Effect of quantum defects on superconductivity

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The IR renormalizations which arise because of an interaction of electrons with quantum defects tend to weaken the influence of these defects on the characteristics of the superconductor. An estimate is derived for the temperature (T_c) of a superconducting transition caused by an interaction of electrons with two-level systems (2LSs) which arise when quantum defects are trapped by immobile heavy impurities. An increase in the asymmetry of the 2LSs due to their interaction with each other causes T_c to approach a constant value with increasing concentration of 2LSs.

1. INTRODUCTION

Light impurities (hydrogen isotopes and helium) which have entered a metal matrix can tunnel from one interstice to another and thereby retain their mobility at low temperatures. The typical band width ε_0 for hydrogen is 0.1–10 K in several transition metals.[1-3] The influence of such defects (treated as internal degrees of freedom of the crystal) on superconductivity was studied by one of us in Ref. 4. It was shown that an inelastic scattering of electrons by quantum defects ("defectons") leads to an additional attraction between electrons and to a broadening of the superconducting gap in the spectrum of electronic excitations.

However, the IR divergences[5] which result from an interaction of electrons with defectons were not taken into consideration in Ref. 4. The renormalizations mandated by these divergences give rise to a nontrivial temperature dependence of the kinetic coefficients of a normal metal which contains defectons.[6-8] Below we examine how the IR renormalizations would affect a defecton superconductivity.

As the temperature is lowered, the system of defectons undergoes a clustering, as the result of an interaction of defectons with each other. A stratification occurs in two phases with high and low defecton concentrations. Where the defecton concentration is low, the defectons may be trapped by immobile heavy impurities or other lattice defects.[9]

We will examine how this clustering of defectons influences the defecton superconductivity. We will also study the superconductivity which stems from an interaction of electrons with the two-level systems which arise upon the trapping of defectons by heavy impurities in several cases.

2. INFRARED RENORMALIZATIONS

The Hamiltonian of the system of noninteracting defectons is

$$\mathscr{H}_d=\sum_{\mathbf{k}} \omega(\mathbf{k})c^+(\mathbf{k})c(\mathbf{k}), \tag{1}$$

where $\omega(\mathbf{k})$ is the defecton dispersion law, the operators $c^+(\mathbf{k})$ and $c(\mathbf{k})$ create and annihilate defectons, and the summation is over the first Brillouin zone.

The Hamiltonian of the electron-defecton interaction is

$$\mathscr{H}_{e-d}=\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V_0(\mathbf{q})a^+(\mathbf{k}'+\mathbf{q})a(\mathbf{k}')c^+(\mathbf{k}-\mathbf{q})c(\mathbf{k}), \tag{2}$$

where $V_0(\mathbf{q})$ is a seed vertex of the electron-defecton interaction, and the operators $a^+(\mathbf{k})$ and $a(\mathbf{k})$ perform a second quantization of the electrons.

Incorporating interaction (2) leads to IR divergences.[5] The summation of these divergences leads, in a parquet approximation corresponding to that developed in Ref. 10, to the following result for the complete vertex of the electron-defecton interaction, $V(\mathbf{q})$, and the renormalized defecton Green's function $\Psi(\mathbf{k},\varepsilon_n)$ in a normal metal:

$$V(\mathbf{q})=V_0(\mathbf{q})\,[E_0/\max(T,\varepsilon_0)]^g, \tag{3}$$

$$\Psi(\mathbf{k},\varepsilon_n)=[i\varepsilon_n-\tilde{\omega}(\mathbf{k})+\zeta]^{-1}[\max(T,\varepsilon_0)/E_0]^g. \tag{4}$$

Here E_0 is the width of the conduction-electron band, T is the temperature, ε_n is the Matsubara frequency, and ζ and $\tilde{\omega}(\mathbf{k})$ are the chemical potential and the renormalized dispersion law for the defectons. For definiteness, we assume that the defectons are fermions. The dimensionless constant g of the electron-defecton interaction is

$$g=2\int \frac{d\mathbf{p}_1\,d\mathbf{p}_2\,|V_0(\mathbf{p}_1-\mathbf{p}_2)|^2}{(2\pi)^6|\nabla\varepsilon(\mathbf{p}_1)|\,|\nabla\varepsilon(\mathbf{p}_2)|}, \tag{5}$$

where the integration is over the Fermi surface, and $\varepsilon(\mathbf{p})$ is the electron dispersion law. In order of magnitude we have $g\sim N^2(0)V_0^2$, where $N(0)$ is the density of electron states at the Fermi surface. For metals, g is typically on the order of 0.1–1, and the parquet approximation is valid if $g\ll 1$, in which case we have $g^{3/2}\ln[E_0/\max(T,\varepsilon_0)]\ll 1$.

