Diffusion of charged particles in a large-scale stochastic magnetic field

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The diffusion of magnetized charged particles, considered as test particles, in a turbulent plasma is analyzed. It is assumed that there is a regular, quasiuniform magnetic field $\mathbf{B}_0$ in the plasma. There are also large-scale stochastic fluctuations of the field, $\widetilde{\mathbf{B}}$, and of the velocity of the medium, $\mathbf{u}$, which satisfy the MHD equations. The length scales of the velocity and field fluctuations are greater than the local transport mean free path of the test particles. The drift kinetic equation for the particle distribution function is averaged over an ensemble of realizations of statistically uniform and isotropic fluctuations of the magnetic field and the velocity of the medium. No upper limit is imposed on the amplitudes. A renormalization method is used to calculate the average diffusion tensor. This method leads to a system of transcendental equations for the components of the diffusion tensor. These equations can be studied analytically and numerically. Some simple analytic expressions are derived to describe the various regimes of the particle diffusion across the regular magnetic field.

1. STATEMENT OF THE PROBLEM

In many systems, both in the laboratory and in astrophysical settings, a transport of charged particles occurs in a random magnetic field $\widetilde{\mathbf{B}}(\mathbf{r}, t)$ whose variations occur over a length scale much larger than the local transport mean free path of the particles with respect to scattering by small-scale electromagnetic fields (Coulomb or plasma fields). Quite frequently, there is a regular (quasiuniform, with shear disregarded) magnetic field $\mathbf{B}_0$ in the system. This situation is particularly common in the diffusion of various impurities, both thermal and nonequilibrium, in turbulent magnetized plasmas with broad spectra of fluctuations of the magnetic field and of the velocity of the medium, $\mathbf{u}(\mathbf{r}, t)$.

If the Larmor radius of the particles is small in comparison with the transport mean free path, the local diffusion of the particles is sharply anisotropic. The particles move primarily along the local magnetic field, deviating from this direction because of drift. On the other hand, the global transport over distances greater than the correlation length of the random field, over times much longer than the correlation time of the large-scale fields, and/or in a system with transverse motions of the plasma may reduce to a diffusion which is approximately isotropic. There is accordingly the problem of the relationship between the local and global diffusion tensors. The transverse diffusion coefficient (transverse with respect to the quasiuniform magnetic field $\mathbf{B}_0$) is a particularly important question. This problem is pertinent to transport in fusion devices[1-3] and to the analysis of the propagation of cosmic rays and of elements synthesized in active processes under astrophysical conditions.[4] For example, let us estimate the local transverse diffusion coefficient $\varkappa_\perp$ for relativistic particles in the local galaxy. The classical transport theory gives the following expression for magnetized particles:

$$\varkappa_\perp \approx \varkappa_\parallel (r_0/\Lambda_\parallel)^2, \tag{1}$$

where $\varkappa_\parallel$ is the diffusion coefficient along the magnetic field, r_0 is the Larmor radius, and $\Lambda_\parallel$ is the longitudinal transport mean free path of the particles. Adopting $B_0 \approx 3\cdot 10^{-6}$ G and $\Lambda_\parallel \approx 10^{18}$ cm (Ref. 4), we find the magnetization factor $\Lambda_\parallel / r = \omega_B \tau \approx 5\cdot 10^6$ $[(1\text{ GeV})/E]$, where E is the total energy of the particle, and $\tau = 1/\nu$ is the mean free time with respect to scattering. It follows from these estimates that the local diffusion of cosmic rays ($E \sim 1$ GeV) should be sharply anisotropic: $\varkappa_\perp / \varkappa_\parallel \sim 10^{-13}$. Analysis of the chemical composition indicates that the mixing of cosmic rays in the local galaxy is nearly isotropic,[4] so the global diffusion tensor is, too.

To correctly calculate the global diffusion tensor, we need to take account of both the turbulent velocity field of the medium and the stochastic component of the large-scale magnetic field. In the local galaxy, this stochastic component is comparable in magnitude to the regular field of the spiral arms. The stochastic nature of the large-scale field results in a particle transport across $\mathbf{B}_0$, as the result of a deviation of the local field from the mean field. The transverse components of the turbulent velocity field (and the associated electric fields of the ideally conducting plasma) play the same role.[5] Ptuskin and Chuvil'gin[6] have calculated the transverse diffusion coefficient by perturbation theory ($\widetilde{B}/B_0 \ll 1$).

The problem we are taking up here is that of deriving a theory without any restrictions on the amplitude of the random field ($\widetilde{B}/B_0 \sim 1$) and with a simultaneous and self-consistent incorporation of turbulent motions. To solve this problem we use a renormalization method.[1-3,5,7-9] In Sec. 2, this renormalization method is applied to the drift kinetic equation, and transcendental algebraic equations are derived. These equations can be used for numerical or analytic calculations of the diffusion coefficients for magnetized test particles in the direction across the mean magnetic field in systems with a statistically uniform and isotropic turbulence. The diffusion coefficients are expressed in terms of binary correlation functions of the fields. It is assumed that the percolation transport of particles (see the review in Ref. 9, for example) is slight. Percolation transport can play a major role for systems in which the longitudinal correlation length of the fluctuations is considerably larger than the cor-

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responding transverse length.[8] In this case the particle transport is determined by a relatively small number of long field lines. Percolation is of only minor importance, on the other hand, for systems with an isotropic turbulence, with dimensions much greater than the correlation length of the fluctuations. In this case the limitation on the topology of the random magnetic field reduces to the condition that the correlation of field lines at scales greater than the correlation length be destroyed rapidly. A description of systems with long correlations requires more in the way of statistical information than the binary correlations which we are using here.

The relationship between the correlation functions for fluctuations of the magnetic field and of the turbulence-velocity field is required for calculations on the diffusion. This relationship is derived in the Appendix, where a turbulent renormalization of the viscosity is taken into account.

