Accounting for Birch's observed anisotropy of the universe: cosmological rotation?

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We have investigated the effect of spacetime curvature on the relative position angle Δ between the direction of maximum elongation of a radio source and the direction of the integrated plane of polarization for emission that has propagated from a source to the observer. A more detailed analysis has been carried out for a cosmological model with global rotation (Gödel-type metric). In contrast to recent ideas about the origin of the dipole anisotropy in Δ first proposed by Birch and ascribed by him to the global rotation of the universe, we have found that such rotation cannot induce the sort of anisotropy described by Birch, with $\Delta \propto \cos\theta$ (θ is the angle between a ray and the rotation axis). Instead, we expect anisotropy with $\Delta \propto \sin^2\theta$. We therefore conclude that the Birch effect, if real, cannot be due to the rotation of the universe.

1. INTRODUCTION

Cosmological models with rotation have been discussed in the recent literature. The impetus for interest in such models came from the observations of Birch,[1] who discovered in a sample of 94 radio galaxies that there is a notable asymmetry in the angle Δ between the dominant direction of a source's magnetic field and its maximum elongation (hereafter we refer to Δ as the relative position angle, or RPA), which depends on the location of the source relative to a certan plane in space. The dipole anisotropy identified statistically by Birch—with $\Delta \propto \cos\theta$ (θ is the angle between the direction of a ray and the anisotropy axis)—he ascribed to the global rotation of the universe at a rate $\sim 10^{-13}$ rad/yr. Various opinions about the reality of the Birch effect were subsequently voiced,[2–5] but to this day the effect has not been convincingly refuted.

Because the emission from radio sources is generated by the synchrotron mechanism, Δ is in fact the angle between the dominant direction of the radio polarization vector and the direction of maximum source elongation. In most radio sources, that dominant polarization vector is either parallel or perpendicular to the direction of source extension,[6] consistent with astrophysical models of radio sources (see, e.g., Ref. 7). For the sake of convenience, then, we may redefine Δ at the source to be zero. If an observer were to detect some nonzero RPA, that would then suggest the existence of some mechanism—possibly even cosmological rotation—leading to rotation of the polarization vector, the direction of elongation, or possibly both, as the radiation traveled from the source to the observer.

It has recently been shown[8] that the polarization vector does indeed rotate (relative to a local coordinate basis) in rotating cosmological models, and that the rotation behaves in a manner similar to that proposed by Birch. This rotation results from deformation of the beam of rays defining the image of the source as it propagates in curved space.[9] There is no such effect in a Friedmann space, but it can show up in a rotating space, so the results obtained by Korotkiĭ and Obukhov[8] still beg the question of what happens to RPA in a rotating universe. Our purpose here is to provide an answer to that question.

2. REPRESENTATIVE BEAM AND ITS PROPERTIES

To analyze variations in the RPA, we introduce a modeling beam (a *representative beam*, or RB) that carries the image of the source, and we investigate the behavior of the so-called optical scalars that characterize the beam geometry. Bearing in mind the simple optical analogy of rays traveling down a telescope, we construct the RB using on- and off-axis rays leaving all source points and focused by the telescope objective. In the geometrical optics approximation, such a beam will yield the same image of the source, in terms of its geometry, as the actual beam. Other properties of the RB are:

1) the beam is a narrow one, by virtue of the small size of the source relative to the distance to the observer, and to the radius of curvature of space;

2) the beam is comprised of nonrotating rays; we neglect rotation internal to the source;

3) for simplicity, the beam geometry is assumed to be elliptical, with principal semiaxes a and b; we call the corresponding directions the extremal directions of the beam cross section.