According to Ref. 5, the renormalized width of the defecton band, $\tilde{\varepsilon}_0$, has the behavior

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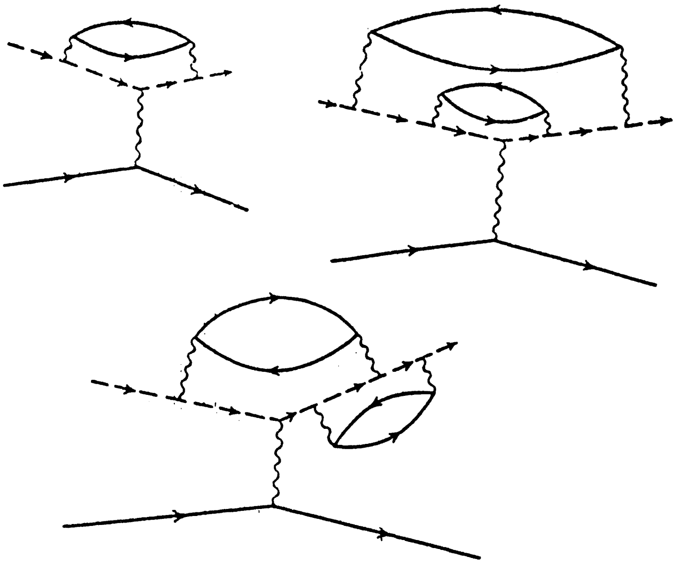

FIG. 1. Feynman diagrams for the vertex of the electron-defecton interaction in a normal metal. Dashed line—seed Green's function of defectons; solid line—electron Green's function; wavy line—$V_0(\mathbf{q})$.

$$\tilde{\varepsilon}_0=\varepsilon_0[\max(T,\ \varepsilon_0)/E_0]^{\beta}, \tag{6}$$

where $\beta\sim g$. The estimates above, found from experimental data, naturally refer to the renormalized value $\tilde{\varepsilon}_0$.

The first terms in the series of the parquet perturbation theory for the quantities $V(\mathbf{q})$ and $\Psi(\mathbf{k},\varepsilon_n)$ are shown in Figs. 1 and 2, a and b, respectively.

It was shown in Ref. 6 that in order to calculate the IR renormalizations in a superconductor one must consider—along with the loops formed by the normal electron Green's functions $G(\mathbf{p},\varepsilon_n)$—the loops which contain anomalous electron Green's functions $F^+(\mathbf{p},\varepsilon_n)$ and $F(\mathbf{p},\varepsilon_n)$ (Fig. 2c). As a result, the expression $\max(T,\varepsilon_0)$ in (3) and (4) is replaced by $\max(T,\varepsilon_0,\Delta)$, where Δ is the superconducting gap in the spectrum of electron excitations.

3. DEFECTON SUPERCONDUCTIVITY

As was mentioned above, the electron-defecton interaction gives rise to an effective attraction between electrons. We recall that immobile impurities in an isotropic superconductor have no effect on the superconductivity.

Figure 3 illustrates the contributions of defectons to the eigenenergy parts $\Sigma_1(\mathbf{p},\varepsilon_n)$ and $\Sigma_2^+(\mathbf{p},\varepsilon_n)$ of the electron Green's functions $G(\mathbf{p},\varepsilon_n)$ and $F^+(\mathbf{p},\varepsilon_n)$.

In the case with electron-phonon and electron-defecton interactions, the Éliashberg equations[11] become

