Nonlinear effects in photon emission by an electron in the field of a circularly polarized electromagnetic wave

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We consider the process of inverse Compton scattering of an intense circularly polarized laser wave focused on a beam of relativistic longitudinally polarized electrons. We investigate regions of nonlinear effects corresponding to values of the intensity parameter obeying not only $x^2 < 1$ but also $x^2 \geqslant 1$ ($x^2 = -e^2 A_\mu^2/m^2$, where A_μ is the vector potential of the wave). In each of these regions we construct for a neodymium laser the spectra and the energy dependence of the degree of circular polarization of the emitted photon for various values of the polarization of the electron and the wave. For $x^2 < 1$, when the radiation is dominated by the first harmonic, we take into account the second, as well as the third, harmonic. We show that for $x^2 > 1$ the process of emission of a hard photon is essentially nonlinear. The calculations are based on direct evaluation of QED matrix elements in the diagonal spin basis.

It was shown in Ref. 1 that in the proposed linear accelerators with colliding e^+e^- beams, colliding γe and $\gamma\gamma$ beams could be achieved with approximately the same energy and luminosity as the original e^+e^- beams. The intense beams of hard γ quanta required for this would be obtained by inverse Compton scattering of an intense laser discharge, focused on the electron beam. For a discharge of sufficiently high power in the conversion region,[1] processes requiring the simultaneous absorption from the wave of several photons become significant

$$n\gamma_0+e^-\to e^-+\gamma,\quad n>1, \tag{1}$$

$$s\gamma_0+\gamma\to e^++e^-,\quad s>1. \tag{2}$$

The first of these nonlinear processes leads to a broadening of the spectrum of the high energy photons, while the second effectively lowers the threshold for e^+e^- pair production.[2] The processes (1) and (2) were studied systematically in Ref. 3. In Ref. 4 phenomena arising in the collision of polarized electrons with photons of a circularly polarized electromagnetic wave were analyzed; nonlinear effects were studied in the region $x^2 < 1$ of the intensity parameter

$$x^2=-e^2\bar{A}_\mu^2/m^2, \tag{3}$$

where A_μ is the vector potential of the wave. Left uninvestigated was the more interesting region of nonlinear effects, determined by the condition $x^2 \geqslant 1$ (see Ref. 1). In this region radiation, processes which proceed via simultaneous absorption of several photons from the wave and which cannot be described by the first few terms of the perturbation-theory series become essential.

The purpose of the present work is the study of nonlinear effects of higher order than were considered in Ref. 4 for $x^2 < 1$, as well as investigation of radiation processes in the region $x^2 \geqslant 1$.

In contrast to Refs. 3 and 4 we approach the process under consideration by direct calculation of the QED matrix elements[5,6] in the diagonal spin basis,[7–10] in which the little Lorentz group, common to the initial and final states of the particle, is realized.[6,11]

In the diagonal spin basis the vectors s and s' of the initial and final electrons lie in the hyperplane determined by their 4-momenta p and p' ($v = p/m$, $v' = p'/m$):[7,8]

$$s=\frac{(vv')v-v'}{[(vv')^2-1]^{1/2}},\qquad s'=-\frac{(vv')v'-v}{[(vv')^2-1]^{1/2}}. \tag{4}$$

Within the framework of this approach the process can be described with the interaction of the electron with the field of the electromagnetic wave taken into account exactly. According to Ref. 12 the S-matrix element for the transition of the electron from the state $\psi_p = \psi^\delta(p,s)$ to the state $\psi_{p'} = \psi^{\pm\delta}(p',s')$ ($\delta = \pm 1$) with the emission of a photon of 4-momentum $k' = (\omega', \mathbf{k}')$ and circular polarization vector $e_{\lambda'}$ ($\lambda' = \pm 1$) is given by the expression

$$S_{fi}=-ie\int \bar{\psi}_{p'}\hat{e}_{\lambda'}{}^*\psi_p \exp(ik'x)\,(2\omega')^{-1/2}\,d^4x, \tag{5}$$

where ψ_p and $\psi_{p'}$ are the exact electron wave functions in the field of the circularly polarized electromagnetic wave,[12] corresponding to the vector potential

$$A=a_1\cos kx+\lambda a_2\sin kx,\quad \lambda=\pm1, \tag{6}$$

where k is the wave vector, $k^2 = 0$, $a_1k = a_2k = a_1a_2 = 0$, $a_1^2 = a_2^2 = a^2$, λ is the helicity.