2. AVERAGING OF THE DRIFT KINETIC EQUATION

We start from the drift kinetic equation for the distribution function of magnetized particles, $f(\mathbf{r}, p, \mu, t)$, in the approximation of a zero gyroradius. We write this equation in the form

$$\frac{\partial f}{\partial t}+[(\mathbf{vb}-\mathbf{ub})b_\alpha+u_\alpha]\frac{\partial f}{\partial r_\alpha}=-\nu(f-\bar{f}). \tag{2}$$

Here $\mathbf{u}(\mathbf{r}, t)$ is the turbulence-velocity field of the medium, which is specified by the binary correlation tensor

$$K_{\alpha\beta}(\rho, \tau)=\langle u_\alpha(\mathbf{r}, t)u_\beta(\mathbf{r}', t')\rangle, \ \rho=\mathbf{r}-\mathbf{r}', \ \tau=t-t', \tag{3}$$

where $\mathbf{b}$ is a unit vector along the overall magnetic field. This unit vector is given by

$$\mathbf{b}=\frac{\mathbf{B}_0+\tilde{\mathbf{B}}}{|\mathbf{B}_0+\tilde{\mathbf{B}}|}, \tag{4}$$

where $\mathbf{B}(\mathbf{r}, t)$ is the turbulent magnetic field, with length scales $l(\Lambda_\parallel < l \lesssim L)$. We shall average over these length scales below. These length scales also characterize the velocity $\mathbf{u}$. The field $\mathbf{B}_0$ is regular; the length scale of its variations, $R \gg L$, is on the order of the dimensions of the plasma system under consideration. To keep the calculations from becoming too complex, we have omitted from Eq. (2) all terms which describe changes in the energy of the particles. These effects are extremely small if we assume that the motion of the medium is incompressible:

$$\operatorname{div}\mathbf{u}=0. \tag{5}$$

Where necessary, we can drop this condition and take the acceleration terms into account, as was done in Ref. 5. The interaction of the particles with magnetic fields with length scales smaller than or on the order of the particle gyroradius is modeled by the right side of Eq. (2), where ν is the rate at which particles are scattered by small-scale fields. The superior bar on f means an average over the pitch angle θ ($\mu = \cos\theta$). We have omitted from Eq. (2) a term which describes the focusing of the pitch angles of the particles by the large-scale field fluctuations, since the scattering rate ν is much higher than the corresponding rate of change of the pitch angle, $v \operatorname{div}\mathbf{b}$. The conditions for the applicability of Eq. (2) are discussed in detail in Ref. 6. We seek a result by taking an average of Eq. (2) over an ensemble of turbulent pulsations in the form

$$\frac{\partial F}{\partial t}+\mathbf{V}\frac{\partial F}{\partial \mathbf{r}}-\chi_{\alpha\beta}(\mathbf{p})\frac{\partial^2 F}{\partial r_\alpha\,\partial r_\beta}=-\nu(F-\bar{F}), \tag{6}$$

where $F(\mathbf{r},\mathbf{p},t) = \langle f(\mathbf{r},\mathbf{p},t)\rangle$ is an average distribution function.

Note that the function $f(\mathbf{r}, p, \theta, t)$, which has not been averaged over the ensemble, and the function $F(\mathbf{r}, \mathbf{p}, t)$, which has been averaged, depend on different angles determining the orientation of the momentum $\mathbf{p}$. The function f depends on the local pitch angle θ, while it is independent of the local (fast) gyrophase. Equation (2) is the result of taking an average over this gyrophase. After an average is taken over the turbulence fields, the direction of the momentum is characterized by the angle ϑ, which is the angle between the momentum and the mean magnetic field $\mathbf{B}_0$, and also by the azimuthal angle φ, reckoned around $\mathbf{B}_0$. The azimuthal angle φ is not a fast variable. The distribution function may be a function of this angle by virtue of either an azimuthal anisotropy of the turbulence or a gradient in the particle distribution which is not along the direction of $\mathbf{B}_0$.

In Eq. (6),

$$V_\alpha=\langle(\mathbf{vb}-\mathbf{ub})b_\alpha+u_\alpha\rangle=\langle v_\alpha^{eff}\rangle \tag{7}$$

is the mean drift velocity of the particle. The term with the second derivative, $\chi_{\alpha\beta}(\mathbf{p})(\partial^2 F/\partial r_\alpha \partial r_\beta)$, is written as the result of an averaging of the term

$$\left\langle (v_\alpha^{eff}-V_\alpha)\frac{\partial f}{\partial r_\alpha}\right\rangle=-\chi_{\alpha\beta}(\mathbf{p})\frac{\partial^2 F}{\partial r_\alpha\,\partial r_\beta} \tag{8}$$

in (2).

The tensor $\chi_{\alpha\beta}(\mathbf{p})$ is not the overall diffusion tensor. It describes only the turbulence component of the diffusion. Below we will calculate it by a self-consistent approach. First, however, we write the mean velocity V_α in a convenient form. Writing

$$\varepsilon=\langle\tilde{B}^2/(B_0^2+\tilde{B}^2)\rangle, \tag{9}$$

we write the correlation function of the unit vectors taken at one point as follows:

$$\langle b_\alpha b_\beta\rangle=(1-\varepsilon)b_{0\alpha}b_{0\beta}+(\varepsilon/2)\delta_{\alpha\beta}^{\perp}, \tag{10}$$

where $b_{0\alpha}$ is a unit vector along the field $\mathbf{B}_0$. Using (10), we find

$$V_\alpha=(1-\varepsilon)b_{0\alpha}v_\parallel+(\varepsilon/2)v_\alpha^{\perp}, \tag{11}$$

where $\parallel$ and $\perp$ now refer to the direction of $\mathbf{B}_0$. The parameter ε here varies over the interval $0 \leqslant \varepsilon \leqslant 1$. It characterizes the contribution of the turbulent component of the magnetic field. Here also, $\delta_{\alpha\beta}^{\perp} = \delta_{\alpha\beta} - b_{0\alpha}b_{0\beta}$, where $\delta_{\alpha\beta}$ is the three-dimensional Kronecker delta.

