hbar}{m_2c}\mathbf{A}_{ext}(\mathbf{r}_2)\frac{\partial}{\partial \mathbf{r}_2}+\frac{e^2}{2m_1c^2}\mathbf{A}_{ext}^2(\mathbf{r}_1)+\frac{e^2}{2m_2c^2}\mathbf{A}_{ext}^2(\mathbf{r}_2)-\frac{e^2}{\varkappa|\mathbf{r}_1-\mathbf{r}_2|}\right\}\psi(\mathbf{r}_1,\mathbf{r}_2)=E\psi(\mathbf{r}_1,\mathbf{r}_2). \tag{1}$$

Here m_1, $\mathbf{r}_1$ and m_2, $\mathbf{r}_2$ are the effective mass and radius vector of an electron and hole, respectively, and $\varkappa$ is the empirical value of the permittivity.

In Ref. 6 it was shown that in a constant magnetic field $\mathbf{B}=\operatorname{curl}\mathbf{A}$ it is possible to introduce a vector operator

$$\hat{\mathbf{P}}=-i\hbar\frac{\partial}{\partial \mathbf{r}_1}-i\hbar\frac{\partial}{\partial \mathbf{r}_2}+\frac{e}{c}(\mathbf{A}(\mathbf{r}_1)-\mathbf{A}(\mathbf{r}_2))-\frac{e}{c}[\mathbf{B},\mathbf{r}_1-\mathbf{r}_2], \tag{2}$$

which is the exciton momentum operator. Introducing the new variables $\mathbf{R}=(m_1\mathbf{r}_1+m_2\mathbf{r}_2)/(m_1+m_2)$ and $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$ (so that $\mathbf{R}$ is the radius vector of the center of mass of the exciton) and choosing $\mathbf{A}_{ext}(x)=[\mathbf{B},\mathbf{x}]/2$, we can rewrite the operator $\hat{\mathbf{P}}$ as

$$\hat{\mathbf{P}}=-i\hbar\frac{\partial}{\partial \mathbf{R}}-\frac{e}{2c}[\mathbf{B},\mathbf{r}]. \tag{2'}$$

We seek the solutions of Eq. (1) that are simultaneous-

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ly eigenfunctions of the operator (2′) (i.e., they describe an exciton with a definite momentum). The corresponding wave function ψ_p can be represented in the form

$$\psi_p(\mathbf{r}_1,\mathbf{r}_2)=\psi_p(\mathbf{R},\mathbf{r})=\exp\left\{\frac{i}{\hbar}\mathbf{R}\left(\mathbf{P}+\frac{e}{2c}[\mathbf{B},\mathbf{r}]\right)\right\}\tilde{\psi}_p(\mathbf{r}). \tag{3}$$

After the substitution

$$\tilde{\psi}_p(\mathbf{r})=\exp\left\{i\frac{\mu}{2\hbar\delta B^2}(\mathbf{r}[\mathbf{B}[\mathbf{P},\mathbf{B}]])\right\}\tilde{\tilde{\psi}}_p(\mathbf{r})$$

and the translation $\mathbf{r}'=\mathbf{r}+c[\mathbf{P},\mathbf{B}]/eB^2$, Eq. (1) assumes the form

$$\left\{-\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial\mathbf{r}'^2}-\frac{ie\hbar}{2\delta c}[\mathbf{B},\mathbf{r}']\frac{\partial}{\partial\mathbf{r}'}+\frac{e^2}{8\mu c^2}[\mathbf{B},\mathbf{r}']^2-\frac{e^2}{\varkappa}\left|\mathbf{r}'-\frac{c}{eB^2}[\mathbf{P},\mathbf{B}]\right|^{-1}\right\}\tilde{\tilde{\psi}}(\mathbf{r}')=\left(E-\frac{1}{2MB^2}(\mathbf{PB})^2\right)\tilde{\tilde{\psi}}_p(\mathbf{r}'),$$

$$\mu=\frac{m_1m_2}{m_1+m_2},\quad \delta=\frac{m_1m_2}{m_1-m_2},\quad M=m_1+m_2. \tag{4}$$

Now the Schroedinger equation is written in a form such that the motion of the exciton as a whole (with momentum $\mathbf{P}$) is manifested only in the shift of the energy and the Coulomb potential (note that while the dependence of the wave function and energy on the projection $P_\parallel^2=(\mathbf{P}\cdot\mathbf{B})^2/B^2$ of the momentum in the direction of the field is trivial, the dependence on the component orthogonal to the field $P_\perp^2=[\mathbf{P},\mathbf{B}]^2/B^2$ is not trivial).

