Transverse spin dynamics of polarized electron systems

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(Submitted 17 September 1991)
Zh. Eksp. Teor. Fiz. **101**, 734–749 (February 1992)

We employ the method of kinematic virial corrections in the Boltzmann equation to derive the equations of motion for the transverse components of the magnetic moment in polarized electron systems in the classical temperature range (dynamic polarization by an external magnetic field). We also calculate the spectrum of transverse spin fluctuations, study the condition in which slowly decaying spin waves can propagate in a degenerate electron gas, and obtain a spatial correlation function for the fluctuations of transverse magnetization, which in typical experimental conditions has a pronounced quasione-dimensional nature. Finally, we consider the possibilities for observing in experiments the related effects in a Maxwellian electron plasma and in semiconductors such as InSb, HgCdTe, PbTe, and PbSnTe.

1. INTRODUCTION

Electron spin resonance (ESR) studies have not lost their significance even today. As a rule, the transverse spin dynamics of electrons has been studied in highly degenerate systems, such as metals and alloys. For such systems the equations of motion for the transverse magnetization are ordinarily derived on the basis of the Landau transport equation for a Fermi liquid (see Silin's Appendix in Ref. 1). In the spatially homogeneous case such a description, which essentially coincides with the Bloch equations, leads for the spin-mode spectrum to Larmor precession of the electron spins and a set of cyclotron frequencies. For a one-component electron Fermi liquid, the exchange interaction does not lead to renormalization of the Larmor frequency. However, for spatially inhomogeneous perturbations of the magnetic moment, electron–electron interactions play a decisive role, significantly renormalizing the spin diffusion coefficient (the Leggett–Rice effect[2]). In magnetic fields that are not too weak the Fermi-liquid interaction of electrons generates slowly decaying spin oscillations, which have been observed experimentally in alkali metals.[3]

The existence of transverse spin waves does not necessarily require an external magnetic field **H**. Polarization of the electron spins, that is, a change in the symmetry of the system, can also be achieved by various dynamical methods, such as optical pumping[4] and injection of polarized electrons. An interesting example is the propagation of spin-wave fluctuations in an electron system with spontaneously broken symmetry, a model of a ferromagnetic metal.[5]

Lately, however, there has been a surge of theoretical and experimental work on collective magnetic phenomena in nondegenerate rarefied systems, such as the Boltzmann gases H↑, D↑, and ^{3}He↑ (see, e.g., Ref. 6). The short-range interaction between the gas particles has proved to be of fundamental importance in deriving the macroscopic equations that describe the magnetic dynamics of such systems. Quantitatively, the entire effect of the interaction is expressed in terms of the exact two-particle scattering amplitude and, in particular, contains terms proportional to the forward-scattering amplitude. This scheme would seem at first glance to be unsuitable for describing an electron gas with Coulomb interaction. Indeed, calculation, say, of the second virial coefficient in the Bethe-Uhlenbeck method[7] is hindered by the divergence of the term containing the zero-angle scattering amplitude, which implies, as is known, summation of the entire series of loop diagrams and introduction of Debye screening. Macroscopically this is expressed in the fact that the second virial correction to the free energy of the system is proportional to $N^{3/2}$ instead of N^2, where N is the electron number density.

Moreover, it is known (see, e.g., Ref. 7) that the exchange corrections in the thermodynamics of an electron gas are quadratic in N, as in the case of short-range forces. This is direct indication that, notwithstanding the long-range nature of the Coulomb interaction, the exchange interaction between electrons is "local." Since collective phenomena in spin dynamics are caused by exchange effects, this raises hopes of the possibility of using the conventional gaskinetic scheme of Refs. 8 and 9 to describe an electron gas.

In this paper we apply the method of kinematic virial corrections in a transport equation to the dynamics of transverse magnetization in a Boltzmann electron gas. Hence, we are dealing with macroscopic quantum phenomena in a classical system of electrons. Here we consider both the dynamical spin polarization in the absence of an external magnetic field and the properties of an electron gas in a constant magnetic field, where the Lorentz force and the orbital motion of particles play an important role. We also study situations where the exchange interaction between electrons leads to slowly decaying spin waves. Finally, we discuss the possibilities of observing the effects in experiments involving semiconductors and ionized gases.

2. THE INTERACTION FUNCTION

The method of kinematic virial corrections consists in retaining on the left-hand side of the transport equation of the terms responsible for the interaction between the particles.[9] These terms, in contrast to the collision integral, do not vanish in the process of deriving the hydrodynamic Euler and continuity equations from the transport equation, that is, in the process of integrating over the phase space and allowing for the conservation of the total number of particles and the total momentum. It is these terms that provide the virial expansion of the speed of sound propagation in a gas when the dispersion law is found via the transport equation.

Formally, the kinetic virial corrections in the transport equation are equivalent to the existence of a self-consistent field of the Fermi-liquid type. Here the transport equation

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can be represented in a form typical of the theory of a Fermi liquid, and $\delta\hat{\varepsilon}$, which acts as the energy of a particle in the self-consistent field of the other particles of the gas, in the second virial coefficient approximation is also a linear functional of the distribution function (the density matrix) $n_{p\sigma}$. Since in this paper we are interested only in the dynamics of transverse magnetization, which in the linear approximation is coupled neither with the density oscillations nor with longitudinal spin fluctuations, we can without loss of generality restrict our investigation to the spin part of the function $\delta\hat{\varepsilon}$ (see Refs. 6 and 9):

$$\delta\varepsilon_{p\sigma}=2\sigma\,\mathrm{Sp}_{\sigma'}\sum_{p'}\zeta(\mathbf{p},\mathbf{p}')\sigma' n_{p'\sigma'}, \tag{2.1}$$

where σ are the Pauli matrices. This formula can serve as a phenomenological definition of the interaction function $\zeta(\mathbf{p},\mathbf{p}')$.