Using the methods described in Refs. 8–10 one can show that the matrix elements (5) have in the diagonal spin basis the form:

$$S_{fi}=-\frac{ie(4\pi)^{1/2}}{(2\omega' 2q_0 2q_0')^{1/2}}\sum_{n=1}^{\infty}\mathscr{M}^{(n)}_{\pm\delta,\delta}\,(2\pi)^4\delta^4(nk+q-q'-k'), \tag{7}$$

$$\mathscr{M}^{(n)}_{-\delta,\delta}=-\frac{1}{2}\lambda'(-\lambda)^n x\Big\{-\frac{2(1-u/u_n)}{(vv'+1)^{1/2}}(J_{n-1}+J_{n+1})+\frac{1}{2(u+1)}\Big[\frac{(u+2)^2}{(vv'+1)^{1/2}}-\delta\lambda'\frac{u^2}{(vv'-1)^{1/2}}\Big]J_{n+\lambda\lambda'}\Big\}, \tag{8}$$

$$\mathscr{M}^{(n)}_{\delta,\delta}=\frac{1}{2}\delta\lambda'(-\lambda)^n x\Big[\frac{u}{u_n}\Big(1-\frac{u}{u_n}\Big)\Big]^{1/2}\Big[\frac{u+2}{u}\Big(\frac{vv'-1}{vv'+1}\Big)^{1/2}-\delta\lambda'\Big]\Big[\frac{(vv'-1)^{1/2}}{(1+x^2)^{1/2}}(J_{n-1}+J_{n+1})-\frac{uu_n(1+x^2)^{1/2}}{2(u+1)(vv'-1)^{1/2}}J_{n+\lambda\lambda'}\Big], \tag{9}$$

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$$q=p+\frac{x^2m^2}{2kp}k, \quad q'=p'+\frac{x^2m^2}{2kp'}k,$$

$$u=\frac{kk'}{kp'}, \quad u_n=\frac{2nkp}{m_*^2},$$

$$q^2=q'^2=m_*^2=m^2(1+x^2),$$

$$2(vv'-1)=\frac{uu_n}{u+1}\left[1+x^2\left(1-\frac{u}{u_n}\right)\right],$$

$$nk+q=k'+q', \quad J_{n+\lambda\lambda'}=\frac{1+\lambda\lambda'}{2}J_{n-1}+\frac{1-\lambda\lambda'}{2}J_{n-1},$$

$$z_n=\frac{2nx}{(1+x^2)^{1/2}}\left[\frac{u}{u_n}\left(1-\frac{u}{u_n}\right)\right]^{1/2},$$

where $\mathscr{M}_{\delta\delta}^{(n)}$ and $\mathscr{M}_{-\delta\delta}^{(n)}$ are the amplitudes for the emission of the nth harmonic corresponding to transitions without and with electron spin flip, q and q' are the 4-vectors of the electron quasimomenta, $q=(q_0,\mathbf{q})$, $q'=(q_0',\mathbf{q}')$, J_n is the Bessel function of order n and of argument (z_n).

It is not hard to verify that the amplitudes (8) and (9) have kinematic singularities: they vanish for $u=u_n$ and $n>1$ $[\mathscr{M}_{\pm\delta\delta}^{(n)}(u=u_n)=0]$. The reason for this behavior will be clarified below.

Knowledge of the diagonal amplitudes (8) and (9) permits any other choice of amplitudes, including amplitudes with experimentally observable polarization states, which for ultrarelativistic particles are, as a rule, helicity amplitudes. The passage from the diagonal amplitudes $\mathscr{M}_{\pm\delta,\delta}^{(n)}$ (8) and (9) to the helicity amplitudes $\mathscr{M}_{\pm\delta,\lambda_e}^{(n)}$ is accomplished with the help of the $d^{1/2}$ functions of finite rotations[13]

$$M_{\pm\delta,\lambda_e}^{(n)}=\mathscr{M}_{\pm\delta,\sigma}^{(n)}d_{\sigma\lambda_e}^{1/2}(\alpha),$$

$$d_{\sigma\lambda_e}^{1/2}=\begin{pmatrix}\cos(\alpha/2) & -\sin(\alpha/2)\\ \sin(\alpha/2) & \cos(\alpha/2)\end{pmatrix}, \tag{10}$$

where α is the angle between the spin projection axis in the diagonal spin basis and the direction of motion of the incident electron beam. The quantities $\sin\alpha$ and $\cos\alpha$ are expressed in terms of the invariant variable u as follows

$$\cos\alpha=\left[\frac{u}{u_n}(u_n+2)\right]^{1/2}\Big/[uu_n+4(u+1)]^{1/2},$$
$$\sin\alpha=2\left(1-\frac{u}{u_n}\right)^{1/2}\Big/[uu_n+4(u+1)]^{1/2}. \tag{11}$$

Since, as a rule, the polarization of the final electrons is of no interest, it is necessary to add the squares of the amplitudes $\mathscr{M}_{\delta,\lambda_e}^{(n)}$ and $\mathscr{M}_{-\delta,\lambda_e}^{(n)}$. As a result we obtain

$$\sum_\delta (M_{\pm\delta,\lambda_e}^{(n)})^2=\Big[\sum_\delta(\mathscr{M}_{\pm\delta,\delta}^{(n)})^2+\lambda_e\cos\alpha[(\mathscr{M}_{++}^{(n)})^2-(\mathscr{M}_{--}^{(n)})^2$$
$$+(\mathscr{M}_{-+}^{(n)})^2-(\mathscr{M}_{+-}^{(n)})^2]$$
$$+2\lambda_e\sin\alpha(\mathscr{M}_{++}^{(n)}\mathscr{M}_{+-}^{(n)}+\mathscr{M}_{-+}^{(n)}\mathscr{M}_{--}^{(n)})\Big]\Big/2.$$