The equation (4) cannot be solved exactly, and in order to find $\tilde{\tilde{\psi}}_p(\mathbf{r}')$ we proceed as follows. We assume that the field $\mathbf{B}$ is oriented along the z axis, and we transfer to a cylindrical coordinate system $\mathbf{r}=(\rho,\varphi,z)$, where $\rho^2=x^2+y^2$. We now represent the wave function $\tilde{\tilde{\psi}}$ as an expansion in the functions $\eta_{n_1n_2}(\rho,\varphi)$, which are the z-independent, where $z=z_1-z_2$, solutions of Eq. (4) without the Coulomb term:[7,8]

$$V^p=-\frac{e^2}{\varkappa}\left\{z^2+\left[\boldsymbol{\rho}-[\mathbf{P},\mathbf{B}]\frac{c}{eB^2}\right]^2\right\}^{-1/2}$$

[i.e., the functions $\eta_{n_1n_2}(\rho,\varphi)$ describe a pair of noninteracting charged particles in an external magnetic field in the center-of-mass system]:

$$\tilde{\tilde{\psi}}_p(\rho,\varphi,z)=\sum_{n_1,n_2=0}^{\infty}\eta_{n_1n_2}(\rho,\varphi)\varphi^p_{n_1n_2}(z), \tag{5}$$

$$\eta_{n_1n_2}(\rho,\varphi)=\left(\frac{n_2!}{2\pi L^2 n_1!}\right)^{1/2}\exp(i(n_1-n_2)\varphi)\times\exp\left(-\frac{\rho^2}{4L^2}\right)\left(\frac{\rho^2}{2L^2}\right)^{(n_1-n_2)/2}L_{n_2}^{n_1-n_2}\left(\frac{\rho}{2L^2}\right). \tag{6}$$

Here $L=(\hbar c/eB)^{1/2}$ is the Larmor radius, n_1 and n_2 enumerate the Landau levels of the electron and hole, respectively, and $L_{n_2}^{n_1-n_2}(x)$ are Laguerre polynomials (for $n_1-n_2<0$, as usual, an analytic continuation must be made). The functions $\eta_{n_1n_2}$ form a complete orthonormal basis.

Substituting the expansion (5) and (6) into Eq. (4), left-multiplying the obtained equation by $\eta^*_{n_1n_2}(\rho,\varphi)$, and integrating over ρ and φ, we obtain the following equation for the expansion coefficients $\varphi_{n_1n_2}(z)$:

$$\sum_{n_1n_2}\left\{\left(E-\frac{eB\hbar}{m_1c}(n_1+{}^1/_2)-\frac{eB\hbar}{m_2c}(n_2+{}^1/_2)-\frac{P_\parallel^2}{2M}+\frac{\hbar^2}{2\mu}\frac{d^2}{dz^2}\right)\delta_{n_1\bar{n}_1}\delta_{n_2\bar{n}_2}-\frac{1}{2\pi}\left(\frac{n_2!\,\bar{n}_2!}{n_1!\,\bar{n}_1!}\right)^{1/2}\int_0^{2\pi}d\varphi\int_0^{\infty}d\lambda\exp[i(n_1-n_2-\bar{n}_1+\bar{n}_2)\varphi]e^{-\lambda}\times\lambda^{(n_1-n_2-\bar{n}_1+\bar{n}_2)/2}L_{n_2}^{n_1-n_2}(\lambda)L_{\bar{n}_2}^{\bar{n}_1-\bar{n}_2}(\lambda)V^p(\rho,\varphi,z)\right\}\varphi^p_{n_1n_2}(z)=0,$$

$$P_\perp^2=P_x^2+P_y^2,\quad P_\parallel^2=P_z^2,\quad \lambda=\frac{\rho^2}{2L^2}. \tag{7}$$

The integration in Eq. (7) is effectively limited to the region $0<\lambda\lesssim 1$. If the condition

$$z^2+P_\perp^2L^4\hbar^{-2}\gg L^2 \tag{8}$$

is satisfied, then it makes sense to expand the potential $V^p(\rho,\varphi,z)$ in Eq. (7) in a series in powers of $\rho(z^2+L^4p_\perp^2\hbar^{-2})^{-1/2}$. In the zeroth order of the expansion the potential does not depend on ρ and φ and the integration in Eq. (7) is trivial:

$$\sum_{n_1n_2}\left\{\tilde{E}+\frac{\hbar^2}{2\mu}\frac{d^2}{dz^2}+\frac{e^2}{\varkappa}\frac{1}{(z^2+P_\perp^2L^4\hbar^{-2})^{1/2}}\right\}\varphi_{n_1n_2}(z)\delta_{n_1\bar{n}_1}\delta_{n_2\bar{n}_2}=0, \tag{9}$$

where $\tilde{E}$ is the Coulomb binding energy:

$$\tilde{E}=E-\frac{eB\hbar}{m_1c}(n_1+{}^1/_2)-\frac{eB\hbar}{m_2c}(n_2+{}^1/_2)-\frac{P_\parallel^2}{2M}. \tag{10}$$

Carrying out the obvious summation in Eq. (9) we can see that the system of equations (7) has been reduced to a decoupled form, and each term in the expansion (5) becomes a particular solution of Eq. (4) with a definite momentum. It also follows from Eq. (9) that in the approximation employed the one-dimensional functions $\varphi_{n_1n_2}(z)$ do not depend on n_1 and n_2, so that in what follows we shall omit these indices from $\varphi(z)$.