From the microscopic view the structure of the interaction function $\zeta(\mathbf{p},\mathbf{p}')$ in a Boltzmann electron gas differs entirely from that of a highly degenerate Fermi liquid. Generally, $\zeta(\mathbf{p},\mathbf{p}')$ contains not only the forward scattering amplitude but terms describing the scattering by arbitrary angles and temperature-dependent terms. (Of course, in the limit as $T\to 0$ the whole quantitative description reduces the equations of the low-density Fermi liquid theory.[10-12]) Thus, the problem of determining the spectrum of collective spin modes is reduced to that of finding $\zeta(\mathbf{p},\mathbf{p}')$ and solving a linearized transport equation with the self-consistent field (2.1).

In the case of an electron system there is a temptation to attempt to consider an interaction of the Yukawa type (Debye screening) and to use the results of Refs. 6 and 9, applicable to arbitrary short-range forces, for calculating $\zeta(\mathbf{p},\mathbf{p}')$. We will not do this here for at least three reasons. First, calculating the exchange corrections in thermodynamics[7] requires no Debye screening since there is no divergence even when a purely Coulomb potential is employed. Second, strictly speaking, introduction of Debye screening requires rigorous substantiation, because the Bethe–Uhlenbeck method, used in Refs. 6 and 9, considers the scattering of two particles in a vacuum in the absence of a background or collective effects of any type and the final result is expressed in terms of the initial two-particle scattering amplitude in a vacuum. Third, by considering a purely Coulomb interaction between two electrons we have in mind the possibility of applying some of the results to systems with unneutralized electric charge, which at present are being extensively studied in experiments. In other words, within the framework of the Bethe–Uhlenbeck method we attempt to use a purely Coulomb interaction between electrons as the initial interaction and avoid, as long as possible, introducing screening or any other renormalization schemes. The validity criteria for such an approximation emerge automatically.

Note that in Ref. 13 the fluctuations in the transverse magnetization in semiconductors with nonequilibrium-oriented spins were considered on the basis of a model Hamiltonian that essentially corresponded to a delta-like interaction between the electrons. The results were applied, among other things, to a nondegenerate system of electrons. In reality, the transverse spin dynamics of a weakly nonideal Boltzmann electron gas can never be described within the framework of such a model. Even if we were to allow for Debye screening, using a model with point-like interaction would presuppose that

$$e^2N^{1/3}\ll T\ll\hbar\omega_L,\quad \varepsilon_F\ll T\ll\hbar\omega_L, \tag{2.2}$$

where ω_L is the electron plasma frequency, and ε_F the Fermi energy. These inequalities cannot, obviously, be met simultaneously.

The first obstacle encountered when following the Bethe–Uhlenbeck method[7] is the "irregular" asymptotic form of the wave function in Coulomb scattering:[14]

$$R_{lk}(r)\approx\frac{\text{const}}{r}\sin\left(kr-\frac{\ln 2kr}{ka}-\frac{\pi l}{2}+\delta_l\right), \tag{2.3}$$

where k and l are the wave number and orbital angular momentum of the relative motion of two electrons, δ_l the scattering phase, and $a=\hbar^2/me^2$ the Bohr radius. Indeed, in comparison to the asymptotic form of the wave function for short-range potentials, R_{lk} of (2.3) contains an additional logarithmic term. But in the intermediate asymptotic region, up to exponentially large distances ($1\ll kr\ll\exp(ka)$), the logarithmic term in (2.3) can be ignored, which enables us to directly use the results of calculations given in Ref. 7. Clearly, such a region exists if the temperature of the electron gas is fairly high:

$$T\gg\mathrm{Ry},\quad \mathrm{Ry}=me^4/\hbar^2. \tag{2.4}$$

In these conditions we can employ the formulas of Refs. 6 and 9 that link the interaction function with the two-particle scattering amplitude.

The function $\zeta(\mathbf{p},\mathbf{p}')$ introduced by (2.1) has in this case the following form:

$$\zeta(\mathbf{p},\mathbf{p}')=\frac{A_+(\mathbf{q})-A_-(\mathbf{q})}{4},\quad \mathbf{q}=\frac{\mathbf{p}-\mathbf{p}'}{2},$$

$$A_\pm(\mathbf{q})=-\frac{4\pi\hbar^2}{m}\Big\{\mathrm{Re}\,f_\pm(0,q)+\frac{mT}{\hbar}\Big[\mathrm{Re}\,f_\pm(\theta,q)\frac{\partial}{\partial q}\mathrm{Im}\,f_\pm(\theta,q)-\mathrm{Im}\,f_\pm(\theta,q)\frac{\partial}{\partial q}\mathrm{Re}\,f_\pm(\theta,q)\Big]\Big\}, \tag{2.5}$$

where f_+ and f_- are the triplet and singlet scattering amplitudes for two electrons, and θ is the scattering angle in the center-of-mass frame of reference. It is easy to verify that in the temperature range (2.4) we can ignore the contribution of the terms quadratic in $f_\pm$ to the functions $A_\pm(\mathbf{q})$ of (2.5). Bearing this in mind and performing simple transformations, we finally get

$$\zeta(\mathbf{p},\mathbf{p}')=\frac{\pi\hbar^2}{m}\mathrm{Re}\,f_0(\pi,q), \tag{2.6}$$

where $f_0(\theta,q)$ is the usual scattering amplitude for a charged particle in a repulsive Coulomb field,[14]

$$f_0(\theta,q)=-\frac{1}{2q^2\sin^2\theta/2}\exp\left[-\frac{2i}{q}\ln\left(\sin\frac{\theta}{2}\right)\right]\frac{\Gamma(1+i/q)}{\Gamma(1-i/q)} \tag{2.7}$$

(we have employed dimensionless Coulomb units).

Thus, in the given approximation the exchange part of

the interaction function $\zeta(\mathbf{p},\mathbf{p}')$ is determined by the "backward" scattering amplitude $f_0(\pi,q)$. The function $f_0(\theta,q)$ has no singularities at $\theta=\pi$, which is a manifestation of the local nature of the exchange interaction and makes it possible to do without Debye screening. Performing the same procedure in calculating the interaction function averaged over the spin directions (of no interest to us here), we would obtain, instead of (2.6), an expression containing $\mathrm{Re}\, f_0(0,q)$. But the Coulomb scattering amplitude is divergent at $\theta=0$, which means that this method cannot be used in this case and that screening must be introduced in all the approximations of the theory. Note that, for high-order terms in $\mathrm{Ry}/T\ll 1$, we must cut off the Coulomb interaction when calculating the exchange function $\zeta(\mathbf{p},\mathbf{p}')$. For instance, allowing for terms in (2.5) that are quadratic in $f_\pm$ presupposes Debye screening of the electron-electron interaction.