Then the differential emission probability per unit volume and unit time is determined by the following expression:

$$\frac{dW}{du}=\frac{e^2m^2n_e}{8q_0(u+1)^2}\sum_{n=1}^{\infty}(F_{0n}+\lambda\lambda_eG_{0n}+\lambda\lambda'F_{2n}+\lambda_e\lambda'G_{2n}),$$

$$F_{0n}=-4J_n^2+x^2\left(2+\frac{u^2}{u+1}\right)(J_{n-1}^2+J_{n+1}^2-2J_n^2), \tag{12}$$

$$G_{0n}=x^2\frac{(2+u)u}{u+1}\left(1-2\frac{u}{u_n}\right)(J_{n-1}^2-J_{n-1}^2),$$

$$F_{2n}=x^2\left(2+\frac{u^2}{u+1}\right)\left(1-2\frac{u}{u_n}\right)(J_{n-1}^2-J_{n+1}^2),$$

$$G_{2n}=\frac{u}{u+1}[-4J_n^2+x^2(2+u)(J_{n-1}^2+J_{n+1}^2-2J_n^2)],$$

where n_e is the electron density in the beam. The expression under the summation sign in Eq. (12) determines the probability of emission of the nth harmonic in the case when the laser and the emitted photons, as well as the initial electron, are in helicity polarization states. We note that for $x^2=0$ Eq. (12) (in contrast to the similar expression of Ref. 4) coincides with the results obtained for ordinary Compton scattering in Ref. 14. Moreover, representing the result in the form (12) is more convenient for numerical calculations and makes it possible to avoid the difficulties for $x^2>1$ noted in Ref. 2.

With the help of (12) the degree of circular polarization of the final state photon λ_f is determined as follows:[12]

$$\lambda_f=\sum_{n=1}^{\infty}(\lambda F_{2n}+\lambda_eG_{2n})\Big/\sum_{n=1}^{\infty}(F_{0n}+\lambda\lambda_eG_{0n}). \tag{13}$$

For $x^2<1$ the main contribution to the probability (12) comes from the first few harmonics. In Ref. 4 the emission of only the first two harmonics was considered. In the present work we consider in addition to the first two also the emission of the 3rd harmonic. Following Ref. 4, we expand the probability in the parameter $\Delta=x^2/(1+x^2)$, and not in x^2, and we expand only the Bessel functions, retaining the exact formulas for the quantities u_n. In this way we are able to correctly identify the above-indicated kinematic singularities. In summary we obtain for the first three harmonics:

$$\begin{aligned}F_{01}=x^2\Big\{&2+\frac{u^2}{1+u}-4\frac{u}{u_1}\left(1-\frac{u}{u_1}\right)\\&+4\Delta\frac{u}{u_1}\left(1-\frac{u}{u_1}\right)\left[1+\frac{u^2}{1+u}\right.\\&\left.-\frac{u}{u_1}\left(1-\frac{u}{u_1}\right)\right]+\Delta^2\left(\frac{u}{u_1}\right)^2\left(1-\frac{u}{u_1}\right)^2\\&\times\left[\frac{7}{2}+\frac{15}{4}\frac{u^2}{1+u}-\frac{5}{3}\frac{u}{u_1}\left(1-\frac{u}{u_1}\right)\right]\Big\},\end{aligned}$$

$$\begin{aligned}G_{01}=x^2\frac{(2+u)u}{1+u}\left(1-2\frac{u}{u_1}\right)\Big[&1-2\Delta\frac{u}{u_1}\left(1-\frac{u}{u_1}\right)\\&+\frac{5}{4}\Delta^2\left(\frac{u}{u_1}\right)^2\left(1-\frac{u}{u_1}\right)^2\Big],\end{aligned} \tag{14}$$

$$\begin{aligned}F_{21}=x^2\left(2+\frac{u^2}{1+u}\right)\left(1-2\frac{u}{u_1}\right)\Big[&1-2\Delta\frac{u}{u_1}\left(1-\frac{u}{u_1}\right)\\&+\frac{5}{4}\Delta^2\left(\frac{u}{u_1}\right)^2\left(1-\frac{u}{u_1}\right)^2\Big].\end{aligned}$$

$$
\begin{aligned}
G_{21}={}&x^2\frac{u}{1+u}\Big\{2+u-4\frac{u}{u_1}\Big(1-\frac{u}{u_1}\Big)\\
&-4\Delta\frac{u}{u_1}\Big(1-\frac{u}{u_1}\Big)\Big[1+u-\frac{u}{u_1}\Big(1-\frac{u}{u_1}\Big)\Big]\\
&+\Delta^2\Big(\frac{u}{u_1}\Big)^2\Big(1-\frac{u}{u_1}\Big)^2\Big[\frac{7}{2}+\frac{15}{4}u-\frac{5}{3}\frac{u}{u_1}\Big(1-\frac{u}{u_1}\Big)\Big]\Big\}
\end{aligned}
$$