The RB thus defined is parametrized by specifying the affine parameter s for the rays ($s = s_0 = 0$ at the source and $s = s_1 > 0$ at the observer), and the two parameters y^A ($A = 1,2$), which number the rays; $k^\mu = \partial x^\mu / \partial s$ is an isotropic vector tangent to the rays. Since we are dealing with a pencil beam, the location of any ray in the RB relative to the fiducial ray ($y^A = 0$) can be specified by the connection vector $\xi^\mu = \partial x^\mu / \partial y^A$ ($s, y^A = 0$), where Δy^A is constant for a given ray. The four-velocity u^μ is also defined at any point along a ray. As a rule, world lines of observers are the t coordinates of curves in four-space. The parameters s and y^A are chosen such that $\xi^\mu k_\mu = 0$ at any point on a basis ray, and $\xi_\mu u^\mu = 0$ for $s = 0$ and $s = s_1$.

In addition to the connection vector ξ^μ along a basis ray, we also specify the (complex) polarization vector $t^\mu = 2^{-1/2}(e^\mu + ih^\mu)$, which is comprised of the unit vectors **e** and **h** collinear with the field vectors **E** and **H**. For t^μ, the geometrical optics approximation yields

$$t^\mu k_\mu = 0, \qquad t_{;\nu}{}^{\mu} k^\nu = 0,$$

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and we may also assume that $t_\mu u^\mu = 0$ at $s=0$.

As we noted above, the beam properties are described by the optical scalars

$$\rho=-{}^1/_2 k_{;\mu}{}^{\mu}=k_{\mu;\nu}t^\mu \bar{t}^\nu, \quad \sigma=k_{\mu;\nu}t^\mu t^\nu,$$

where the overbar denotes complex conjugation. The parameter ρ is the relative rate of change of the cross-sectional area ΔS of the beam,

$$\rho=-\frac{1}{2}\frac{d\ln\Delta S}{ds},$$

and σ is the rate of deformation of the beam cross section (see Ref. 10 for a more detailed account of the geometrical meaning of ρ and σ). In the Newman-Penrose formalism,[10,11] the equations for ρ and σ are

$$d\rho/ds=\rho^2+|\sigma|^2+\Phi_{00}, \quad \Phi_{00}=-{}^1/_2 R_{\mu\nu}k^\mu k^\nu,$$
$$d\sigma/ds=2\rho\sigma+\Psi_0, \quad \Psi_0=-C_{\alpha\beta\gamma\delta}k^\alpha t^\beta k^\gamma t^\delta, \tag{1}$$

where $R_{\mu\nu}$ is the Ricci tensor, and $C_{\alpha\beta\gamma\delta}$ is the conformal curvature tensor. At the initial time $s=0$, ρ_0 and σ_0 are generally nonzero, with $\rho_0>0$ if the beam is manifestly convergent. At s_1, the location of the observer and the point at which the rays come to a focus, $\rho(s_1)=\infty$ (which follows from $\Delta S(s_1)=0$) and $\sigma(s_1)=0$. In view of the importance of the latter equality, we outline a proof. From the second of Eqs. (1),

$$\sigma=F^{-2}(s)\left[\sigma_0+\int_0^s\Psi_0(s')F^2(s')ds'\right], \tag{2}$$

where

$$F^2(s)=\exp\left[-2\int_0^s\rho(s')ds'\right]=\Delta S(s)/\Delta S(0).$$

Since Ψ_0, F^2, and σ_0 are bounded, we have

$$\sigma(s)\xrightarrow[s\to s_1]{}\frac{c}{F^2(s)}, \quad c=\sigma_0+\int_0^{s_1}\Psi_0(s)F^2(s)ds.$$

From the first of Eqs. (1) and the fact that Φ_{00} is bounded, we find that near $s=s_1$,

$$d^2F/ds^2=-|\sigma|^2F\approx -c^2/F^3. \tag{3}$$

Since $\Delta S(s_1)=0$, $F(s_1)=0$, and it can be shown that $F(s)\sim O(s_1-s)$ and $s\to s_1$. This asymptotic behavior is incompatible with (3) as long as c is nonzero. We thus conclude that

$$\sigma_0=-\int_0^{s_1}\Psi_0(s)F^2(s)ds$$

and

$$\sigma=-F^{-2}(s)\int_s^{s_1}\Psi_0(s')F^2(s')ds'. \tag{4}$$

Equation (4) is the basic equation that describes the effects of the curvature of spacetime on the characteristics in question (source image geometry, RPA).