The overall diffusion coefficient should be expressed in terms of $\mathbf{V}$ and $\chi_{\alpha\beta}(\mathbf{p})$ by going over to the small-anisotropy approximation in Eq. (6). Setting

$$F=\frac{1}{4\pi}[N(\mathbf{r},p,t)+\delta N(r,p,t)],$$

$$\overline{\delta N(\mathbf{r},p,t)}=0, \quad |\delta N| \ll N, \tag{12}$$

we put Eq. (6) in diffusion form:

$$\frac{\partial N}{\partial t}=D_{\alpha\beta}\frac{\partial^2 N}{\partial r_\alpha\,\partial r_\beta}. \tag{13}$$

where

$$D_{\alpha\beta}=\varkappa_{\alpha\beta}+\overline{\chi_{\alpha\beta}(\mathbf{p})},$$

$$\varkappa_{\alpha\beta}=\varkappa_\perp\delta_{\alpha\beta}^\perp+\varkappa_\parallel b_{0\alpha}b_{0\beta},\quad \varkappa_\perp=\left(\frac{\varepsilon}{2}\right)^2\frac{v^2}{3\nu},\quad \varkappa_\parallel=\frac{v^2}{3\nu}(1-\varepsilon)^2. \tag{14}$$

Moving on to a calculation of the kinetic coefficient $\chi_{\alpha\beta}(\mathbf{p})$, we single out a narrow wave-number interval $\Delta\mathbf{k}$ from the turbulence spectrum. We denote by $\delta\mathbf{u}$ and $\delta\tilde{\mathbf{B}}$ the components of the turbulence field which come from harmonics associated with $\Delta\mathbf{k}$. As a result we find the representation

$$v_\alpha^{eff}=v_\alpha^{\prime eff}+\delta v_\alpha^{eff}, \tag{15}$$

where

$$\delta v_\alpha^{eff}=(v\delta b-\delta u b'-u'\delta b)\,b_\alpha'+(vb'-u'b')\,\delta b_\alpha+\delta u_\alpha,$$

$$\delta b_\alpha=\left(\frac{B_0}{B'}\delta_{\alpha\beta}-\frac{B_0}{B'^3}(\mathbf{B}_0+\tilde{\mathbf{B}}')_\alpha\tilde{B}_\beta'\right)\frac{\delta\tilde{B}_\beta}{B_0}. \tag{16}$$

The interval $\Delta\mathbf{k}$ has been eliminated from the spectrum of the quantities marked with primes.

We now take an average of our original equation, (2), over the entire spectrum of turbulence fluctuations, except those in the interval $\Delta\mathbf{k}$. We denote this averaging by means of a prime ($\langle\ldots\rangle'$). We have

$$\langle f(\mathbf{r},\mathbf{p},t)\rangle'=F(\mathbf{r},\mathbf{p},t),$$

$$\left\langle v_\alpha^{\prime eff}\frac{\partial f}{\partial r_\alpha}\right\rangle'=\mathbf{V}'\frac{\partial F}{\partial \mathbf{r}}+\chi_{\alpha\beta}'\frac{\partial^2 F}{\partial r_\alpha\,\partial r_\beta}. \tag{17}$$

Here $\mathbf{V}'$ and $\chi_{\alpha\beta}'$ differ only slightly from the corresponding quantities in the completely averaged equation, (6). We then write

$$\left\langle \delta v_\alpha^{eff}\frac{\partial f}{\partial r_\alpha}\right\rangle'=\delta\mathbf{V}\frac{\partial F}{\partial \mathbf{r}}+\left\langle(\delta v_\alpha^{eff}-\delta V_\alpha)\frac{\partial f}{\partial r_\alpha}\right\rangle', \tag{18}$$

where

$$\delta V_\alpha=\langle v_\alpha^{eff}\rangle'$$

$$=\left[1-\varepsilon'-\left\langle\frac{B_0^2\tilde{B}'^2}{B'^4}\right\rangle\right]\left(b_{0\alpha}\frac{v\delta\tilde{B}}{B_0}+v_\parallel\frac{\delta\tilde{B}_\alpha}{B_0}\right)+\left(1-\frac{\varepsilon'}{2}\right)\delta u_\alpha. \tag{19}$$

The primes can be omitted everywhere on the right side of this equation. The last term in (18) is the most difficult to estimate. Comparing it with the corresponding term in (8), we write it in the form

$$\left\langle(\delta v_\alpha^{eff}-\delta V_\alpha)\frac{\partial f}{\partial r_\alpha}\right\rangle$$

$$=\frac{v_\gamma\delta\tilde{B}_\gamma}{vB_0}A_{\alpha\beta}^{(1)}\frac{\partial^2 F}{\partial r_\alpha\,\partial r_\beta}+\frac{v_\gamma\delta u_\gamma}{v^2}A_{\alpha\beta}^{(2)}\frac{\partial^2 F}{\partial r_\alpha\,\partial r_\beta}+\ldots$$

$$\ldots+\frac{\delta u_\alpha b_{0\beta}}{v}A^{(6)}\frac{\partial^2 F}{\partial r_\alpha\,\partial r_\beta}, \tag{20}$$

where the unknown coefficients $A_{\alpha\beta}^{(1)},\ldots,A^{(6)}$ are quantities on the same order as $\chi_{\alpha\beta}'$ [as follows from a comparison of (18) with (8) and from the structure of the expressions for $v_\alpha^{\rm eff}$ and $\delta v_\alpha^{\rm eff}$]. The form of (20) is based on the incorporation of all possible tensor combinations which are linear in the small quantities δu_α and $\delta\tilde{B}_\alpha$ involved in the problem at hand. As a result, after averaging our original equation, (2), over the turbulence spectrum, except in the interval $\Delta\mathbf{k}$, we find the equation

$$\frac{\partial F}{\partial t}+\mathbf{V}'\frac{\partial F}{\partial \mathbf{r}}-\chi_{\alpha\beta}'(\mathbf{p})\frac{\partial F^2}{\partial r_\alpha\,\partial r_\beta}+\nu(F-\bar{F})=\hat{L}F. \tag{21}$$

where the perturbation operator $\hat{L}$ is

$$\hat{L}=-\delta V_\alpha\frac{\partial}{\partial r_\alpha}-\left(\frac{v_\gamma\delta\tilde{B}_\gamma}{vB_0}A_{\alpha\beta}^{(1)}+\frac{v_\gamma\delta u_\gamma}{v^2}A_{\alpha\beta}^{(2)}+\frac{v_\alpha\delta\tilde{B}_\beta}{vB_0}A^{(3)}\right.$$

$$\left.+\frac{v_\alpha\delta u_\beta}{v^2}A^{(4)}+\frac{\delta\tilde{B}_\alpha b_{0\beta}}{B}A^{(5)}+\frac{\delta u_\alpha b_{0\beta}}{v}A^{(6)}\right)\frac{\partial^2}{\partial r_\alpha\,\partial r_\beta}. \tag{22}$$