—for the first harmonic;

$$
\begin{aligned}
F_{02}={}&4x^2\Delta\frac{u}{u_2}\Big(1-\frac{u}{u_2}\Big)\Big\{2+\frac{u^2}{1+u}-4\frac{u}{u_2}\Big(1-\frac{u}{u_2}\Big)\\
&-2\Delta\frac{u}{u_2}\Big(1-\frac{u}{u_2}\Big)\Big[4+\frac{3u^2}{1+u}-\frac{16}{3}\frac{u}{u_2}\Big(1-\frac{u}{u_2}\Big)\Big]\Big\},\\
G_{02}={}&4x^2\Delta\frac{u}{u_2}\Big(1-\frac{u}{u_2}\Big)\frac{(2+u)u}{1+u}\Big(1-2\frac{u}{u_2}\Big)\\
&\times\Big[1-4\Delta\frac{u}{u_2}\Big(1-\frac{u}{u_2}\Big)\Big],\\
F_{22}={}&4x^2\Delta\frac{u}{u_2}\Big(1-\frac{u}{u_2}\Big)\Big(2+\frac{u^2}{1+u}\Big)\Big(1-2\frac{u}{u_2}\Big)\\
&\times\Big[1-4\Delta\frac{u}{u_2}\Big(1-\frac{u}{u_2}\Big)\Big],\\
G_{22}={}&4x^2\Delta\frac{u}{u_2}\Big(1-\frac{u}{u_2}\Big)\frac{u}{1+u}\Big\{2+u-4\frac{u}{u_2}\Big(1-\frac{u}{u_2}\Big)\\
&-2\Delta\frac{u}{u_2}\Big(1-\frac{u}{u_2}\Big)\Big[4+3u-\frac{16}{3}\frac{u}{u_2}\Big(1-\frac{u}{u_2}\Big)\Big]\Big\}
\end{aligned}
\tag{15}
$$

—for the second harmonic;

$$
\begin{aligned}
&F_{03}\\
&=\frac{81}{4}x^2\Delta^2\Big(\frac{u}{u_3}\Big)^2\Big(1-\frac{u}{u_3}\Big)^2\Big[2+\frac{u^2}{1+u}-4\frac{u}{u_3}\Big(1-\frac{u}{u_3}\Big)\Big],\\
&G_{03}=\frac{81}{4}x^2\Delta^2\Big(\frac{u}{u_3}\Big)^2\Big(1-\frac{u}{u_3}\Big)^2\frac{(2+u)u}{(1+u)}\Big(1-2\frac{u}{u_3}\Big),\\
&F_{23}=\frac{81}{4}x^2\Delta^2\Big(\frac{u}{u_3}\Big)^2\Big(1-\frac{u}{u_3}\Big)^2\Big(2+\frac{u^2}{1+u}\Big)\Big(1-2\frac{u}{u_3}\Big),\\
&G_{23}=\frac{81}{4}x^2\Delta^2\Big(\frac{u}{u_3}\Big)^2\Big(1-\frac{u}{u_3}\Big)^2\frac{u}{1+u}\Big[2+u-4\frac{u}{u_3}\Big(1-\frac{u}{u_3}\Big)\Big]
\end{aligned}
\tag{16}
$$

—for the third harmonic.

After summing over the polarization of the final photon and integration over the variable $u\,(0\leqslant u\leqslant u_n)$ we obtain the total probabilities, corresponding to the first three harmonics:

$$
W_n=\frac{e^2m^2x^2n_e}{4q_0}\,(F_n+\lambda\lambda_e G_n),\qquad n=1,2,3,
\tag{17}
$$