3. EQUATIONS FOR THE CONNECTION VECTOR AND RPA

The properties of the connection vector ξ^μ make it possible to derive the expansion

$$\xi^\mu=qk^\mu+2^{-1/2}(\bar{v}t^\mu+v\bar{t}^\mu), \quad v=-2^{1/2}(t_\mu\xi^\mu). \tag{5}$$

From (5), $|\xi|^2\equiv-\xi_\mu\xi^\mu=|v|^2$. The phase ψ of v $\times(v=|v|e^{i\psi})$ is the angle between the connection vector and the real polarization vector e^μ (ψ is reckoned in the direction from e^μ to h^μ).

Next, we deal with the connection vectors, which are produced to the boundary of the RB. Points on the boundary of the two-dimensional beam profile are parametrized by the single angular parameter φ. The quantity v becomes a function of s and φ. Making use of the laws of transport of t^μ and ξ^μ along k^μ ($t^\mu_{;\nu}k^\nu=0$, $\xi^\mu_{;\nu}k^\nu=k^\mu_{;\nu}\xi^\nu$), it is straightforward to obtain the transport law for v along the beam. With the definitions

$$\hat{V}=\begin{pmatrix}v\\ \bar{v}\end{pmatrix}, \quad \hat{A}=\begin{pmatrix}\rho & \sigma\\ \bar{\sigma} & \rho\end{pmatrix}$$

we have

$$d\hat{V}/ds=-\hat{A}\hat{V}, \quad \hat{V}(0)=\hat{V}_0(\varphi). \tag{6}$$

The formal solution of (6) is

$$\hat{V}(s,\varphi)=\hat{B}(s)\hat{V}_0(\varphi), \tag{7}$$

where the 2×2 matrix $\hat{B}(s)$ is equal to the matrix $\hat{A}$ to an integer power, whereupon $B_{21}=\bar{B}_{12}$ and $B_{22}=\bar{B}_{11}$. Furthermore,

$$|v|^2={}^1/_2\hat{V}_0{}^+\hat{B}^+\hat{B}\hat{V}_0=(B_{11}B_{21}v_0{}^2+B_{12}B_{22}\bar{v}_0{}^2$$
$$+(B_{11}B_{22}+B_{12}B_{21})|v_0|^2). \tag{8}$$

We will be interested in the extremal connection vectors, which are produced to the extremal points of the beam contour. The corresponding values of φ may be obtained from the equation $d|v|^2/d\varphi=0$, where $|v|^2$ is given by (8) (v_0 depends on φ). We have assumed that the RB is initially elliptical in cross section, with semiaxes a and b; we may also assume, with no loss of generality, that the polarization vector e^μ is initially parallel to the major axis of the ellipse. Then by virtue of the fact that the vector ξ^μ is initially orthogonal to u^μ, we can write

$$\xi_0{}^\mu=a\cos\varphi\cdot e_0{}^\mu+b\sin\varphi\cdot h_0{}^\mu.$$

This then implies that

$$v_0=a\cos\varphi+ib\sin\varphi, \quad |v_0|^2=a^2\cos^2\varphi+b^2\sin^2\varphi. \tag{9}$$