Essentially, condition (2.4) coincides with the quantum-mechanical criterion for applying perturbation theory to a Coulomb potential. For this reason the interaction function is determined with a good accuracy by the Born approximation. Then, combining (2.6) and (2.7), we immediately obtain

$$\zeta(\mathbf{p},\mathbf{p}')=-\pi\left(\frac{e\hbar}{|\mathbf{p}-\mathbf{p}'|}\right)^2. \tag{2.8}$$

This expression for the exchange part of the interaction function will be used in what follows to find the spectrum of transverse spin fluctuations in an electron gas.

3. THE TRANSPORT EQUATION AND THE SPIN-MODE SPECTRUM

The dynamical equation for an electron system in the long-wave limit can be obtained from the quasiclassical transport equation with kinematic virial correction,[9] which is commonly written as

$$\frac{\partial\hat{n}}{\partial t}+\frac{1}{2}\left(\frac{\partial\hat{n}}{\partial\mathbf{r}}\frac{\partial\hat{\varepsilon}}{\partial\mathbf{p}}+\frac{\partial\hat{\varepsilon}}{\partial\mathbf{p}}\frac{\partial\hat{n}}{\partial\mathbf{r}}\right)-\frac{1}{2}\left(\frac{\partial\hat{n}}{\partial\mathbf{p}}\frac{\partial\hat{\varepsilon}}{\partial\mathbf{r}}+\frac{\partial\hat{\varepsilon}}{\partial\mathbf{r}}\frac{\partial\hat{n}}{\partial\mathbf{p}}\right)$$
$$+\frac{i}{\hbar}[\hat{\varepsilon},\hat{n}]_\sigma+\frac{1}{2}\frac{e}{c}\left\{\left[\frac{\partial\hat{\varepsilon}}{\partial\mathbf{p}}\mathbf{H}\right]\frac{\partial\hat{n}}{\partial\mathbf{p}}+\frac{\partial\hat{n}}{\partial\mathbf{p}}\left[\frac{\partial\hat{\varepsilon}}{\partial\mathbf{p}}\mathbf{H}\right]\right\}=\mathrm{St}\,\hat{n}, \tag{3.1}$$

where the self-consistent electron energy $\hat{\varepsilon}$ contains, among other things, terms of the form (2.1). As noted earlier, we are interested here only in the equations of motion for the transverse spin components, which in the linear approximation are not coupled with the fluctuations in the electron number density. For this very reason we have dropped the self-consistent electric field from the transport equation (3.1), and there is no need to add the Maxwell equations to (3.1), as is commonly done in plasma-wave theory.

It is easy to see that for a weakly collisional electron gas with

$$T\gg e^2N^{1/3}, \tag{3.2}$$

we can ignore the interaction when calculating the group velocity $\partial\hat{\varepsilon}/\partial\mathbf{p}$ and restrict our attentions to $\mathbf{v}=\mathbf{p}/m$. In addition to this, with the same accuracy we can ignore in Eq. (3.1) the gradient terms of the form $(\partial\hat{\varepsilon}/\partial\mathbf{r})\cdot(\partial\hat{n}/\partial\mathbf{p})$ in comparison to the term $\mathbf{v}\cdot\nabla\hat{n}$. We must retain, however, the homogeneous spin commutator $[\hat{\varepsilon},\hat{n}]_\sigma$, which, strictly speaking, determines the transverse spin dynamics of an electron gas. This imposes an upper bound on the size of the spatial gradient:

$$k\Lambda\ll\alpha,\quad \Lambda=\frac{\hbar}{(mT)^{1/2}}, \tag{3.3}$$

where $\mathbf{k}$ is the wave vector corresponding to the spatial inhomogeneity, and we have introduced the degree of polarization of the electron gas,

$$\alpha=\frac{N_+-N_-}{N},\quad N_++N_-=N \tag{3.4}$$

with N_+ and N_- the number densities of electrons with spins direction parallel and antiparallel to the polarization vector. Note that the "hydrodynamic" condition (3.3) automatically agrees with the quantum mechanical criterion of applicability of the quasiclassical transport equation, $k\Lambda\ll 1$. After the above simplifications have been enforced, Eq. (3.1) for the off-diagonal elements of the polarization density matrix $\hat{n}$ (the transverse spin components) takes the following form:

$$\frac{\partial\hat{n}}{\partial t}+\mathbf{v}\nabla\hat{n}+\frac{i}{\hbar}[\hat{\varepsilon},\hat{n}]_\sigma+\frac{e}{c}[\mathbf{vH}]\frac{\partial\hat{n}}{\partial\mathbf{p}}=\mathrm{St}\,\hat{n}. \tag{3.5}$$

Below we consider the long-wave solutions to Eq. (3.5) for various types of spin-polarized electron systems.

3.1. Quasiequilibrium polarization at $H=0$

Methodologically it is convenient to begin with the case where the electron spins are polarized by a dynamical method of some sort in the absence of an external magnetic field. Such dynamical methods may, for instance, be optical pumping, injection of polarized electrons, or polarization of the system by an external magnetic field followed by rapidly switching off the field. Subsequent depolarization of the spin system occurs much more slowly than the onset of equilibrium in the electron momenta, since the longitudinal magnetization varies because of the relativistically weak dipole-dipole interaction, while relaxation in the energies is determined by the strong Coulomb scattering. This means that in the course of time intervals shorter than the time t_s that it takes the longitudinal magnetization to relax we have a spin-polarized system of electrons, that, as far as the momentum distribution is concerned are in a state of thermodynamic equilibrium. In other words, the electron gas is described, to within fluctuations, by an equilibrium Maxwell-Boltzmann distribution function. In certain cases this situation can be realized directly in the process of dynamical polarization (say, during optical pumping).