We now return to the condition (8). It is easy to see that for $P_\perp L^2\hbar^{-2}\gg 1$ the potential can be expanded in a rapidly converging series for any value of z. If, however, this condition is not satisfied, the inequality (8) holds only for $z^2\gg\tilde{z}^2$, where $\tilde{z}^2=\max\{0,L^2(1-P_\perp^2L^2\hbar^{-2}\}$, and it is not satisfied for $z^2<\tilde{z}^2$. Nonetheless, if $\tilde{z}$ is much less than the Bohr radius $a_B=\varkappa\hbar^2/\mu e^2$, Eq. (9) holds almost in the entire region where the wave function is concentrated and we can extrapolate Eq. (9) into the region $z\lesssim\tilde{z}$ [although strictly speaking Eq. (9) cannot be employed in so doing, this assumption is considered to be satisfactory[10,11]. If, however, the inequality $\tilde{z}\ll a_B$ is not satisfied, then Eq. (9) describes only the peripheral region. [Note that the condition $\tilde{z}\ll a_B$ can also be interpreted differently: It means that the Coulomb binding energy of the exciton is small compared with the cyclotron precession energy of the particle and hole. The condition (8) is an extension of the adiabatic condition $L^2\ll a_B^2$ to the case $P_\perp\neq 0$ (Refs. 10 and 11).]

Now, solving Eq. (9) similarly to the manner in which this was done in Refs. 9 and 12, in the region of the discrete spectrum, described by the quantum number n_c, $n_c = 0,1,2,...$, we obtain the following values of the wave function $\varphi(z)$ at the point $z = 0$, which are the only wave functions we require below:

$$|\varphi_0(z=0)|^2 = \frac{2}{a_B} \ln \frac{a_B}{2P_\perp L^2 \hbar^{-1}},$$

$$|\varphi_{n_c}(z=0)|^2 = \frac{2}{n_c^3 a_B^3} P_\perp^2 L^4 \hbar^{-2}, \quad n_c = 1, 2, \ldots \tag{11}$$

The binding-energy eigenvalues are

$$\bar{E}_0 = -\frac{\alpha\mu c^2}{2\varkappa^2}\left(2\ln\frac{a_B}{2L^2P_\perp\hbar^{-1}}\right)^2,$$

$$\bar{E}_{n_c} = -\frac{\alpha^2\mu c^2}{2\varkappa^2}\left[n_c + \left(\ln\frac{a_B n_c}{2L^2P_\perp\hbar^{-1}}\right)^{-1}\right]^{-2}, \quad n_c = 1, 2, \ldots, \tag{12}$$

where $\alpha = e^2/\hbar c$ is the fine structure constant.

In the calculation of Eqs. (11) and (12) it was assumed that $\ln(a_B/L^2P\hbar^{-1}) \gg 1$. Moreover, if $L^2P_\perp^2\hbar^{-2} \ll 1$ holds, then regularization is required; this consists of replacing the term $L^2P_\perp\hbar^{-1}$ in Eqs. (11) and (12) by $(L^2 + L^4P_\perp^2\hbar^{-2})$ (Ref. 9).

We now evaluate the permittivity tensor. Using the standard procedure,[13] we can write the general expression

$$\varepsilon_{ij}(\omega, \mathbf{k}) = \varepsilon_0\delta_{ij} - \frac{16\pi}{\hbar V E_0^2}\sum_l \frac{F_i^{(l)}(-\mathbf{k})F_j^{(l)}(\mathbf{k})}{\omega - \omega^{(l)} + i0}. \tag{13}$$

Here V is the volume of the crystal, l is the set of quantum numbers (defined below), $\omega^{(l)}$ is the frequency of the transition from the valence band into the conduction band, ε_0 is the background permittivity, and the vector potential $\mathbf{A}(\mathbf{r},t)$ of the perturbing field is taken in the form

$$\mathbf{A}(\mathbf{r}, t) = \frac{\mathbf{E}_0}{\omega}\sin(\omega t - \mathbf{kr}).$$

For $F_i^{(l)}(\mathbf{k})$ we must take the function

$$F_i^{(l)}(\mathbf{k}) = \frac{eE_0}{2i\omega m}\int U_{10}^*(\mathbf{R})\left(-i\hbar\frac{\partial}{\partial \mathbf{R}_i} + \frac{e}{c}A_i^{ext}(\mathbf{R})\right) \times \exp(-i\mathbf{kR})\psi^p_{n_1 n_2 n_c}(\mathbf{R})U_{20}(\mathbf{R})d\mathbf{R}, \tag{14}$$

which is the amplitude for photoproduction of a Wannier–Mott exciton (generally speaking, not lying on the mass shell, $\omega^2/c^2 \neq \mathbf{k}^2$).