Differentiating $|v|^2$ with respect to φ and making use of (9), we find that the angle φ_e corresponding to the extremal connection vector for given s is

$$\operatorname{tg}2\varphi_e=\frac{2ab(\operatorname{Re}X\operatorname{Im}Y-\operatorname{Im}X\operatorname{Re}Y)}{b^2|Y|^2-a^2|X|^2}, \tag{10}$$

where $X=B_{11}+B_{12}$ and $Y=B_{11}-B_{12}$ satisfy

$$dX/ds = -\rho X - \sigma \bar{X}, \quad dY/ds = -\rho Y + \sigma \bar{Y}, \quad X(0) = Y(0) = 1. \tag{11}$$

The RPA is the angle between the polarization vector e^μ and the connection vector produced to the extremal point. As noted above, it equals the phase ψ of v. In the present instance, we must take v at given s and with $\varphi = \varphi_e(s)$. For the tangent of the RPA, which we denote by Δ, we than have

$$\operatorname{tg} \Delta = \operatorname{Im} v(s,\varphi_e(s))/\operatorname{Re} v(s,\varphi_e(s)). \tag{12}$$

Using (10), some manipulation of (12) yields

$$\operatorname{tg} 2\Delta = \operatorname{Im}(a^2X^2 - b^2Y^2)/\operatorname{Re}(a^2X^2 - b^2Y^2). \tag{13}$$

If ρ and σ are known, then (13) and (11) together completely solve the present problem, specifically, finding a quantity that properly corresponds to the one utilized by Birch.

4. COSMOLOGICAL MODEL WITH ROTATION

Equation (13) will now be used to calculate the effect of interest in a rather general cosmological model with rotation, which is described by the metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - R^2(t)\left[(dx^1)^2 + ka^2(x^1)(dx^2)^2 + (dx^3)^2\right] - 2R(t)p^{1/2}a(x^1)dx^2dt, \tag{14}$$

where $a(x^1) = \exp(mx^1)$, $k > 0, m > 0$, and p are constants, and $t = x^0$. The metric (14) is a natural time-dependent generalization of Gödel-type cosmological models. One obtains the Gödel metric when R is constant and $k/p = -1/2$. It can be shown that when $k > 0$, the model has none of the closed timelike curves that exist in the Gödel metric. In the model represented by (14), the local rotation of matter and observers comoving with it at four-velocity $u^\mu = \delta_0^\mu$ may be characterized by the rotation tensor

$$\Omega_{\mu\nu} = -Rmp^{1/2}\delta_{[\mu}{}^1\delta_{\nu]}{}^2$$

and the angular velocity

$$\Omega = (\tfrac{1}{2}\Omega_{\mu\nu}\Omega^{\mu\nu})^{1/2} = \frac{m}{2R}\left(\frac{p}{p+k}\right)^{1/2}.$$

Rather than working with the metric (14), it will be more convenient to use the stationary metric conformal to it,

$$d\tilde{s}^2 = \tilde{g}_{\mu\nu}dx^\mu dx^\nu = d\tau^2 - (dx^1)^2 - ka^2(x^1)(dx^2)^2 - (dx^3)^2 - 2p^{1/2}a(x^1)dx^2d\tau, \tag{15}$$

where

$$\tilde{g}_{\mu\nu} = R^{-2}g_{\mu\nu}, \qquad \tau = \int R^{-1}\,dt.$$

We wish to map the RB and all associated structures from the space (14) to the space (15). The basic steps are as follows.

a) The congruence of rays in the RB is mapped to the congruence of $\widetilde{\mathrm{RB}}$ in such a way that the rays of $\widetilde{\mathrm{RB}}$ pass through the same points x^α as the corresponding rays of RB. The rays of $\widetilde{\mathrm{RB}}$ will obviously be isotropic. We parametrize $\widetilde{\mathrm{RB}}$ as

$$s' = \int_0^s R^{-2}(t(s))ds. \tag{16}$$

The congruence of the rays in the RB, $x^\mu(s,y^A)$, is then transformed into the congruence in $\widetilde{\mathrm{RB}}$, $x^\mu(s(s^1),y^A) = \tilde{x}^\mu(s^1,y^A)$.