The diagonal elements of the polarization matrix $\hat{n}$ are the occupation numbers for electrons whose spins are directed parallel and antiparallel to the magnetization vector. These numbers are determined by the attained degree of polarization α and must be assumed given, since in the linear approximation the perturbations of the off-diagonal elements of $\hat{n}$ have no effect on the occupancy of states with different spin projections. The matrix $\hat{n}$ is, of course, a linear operator of the Pauli matrices, but only its spin part contributes to the equations of motion for transverse magnetization. Accordingly, the spin part of $\hat{n}$ can be written as

$$n_{p\sigma}=(\alpha n_p^{(0)}\mathbf{m}+\boldsymbol{\lambda}_p)\boldsymbol{\sigma},\quad \boldsymbol{\lambda}_p\perp\mathbf{m}, \tag{3.6}$$

where the first term corresponds to the equilibrium state, λ_p describes the perturbation in the transverse components of the spin, $\mathbf{m}$ is the unit vector directed along the vector of spin polarization, and $n_p^{(0)}$ is the Maxwellian distribution function,

$$n_p^{(0)}=\frac{N}{2}\left(\frac{2\pi\hbar^2}{mT}\right)^{3/2}\exp\left(-\frac{p^2}{2mT}\right). \tag{3.7}$$

The fluctuating macroscopic magnetic moment $\mathbf{M}$ can easily be calculated as

$$\mathbf{M}=\beta N\alpha\mathbf{m}+\delta\mathbf{M}_\perp,\quad \delta\mathbf{M}_\perp=2\beta\int\lambda_p\frac{d^3p}{(2\pi\hbar)^3}, \tag{3.8}$$

where β is the electron magnetic moment.

Substituting (3.6) into (2.1), we find the spin part of the self-consistent electron energy, which provides the only contribution to the commutator $[\hat{\varepsilon},\hat{n}]_\sigma$ in the transport equation (3.5):

$$\delta\varepsilon_{p\sigma}=4\sigma\int\zeta(\mathbf{p},\mathbf{p}')(\alpha n_{p'}^{(0)}\mathbf{m}+\lambda_{p'})\frac{d^3p'}{(2\pi\hbar)^3}. \tag{3.9}$$

Then for $H=0$ Eq. (3.5) can be transformed into

$$\frac{\partial}{\partial t}\lambda_p+(\mathbf{v}\nabla)\lambda_p+\frac{8\alpha}{\hbar}\int\zeta(\mathbf{p},\mathbf{p}')\{n_p^{(0)}[\mathbf{m}\lambda_{p'}] -n_{p'}^{(0)}[\mathbf{m}\lambda_p]\}\frac{d^3p'}{(2\pi\hbar)^3}=\mathrm{St}\,\lambda_p. \tag{3.10}$$

The linearized transport equation (3.10) together with definition (3.8) constitute the basis for the "hydrodynamic" description of the transverse dynamics of macroscopic magnetization.

To determine the spectrum of transverse spin fluctuations we have found it expedient to look for the solution to Eq. (3.10) for monochromatic perturbations of the distribution function in the form

$$\lambda_p\propto\exp(i\mathbf{kr}-i\omega t), \tag{3.11}$$

where ω and $\mathbf{k}$ are the frequency and wave vector of a spin model. Combining this with (3.10) and (3.11) immediately yields

$$(\omega-\mathbf{kv})\lambda_-(\mathbf{p})+\frac{8\alpha}{\hbar}\int\zeta(\mathbf{p},\mathbf{p}')\{n_p^{(0)}\lambda_-(\mathbf{p}') -n_{p'}^{(0)}\lambda_-(\mathbf{p})\}\frac{d^3p'}{(2\pi\hbar)^3}=\mathrm{St}\,\lambda_-(\mathbf{p}), \tag{3.12}$$

where we have introduced the circular variable $\lambda_-=\lambda_x-i\lambda_y$. The common approach to considering relaxation processes in an electron system is to employ the Landau local collision integral,[15] which imposes additional requirements on the size of a spatial gradient

$$kr_D\ll 1, \tag{3.13}$$

where r_D is the Debye screening radius. If the degree of polarization of the electron gas is fairly high, that is, if

$$1\geqslant\alpha\gg\hbar\omega_L/T, \tag{3.14}$$

then Eq. (3.12) with the local collision integral $\mathrm{St}\,\lambda_-(\mathbf{p})$ is valid only if conditions (3.13) are met. In the opposite case where

$$1\gg\hbar\omega_L/T\gg\alpha, \tag{3.15}$$

condition (3.3) serves as the criterion for the applicability of the results, and this automatically presupposes that inequalities (3.13) are satisfied.

For the transport equation (3.12) with a local collision integral we seek the long-wave solution via the Chapman–Enskog method. In the given case we can apply the results of Refs. 6 and 8 to the distribution function (2.7) and arrive at the following dispersion law for transverse spin fluctuations:

$$\omega=\omega_k=\frac{D_0k^2}{1+(D_0N\alpha\gamma)^2}(D_0N\alpha\gamma-i), \tag{3.16}$$

where D_0 is the coefficient of spin diffusion in the absence of polarization, and the exchange parameter γ of the theory depends only on temperature,

$$\frac{1}{\gamma}=\frac{2A}{3\pi}\frac{T^2}{e^2\hbar}. \tag{3.17}$$

The numerical factor A is determined from the following system of integral equations:

$$A=2\pi\int_0^\infty s^4g(s)\exp(-s^2)ds,$$
$$\int K(\mathbf{s},\mathbf{s}')\left[g(s)-\frac{\mathbf{ss}'}{s^2}g(s')\right]d^3s'=1, \tag{3.18}$$
$$K(\mathbf{s},\mathbf{s}')=\frac{\exp(-s'^2)}{(\mathbf{s}-\mathbf{s}')^2}.$$

In establishing the solution to (3.18) it has proved convenient to use the following relation:

$$\int K(\mathbf{s},\mathbf{s}')d^3s'=2\pi^{3/2}\frac{\exp(-s^2)}{s}\int_0^s\exp(t^2)dt=\Phi(s). \tag{3.19}$$