We take the final average over the ensemble of realizations of $\delta\tilde{B}_\alpha$ and δu_α by making use of standard perturbation theory:

$$\langle F\rangle=F,\ F=F+\delta F,$$

$$\frac{\partial F}{\partial t}+\mathbf{V}'\frac{\partial F}{\partial \mathbf{r}}-\chi_{\alpha\beta}'\frac{\partial^2 F}{\partial r_\alpha\,\partial r_\beta}+\nu(F-\bar{F})=\langle\hat{L}\delta F\rangle. \tag{23}$$

$$\frac{\partial\delta F}{\partial t}+\mathbf{V}'\frac{\partial\delta F}{\partial \mathbf{r}}-\chi_{\alpha\beta}'\frac{\partial^2\delta F}{\partial r_\alpha\,\partial r_\beta}+\nu(\delta F-\delta\bar{F})=\hat{L}F. \tag{24}$$

It is sufficient to solve Eq. (24) for time scales and length scales shorter than or on the order of the corresponding correlation scales. Here we use the assumption that there are no field lines which preserve correlations over a large scale (and which might lead to a percolation transport of particles).

We solve Eq. (24) with the help of the Green's function $G(\mathbf{r},\mathbf{p},\mathbf{t};\mathbf{r}',\mathbf{p}',t')$, which satisfies the equation

$$\frac{\partial G}{\partial t}+\mathbf{V}\frac{\partial G}{\partial \mathbf{r}}-\chi_{\alpha\beta}\frac{\partial^2 G}{\partial r_\alpha\,\partial r_\beta}$$

$$+\nu(G-\bar{G})=\delta(\mathbf{p}-\mathbf{p}')\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'). \tag{25}$$

Here the average velocity and the diffusion tensor have been replaced by the exact values. Using Fourier coordinate and time transforms, we find

$$G_{\mathbf{k}\omega}(\mathbf{p},\mathbf{p}')$$

$$=\frac{\nu}{4\pi[\nu-i\omega+ik_\alpha V_\alpha+k_\alpha k_\beta\chi_{\alpha\beta}(\mathbf{p})][\nu-i\omega+ik_\alpha V_\alpha'+k_\alpha k_\beta\chi_{\alpha\beta}(\mathbf{p}')]}$$

$$\times\left(1-\overline{\frac{\nu}{\nu-i\omega+ik_\alpha V_\alpha+k_\sigma k_\gamma\chi_{\sigma\gamma}}}\right)^{-1}$$

$$+\frac{\delta(\cos\vartheta-\cos\vartheta')\delta(\varphi-\varphi')}{\nu-i\omega+ik_\alpha V_\alpha+k_\alpha k_\beta\chi_{\alpha\beta}(\mathbf{p})}. \tag{26}$$

Here, as before, the superior bar means an average over the angles (ϑ and φ) specifying the direction of the momentum of the particle. This averaging, like all the subsequent calculations, is carried out under the assumption that the wavelengths of all harmonics of the stochastic field are large in comparison with $\Lambda_\parallel = v/\nu$. As a result we find the inequality $|\mathbf{kv}| \ll \nu$, and we then find

$$\overline{\frac{\nu}{\nu-i\omega+ik_\alpha V_\alpha+k_\sigma k_\tau \chi_{\sigma\tau}}} = \frac{\nu}{\nu'}\left(1-\frac{k_\parallel^2\varkappa_\parallel}{\nu'}-\frac{k_\perp^2\varkappa_\perp}{\nu'}-\frac{k_\alpha k_\beta \overline{\chi_{\alpha\beta}}}{\nu'}\right),$$

where $\nu' = \nu - i\omega$. We finally find

$$\left(1-\overline{\frac{\nu}{\nu-i\omega+ik_\alpha V_\alpha+k_\sigma k_\tau\chi_{\sigma\tau}}}\right)^{-1} = \frac{\nu'}{-i\omega+k_\parallel^2\varkappa_\parallel+k_\perp^2\varkappa_\perp+k_\sigma k_\tau\overline{\chi_{\sigma\tau}}}. \quad (27)$$

The Green's function in (26) has thus been completely determined, and we can write the solution of Eq. (23) as

$$\delta F(\mathbf{r},\mathbf{p},t)=\int G(\mathbf{r},\mathbf{p},t;\mathbf{r}',\mathbf{p}',t')\tilde{L}'F'\,d^3r'\,dt'\,d\Omega', \quad (28)$$

where $d\Omega'$ is an element of the solid angle specifying the direction of $\mathbf{p}'$.

Turning now to an evaluation of the right-hand side of Eq. (23), $\langle\tilde{L}\delta F\rangle$, we first extract from (22) the operator with the first derivative, which dominates the diffusion tensor:

$$\left\langle \delta V_\alpha \frac{\partial}{\partial r_\alpha}\int G(\mathbf{r}-\mathbf{r}',t-t')\delta V_\beta'\right\rangle \frac{\partial F'}{\partial r_\beta'}d^3r'\,dt'\,d\Omega' = \Delta\chi_{\alpha\beta}^{(1)}\frac{\partial^2 F}{\partial r_\alpha\,\partial r_\beta}, \quad (29)$$

where

$$\Delta\chi_{\alpha\beta}^{(1)}(\mathbf{p}) = -\int\frac{\partial G(\rho,\tau)}{\partial\rho_\gamma}\rho_\alpha\langle\delta V_\gamma\delta V_\beta'\rangle d^3\rho\,d\tau\,d\Omega' = \int\langle\delta V_\gamma\delta V_\beta'\rangle_{\mathbf{k},\omega}\frac{\partial}{\partial k_\alpha}(k_\gamma G_{-\mathbf{k},-\omega})\frac{d^3k\,d\omega}{(2\pi)^4}d\Omega'. \quad (30)$$