b) The expression for the tangent vector $\tilde{k}^\mu$ is

$$\tilde{k}^\mu = \frac{\partial \tilde{x}^\mu}{\partial s'} = \frac{\partial x^\mu}{\partial s}\frac{ds}{ds'} = R^2k^\mu, \quad \tilde{k}_\mu = k_\mu. \tag{17}$$

It is not difficult to show that $\tilde{k}^\mu$ is a geodesic vector field. In $\widetilde{\mathrm{RB}}$, we define the connection vector

$$\tilde{\xi}^\mu = \frac{\partial \tilde{x}^\mu}{\partial y^A}(s',0)\Delta y^A = \frac{\partial x^\mu}{\partial y^A}\Delta y^A = \xi^\mu. \tag{18}$$

c) On the basis ray in $\widetilde{\mathrm{RB}}$ we define the vector

$$\tilde{t}^\mu = Rt^\mu + b(s)R^2k^\mu, \quad \tilde{t}_\mu = R^{-1}t_\mu + bk_\mu, \tag{19}$$

where $b(s)$ satisfies

$$\frac{db}{ds} = \frac{dR}{ds}R^{-2}t^0, \qquad b(0) = 0.$$

Then $\tilde{t}^\mu$ satisfies the relations

$$\tilde{t}_\mu\tilde{k}^\mu = 0, \quad \tilde{t}_{\mu;\nu}\tilde{k}^\nu = 0.$$

d) The optical scalars and the scalar Ψ_0 transform as

$$\tilde{\rho} = R^2[\rho + (dR/ds)R^{-1}k^0], \quad \tilde{\sigma} = \sigma R^2, \quad \tilde{\Psi}_0 = R^4\Psi_0. \tag{20}$$

The equations for $\tilde{\rho}$ and $\tilde{\sigma}$ retain the same form as (1), except that Ψ_0 and Φ_{00} must be replaced by $\tilde{\Psi}_0$ and $\tilde{\Phi}_{00} = -\frac{1}{2}\tilde{R}_{\mu\nu}\tilde{k}^\mu\tilde{k}^\nu$.

e) Finally, we have

$$\tilde{v} = -2^{1/2}(\tilde{\xi}^\mu\tilde{t}_\mu) = R^{-1}v. \tag{21}$$

Taking (20) into account, the equations for $\tilde{v}$ retain the same form as in (6). Note that by virtue of (21), the phase of $\tilde{v}$ is the same as that of v. Since the RPA may be expressed in terms of the phase of v, its invariance under a conformal transformation makes it possible to work with the simpler conformal metric. From here on we work only with the stationary metric (15), and drop the tilde.

In the space (15) we choose a reference frame of the form

$$l^\mu = 2^{-1/2}(\delta_0{}^\mu + \delta_3{}^\mu), \quad n^\mu = 2^{-1/2}(\delta_0{}^\mu - \delta_3{}^\mu), \tag{22}$$

$$m^\mu = 2^{-1/2}\left[\left(\frac{p}{p+k}\right)^{1/2}\delta_0{}^\mu + \frac{\delta_2{}^\mu}{a(p+k)^{1/2}} + i\delta_1{}^\mu\right].$$

With this basis set, the only nonvanishing scalar out of all the basis components Ψ_A $(A = 0,1,\ldots,4)$ of the Weyl tensor in the Newman–Penrose formalism is

$$\Psi_2 = \frac{m^2k}{6(p+k)} = \frac{2}{3}\frac{k}{p}\Omega^2. \tag{23}$$

Here Ω is the angular velocity at which the space (15) is rotating.