Numerical integration of (3.18) yields $A\approx 2.51$. The spin diffusion coefficient D_0 can be defined as the coefficient of self-diffusion in a two-component mixture of electrons with spins "up" and "down." In the simplest gaskinetic approximation,[15] D_0 can be calculated by the following formula:

$$D_0=\frac{3}{16\pi^{1/2}}\frac{T^{5/2}}{m^{1/2}e^4NL},\quad L=\ln\frac{r_D}{\Lambda}. \tag{3.20}$$

Equation (3.16) immediately makes evident the difference between the spectrum of transverse-magnetization fluctuations and the purely diffusion mode ($-iD_0k^2$), the latter emerging only if we completely ignore exchange effects. The measure of this difference is the parameter

$$D_0N\alpha\gamma=\frac{9\pi^{1/2}}{32A}\frac{\alpha}{L}\left(\frac{T}{\mathrm{Ry}}\right)^{1/2}, \tag{3.21}$$

which, to within the Coulomb-logarithm term, is density independent and, hence, reflects the fact that the electron exchange interaction is local. If the temperature is sufficiently high, that is, if

$$T\gg\mathrm{Ry}\left(\frac{L}{\alpha}\right)^2\geqslant\mathrm{Ry}, \tag{3.22}$$

the real part of the spectrum (3.16) is found to be much greater than the damping $\mathrm{Im}\,\omega$, and instead of diffusive spreading we have weakly damped oscillations of the transverse magnetization with the following dispersion law:

$$\omega_k = \frac{8A}{3}\frac{1}{\alpha}\frac{T}{\hbar}(kr_D)^2. \tag{3.23}$$

It is easy to see that although the spin waves are "hydrodynamic" in nature (small wave vectors), within the range of applicability of (3.23) defined by the inequalities (3.3) and (3.13)–(3.15) there exist high-frequency zero-sound modes, for which $\omega\tau_e \gg 1$, where τ_e is the electron relaxation time.

The presence in the electron system of collective spin modes automatically implies the existence of long-range transverse-magnetization correlations, notwithstanding the local nature of the initial exchange interaction. We define the correlation function for the fluctuations of the transverse magnetic moment in the ordinary manner:

$$S_{ik}(t, r) = \langle \delta M_i(t_1, \mathbf{r}_1)\delta M_k(t_2, \mathbf{r}_2)\rangle, \quad i, k = x, y, \qquad t \equiv t_1 - t_2, \quad r \equiv |\mathbf{r}_1 - \mathbf{r}_2|. \tag{3.24}$$

Actually we are interested here only in the simultaneous correlator $S_{ik}(0,r)$.

The appearance of a macroscopic variation $\delta\mathbf{M}_\perp(\mathbf{r})$ in the electron gas leads to an increase in the total free energy, and the size of this increase can be written as

$$\Delta F = \frac{1}{2}\int \varphi(r)\delta\mathbf{M}_\perp(\mathbf{r}_1)\delta\mathbf{M}_\perp(\mathbf{r}_2)\,d^3r_1\,d^3r_2. \tag{3.25}$$

At high temperatures the fluctuations may be considered classical.[7] Therefore, averaging (3.25), we immediately get

$$S_{ik}(k)\varphi(k) = T\delta_{ik}, \tag{3.26}$$

where $S_{ik}(k)$ and $\varphi(k)$ are the Fourier components of $S_{ik}(0,r)$ and $\varphi(r)$, respectively. The function $\varphi(k)$ is uniquely determined by the spin-mode spectrum:

$$\varphi(k) = \frac{\hbar\omega_k}{2\beta^2 N\alpha}. \tag{3.27}$$

Combining (3.23), (3.26), and (3.27), we find the structure factor $S_{ik}(k)$ and, performing the inverse Fourier transformation, we finally get

$$S_{ik}(0,r) = \frac{3}{4A}\frac{(\beta\alpha Ne)^2}{Tr}\delta_{ik}, \quad r \gg \max\left\{r_D, \frac{\Lambda}{\alpha}\right\}. \tag{3.28}$$

Thus, we have found that over large distances the transverse spin correlations in an electron gas fall off very slowly. As r grows, (3.28) decreases in a manner similar to the magnetization correlations in a cubic ferromagnet with localized spins, which is a direct consequence of the quadratic dispersion law for spin waves.

The spectrum of spin fluctuations and the correlations of macroscopic magnetization were considered here using the transport equation with kinetic virial corrections. When the Born approximation (2.7) is valid, such an equation can, of course, be obtained "microscopically" within the perturbation-theory framework, as suggested in Ref. 16.

3.2. Electron gas in a magnetic field

It is much more interesting from the practical view to study the case where a system of electrons is in an external magnetic field and the degree of polarization of the spins is determined by the field strength. In thermodynamic equilibrium we, obviously, have the following formula for the degree of polarization:

$$\alpha = \mathrm{th}(\beta H/T). \tag{3.29}$$

Here we study the possibility of slowly decaying spin waves propagating in an electron gas placed in an external magnetic field. If the requirements (3.22) are met, the collisional dissipation terms can be ignored, with the result that the spectrum of transverse-magnetization fluctuations is determined by a collisionless transport equation, that is, Eq. (3.5) in which we must put $\mathrm{St}\,\hat{n} = 0$. Of course, there is also collisionless Landau damping, which, however, is exponentially small in the range of wave vectors specified by (3.3) and (3.13).