In Eq. (14) $U_{10}(\mathbf{R})$ and $U_{20}(\mathbf{R})$ are Bloch functions of the edges of the valence and conduction bands, $l = (n_1, n_2, n_c, \mathbf{P})$, m is the electron mass, and $\psi^p_{n_1 n_2 n_c}(\mathbf{R})$ is the wave function of an exciton with momentum $\mathbf{P}$, taken at $\mathbf{r}_1 = \mathbf{r}_2 (\mathbf{r} = 0)$:

$$\psi^p_{n_1 n_2 n_c}(\mathbf{R}) = \exp\left(\frac{i}{\hbar}\mathrm{RP}\right)\eta_{n_1 n_2}(\rho = 0, \varphi)\varphi^p_{n_c^*}(z=0) \tag{15}$$

[see Eqs. (6) and (11)]. In the integration in Eq. (14) there appears a Kronecker delta function $\delta_{k\hbar,P}$, which relates, via the law of conservation of momentum, the momentum of the exciton with the momentum of the photon.

We confine our attention to cubic isotropic crystals. Although it is not obvious, the permittivity tensor can be approximately represented in the form $\varepsilon_{ij}(\omega,\mathbf{k}) = \delta_{ij}\varepsilon(\omega,\mathbf{k})$ even in the presence of an external field.

We clarify this. Since $\int_{V_0} U_{10}^*(\mathbf{R})U_{20}(\mathbf{R})d\mathbf{R} = 0$, where V_0 is the unit-cell volume, and the field $\mathbf{A}_{ext}(\mathbf{R})$ does not vary much within the cell, the contribution of the term proportional to $\mathbf{A}_{ext}(\mathbf{R})$ in the expression (14) can be neglected. Then the second term in Eq. (13) will be proportional to q_iq_j, (Ref. 5), where

$$q_i = \int_{V_0} U_{10}^*(\mathbf{R})\left(-i\hbar\frac{\partial}{\partial R_i}\right)U_{20}(\mathbf{R})d\mathbf{R}.$$

But in an isotropic cubic crystal the tensor q_iq_j is proportional to δ_{ij} (see Ref. 13, Sec. 13).

This approach is different from the common approach, when spatial dispersion is related primarily to the nontrivial tensor structure of ε_{ij}. Here, however, spatial dispersion is manifested in the fact that the scalar part of ε_{ij} depends not on $|\mathbf{k}|$ but rather on two components of the momentum—parallel and orthogonal to the field: $\varepsilon(\omega,\mathbf{k}) = \varepsilon(\omega, k_\parallel^2, k_\perp^2)$.

This approximation is sufficient for our purposes. If, however, one attempts to take into account the tensor structure of ε_{ij} more accurately, then the results presented below will be valid for each mode of the electromagnetic wave.

Finally, for $\varepsilon(\omega,\mathbf{k})$ we obtain the following expression:

$$\varepsilon(\omega,\mathbf{k}) = \varepsilon_0 - \frac{4e^2E_g}{\mu\omega^2L^2}\sum_{n_1n_2n_c}\left(\frac{n_2!}{n_1!}\right)\left(\frac{k_\perp^2L^2}{2}\right)^{n_1-n_2} \times \exp\left(-\frac{k_\perp^2L^2}{2}\right)\left\{L_{n_2}^{n_1-n_2}\left(\frac{k_\perp^2L^2}{2}\right)\varphi_{n_c}(0)\right\}^2 (\omega\hbar - \omega_{n_1n_2n_c}(k)\hbar + i\cdot 0)^{-1}. \tag{16}$$

The energy $\hbar\omega_{n_1n_2n_c}(\mathbf{k})$ of a transition from the valence band into the conduction band is equal to

$$\hbar\omega_{n_1n_2n_c}(\mathbf{k}) = E_g + \frac{eB\hbar}{m_1c}(n_1 + {}^1/_2) + \frac{eB\hbar}{m_2c}(n_2 + {}^1/_2) + \frac{\hbar^2k_\parallel^2}{2M}$$

$$-\bar{E}_{n_c}(\hbar^2k_\perp^2) = \hbar\bar{\omega}_{n_1n_2n_c}(k_\perp^2) + \frac{\hbar^2k_\parallel^2}{2M}, \tag{17}$$

where E_g is the energy gap between the valence and conduction bands in the absence of a field.

Thus taking into account the Coulomb interaction between the electron and the hole has resulted in a singularity stronger than the square-root singularity obtained in the case of photoproduction of a noninteracting pair.[5] The pole character of the singularity is explained by the appearance of the bound state, which has a smaller number of degrees of freedom than a free (noninteracting) pair.