$$
\begin{aligned}
F_1={}&\Big(1-\frac{4}{u_1}-\frac{8}{u_1^2}\Big)\ln(1+u_1)+\frac{1}{2}+\frac{8}{u_1}-\frac{1}{2(1+u_1)^2}\\
&-\Delta\Big[2+\frac{44}{3u_1}-\frac{16}{u_1^2}-\frac{16}{u_1^3}-\frac{2}{1+u_1}\\
&-\frac{8}{u_1}\Big(1+\frac{1}{u_1}-\frac{3}{u_1^2}-\frac{2}{u_1^3}\Big)\ln(1+u_1)\Big]\\
&+\Delta^2\Big[\frac{5}{16}-\frac{8}{3u_1}-\frac{769}{24u_1^2}-\frac{377}{12u_1^3}+\frac{20}{u_1^4}+\frac{10}{u_1^5}+\frac{1}{u_1^2}\Big(\frac{31}{2}+\frac{49}{u_1}\\
&+\frac{89}{4u_1^2}-\frac{25}{u_1^3}-\frac{10}{u_1^4}\Big)\ln(1+u_1)\Big],\\
G_1={}&\Big(1+\frac{2}{u_1}\Big)\ln(1+u_1)-\frac{5}{2}+\frac{1}{1+u_1}-\frac{1}{2(1+u_1)^2}\\
&-\Delta\Big[\frac{1}{3}\mp\frac{4}{u_1}-\frac{8}{u_1^2}-\frac{1}{1+u_1}-\frac{2}{u_1}\Big(1-\frac{4}{u_1^2}\Big)\ln(1+u_1)\Big]\\
&+\Delta^2\Big[\frac{1}{48}-\frac{25}{4}\frac{1}{u_1}+\frac{15}{8u_1^2}+\frac{20}{u_1^3}+\frac{45}{2}\frac{1}{u_1^4}\\
&-\frac{5}{u_1^3}\Big(2+\frac{25}{4u_1}+\frac{9}{2u_1^2}\Big)\ln(1+u_1)\Big],\\
F_2={}&4\Delta\Big\{\frac{1}{2}+\frac{2}{3u_2}-\frac{16}{u_2^2}-\frac{16}{u_2^3}-\frac{1}{2(1+u_2)}\\
&-\frac{1}{u_2}\Big(1-\frac{6}{u_2}-\frac{24}{u_2^2}-\frac{16}{u_2^3}\Big)\ln(1+u_2)\\
&-\Delta\Big[\frac{1}{2}-\frac{58}{15u_2}-\frac{113}{3u_2^2}+\frac{34}{3u_2^3}\\
&+\frac{128}{u_2^4}+\frac{64}{u_2^5}+\frac{1}{u_2^2}\Big(20+\frac{40}{u_2}-\frac{70}{u_2^2}-\frac{160}{u_2^3}-\frac{64}{u_2^4}\Big)\ln(1+u_2)\Big]\Big\},\\
G_2={}&4\Delta\Big\{\frac{1}{6}+\frac{2}{u_2}-\frac{4}{u_2^2}-\frac{1}{2(1+u_2)}-\frac{1}{u_2}\Big(1-\frac{4}{u_2^2}\Big)\ln(1+u_2)\\
&-\Delta\Big[\frac{1}{15}-\frac{2}{3u_2}+\frac{6}{u_2^2}+\frac{64}{u_2^3}+\frac{72}{u_2^4}\\
&-\frac{1}{u_2^3}\Big(32+\frac{100}{u_2}+\frac{72}{u_2^2}\Big)\ln(1+u_2)\Big]\Big\},\\
F_3={}&\frac{81}{4}\Delta^2\Big[\frac{1}{12}-\frac{8}{15u_3}-\frac{5}{2u_3^2}+\frac{19}{u_3^3}+\frac{48}{u_3^4}+\frac{24}{u_3^5}\\
&+\frac{1}{u_3^2}\Big(2-\frac{4}{u_3}-\frac{41}{u_3^2}-\frac{60}{u_3^3}-\frac{24}{u_3^4}\Big)\ln(1+u_3)\Big],\\
G_3={}&\frac{81}{4}\Delta^2\Big[\frac{1}{60}-\frac{1}{6u_3}+\frac{3}{2u_3^2}+\frac{16}{u_3^3}+\frac{18}{u_3^4}\\
&-\frac{1}{u_3^3}\Big(8+\frac{25}{u_3}+\frac{18}{u_3^2}\Big)\ln(1+u_3)\Big].
\end{aligned}
$$

The expressions (17) will be used below for normalization of the spectra determining the energy dependence of the nonlinear effects in inverse Compton scattering for $x^2<1$.

We note that taking additionally into account the emission of the third harmonic, whose probability is proportional to Δ^2 [see (16)], results in the appearance of terms containing Δ^2 in the expressions (14), (15) and (17). These are the main differences between our results for the probabilities of emission of the first two harmonics and the similar expressions in Ref. 4. In addition, the expansion in the parameter Δ, at $x^2<1$, for the differential and total probabilities in Ref. 4 has a number of inaccuracies. Some of them were corrected in Ref. 15.

In order to obtain the energy distribution dW/dy of the produced photons, where $y=\omega'/E$, E—the electron energy, for the case of head-on collision of relativistic electrons with photons of the laser wave, it is necessary to make in (12) the replacement: $u\to y/(1-y)$.[3,4] The range $0\leqslant u\leqslant u_n$ of the variable u corresponds to the range $0\leqslant y\leqslant y_n$ of the variable y, where

$$y_n=\frac{u_n}{1+u_n}=\frac{n\varkappa}{n\varkappa+1+x^2},\qquad \varkappa=\frac{2kp}{m^2}=\frac{4\omega E}{m^2}.$$

As we compare the maximum possible energy of the photons produced in ordinary Compton scattering ($n=1, x^2=0$) with the energy calculated with nonlinear effects taken into account ($x^2\neq 0$), we see that the photons of the first harmonic ($n=1$) have lower maximum possible energy. On the other hand the energy of the γ quanta emitted upon absorption of several photons ($n>1+x^2$) exceeds the energy attainable in ordinary Compton scattering.