The transport equation (3.5) must incorporate in the self-consistent energy $\hat{\varepsilon}$ not only the terms of the form (3.9) but also the Zeeman energy of the interaction of the electron spin with an external magnetic field,

$$\delta\varepsilon_{p\sigma}^{(z)} = -\beta\boldsymbol{\sigma}\mathbf{H}. \tag{3.30}$$

In finding the spin-mode spectrum it has proved convenient to introduce cylindrical coordinates, $p_\perp$, ϕ, and p_z, with the z axis directed along the magnetic field. After performing transformations similar to (3.6) and (3.9)–(3.12), from Eq. (3.5) we get

$$(\omega - \Omega_H - \mathbf{kv})\lambda_-(\mathbf{p}) + \frac{8\alpha}{\hbar}\sum_{p'}\zeta(\mathbf{p},\mathbf{p}')\{n_p^{(0)}\lambda_-(\mathbf{p}') - n_{p'}^{(0)}\lambda_-(\mathbf{p})\} + i\Omega_C\frac{\partial}{\partial\varphi}\lambda_-(\mathbf{p}) = 0, \tag{3.31}$$

where we have introduced the Larmor frequency $\Omega_H = 2\beta H/\hbar$ and the cyclotron frequency $\Omega_C = eH/mc$. For a gas consisting of free electrons the frequencies Ω_H and Ω_C, of course, coincide. However, bearing in mind that we are interested in applying the results to semiconductors, where the charge carriers and hence the cyclotron frequency are characterized by the effective electron mass rather than the "bare" electron mass, we will distinguish between Ω_H and Ω_C. Solving Eq. (3.31) by the method of successive approximations in powers of the small wave vector $\mathbf{k}$ (see, e.g., Ref. 6), in second order we get

$$\omega_k = \Omega_H - \frac{2}{N}\sum_p \mathbf{kv} f_k(\mathbf{p}) n_p^{(0)}, \qquad \frac{8\alpha}{\hbar}\sum_{p'}\zeta(\mathbf{p},\mathbf{p}')n_{p'}^{(0)}[f_k(\mathbf{p}') - f_k(\mathbf{p})] + i\Omega_C\frac{\partial}{\partial\varphi}f_k(\mathbf{p}) = \mathbf{kv}. \tag{3.32}$$

It is absence of a magnetic field, the spectrum of spin waves (3.23) is isotropic, that is, independent of the direction of the wave vector $\mathbf{k}$, which is a direct consequence of the exchange approximation. By switching on the external magnetic field we introduce relativistic terms related to the Lorentz force, which invariably leads to an anisotropy in the dispersion law. In view of this, it is useful to look for the distribution function (in our case the function f_k) in the form of a sum of the exchange, "isotropic," part and the anisotropic additional term z_k due to the external magnetic field:

$$f_k(\mathbf{p}) = -\frac{2\pi^{3/2}}{\alpha}\frac{T}{\hbar\omega_L^2}[g(s)\mathbf{kv} + z_k(\mathbf{p})]. \tag{3.33}$$

Here the function $g(s)$ is defined in Eqs. (3.18) and substituting the first term in (3.33) into (3.32) leads to the exchange formula (3.23). Combining (3.32) and (3.33) also immediately results in the following expression for z_k:

$$\frac{8\alpha}{\hbar}\sum_{p'}\zeta(\mathbf{p},\mathbf{p}')n_{p'}^{(0)}[z_k(\mathbf{p}')-z_k(\mathbf{p})]+i\Omega_c\frac{\partial}{\partial\varphi}z_k(\mathbf{p})$$
$$-i\Omega_c k_\perp v_\perp g(s)\sin\varphi=0. \tag{3.34}$$

Performing an order-of-magnitude estimate of the various terms in Eq. (3.34), one can easily see that because of the presence of the small parameter

$$\gamma\equiv\frac{1}{4\pi^{1/2}}\left(\frac{\hbar\omega_L}{T}\right)^2\ll 1 \tag{3.35}$$

which is characteristic of an electron gas, the integral terms in (3.34) are always small and can be ignored to lowest order in $\gamma\ll 1$. This automatically leads to

$$z_k(\mathbf{p})=-\mathbf{k}_\perp\mathbf{v}_\perp g(s)\equiv-\mathbf{k}_\perp\mathbf{v}g(s), \tag{3.36}$$

and, hence,

$$f_k(\mathbf{p})=-\frac{2\pi^{1/2}}{\alpha}\frac{T}{\hbar\omega_L^2}g(s)\mathbf{k}_z\mathbf{v}. \tag{3.37}$$

Substituting (3.37) into (3.32) finally yields

$$\omega_k=\Omega_H+\frac{8A}{3}\frac{1}{\alpha}\frac{T}{\hbar}(k_z r_D)^2$$
$$=\Omega_H+\frac{8A}{3}\frac{T}{\hbar}\operatorname{cth}\frac{\beta H}{T}k^2r_D^2\cos^2\theta,\quad A\approx 2{,}51. \tag{3.38}$$

The exact solution of Eq. (3.32) and the calculation of higher-order terms in the expansion of the spin-wave spectrum ω_k in a power series in $\gamma\ll 1$ can be found in the Appendix.

Thus we have found the highly anisotropic spectrum of transverse magnetization fluctuations in an electron gas placed in an external magnetic field. In the principal approximation, as the result (3.38) clearly shows, this spectrum has a pronounced one-dimensional nature, which, of course, greatly affects the behavior of the correlation function for the macroscopic magnetic moment. Calculations similar to (3.24)–(3.28) yield

$$S_{ik}(0,\mathbf{r})=\frac{3\pi}{2A}\frac{(\beta N\alpha e)^2}{T}r_H\exp\left(-\frac{|z|}{r_H}\right)\delta(x)\delta(y)\delta_{ik},$$
$$r_H^2=\frac{8A}{3\alpha}\frac{T}{\hbar\Omega_H}r_D^2. \tag{3.39}$$

In contrast to (3.28), we have found correlations only parallel to the magnetic field, and these decrease exponentially rapidly as the distance along the z axis grows. The correlation radius, however, proves to be extremely large, $r_H\gg r_D$, under the natural conditions $\hbar\Omega_H\ll T$, so that the correlation zone always contains a macroscopically large number of electrons.

4. EXPERIMENTAL POSSIBILITIES

To observe macroscopic quantum effects in a classical electron gas, an example of which is the propagation of collective spin waves, is highly intriguing. Experimentally this appears possible. Phenomena similar in their physics and related to exchange interaction have been observed even in experiments involving low density gases of neutral particles[17–20] (NMR studies of gaseous H↑ and ^{3}He↑ and dilute ^{3}He↑–^{4}He solutions). These experiments studied the fluctuations in nuclear magnetization, which required higher sensitivity from the measuring apparatus. Nuclear spin modes have also been observed in liquid ^{3}He↑ in both damped and long-lived regimes.[21,22] Various magnetic resonance methods, such as NMR in the gradient of an external magnetic field, spin echo, and spin-mode excitation in a nonuniform variable field, have been employed in detecting collective modes. Similar approaches can be used in EPR to record spin waves propagating in electron systems. Spin waves in the degenerate electron Fermi liquid in metals have been discovered in ingenious experiments in which light was transmitted through thin films of alkali metals.[3] Thus, there exists a fairly large number of experimental methods that allow identifying spin oscillations in an electron gas.