3. In the case $\varepsilon_{ij}(\omega,k) = \delta_{ij}\varepsilon(\omega,\mathbf{k})$, the dispersion equation assumes the form $\mathbf{k}^2 = (\omega^2/c^2)\varepsilon(\omega,\mathbf{k})$. Near a resonance, described by the quantum numbers n_1, n_2, and n_c [i.e., when the corresponding term dominates in the sum on the right-hand side of Eq. (13)], we have

$$\frac{\varkappa\omega^2}{c^2} - \mathbf{k}^2 = \frac{\lambda_{n_1n_2n_c}(k_\perp^2)}{\omega\hbar - \omega_{n_1n_2n_c}(k)\hbar + i\cdot 0}, \tag{18}$$

$$\lambda_{n_1n_2n_c}(k_\perp^2) = \frac{4e^2E_g}{L^2\mu c^2}\left(\frac{n_2!}{n_1!}\right)\left(\frac{k_\perp^2L^2}{2}\right)^{n_1-n_2}$$

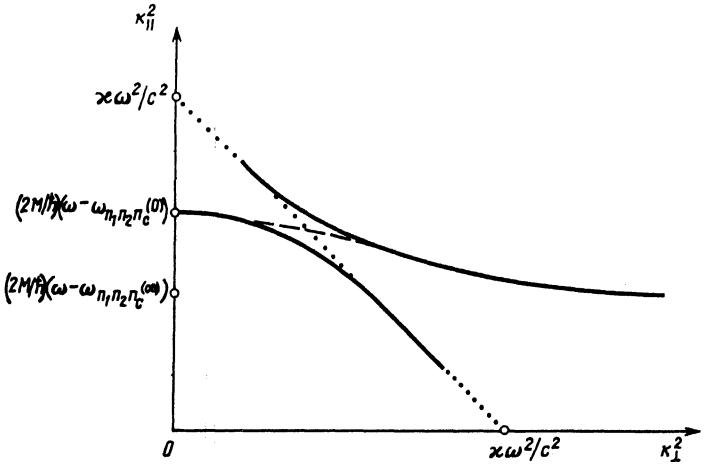

FIG. 1. Dispersion curves of a polariton (solid lines) in a neighborhood of a separate resonance $n_1n_2n_c$ in the $(k_\perp^2,k_\parallel^2)$ plane. The dashed and dotted lines represent the dispersion curves of a bare exciton and a bare photon, respectively.

$$\times\exp\left(-\frac{k_\perp^2L^2}{2}\right)|\varphi_{n_c}(0)|^2\left(L_{n_2}^{n_1-n_2}\left(\frac{k_\perp^2L^2}{2}\right)\right)^2, \qquad (19)$$

where $\varkappa$ is the background permittivity, which includes all terms on the right-hand side of Eq. (13) (including ε_0), except for the singular term on the right-hand side of Eq. (18). We assume that $\varkappa$ has the same value as in the expression for the Coulomb binding energy. The equation (18) describes a polariton (or "photon in a medium")—a mixed state of a photon and an exciton.

Note that a separate resonance $n_1n_2n_c$ can be studied in Eq. (13) only if n_c is sufficiently small—it is evident from Eq. (12) that as n_c increases the bound states become more dense $[E_{n_c+1}-E_n\to 0]$ and at some value $n_c=\tilde{n}_c$ a quasicontinuum forms.[3,9]

Figure 1 shows the solutions of Eq. (18) in the $(k_\perp^2,k_\parallel^2)$ plane, $\omega=$ const, which we shall call the dispersion curves of the polariton. On each branch it is possible to distinguish a region where the dispersion curve is close to that of the bare photon, $\varkappa\omega^2/c^2-k_\perp^2-k_\parallel^2=0$ (photon-dominant region), and a region close to the dispersion curve of a bare exciton, $\omega=\omega_{n_1n_2n_c}(k)=\omega_{n_1n_2n_c}(k_\perp^2,k_\parallel^2)$ (exciton-dominated region).

If the group velocity $V_\parallel^{gr}$ of the polariton is calculated along the lines of force and in a direction orthogonal to the field, then it turns out that in the exciton-dominant region (as $k_\perp\to\infty$ for the upper branch and $k_\perp\to 0$ for the lower branch) the following relation holds:

$$V_\parallel^{gr}=\left(\frac{\partial\omega}{\partial k_\parallel}\right)_{k_\perp}=\frac{\hbar}{M}k_\parallel, \quad V_\perp^{gr}=\left(\frac{\partial\omega}{\partial k_\perp}\right)_{k_\parallel}\to 0 \qquad (20)$$

(the complete expression is given in Ref. 9a), i.e., in the vicinity of a resonance a polariton moves along the lines of the magnetic field (this is completely natural, since near resonance the polariton is in the exciton-dominant state and consists of a pair of charged particles). It also follows from Eq. (20) that the momentum $\mathbf{k}$ of the polariton does not always indicate the true direction of propagation of the polariton. Such a situation, when the phase velocity is different from the group velocity, can arise if a photon propagating in a magnetic field comes into resonance, i.e., $(\omega-\tilde{\omega}_{n_1n_2n_c}(k_\perp^2)-\hbar k_\parallel^2/2M)$ becomes small. In the process, the trajectory of the photon bends along the field lines. We illustrate this for the following example.