To evaluate the latter expression we use (19), (26), (27), and the inequalities $\omega/\nu\sim u\Lambda_\parallel/vL\ll 1$, $kV/\nu\sim\Lambda_\parallel/L\ll 1$, and $k_\sigma k_\lambda\bar{\chi}_{\sigma\lambda}/\nu\sim(\Lambda_\parallel/L)^2\ll 1$. Using the notation $D_{\alpha\beta}$ for the overall diffusion coefficient [see (14)], and taking an average of $\Delta\chi_{\alpha\beta}^{(1)}(\mathbf{p})$ over the angles specifying the direction of the vector $\mathbf{p}$, we find

$$\overline{\Delta\chi_{\alpha\beta}^{(1)}} = \left(1-\frac{\varepsilon}{2}\right)^2\int\frac{\tilde{K}_{\alpha\beta}(\mathbf{k},\omega)}{i\omega+k_\mu k_\nu D_{\mu\nu}}\frac{d^3k\,d\omega}{(2\pi)^4} + \varkappa_\parallel\frac{1-\varepsilon-\langle B_0^2\tilde{B}'^2/B'^4\rangle}{(1-\varepsilon)^2}\int\frac{\langle\delta\tilde{B}_\alpha\delta\tilde{B}_\beta'\rangle_{\mathbf{k},\omega}}{B^2}\times\left\{1-\frac{k_\parallel^2\varkappa_\parallel}{i\omega+k_\mu k_\nu D_{\mu\nu}}\right\}\frac{d^3k\,d\omega}{(2\pi)^4} + \varkappa_\parallel\frac{1-\varepsilon-\langle B_0^2\tilde{B}'^2/B^4\rangle}{(1-\varepsilon)^2}\frac{\langle\delta\tilde{B}^2\rangle}{B_0^2}b_{0\alpha}b_{0\beta}. \quad (31)$$

The first integral on the right side here shows the contribution from turbulence velocities described by the binary correlation function $\tilde{K}_{\alpha\beta}(\mathbf{k},\omega)$ [see (A19) in the Appendix]. The other terms stem primarily from the large-scale stochastic magnetic field. The correlation function for the magnetic fields, $\langle\delta\tilde{B}_\alpha\delta\tilde{B}'_\beta\rangle_{\mathbf{k},\omega}$, can be expressed without difficulty in terms of the velocity correlation function with the help of Eq. (A23) from the Appendix. The other contributions from the operator with the second derivative in (22) either vanish or contain factors on the order of $(\Lambda_\parallel/\bar{L})$ and $(\Lambda_\parallel/\bar{L})^2$. Here $\bar{L}$ is an average value of the length scale of the random magnetic field, defined by

$$1/\bar{L}^2 = B_0^{-2}(2\pi)^{-4}\int k^2\langle\tilde{B}^2\rangle_{\mathbf{k},\omega}\,d^3k\,d\omega. \quad (32)$$

If all harmonics in the spectrum $\langle\tilde{B}^2\rangle_{\mathbf{k},\omega}$ satisfy the condition $l\gg\Lambda_\parallel$, then these factors are small, regardless of the shape of the spectrum, and we can ignore all terms other than (29). Taking these conditions into account, we find $\overline{\Delta\chi_{\alpha\beta}} = \overline{\Delta\chi_{\alpha\beta}^{(1)}}$. Using (14), and carrying out the integration over the entire wave-number spectrum in Eq. (31), we find a self-consistent equation for the transverse diffusion coefficient:

$$D_{\alpha\beta}^\perp = \varkappa_\perp\delta_{\alpha\beta}^\perp + \left(1-\frac{\varepsilon}{2}\right)^2\int\frac{\langle u_\alpha u_\beta'\rangle_{\mathbf{k},\omega}}{i\omega+k_\parallel^2D_\parallel+k_\perp^2D_\perp}\frac{d^3k\,d\omega}{(2\pi)^4} + \varkappa_\parallel\frac{1-\varepsilon-\langle B_0^2\tilde{B}^2/B^4\rangle}{(1-\varepsilon)^2}\times\int\frac{\langle\tilde{B}_\alpha\tilde{B}_\beta'\rangle_{\mathbf{k},\omega}}{B_0^2}\left(1-\frac{k_\parallel^2\varkappa_\parallel}{i\omega+k_\parallel^2D_\parallel+k_\perp^2D_\perp}\right)\frac{d^3k\,d\omega}{(2\pi)^4}, \quad (33)$$

where

$$D_\parallel = \varkappa_\parallel\left[1+\frac{\langle\tilde{B}^2\rangle}{B_0^2}\frac{1-\varepsilon-\langle B_0^2\tilde{B}^2/B^4\rangle}{(1-\varepsilon)^2}\right]. \quad (34)$$

We wish to stress that these relations are, for given correlation functions, transcendental algebraic equations for the components of the diffusion tensor. They are valid over the entire range $0\leqslant\varepsilon\leqslant 1$, i.e., without any restriction on the amplitude of the magnetic field or on the amplitude of the turbulence fluctuations of the velocity which are transverse with respect to the field.

3. ANALYSIS OF RESULTS

Let us examine the transport of particles by a weak Alfvén turbulence ($\varepsilon = \langle\tilde{B}^2\rangle/B_0^2 = \langle u^2\rangle/v_{\rm ph}^2 \ll 1$). If the local transport mean free path is sufficiently small, we have $v_{\rm ph}L/v\Lambda_\parallel \gg 1$. We then have $D_\parallel\approx\chi_\parallel\approx v\Lambda_\parallel/3$, and $\omega = |\mathbf{kv}_A| \gg k_\parallel^2 D_\parallel + k_\perp^2 D_\perp$. Using (A23), we find from (33)

$$D_\perp\approx 2\varepsilon\varkappa_\parallel\ll\varkappa_\parallel. \quad (35)$$

In this case the anomalous transport across a uniform magnetic field is proportional to the square of the amplitude of the turbulence component of the field.