Much more important is whether a sample suitable for the experiments can be chosen. Qualitatively, exchange renormalization of the transverse spin diffusion coefficient takes place for all values of the parameters characterizing the state of the electron system (as is generally the case with all paramagnetic liquids and gases). Hence, we cannot exclude the possibility that the effects considered here may manifest themselves in a broad variety of conducting media. However, it would be preferable by far to have a sample that satisfies the criteria of applicability of the theory constructed here, since it would then be possible to estimate the expected effects quantitatively. The criterion for the validity of our calculations imposes a stringent lower bound (2.4) on the temperature of the system. An electron plasma is certain to satisfy these conditions because the ionization potential of all the atoms is on the order of, or greater than, Ry. From the practical view this means such high temperatures that high values of the degree of polarization α remain in doubt. For this reason, in ordinary conditions the parameter $D_0N\alpha\gamma$ of (3.21), which serves as a measure of the effects, is small, and this interferes with the experimental identification of the phenomenon (although it does not make it impossible).

The situation may be quite different, however, for the system of charge carriers (electrons and holes) in semiconductors. The electrostatic screening of the Coulomb interaction and the renormalization of the carrier mass (effective mass) make the criterion (2.4) considerably less stringent. Indeed, instead of (2.4) we obviously have

$$T\gg\frac{m^*e^4}{\hbar^2\varepsilon^2}\equiv\mathrm{Ry}^*=\frac{1}{\varepsilon^2}\frac{m^*}{m}\mathrm{Ry}, \tag{4.1}$$

where m^* is the effective carrier mass, and ε the dielectric constant. Hence, by selecting a semiconductor with a maximum possible value of ε and a minimal m^* we can have a temperature range defined by (4.1) that is suitable for experiments. Such semiconductors abound. Some of the more common examples with a fcc lattice (so that all the calculations for an isotropic electron spectrum are applicable) are InSb ($m^*\approx 1.3\times10^{-2}m$, $\varepsilon\approx 20$) and HgCdTe [$m^*\approx(2\times10^{-3}$–$1{,}1\times10^{-2})m$; $\varepsilon\approx 20$].[23] For these cases the size of Ry*, which determines the temperature range where the theory is valid, is Ry*(InSb) = 10 K and Ry*(HgCdTe) = 2–9 K, respectively. Such weak temperature restrictions make it possible to achieve fairly high values of the parameter $D_0N\alpha\gamma$ of (3.21), which provides a fairly

good chance for experimental studies of exchange collective effects in transverse spin dynamics. It even becomes possible to detect weakly decaying spin waves (magnons), which, of course, is the most interesting consequence of the theory. For instance, in semiconductors with Ry* ≈ 2 K at $T = 12$ K, the criterion (3.32) for spin fluctuations to decay slowly (where Ry should be replaced by Ry*) reduces to the condition $H \gg 6$ T, which raises hopes that magnetic oscillations with the spectrum (3.38) will be discovered.

There is a sizable group of semiconductors with a still lower value of Ry*. Unfortunately, these often possess a different crystal symmetry, so that the formulas for the dispersion law of the spin modes cannot be applied directly and must be found for each given electron spectrum separately. But the criteria for the applicability of the theory, (2.4), and smallness of collision absorption, (3.22), can in the majority of cases be used for order-of-magnitude estimates. For instance, numerical estimates for PbTe ($m^* \approx 0.2m$; $\varepsilon \approx 10^2$–10^3) and PbSnTe ($m^* \approx 10^{-2} m$; $\varepsilon \approx 10^2$) crystals yield amazingly weak restrictions on the temperature: Ry*(PbTe) $\approx 6 \times 10^{-2}$–6 K and Ry*(PbSNTe) ≈ 0.3 K, respectively.[23] In these conditions the slowly decaying spin waves could propagate in PbSnTe at $T \approx 3$K and in magnetic fields $H \gg 1$ T. In samples with Ry* $\approx 6 \times 10^{-2}$ K, long-lived magnons could appear at $T \approx 0.6$ K and $H \gg 0.3$ T. Some data on collective spin waves in a Maxwellian electron plasma was given by the present authors in Ref. 24.

All the results listed here for semiconductors refer to pure samples without magnetic impurities. Hence, the effects are related to a purely electron–electron interaction. But if the sample contains a large number of magnetic impurities, new branches of collective spin modes may appear due to exchange effects resulting from the scattering of band charge carriers on impurities.[25]

ACKNOWLEDGMENTS

The authors are grateful to I. M. Tsidil'kovskiĭ for pointing out the experimental data on the dielectric constants and effective carrier masses for a number of semiconductors. One of the authors (E.P.B.) would like to thank Ch. Campbell and M. Ristig for the hospitality shown him at the universities of Minneapolis and Cologne, where some of this paper was prepared.

APPENDIX

It is obvious from the transport equation that at $\mathbf{k} \| \mathbf{H}$ the cyclotron terms generated by the Lorentz force have no effect on the dispersion law for the spin modes, so that we again have the former spectrum (3.23) augmented by the Larmor gap Ω_H at $k = 0$. Hence, the "anisotropic" part $z_k(\mathbf{p})$ of the distribution function (3.33) must be proportional to $k_\perp$. It is conveniently written as $z_k(\mathbf{p}) = k_\perp v_\perp u(\mathbf{s})$, with $p^2 = 2mTs^2$, after which Eq. (3.34) assumes the form

$$\mu\Phi(s)u(\mathbf{s}) - \mu\sum_{s'} \frac{\exp(-s'^2)}{(\mathbf{s}-\mathbf{s}')^2} \frac{s_\perp'}{s_\perp} u(\mathbf{s}') - ig(s)\sin\varphi + i\frac{\partial}{\partial\varphi} u(\mathbf{s}) = 0, \tag{A1}$$

where the functions $\Phi(s)$ and $g(s)$ have been defined in Eqs. (3.18) and (3.19) and we have introduced the dimensionless parameter

$$\mu = \frac{1}{2\pi^{3/2}} \alpha \frac{\omega_L}{\Omega_H} \frac{\hbar\omega}{T}, \quad \omega_L^2 = \frac{4\pi N e^2}{m}. \tag{A2}$$