We introduce a constant magnetic field $\mathbf{B}(\mathbf{r})$, where $|\mathbf{B}(\mathbf{r})|=B=$ const, for which the lines of force form concentric circles and the z axis is the symmetry axis (it is sufficient that these conditions hold in a bounded region of the crystal). A semiconductor placed in such a field is, on the basis of the geometric-optics approximation, a medium with spatially variable permittivity. The subsequent analysis is performed in this approximation (it is admissible if the magnitude of the magnetic field satisfies certain restrictions).

We now study a polariton propagating in a plane orthogonal to the z axis in a semiconductor crystal in such a field (in view of the symmetry of the problem k_z remains equal to zero and the projections $k_\parallel$ and $k_\perp$ of the momentum, which lie in this plane, vary with time). If the distance between the center of the wave packet and the z axis increases, the projection $k_\perp$ of the momentum increases and the point on the upper branch of the dispersion curve (see Fig. 1) shifts to the right and down along this branch.

Suppose that initially the polariton was in the photon-dominant state (upper left-hand part of the curve). With time, instead of reaching in momentum space the point $k_\parallel^2=0$, $k_\perp^2=\varkappa\omega^2/c^2$ (as would happen if the polariton evolved along the dispersion curve of the bare photon $\varkappa\omega^2/c^2-k_\parallel^2=k_\perp^2$ in momentum space and correspondingly propagated rectilinearly in configuration space), it shifts along the top branch of the dispersion curve in Fig. 1 to the exciton-dominant region $k_\perp^2\to\infty$, $k_\parallel^2\to(2M/\hbar)[\omega-\tilde{\omega}_{n_1n_2n_cc}(\infty)]$. This process can be interpreted as trapping of a photon by a curvilinear magnetic field: to the extent that the photon moves adiabatically, it transforms, via the formation of a mixed state, into an exciton and at the same time its trajectory bends along the lines of force of the field (since $V_\perp\to 0$ and $V_\parallel\to$ const).

4. We now give a quantitative description of this process. In the geometric-optics approximation the differential equation describing refraction, as also the equation of motion of a wave packet, is the same as in vacuum:[2,3]

$$dk_\perp=k_\parallel d\varphi, \quad dr_{gr}=V_\perp^{gr}dt, \quad r_{gr}d\varphi=V_\parallel^{gr}dt. \qquad (21)$$

Here r_{gr} and φ are the polar coordinates of the center of the

wave packet, and the origin of the coordinates is also the center of the lines of force.

After simple substitutions into Eq. (21) we obtain first

$$dr_{gr}=\frac{r_{gr}V_{\perp}^{gr}}{V_{\parallel}^{gr}k_{\parallel}}dk_{\perp}.$$

(Because of the definition $V_{\perp}^{gr}=(\partial\omega/\partial k_{\perp})_{k_{\parallel}}$, $V_{\parallel}^{gr}=(\partial\omega/\partial k_{\parallel})_{k_{\perp}}$, this equation can be rewritten as $r_{gr}dk_{\parallel}+k_{\parallel}dr_{gr}=0$, which expresses conservation of angular momentum.)

Now, using Eq. (15) we obtain the following solution of the system (21):

$$\begin{aligned}
&r_{gr}(k_{\perp}^{2})=r_{gr}^{(0)}\frac{k_{\parallel}^{(0)}}{k_{\parallel}(k_{\perp}^{2})},\\
&\varphi(k_{\perp}^{2})=\int_{k_{\perp}^{(0)}}^{k_{\perp}}\frac{d\chi}{k_{\parallel}(\chi^{2})},\\
&t(k_{\perp}^{2})=\frac{\varkappa\omega r_{gr}^{(0)}k_{\parallel}^{(0)}}{c^{2}}\int_{k_{\perp}^{(0)}}^{k_{\perp}}\frac{d\chi}{k_{\parallel}^{3}(\chi^{2})}\\
&\times\frac{1+\frac{c^{2}}{2\varkappa\omega}\left(\frac{\varkappa\omega^{2}}{c^{2}}-k_{\parallel}^{2}(\chi^{2})-\chi^{2}\right)\left(\omega-\tilde{\omega}_{n_1n_2n_c}(\chi^{2})-\frac{\hbar}{2M}k_{\parallel}^{2}(\chi^{2})\right)^{-1}}{1+\frac{\hbar}{2M}\left(\frac{\varkappa\omega^{2}}{c^{2}}-k_{\parallel}^{2}(\chi^{2})-\chi^{2}\right)\left(\omega-\tilde{\omega}_{n_1n_2n_c}(\chi^{2})-\frac{\hbar}{2M}k_{\parallel}^{2}(\chi^{2})\right)^{-1}}.
\end{aligned}\tag{22}$$