The geodesic equations for k^μ are integrable.[8] Expressing k^μ in terms of the basis vectors (22),

$$k^\mu = d_1l^\mu + d_2n^\mu + d_3m^\mu + \bar{d}_3\bar{m}^\mu,$$

the coefficients d_i are

$$d_1=2^{-1/2}(q_0+q_3),\quad d_2=2^{-1/2}(q_0-q_3),$$

$$d_3=-2^{-1/2}\left[\left(\frac{p}{p+k}\right)^{1/2}\left(q_0-\frac{q_2}{ap^{1/2}}\right)+im(q_1+x^2q_2)\right],\qquad (24)$$

where the q_i ($i=0,...,3$) are constants along the geodesic, with q_0 being the frequency of emission as seen by an observer at rest, and q_3/q_0 is the cosine of the angle between the direction of the ray and the x^3 axis.

The components of the real polarization vector e^μ may be conveniently written in the form

$$e^\mu=c_1l^\mu+c_2n^\mu+c_3m^\mu+\bar{c}_3\bar{m}^\mu.$$

Integrating the equation of parallel transport for e^μ along k^μ and exploiting the fact that e^μ is a unit vector, we obtain

$$\begin{gathered}c_1={}^1/_2q_4(q_0/q_3+1)+P\cos(\omega s-\alpha),\\ c_2={}^1/_2q_4(q_0/q_3-1)+P\cos(\omega s-\alpha),\\ c_3=2^{-1/2}D/\bar{d}_2,\end{gathered}\qquad (25)$$

where

$$P=\left[\frac{1}{2}(q_0^2-q_3^2)\right]^{1/2}\Big/|q_3|.$$

$$\omega=\frac{m}{2}\left(\frac{p}{p+k}\right)^{1/2}|q_3|=\Omega|q_3|,$$

$$D=\frac{q_4(q_0^2-q_3^2)}{2^{1/2}q_3}+q_0P\cos(\omega s-\alpha)+i|q_3|P\sin(\omega s-\alpha),$$

q_4 is constant along a ray, and amounts to the cosine of the angle between the polarization vector and the x^3 axis. The angle α in (25) is related to the initial orientation of the polarization vector relative to the basis m^μ. The expansion coefficients of the complex polarization vector t^μ in the basis (22) ($t^\mu=c_1l^\mu+c_2n^\mu+c_3m^\mu+c_4\bar{m}^\mu$) are comprised of the corresponding coefficients for the vectors e^μ and h^μ; for example, $c_3=2^{-1/2}(c_{3e}+ic_{3h})$, etc. The expressions for D_e and D_h differ by the constant q_4 and the angle α. The orthogonality of e^μ and h^μ implies that $\alpha_e=\alpha_h+(\pi/2)\operatorname{sgn}q_3$.

Finally, we can use the foregoing results to obtain an expression for Ψ_0, which enters into Eq. (2) for σ:

$$\begin{aligned}\Psi_0&=6\Psi_2(d_1c_2-d_3c_4)^2=6\Psi_2[d_1c_2-2^{-1/2}(\bar{D}_e+i\bar{D}_h)]^2\\&=\frac{km^2}{p+k}(q_0^2-q_3^2)\exp[\pm2i(\omega s-\alpha)]\equiv A\exp[\pm2i(\omega s-\alpha)].\end{aligned}\qquad (26)$$

The upper sign in the exponential corresponds to $q_3>0$, and the lower to $q_3<0$.

5. CALCULATION OF THE RPA

Before calculating the RPA Δ in the cosmological model with rotation, let us analyze the general equation (13). The latter implies that Δ will not vary if the quantities X and Y are real, which will be the case [see (11)] if the scalar σ is real; that, in turn will be true when the scalar Ψ_0 is real. In particular, the RPA will not vary at all in a Friedmann (conformally flat) universe, for which $\Psi_0=0$. Likewise, the RPA will not vary in a rotating universe with the metric (14) for $k>0$. In the space with metric (14) with $k>0$, or (15), we will have the same result for ray propagation along the rotation axis x^3, where $q_0^2=q_3^2$.