If the longitudinal mean free path of the particles is long enough that the condition $v\Lambda_\parallel\gg v_A L$, holds, then by ignoring the damping of the Alfvén modes, and integrating over frequency and over the angles of the vector $\mathbf{k}$ in the second term on the right side of (33), we find

$$\int \frac{\langle u_\alpha u_\beta' \rangle_{\mathbf{k},\omega}}{i\omega + k_\parallel^2 D_\parallel + k_\perp^2 D_\perp} \frac{d^3k\, d\omega}{(2\pi)^4}$$

$$= \pi\delta_{\alpha\beta}^{\perp} \int \frac{k^2\, dk}{(2\pi)^3} T(\mathbf{k}) \frac{1}{k^2(D_\parallel - D_\perp)}$$

$$\times \operatorname{Re} \int_0^1 \left[\frac{D_\parallel}{(D_\parallel - D_\perp)} + \frac{v_A^2}{k^2 (D_\parallel - D_\perp)^2} + \tilde{x} \right]^{-1} dx, \quad (36)$$

where $\tilde{x} = x + iv_A/k(D_\parallel - D_\perp)$. For the spectra of fluctuations with the length scale L from (36) we have

$$\int \frac{\langle u_\alpha u_\beta' \rangle_{\mathbf{k},\omega}}{i\omega + k_\parallel^2 D_\parallel + k_\perp^2 D_\perp} \frac{d^3k\, d\omega}{(2\pi)^4} \approx \varepsilon v_A L b^* \delta_{\alpha\beta}^{\perp}. \quad (37)$$

where b^* is a numerical factor on the order of unity. Correspondingly, we can find an estimate of the third term on the right side of (33). As a result, for particles which have a longitudinal diffusion coefficient in the interval $v_A L \ll \varkappa_\parallel \ll v_A L \varepsilon^{-1}$, we find

$$D_\perp \approx \varepsilon v_A L. \quad (38)$$

If $\varkappa_\parallel \gg v_A L\varepsilon^{-1}$, then the fluctuations for such particles can be assumed quasistatic. In this case we have

$$D_\perp \approx {}^3/_4 \varepsilon^2 \varkappa_\parallel + {}^1/_4 \varepsilon\pi (D_\perp \varkappa_\parallel)^{1/2}. \quad (39)$$

Hence

$$D_\perp \approx g\varepsilon^2 \varkappa_\parallel, \quad (40)$$

where the numerical factor is $g \approx 1.8$ (in practice, this factor may depend on the fluctuation spectrum). These results are in qualitative agreement with the corresponding regimes studied in Ref. 6. The transport of charged particles by small-amplitude quasistatic fluctuations is proportional to the fourth power of the amplitude.

We have been discussing particle transport across a magnetic field by a weak Alfvén turbulence. It is pertinent to note here that the second term in Eq. (33) has the same form as the corresponding term which describes turbulent transport in a system without large-scale fluctuations of the magnetic field, but with a strong regular field.[10] We could thus examine the transport of particles by velocity fluctuations with a longitudinal component. In this case, the longitudinal diffusion coefficient is renormalized at $\varkappa_\parallel \ll v_{\rm ph} L\varepsilon^{1/2}$. In this region of parameter values we find $\varkappa_\parallel \approx b^* v_{\rm ph} L\varepsilon^{1/2}$. The transverse-transport regime in (35) persists, but instead of (38) we have a regime with a weak $\varkappa_\parallel$ dependence of $D_\perp$. This regime later becomes (40).

Finding a description of particle transport by a strong turbulence requires a numerical solution of transcendental algebraic equations (33). For typical turbulence spectra, the results lead to the conclusion that the diffusion is rendered isotropic by a strong turbulence: $D_\perp \sim D_\parallel$.

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APPENDIX

The fields $\mathbf{u}$, $\tilde{\mathbf{B}}$, and $\mathbf{B}_0$, in Eq. (2) are not independent. They are coupled by the magnetic-induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \operatorname{rot}[\mathbf{uB}] + \eta_{\alpha\beta}^m \frac{\partial^2 \mathbf{B}}{\partial r_\alpha\, \partial r_\beta}. \quad (A1)$$

Here $\eta_{\alpha\beta}^m$ is the local magnetic-viscosity tensor. Since the turbulent magnetic viscosity is generally far larger than $\eta_{\alpha\beta}^m$, we will treat the latter quantity as a "seed," which need not be taken into account precisely. We choose it to be of the form $\eta_{\alpha\beta}^m = \eta_m \delta_{\alpha\beta}$, where $\eta_m = c^2/4\pi\sigma$ is determined by the electrical conductivity of the plasma, σ. Incorporating a possible anisotropy and a possible gyrotropy of the turbulence, we write the Fourier transform of the correlation velocity tensor in the most general form:

$$K_{\alpha\beta}(\mathbf{k}, \omega) = T_{\alpha\beta}(\mathbf{k}, \omega) + iC_{\alpha\beta}(\mathbf{k}, \omega), \quad (A2)$$

where $T_{\alpha\beta}(\mathbf{k}, \omega) = T_{\beta\alpha}(\mathbf{k}, \omega) = T_{\alpha\beta}(-\mathbf{k}, \omega)$ is a symmetric real tensor which is invariant under the replacement of $\mathbf{k}$ by $-\mathbf{k}$, and $C_{\alpha\beta}(\mathbf{k}, \omega) = -C_{\alpha\beta}(-\mathbf{k}, \omega) = C_{\beta\alpha} \times (-\mathbf{k}, \omega)$ is an antisymmetric, negatively noninvariant tensor. If $C_{\alpha\beta} \neq 0$, the turbulence is gyrotropic.