Here r_{gr}, φ, and the time t are given as functions of the parameter $k_{\perp}^{2}$, determining the position of the polariton on the upper branch of the dispersion curve (see Fig. 1), the initial values are designated with an index (0), and the function $k_{\parallel}^{2}(\chi^{2})$ has the form

$$\begin{aligned}
k_{\parallel}^{2}(k_{\perp}^{2})=&-\left[\frac{1}{2}\left(k_{\perp}^{2}-\frac{\varkappa\omega^{2}}{c^{2}}\right)+\frac{M}{\hbar}(\tilde{\omega}_{n_1n_2n_c}(k_{\perp}^{2})-\omega)\right]\\
&+\left\{\left[\frac{1}{2}\left(k_{\perp}^{2}-\frac{\varkappa\omega^{2}}{c^{2}}\right)\right.\right.\\
&\left.\left.-\frac{M}{\hbar}(\tilde{\omega}_{n_1n_2n_c}(k_{\perp}^{2})-\omega)\right]^{2}+\frac{2\lambda_{n_1n_2n_c}(k_{\perp}^{2})M}{\hbar^{2}}\right\}^{1/2}.
\end{aligned}\tag{23}$$

It follows from Eq. (21) that for $\omega-\omega_{n_1n_2n_c}(k_{\perp}^{2}\to\infty)>0$, $k_{\parallel}$ does not vanish on the upper branch of the dispersion curve for any value of $k_{\perp}$ and the point $k_{\parallel}$ ($k_{\perp}^{2}\to\infty$) lies above the abscissa axis. The integrals in Eq. (22), determining φ and t, grow without bound as $k_{\perp}\to\infty$, while the radius vector approaches a finite limit: $r_{gr}(\infty)=r_{gr}^{(0)}k_{\parallel}^{(0)}/k_{\parallel}(\infty)=r_{gr}^{(0)}k_{\parallel}^{(0)}[(2M/\hbar)\times(\omega-\tilde{\omega}_{n_1n_2n_c}(\infty))]^{-1/2}$. This dependence of φ, t, and r_{gr} on $k_{\perp}$ indicates that the motion of a point corresponding to the state of a polariton to the right and downwards along the top branch of the dispersion curve corresponds to motion along a spiral, converging to a circle of finite radius $r_{gr}(\infty)$.

This effect can be realized with the following very simple scheme: A photon is incident normally on a flat surface of a semiconductor, placed in a field with the configuration described above (the z axis lies in this plane, so that $k_z=0$). Then, at first (in the photon-dominant state) the wave packet propagates almost rectilinearly, and then (at the transition into the exciton-dominant state) its trajectory becomes curved.

The conclusion that the photon is captured is valid if two conditions are satisfied: $\varkappa\omega^{2}/c^{2}>(2M/\hbar)(\omega-\tilde{\omega}_{n_1n_2n_c}(0))$ (indicating the existence of a photon-dominant region on the upper branch of the dispersion curve) and $[\omega-\tilde{\omega}_{n_1n_2n_c}(\infty)]>0$ (i.e., $k_{\parallel}(k_{\perp}^{2}\to\infty)>0$), which was assumed when the figure was constructed. A detailed analysis shows[9a] that in a real semiconductor both conditions are satisfied only for $n_c>2$. This means that in order for trapping to occur the initial state of the photon must lie in the gap between two neighboring Coulomb levels, for example, $\omega-\tilde{\omega}_{n_2n_3}(0)\leqslant\hbar k_{\parallel}^{2}/2M\leqslant\omega-\tilde{\omega}_{n_1n_2}(0)$ (in order to decrease the value of n_c at which the inequalities have a simultaneous solution, it is necessary to take a semiconductor for which the relation $\varkappa E_g^{2}/M$ is maximum[9]). It was assumed above that $E_g=1.5$ eV, $\varkappa=20$, $m_1=m_2=0.08m$, and m is the electron mass.

In the case when the inequality $\omega-\tilde{\omega}_{n_1n_2n_c}(\infty)<0$ holds, the top branch intersects the abscissa axis at some point $k_{\perp}^{\max}$, $k_{\parallel}(k_{\perp}^{\max})=0$. Then the integral determining the time diverges, when the upper limit of integration approaches $k_{\perp}^{\max}$, and the integral determining the angle φ approaches a finite limit $\varphi(k_{\perp}^{\max})$; r_{gr} with $k_{\parallel}=0$ also approaches infinity. This behavior means that for $\omega-\tilde{\omega}_{n_1n_2n_c}(\infty)<0$ the photon is deflected by a finite angle (greater or less than 2Λ).