Another special case is that in which a ray propagates in the equatorial plane and $q_3=0$; then $\omega=0$ and the scalar Ψ_0 is a complex constant. Except for those cases in which $\alpha=0$ or $\pm\pi/2$, the RPA will vary, corresponding to the fact that the extremal directions of the source beam lie in the equatorial plane and perpendicular to it.

We now treat the general case of arbitrary ray direction. Here we seek a solution to Eq. (1) (and taking (4) into account) with Ψ_0 given by Eq. (26) and constant Φ_{00}:

$$\Phi_{00}=\omega^2\left[1-\frac{2k}{p}\left(\frac{q_0^2}{q_3^2}-1\right)\right].\qquad (27)$$

These equations can be solved, in principle, given Ψ_0 and Φ_{00}, but only with a great deal of effort and by analyzing many special cases. On the other hand, with $\Omega\sim10^{-13}$ rad/yr $=3\cdot10^{-21}$ rad/sec, it is perfectly adequate to use the approximate solution obtained b assuming that $\omega s_1\ll1$. In point of fact, in real dimensional units,

$$\omega s=\Omega\frac{|x^3|}{c}\leqslant\Omega\frac{r}{c}\sim10^{-31}r\,(\mathrm{cm}),$$

so that even at $r\sim10^{28}$ cm, the foregoing inequality holds.

Equation (4) yields

$$|\sigma|\leqslant|\Psi_0|(s_1-s)\leqslant|\Psi_0|s_1\equiv\max|\sigma|.$$

Strictly speaking, it has been assumed here that the cross section of the RB decreases monotonically between the source and observer. Then from (26) and (27) we obtain

$$\frac{\max|\sigma|^2}{|\Phi_{00}|}=\frac{16(k^2/p^2)(q_0^2/q_3^2-1)^2}{1-(2k/p)(q_0^2/q_3^2-1)}\omega^2s_1^2\ll1.$$

This means that in the equation of ρ, we can neglect $|\sigma|^2$ in comparison with Φ_{00}. Furthermore, even the contribution of Φ_{00} to ρ is of order $\omega^2s_1^2$, so that it too can be neglected. Thus, for ρ we can use the equation for flat space ($|\sigma|=\Phi_{00}=0$), and the solution of that equation leads to the function $F=1-s/s_1$. Substituting this function into (4) and bearing in mind that $A\sim O(\omega^2)$ yields to the same order

$$\sigma=-{}^1/_3A\exp(\mp2i\alpha)(s_1-s).\qquad (28)$$

Integrating (11) up to terms that are $O(\omega^2s_1^2)$ yields the functions

$$\begin{aligned}X&=(1-s/s_1)[1+{}^1/_3A(s_1s-s^2/2)\exp(\mp2i\alpha)],\\ Y&=(1-s/s_1)[1-{}^1/_3A(s_1s-s^2/2)\exp(\mp2i\alpha)].\end{aligned}\qquad (29)$$

From (13), we finally obtain

$$\operatorname{tg}2\Delta(s_1)=\mp\frac{1}{3}As_1^2\frac{a^2+b^2}{a^2-b^2}\sin2\alpha.\qquad (30)$$

The minus sign in Eqs. (28)–(30) corresponds to $q_3>0$ and the plus sign to $q_3<0$. It follows from (30) that $\Delta=0$ when $\alpha=0$ or $\pm\pi/2$. Further investigation reveals that these special cases correspond to one of the extremal directions of the source lying in the equatorial plane.

Note that in the present treatment, the RPA depends on specific characteristics of the source—its orientation relative to the rotation axis and the size of the semiaxes a and b. For example, the angle Δ increases as $a-b\to0$ (albeit the error in determining Δ also grows as $(a-b)^{-1}$).