We pass now to a more detailed analysis of the influence of nonlinear effects on the process under consideration. We start from the following initial conditions: the collision is arbitrary, the electrons have energy $E=50$ and 300 GeV, $\omega=1.17$ eV (neodymium laser). For numerical calculations of the energy spectra $(1/W)\ dW/dy$ where $W=\Sigma_{n=1}^{n_{\max}} W_n$ is the total emission probability) and of the degree of circular polarization of the emitted photon λ_f for $x^2<1$ we make use of the expansions (14)–(17). On the other hand, for $x^2\geqslant 1$, when the contribution of a large number of harmonics with different n is essential, we use the exact formulas (12) since a description in terms of the first few terms of the perturbation theory series is inadequate. We note that in that case $n_{\max}$ is determined from the conditions for convergence of the series (12).

The results of the calculation of the energy spectra for different polarizations of the initial electrons (λ_e) and photons of the laser wave (λ) are shown in the graphs of Fig. 1a and 1b, which were constructed for values of the parameter x^2 equal to 0.3 and 3 respectively. As can be seen from this figure, inclusion of the nonlinear effects leads to an essential difference between the calculated spectra and the spectra of ordinary Compton scattering. First, the simultaneous absorption of several photons from the wave leads to a broadening of the spectrum of the high-energy photons, and to the appearance of additional peaks corresponding to the emission of higher order harmonics. This broadening is larger, for one and the same electron energy, the higher the intensity. Thus, for $E=50$ GeV and $x^2=0.3$ the spectrum is bounded from above by $y\approx 0.67$, while in the case of $x^2=3$ it practically vanishes for $y\approx 0.8$ (although an insignificant part of the photons can carry off 87% of the electron energy).

Second, the effective increase in the electron mass $m^2\to m_*^2=m^2(1+x^2)$ leads to a compression of the spectra towards the region of small y, since for each n the spectrum is bounded from above by $y_n=n\varkappa/(1+n\varkappa+x^2)$ and not $n\varkappa(1+n\varkappa)$. An increase of the electron energy decreases the relative compression of the first harmonic (see Fig. 1). For relatively low intensity of the laser wave ($x^2=0.3$) the main contribution to the emission comes from the photons of the first harmonic, the yield of higher harmonic photons is insignificant. For intermediate intensity ($x^2=1$) the broadening of the spectrum due to nonlinear effects is accompanied by an increase in the probability and the yield of harder photons becomes essential.

Lastly, at high intensities ($x^2=3$, $n_{\max}=100$), as can be seen from Fig. 1b, radiation due to nonlinear processes of multiphoton absorption becomes comparable with one-photon processes and even predominant (for $E=50$ GeV). Thus, in the spectra of inverse Compton scattering in the field of a circularly polarized electromagnetic wave for $x^2=0.3$ the emission of the first harmonic dominates, while for $x^2=3$ the emission proceeds mainly via higher harmonics, i.e., the process of emission of the photon becomes essentially nonlinear.

Aiming at a study of the polarization phenomena for each value of the energy, we plotted the energy spectra for the following polarization states of the electron and the laser photon:

$$1\to\lambda_e=0,\ \lambda=1;\ 2\to\lambda_e=1,\ \lambda=-1;\ 3\to\lambda_e=1,\ \lambda=1.$$

In Fig. 1 the corresponding lines are labeled *1*, *2*, and *3* respectively.

Everything that was said above about the behavior of the energy spectra applies to all three lines *1*, *2*, and *3*. As regards their relative arrangement, it is seen from Fig. 1 that the most intense spectra correspond to the case when the spins of the electron and the laser photon are parallel

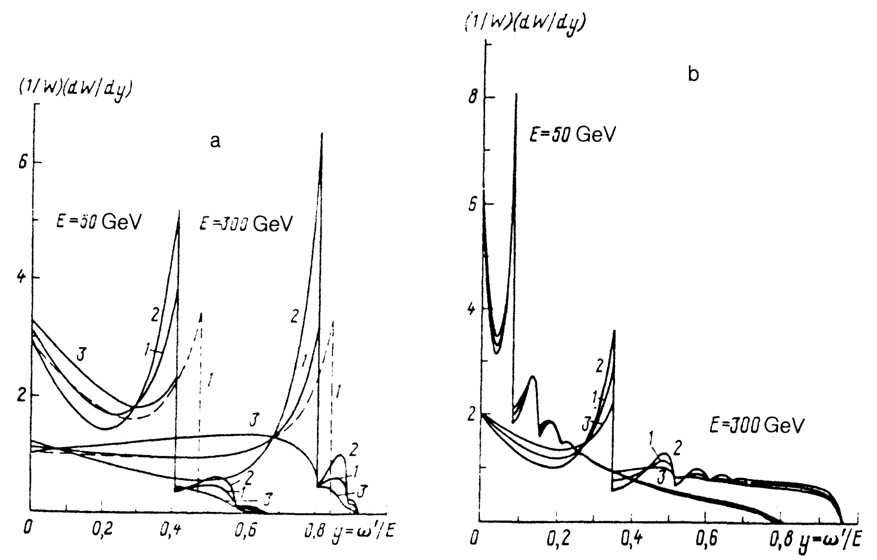

FIG. 1. The spectra of inverse Compton scattering, constructed for various values of the intensity parameter x^2: a$\to x^2=0.3$; b$\to x^2=3$. The curves *1*, *2*, *3* correspond to the following polarization states of the colliding particles: *1*—$\lambda_e=0$, $\lambda=1$; *2*—$\lambda_e=1$, $\lambda=-1$; *3*—$\lambda_e=1$, $\lambda=1$. The dashed lines correspond to the ordinary Compton effect.