5. Another effect related to the pole in the dispersion equation is photon scattering by a nonuniform magnetic field. Let the lower branch of the dispersion curve in Fig. 1 intersect the abscissa axis in the photon-dominant region and let the initial state of the polariton lie in this region. If the polariton moves so that the distance between the center of the wave packet and the z axis decreases (our arguments refer to the field configuration described above), the corresponding point in the momentum space shifts up and to the left along the bottom branch of the dispersion curve (see Fig.

1). However this branch becomes more level as $k_\perp^2$ decreases, so that the linear growth of $k_\parallel^2$, which would correspond to rectilinear motion in the configuration space, is impossible. Instead, $k_\parallel^2$ increases to some maximum value $(k_\parallel^2)_{\max} = k_\parallel^2(k_\perp^2 = 0)$, less than $\varkappa\omega^2/c^2$. After the point $(0, (k_\parallel^2)_{\max})$, at which, according to Eq. (20), $V_\perp^{gr} = 0$ is reached and the point r_{gr} reaches its minimum value, the polariton evolves along the same branch of the dispersion curve down and to the right until the photon-dominant region is reached.

Thus the process described above is nothing more than photon scattering by a nonuniform magnetic field. It is described quantitatively by the same equations as photon trapping [see Eq. (22)], but here $k_\parallel^2(k_\perp^2)$ is determined by the relation (23) with the opposite sign in front of the braces:

$$k_\parallel^2(k_\perp^2) = -\left[\frac{1}{2}\left(k_\perp^2 - \frac{\varkappa\omega^2}{c^2}\right) + \frac{M}{\hbar}\left(\tilde{\omega}_{n_1n_2n_c}(k_\perp^2) - \omega\right)\right] - \left\{\left[\frac{1}{2}\left(k_\perp^2 - \frac{\varkappa\omega^2}{c^2}\right) - \frac{M}{\hbar}\left(\tilde{\omega}_{n_1n_2n_c}(k_\perp^2) - \omega\right)\right]^2 + \frac{2\lambda_{n_1n_2n_c}(k_\perp^2)\,M}{\hbar^2}\right\}^{1/2}. \tag{23'}$$

The maximum penetration depth into the central region of the field $(r_{gr})_{\min}$ and the scattering angle $\sigma(\tilde{r})$ can be calculated as a function of the impact parameter $\tilde{r}$:

$$(r_{gr})_{min} = \frac{\tilde{r}}{k_\parallel(k_\perp=0)}\left\{\frac{\varkappa\omega^2}{c^2}\right\}^{1/2}, \tag{24}$$

$$\sigma(\tilde{r}) = \pi - 2\arcsin\left(\frac{\tilde{r}}{r_{gr}^{(0)}}\right) - 2\int_{[\varkappa\omega^2/c^2 - k_\parallel^{(0)2}]^{1/2}}^{0} \frac{d\chi}{k_\parallel(\chi^2)}. \tag{25}$$

It follows from Eq. (25) that in the limit $\tilde{r}\to 0$ [or, which is the same thing, $(k_\parallel^{(0)})^2 \to 0$] $\sigma(\tilde{r}\to 0) \neq 0$, i.e., a shadow with the angle $2\sigma(0)$ forms behind the center of the field lines (behind the z axis). This result is formal, since it falls outside the limit of applicability of geometric optics.

6. In our calculation of the permittivity and derivation of the equation of motion (21) we made some assumptions (we assumed that the states of the exciton are stationary, we employed the geometric-optics approximation, etc.). Their validity is ultimately determined by certain restrictions on the intensity of the magnetic field and the curvature of the field lines (for given values of the parameters of the semiconductor).

In Sec. 2 we already discussed the adiabatic approximation. For $\varkappa = 20$ and $m_1 = m_2 = 0.08m$ the condition of adiabaticity reduces to the requirement $B > 10^4$ G. Further, the effective-mass approximation gives an upper limit on the field: $B < 10^7$ G.

We likewise assumed everywhere above that the states of the exciton are stationary. In reality, however, the lifetime of the exciton is equal to 10^{-5}–10^{-7} s. This can be taken into account by introducing instead of the term $i\cdot 0$ in the denominator of Eq. (18) the factor $i\Gamma$ (Breit–Wigner broadening) and assuming that ω is a complex number. The arguments about the evolution of a point in the momentum space along the dispersion curve are valid if the broadening can be neglected, i.e., when Γ is much less than the minimum distance between two branches of the dispersion curve. For $B > 10^3$ G this condition is satisfied.[9a]

The condition for the validity of geometric optics can be formulated as follows: the time interval τ, corresponding to bending of the trajectory (transition of the polariton from the photon-dominant into the exciton-dominant state), must be much larger than ω^{-1}. If the radius of curvature of the field lines is equal to 1 mm and longer, $\tau \gg \omega^{-1}$ for field intensity greater than 10^3 G (a detailed analysis is given in Ref. 9a).

Summarizing these restrictions, we find that photon capture and photon scattering in a nonuniform magnetic field are theoretically possible for $B = 10^4$–10^7 G, and the lower limit depends significantly on the characteristics of the semiconductor.

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