6. DISCUSSION

It is immediately obvious that the results obtained here are inconsistent with the behavior deduced by Birch from the observational data. To see this, we represent (30) in the form $f_1(z)f_2(\theta)$, where θ is the angle between the ray direction and the rotation axis, and $z = \delta v/v$ is the redshift. If we retain only those terms in Δ that are at most quadratic in z, then it suffices to find s as a linear function of z. It can be shown that $s = z(H_0R_0q_0\gamma)^{-1}$, where R_0 and H_0 are the radius of the universe and the Hubble constant at the present epoch, and $\gamma = k(k+p)^{-1}(1+p^{1/2}q_2/kq_0)$. Substituting this expression for s into (30) and noting that Δ is a small angle, some rearrangement yields

$$\Delta = \mp\frac{2}{3}\frac{k}{p\gamma^2}\frac{\Omega_0^2}{H_0^2}z^2\frac{a^2+b^2}{a^2-b^2}\sin^2\theta\sin 2\alpha. \qquad (31)$$

In deriving (31), we have used the fact that $q_3/q_0 = \cos\theta$;

$$\Omega_0 = \frac{m}{2R_0}\left(\frac{p}{p+k}\right)^{1/2}$$

is the angular rotation rate of the universe (with the metric (14)) at the present epoch. In general, γ induces an additional angular dependence in Δ. The signs in (30) and (31) mean that the major axis of source's boundary map actually rotates in the same direction as the global rotation. Birch,[1] on the other hand, has not specified $\Delta \propto \cos\theta$ or the z-dependence of Δ, and in the metric (14), the polarization vector rotates by an angle proportional to $\Omega_0 z\cos\theta/H_0$ (Ref. 8).

The reason for the disparity with Birch's result[1] is plain to see if we consider the situation in which a ray propagates along the rotation axis. Taking $q_3 > 0$, the complex and real polarization vectors are then

$$t^\mu = \exp[-i(\beta+\omega s)]m^\mu, \quad -e^\mu m_\mu = 2^{-1/2}\exp[i(\beta+\omega s)]. \qquad (32)$$

Substituting this expression for t^μ into (5), we obtain

$$-(\xi^\mu m_\mu)/|\xi| = -(\xi^\mu m_\mu)/|v| = 2^{-1/2}\exp[i(\alpha+\beta+\omega s)]. \qquad (33)$$

where α is the phase of v, which is constant for propagation along the rotation axis. Equations (32) and (33) imply that the connection vector and polarization vector rotate in the same direction at the same rate, and for a suitable choice of α they coincide.

To put matters differently, we can interpret this situation by saying that the rotation of the universe induces local coordinate systems, and free particles "fall behind" the reference frames. In other words, the direction to one of these particles rotates with respect to the coordinate basis in a sense opposite that of the rotation of the universe. This is also true of the vector joining two geodesics, and of the polarization vector, which undergoes parallel transport along a ray. Extending these results, we might expect that such will be the case in any metric with global rotation, and therefore that the type of anisotropy discovered by Birch, if it is in fact real, cannot be accounted for by rotation of the universe, but must arise instead for some other reason (for example, the existence of a metagalactic magnetic field).

Our investigation leads us to believe that a rotating universe will give rise to anisotropy of the kind described by Eq. (31). That anisotropy is greatest when a ray propagates in the equatorial plane ($\theta = \pi/2$), although even then it will clearly be small. For example, for $H_0 = 50$ km/(sec·Mpc), $z = 1$, and the assumed value $\Omega_0 = 10^{-13}$ rad/yr, we have $\Delta \sim (\Omega_0 z/H_0)^2 = 4\cdot 10^{-6}$.

Let us examine the nature of the anisotropy predicted by Eq. (31) for a ray propagating either in the equatorial plane or at a small angle to it (it also makes sense to consider a radio source whose major axis is oriented at $\alpha \approx 45°$ to the rotation axis). What (31) means is that the polarization vector for any such source will be rotated by an angle Δ relative to the major axis in the direction away from the rotation axis. We hope that this prediction will spark the interest of the astrophysical community to search for anisotropy induced by the global rotation of the universe.

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