We will use the method of Ref. 5 to average Eq. (A1) over the ensemble of turbulence velocities and to calculate the coefficients of the average equation. We seek an equation for the average field $\mathbf{B}_0 = \langle B \rangle$, whose variations occur over a distance substantially larger than L:

$$\frac{\partial B_{0\alpha}}{\partial t} = A_{\alpha\mu\nu} \frac{\partial B_{0\nu}}{\partial r_\mu} + \eta_{\mu\nu}^{tot} \frac{\partial^2 B_{0\alpha}}{\partial r_\mu\, \partial r_\nu}. \quad (A3)$$

Here $A_{\alpha\mu\nu}$ and $\eta_{\mu\nu}^{\rm tot}$ are constant tensor coefficients in the case of a uniform turbulence, and a repeated Greek index means a summation. If there are no special directions in coordinate space, other than the direction of $\mathbf{k}$, we can write the following, in the case of an incompressible medium:

$$T_{\alpha\beta}(\mathbf{k}, \omega) = T(k, \omega)(\delta_{\alpha\beta} - k_\alpha k_\beta / k^2). \quad (A4)$$

$$C_{\alpha\beta}(\mathbf{k}, \omega) = C(k, \omega) e_{\alpha\beta\gamma} k_\gamma. \quad (A5)$$

Equation (A3) then takes the known form[11]

$$\frac{\partial \mathbf{B}_0}{\partial t} = \operatorname{rot}(\alpha \mathbf{B}_0) + \eta^{tot} \Delta \mathbf{B}_0. \quad (A6)$$

The coefficient α, which leads to the generation of a large-scale magnetic field, is zero except in the case of a gyrotropic turbulence $[C(k, \omega) \neq 0]$.

To calculate the coefficients in the more general equation (A3), we again single out a small part $\delta\mathbf{u}$ of the velocity field—a part which contains harmonics of a narrow interval of wave numbers $\Delta\mathbf{k}$. Taking an average over all harmonics in (A1) except in this narrow interval, and designating the magnetic field averaged in this manner by $\tilde{\mathbf{B}}_0$, we find the equation

$$\frac{\partial \tilde{B}_{0\alpha}}{\partial t} = A'_{\alpha\mu\nu} \frac{\partial \tilde{B}_{0\nu}}{\partial r_\mu} + \eta_{\mu\nu}^{\prime tot} \frac{\partial^2 \tilde{B}_{0\alpha}}{\partial r_\mu\, \partial r_\nu} + \operatorname{rot}_\alpha [\delta \mathbf{u} \tilde{\mathbf{B}}_0], \quad (A7)$$

where $A'_{\alpha\mu\nu}$ and $\eta_{\mu\nu}^{\prime\rm tot}$ differ only slightly (to the extent that $\Delta\mathbf{k}$ is small) from the exact coefficients $A_{\alpha\mu\nu}$ and $\eta_{\mu\nu}^{\rm tot}$.

We take an average of the last equation over realizations of $\delta\mathbf{u}$ by perturbation theory. Setting

$$\tilde{\mathbf{B}}_0 = \mathbf{B}_0 + \delta\tilde{\mathbf{B}}, \quad \langle \delta\tilde{\mathbf{B}} \rangle = 0, \quad (A8)$$

we find from (A7) the two equations

$$\frac{\partial B_{0\alpha}}{\partial t}=A'_{\alpha\mu\nu}\frac{\partial B_{0\nu}}{\partial r_\mu}+\eta_{\mu\nu}^{\prime tot}\frac{\partial^2 B_{0\alpha}}{\partial r_\mu\,\partial r_\nu}+\mathrm{rot}_\alpha\langle[\delta\mathbf{u}\times\delta\tilde{\mathbf{B}}]\rangle, \quad (A9)$$

$$\frac{\partial}{\partial t}\delta\tilde{B}_\alpha=A'_{\alpha\mu\nu}\frac{\partial\delta B_{0\nu}}{\partial r_\mu}+\eta_{\mu\nu}^{\prime tot}\frac{\partial^2\delta B_{0\alpha}}{\partial r_\mu\,\partial r_\nu}+\mathrm{rot}_\alpha[\delta\mathbf{u}\times\mathbf{B}_0]. \quad (A10)$$

The length scales of the turbulence fluctuations ($l\lesssim L$) and of the regular field $\mathbf{B}_0$ can be sharply different ($R\gg L$) only if the gyrotropic part $C_{\alpha\beta}$ of correlation tensor (A2) is small in comparison with the nongyrotropic part $T_{\alpha\beta}$. The reason is that the magnetic field is generated at length scales $l<L_c\approx 2\pi\eta^{\rm tot}/\alpha\approx 2\pi L\,\langle u^2\rangle^{1/2}/\alpha$, and the condition $L_c\gg L$ holds at $\alpha\ll\langle u^2\rangle^{1/2}$, where

$$\alpha=\frac{1}{3}\int_0^\infty\langle\mathbf{u}(\mathbf{r},t)\,\mathrm{rot}\,\mathbf{u}(\mathbf{r},t-\tau)\rangle d\tau$$

is a gyrotropy parameter.

Since we are assuming that the gyrotropic term in (A10) is small, we can discard the first term on the right side of this equation and write the solution in the form

$$\delta B_\alpha(\mathbf{r},t)=B_{0\beta}(\mathbf{r},t)\int G_m(\mathbf{r}-\mathbf{r}',t-t')\frac{\partial\delta u_\alpha(\mathbf{r}',t')}{\partial r_\beta'}d^3r'\,dt'$$

$$-\frac{\partial B_{0\alpha}(\mathbf{r},t)}{\partial r_\beta}\int G_m(\mathbf{r}-\mathbf{r}',t-t')\delta u_\beta(\mathbf{r}',t')d^3r'\,dt'. \quad (A11)$$

Here we have used the condition div $\delta\mathbf{u}=0$ and the Green's function G_m which satisfies the equation

$$\frac{\partial G_m}{\partial t}-\eta_{\mu\nu}^{tot}\frac{\partial^2 G_m}{\partial r_\mu\,\partial r_\nu}=\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'). \quad (A12)$$

Substituting solution (A11) into the last term in Eq. (A9), we find the contributions of harmonics from the wave-number interval $\Delta\mathbf{k}$ to the kinetic coefficients, $\Delta A_{\alpha\mu\nu}$ and $\Delta\eta_{\mu\nu}^{\rm tot}$:

$$\Delta A_{\alpha\mu\nu}=-2\int_{(\Delta\mathbf{k})}G_m(\rho,\tau)\frac{\partial}{\partial\rho_\nu}\delta C_{\alpha\mu}(\rho,\tau)d^3\rho\,d\tau, \quad (A13)$$

$$\Delta\eta_{\mu\nu}^{tot}=\int_{(\Delta\mathbf{k})}G_m(\rho,\tau)\delta T_{\alpha\mu}(\rho,\tau)d^3\rho\,d\tau. \quad (A14)$$