Expanding $u(s) \equiv u(s_z, s_\perp, \varphi)$ in a Fourier series in the variable φ and allowing for the structure of the kernel in the integro-differential equation (A1), we can easily see that only the first harmonics (terms of the $\exp(\pm i\varphi)$ type) provide a nonzero contribution to the solution. Moreover, the spin-mode spectrum is determined by the zeroth harmonic (the "isotropic" part) and the first harmonics of $f_k(\mathbf{p})$. Hence, without loss of generality, we can find the function $u(\mathbf{s})$ in the form

$$u(\mathbf{s}) = x(s_z, s_\perp) e^{i\varphi} + y(s_z, s_\perp) e^{-i\varphi}. \tag{A3}$$

Substituting (A3) into (A1), we get

$$\begin{aligned} &\mu\Phi(s)x(s_z, s_\perp)s_i - \mu\int K(\mathbf{s},\mathbf{s}')x(s_z', s_\perp')s_i' d^3s' - \frac{1}{2}g(s)s_i - x(s_z, s_\perp)s_i = 0, \\ &\mu\Phi(s)y(s_z, s_\perp)s_i - \mu\int K(\mathbf{s},\mathbf{s}')y(s_z', s_\perp')s_i' d^3s' + \frac{1}{2}g(s)s_i + y(s_z, s_\perp)s_i = 0, \\ &K(\mathbf{s},\mathbf{s}') = \frac{\exp(-s'^2)}{(\mathbf{s}-\mathbf{s}')^2}, \quad i = x, y. \end{aligned} \tag{A4}$$

The range of the variable i can be broadened to incorporate three variables, x, y, and z, without adding new solutions. This makes it possible to write (A4) in a more symmetric form,

$$\mu\Phi(s)xs^2 - \mu\int K(\mathbf{s},\mathbf{s}')x'\mathbf{s}\mathbf{s}' d^3s' - \frac{1}{2}g(s)s^2 - xs^2 = 0,$$

$$\mu\Phi(s)ys^2 - \mu\int K(\mathbf{s},\mathbf{s}')y'\mathbf{s}\mathbf{s}' d^3s' + \frac{1}{2}g(s)s^2 + ys^2 = 0,$$

with the same $K(\mathbf{s},\mathbf{s}')$. The new equations are isotropic under rotations of vector $\mathbf{s}$, whereby we can look for the solution in the isotropic form $x = x(s)$ and $y = y(s)$. Going over to the function $z = x + y$, we finally arrive at the integral equation

$$\begin{aligned} &z(s)[\mu^2\Phi^2(s) - 1] + \mu^2\int L(\mathbf{s},\mathbf{s}')\frac{\mathbf{s}\mathbf{s}'}{s^2} z(s') d^3s' = g(s), \\ &L(\mathbf{s},\mathbf{s}') = \int K(\mathbf{s},\mathbf{s}'')K(\mathbf{s}'',\mathbf{s}')d^3s'' - K(\mathbf{s},\mathbf{s}')[\Phi(s) + \Phi(s')] \\ &\qquad = L(s, s', \cos\theta), \quad \cos\theta = \frac{\mathbf{s}\mathbf{s}'}{ss'}. \end{aligned} \tag{A5}$$

The energy spectrum (3.32) then assumes the form

$$\begin{aligned} &\omega = \Omega_H + \frac{8}{3\alpha}\frac{T}{\hbar} r_D^2 (k^2 A + k_\perp^2 B_z), \\ &B_z = 2\pi\int_0^\infty s^4 z(s)\exp(-s^2) ds, \end{aligned} \tag{A6}$$

where the numerical factor A is determined via Eqs. (3.18) and is approximately equal to 2.51.

Equations (A5) and (A6) determine the spectrum of transverse spin waves generally for an arbitrary degree of polarization and in the presence of an external magnetic field (i.e., also for quasiequilibrium polarization at $H \neq 0$ and $\mathbf{H} \| \mathbf{m}$). In the most interesting case of thermodynamically

equilibrium polarization, where α is given by formula (3.29), Eq. (A6) simplifies somewhat. Indeed, in this situation the parameter μ is always very small, $\mu \ll 1$. For $\beta H \ll T$, the value of μ coincides with the small parameter γ specified in (3.35), and a further increase in H makes μ even smaller. Expanding (A5) and (A6) in powers of μ, we find that

$$z(s) \approx -g(s) + \mu^2 q(s), \quad B_z \approx -A + \mu^2 B_q,$$
$$q(s) \approx -\Phi^2(s) g(s) - \int L(\mathbf{s}, \mathbf{s}') \frac{\mathbf{s}\mathbf{s}'}{s^2} g(s') d^3 s'. \tag{A7}$$

In the principal approximation in $\mu \ll 1$ the results of (A5) and (A7), naturally, coincide with (3.38).

When quasiequilibrium polarization occurs at $H \neq 0$ in a fairly weak field, that, when

$$\frac{\Omega_H}{\omega_L} \ll \alpha \frac{\hbar \omega_L}{T} \ll 1, \tag{A8}$$

the situation with $\mu \gg 1$ may be realized. In this limiting case the approximate solutions of (A5) and (A6) take the form

$$z(s) \approx \frac{1}{\mu^2} t(s), \quad B_z \approx \frac{1}{\mu^2} B_t,$$
$$\Phi^2(s) t(s) + \int L(\mathbf{s}, \mathbf{s}') \frac{\mathbf{s}\mathbf{s}'}{s^2} t(s') d^3 s' = g(s). \tag{A9}$$

Naturally, the results (A7) and (A9) have meaning only if the degree of polarization, α, which enters into parameter μ, meets the criterion (3.22).

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Translated by Eugene Yankovsky