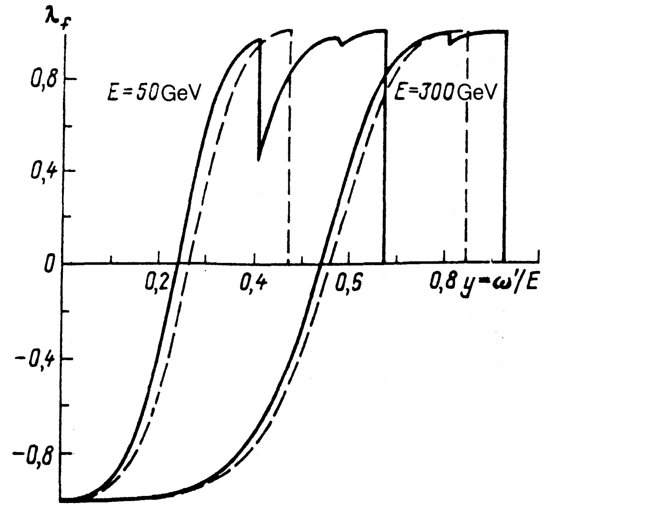

FIG. 2. The energy dependence of the degree of circular polarization of hard γ quanta produced in inverse Compton scattering for $x^2 = 0.3$ for parallel spins of the colliding particles ($\lambda_e = 1, \lambda = -1$)). The dashed line corresponds to ordinary Compton scattering.

($\lambda\lambda_e = -1$), and the least intense spectra correspond to antiparallel spins ($\lambda\lambda_e = 1$), just like in the case of ordinary Compton scattering (see Ref. 1). We note that the difference between the spectra plotted for the three polarization cases of the electron and laser photon considered, while so substantial for small values of the intensity parameter ($x^2 = 0.3$), becomes insignificant for $x^2 = 3$ ($E = 50$ GeV). It then reappears only in connection with an increase in the electron energy (see Fig. 1b for $E = 300$ GeV).

Let us consider the energy dependence of the degree of circular polarization of the emitted photon, represented by the graphs in Fig. 2 and 3. We note first of all that the indicated above kinematic singularities in the behavior of the amplitudes $\mathscr{M}^{(n)}_{\pm\delta,\delta}$ (8) and (9) have spin origin. Indeed, the equality $u = u_n$ corresponds to the emission of a high-energy photon in the direction of the motion of the incident electron beam.[3] In the case of absorption of n photons ($n > 1$) from the wave and exact backward scattering of the hard photon, the total helicity of the systems $e + n\gamma_0$ and $e + \gamma$ before and after the interaction is not conserved. This is precisely the reason for the vanishing of the amplitudes $\mathscr{M}^{(n)}_{\pm\delta,\delta}(u = u_n)$ for $n > 1$, as well as of $\mathscr{M}^{(1)}_{\delta,\delta}(u = u_1)$. The requirement of helicity conservation also leads to the fact that for ordinary Compton scattering we have at the boundary of the spectrum $\lambda_f = -\lambda$ [calculation according to Eq. (13) confirms this result]. As is evident from Figs. 2 and 3, inclusion of nonlinear effects ($x^2 \neq 0$) decreases the degree of circular polarization in the first peak. The contribution of higher harmonics leads to the appearance of additional peaks, and at the boundary of the spectrum (for $n = n_{\max}$), just as in the case of ordinary scattering, the relation $\lambda_f = -\lambda$ is valid since

$$\lambda_f = \lim_{y \to y_{n_{max}}} \frac{\lambda F_{2n} + \lambda_e G_{2n}}{F_{0n} + \lambda\lambda_e G_{0n}} = -\lambda.$$