Integrating (A13) and (A14) over the entire wave-number spectrum, and going over the Fourier representation, we find a system of self-consistent equations for the viscosity coefficient $\eta_{\mu\nu}^{\rm tot}$ and for the coefficient representing the magnetic-field generation, $A_{\alpha\mu\nu}$, which appear in average equation (A3):

$$\eta_{\mu\nu}^{tot}=\int\frac{T_{\mu\nu}(k,\omega)}{i\omega+k_\sigma k_\lambda\eta_{\sigma\lambda}^{tot}}\frac{d^3k\,d\omega}{(2\pi)^4}, \quad (A15)$$

$$A_{\alpha\mu\nu}=2\int\frac{k_\mu C_{\alpha\nu}(k,\omega)}{i\omega+k_\sigma k_\lambda\eta_{\sigma\lambda}^{tot}}\frac{d^3k\,d\omega}{(2\pi)^4}. \quad (A16)$$

It follows from this system of equations that the diffusion tensor for the magnetic field, $\eta_{\mu\nu}^{\rm tot}$, is initially calculated from the first (transcendental) equation. A simple integration over the result found for $\eta_{\mu\nu}^{\rm tot}$ then leads to the third-rank tensor $A_{\alpha\mu\nu}$, which is antisymmetric with respect to its first two indices. For the simple case of a gyrotropic turbulence, (A7), Eqs. (A15) simplify. The diffusion tensor becomes diagonal, $\eta_{\mu\nu}^{\rm tot}=\eta_{\rm tot}\delta_{\mu\nu}$, and the tensor representing the magnetic-field generation can be expressed in terms of a pseudoscalar α: $A_{\beta\mu\nu}=\alpha e_{\beta\mu\nu}$. The coefficients $\eta_{\rm tot}$ and α are found from the system of equations

$$\eta_{tot}=\eta_m+\frac{2}{3}\int\frac{T(k,\omega)}{i\omega+\eta_{tot}k^2}\frac{d^3k\,d\omega}{(2\pi)^4}, \quad (A17)$$

$$\alpha=\frac{2}{3}\int\frac{k^2C(k,\omega)}{i\omega+\eta_{tot}k^2}\frac{d^3k\,d\omega}{(2\pi)^4}. \quad (A18)$$

Since we are not interested here in the generation of a large-scale magnetic field (see Refs. 11 and 12), we are concerned primarily with Eq. (A11), which relates the turbulence fluctuations of the velocity and the magnetic field. Our only comment is that the applicability of the renormalization procedure in the case of a system with a gyrotropy requires a firmer foundation.[13] To simplify the problem of taking an average of the original kinetic equation, (2), we ignore the gyrotropy of the turbulence velocities, and we assume that the velocity field is two-dimensional, directed perpendicular to the uniform magnetic field $\mathbf{B}_0=$ const. We describe the velocity field by means of the correlation tensor

$$K_{\alpha\beta}(\mathbf{k},\omega)=T(k,\omega)(\delta_{\alpha\beta}^{\perp}-k_\alpha^{\perp}k_\beta^{\perp}/k_\perp^2), \quad (A19)$$

where $\mathbf{b}_0$ is a unit vector along $\mathbf{B}_0$, and $\mathbf{k}_\perp$ is the transverse component of the wave vector (transverse with respect to $\mathbf{B}_0$). We are assuming that the motion is incompressible.

In the case under consideration here, the turbulence magnetic field $\tilde{\mathbf{B}}$ is also perpendicular to $\mathbf{B}_0$, and it is related to the velocity fluctuations by

$$\delta\tilde{B}_\alpha(\mathbf{r},t)=B_{0\beta}\int G_m(\mathbf{r}-\mathbf{r}',t-t')\frac{\partial\delta u_\alpha(\mathbf{r}',t')}{\partial r_\beta'}d^3r'\,dt'. \quad (A20)$$

The Green's function G_m contains the renormalized field diffusion tensor $\eta_{\mu\nu}^{\rm tot}$, which must be calculated in terms of the velocity correlation function in (A19) from the system of equations

$$\eta_\parallel^{tot}=\eta^m, \quad (A21)$$

$$\eta_\perp^{tot}=\eta^m+\frac{1}{2}\int\frac{d^3k\,d\omega}{(2\pi)^4}\frac{T(k,\omega)}{i\omega+\eta_\perp^{tot}k_\perp^2+\eta_\parallel^{tot}k_\parallel^2}. \quad (A22)$$

Using (A20), we can easily find the relationship between the correlation tensors of the velocity and of the turbulence magnetic field. Going over to the Fourier representation, we find

$$\langle\tilde{B}_\alpha\tilde{B}_\beta'\rangle_{\mathbf{k},\omega}=B_0^2k_\parallel^2|\tilde{G}_m(\mathbf{k},\omega)|^2\langle u_\alpha u_\beta'\rangle_{\mathbf{k},\omega}, \quad (A23)$$

where, according to (A12), the Fourier transform of the Green's function is

$$\tilde{G}_m(\mathbf{k},\omega)=[-i\omega+k_\sigma k_\lambda\eta_{\sigma\lambda}^{tot}]^{-1}. \quad (A24)$$

If the turbulence is weak and can be treated as a set of quasilinear MHD modes with phase velocities $v_{\rm ph}\gg u$, then $\omega\gg k_\lambda k_\sigma\eta_{\lambda\sigma}^{\rm tot}$. According to (A23) and (A24), we have

$$\frac{\langle\tilde{B}^2\rangle}{B_0^2}\approx\frac{\langle u^2\rangle}{v_0^2}\ll 1. \quad (A25)$$

In the case of a strong turbulence we would have $v_{\rm ph}\approx u$ and $\langle\tilde{B}^2\rangle\approx B_0^2$.

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