It should be noted however that the yield of photons for

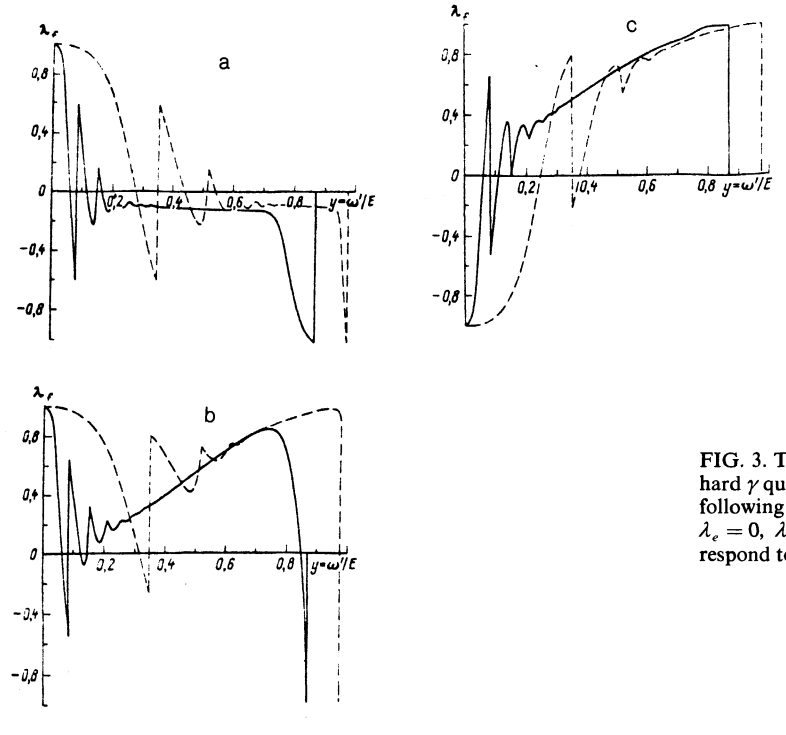

FIG. 3. The energy dependence of the degree of circular polarization of hard γ quanta produced in inverse Compton scattering for $x^2 = 3$ for the following polarization states of the colliding particles: a—$\lambda_e = 0$, $\lambda = 1$; b—$\lambda_e = 1$, $\lambda = -1$; c—$\lambda_e = 1$, $\lambda = 1$. Solid lines correspond to electron energy $E = 50$ GeV, dashed lines—$E = 300$ GeV.

which $\lambda_f \to -\lambda$ is insignificant, since the spectra practically break off for values of y much smaller than $y_{n_{\max}}$. The situation is most favorable in this respect when $\lambda\lambda_e = -1$, corresponding to a large energy interval of the scattered γ quanta with degree of circular polarization $|\lambda_f|$ closest to unity.

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[1] I. F. Ginzburg, G. L. Kotkin, V. G. Serbo, and V. I. Tel'nov, Preprint 81-50 IYaF SO AN SSSR, Novosibirsk, 1981; Yad. Fiz. **38**, 372 (1983) [Sov. J. Nucl. Phys. **38**, 222 (1983)]; Pis'ma Zh. Eksp. Teor. Fiz. **34**, 514 (1981) [JETP Lett. **34**, 491 (1981)]; Preprint 81-102 IYaF SO AN SSSR, Novosibirsk, 1981.

[2] I. F. Ginzburg, G. L. Kotkin, and S. I. Polityko, Yad. Fiz. **37**, 368 (1983); **40**, 1495 (1984) [Sov. J. Nucl. Phys. **37**, 222 (1983); **40**, 949 (1984)].

[3] V. I. Ritus and A. I. Nikishev, Tr. Fiz. Inst. Akad. Nauk SSSR **111**, 16, 56 (1979) [Lebedev Institute Reports].

[4] Ya. T. Grinchishin and M. P. Rekalo, Zh. Eksp. Teor. Fiz. **84**, 1605 (1983) [Sov. Phys. JETP **57**, 935 (1983)].

[5] A. A. Bogush and F. I. Fedorov, Izv. AN BSSR, Ser. fiz.-mat. nauk, No. 2, 26 (1962).

[6] F. I. Fedorov, *The Lorentz Group* [in Russian], Nauka, Moscow, 1979.

[7] S. M. Sikach, *Covariant Methods in Theoretical Physics* [in Russian], Nauka i technika, Minsk, 1981, p. 91.

[8] S. M. Sikach, Izv. AN BSSR, Ser. fiz.-mat. nauk, No. 2, 84 (1984).

[9] M. V. Galynskiĭ and S. M. Sikach, *Covariant Methods in Theoretical Physics* [in Russian], Nauka i technika, Minsk, 1986, p. 91.

[10] M. V. Galynskiĭ, L. F. Zhirkov, S. M. Sikach, and F. I. Fedorov, Zh. Eksp. Teor. Fiz. **95**, 1921 (1989) [Sov. Phys. JETP **68**, 1111 (1989)].

[11] F. I. Fedorov, Teor. Mat. Fiz. 2, 343 (1970) [Theor. Math. Phys. (USSR) **2**, 248 (1970)].

[12] V. B. Berestetskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, *Relativistic Quantum Theory*, Pergamon, Oxford, 1973.

[13] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, Pergamon, Oxford, 1974.

[14] I. F. Ginzbrug, G. L. Kotkin, S. L. Panfil', and V. G. Serbo, Yad. Fiz. **38**, 1021 (1983) [Sov. J. Nucl. Phys. **38**, 614 (1983)].

[15] Ya. T. Grinchishin and M. P. Rekalo, Zh. Eksp. Teor. Fiz. **86**, 783 (1984) [Sov. Phys. JETP **59**, 458 (1984)].

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