$$\begin{aligned} f_1(\mathbf{p},\varepsilon_k)=\pi T\sum_{\varepsilon_m}\int &\frac{d\mathbf{p}_1|M(\mathbf{p}-\mathbf{p}_1)|^2\omega_0^2(\mathbf{p}-\mathbf{p}_1)}{(2\pi)^3|\nabla\varepsilon(\mathbf{p}_1)|[\omega_0^2(\mathbf{p}-\mathbf{p}_1)+(\varepsilon_m-\varepsilon_k)^2]}\\ &\times\frac{i\varepsilon_m}{[\varepsilon_m^2+\Delta^2(\mathbf{p}_1,\varepsilon_m)]^{1/2}}+T\sum_{\varepsilon_m}\int\frac{d\mathbf{k}\,d\mathbf{p}_1|V_0(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^6|\nabla\varepsilon(\mathbf{p}_1)|}\\ &\times\left(\frac{\max(T,\Delta,\varepsilon_0)}{E_0}\right)^{g}\\ &\times\left[\operatorname{th}\frac{\tilde{\omega}(\mathbf{k})-\zeta}{2T}-\operatorname{th}\frac{\tilde{\omega}(\mathbf{k}+\mathbf{p}_1-\mathbf{p})-\zeta}{2T}\right]\\ &\times[i(\varepsilon_k-\varepsilon_m)-\tilde{\omega}(\mathbf{k})+\tilde{\omega}(\mathbf{k}+\mathbf{p}_1-\mathbf{p})]^{-1}\\ &\times\int_{-\infty}^{\infty}\frac{i\varepsilon_m\,d\xi}{\varepsilon_m^2+\xi^2+\Delta^2(\mathbf{p}_1,\varepsilon_m)}, \end{aligned} \tag{7}$$

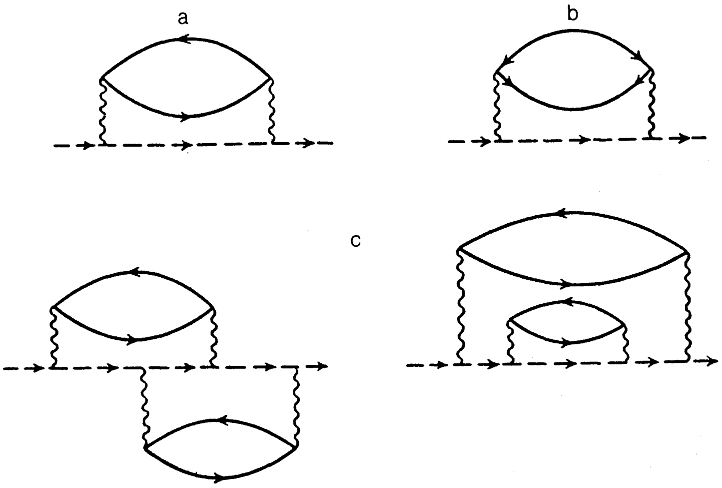

FIG. 2. Feynman diagrams for the defecton Green's function. Only the diagrams in frames a and b arise in a normal metal. In a superconductor, diagrams of the type in frame c also come into play. The solid lines with two outgoing arrows and two incoming arrows correspond to anomalous electron Green's functions $F^+(\mathbf{p},\varepsilon_k)$ and $F(\mathbf{p},\varepsilon_k)$, respectively. The notation is otherwise the same as in Fig. 1.

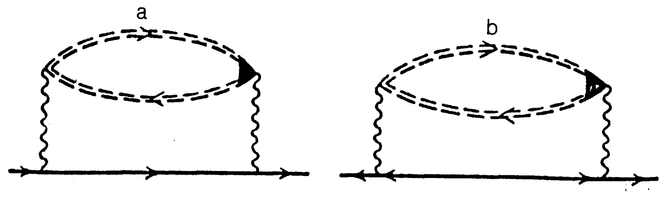

FIG. 3. Feynman diagrams showing the defecton contribution to the eigenenergy parts (a) $\Sigma_1(\mathbf{p},\varepsilon_k)$ and (b) $\Sigma_2^+(\mathbf{p},\varepsilon_k)$ of the electron Green's functions. The double dashed line corresponds to $\Psi(\mathbf{p},\varepsilon_k)$, the blackened triangle with a wavy line corresponds to $V(\mathbf{q})$, and the notation is otherwise the same as in Figs. 1 and 2.

$$\Delta(\mathbf{p},\varepsilon_k)=\frac{\pi T}{1+if_1(\mathbf{p},\varepsilon_k)/\varepsilon_k}\sum_{\varepsilon_m}\int\frac{d\mathbf{p}_1|M(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^3|\nabla\varepsilon(\mathbf{p}_1)|}\times\frac{\omega_0^2(\mathbf{p}-\mathbf{p}_1)}{\omega_0^2(\mathbf{p}-\mathbf{p}_1)+(\varepsilon_k-\varepsilon_m)^2}\,\frac{\Delta(\mathbf{p}_1,\varepsilon_m)}{[\varepsilon_m^2+\Delta^2(\mathbf{p}_1,\varepsilon_m)]^{1/2}}$$
$$-\frac{T}{1+if_1(\mathbf{p},\varepsilon_k)/\varepsilon_k}\sum_{\varepsilon_m}\int\frac{d\mathbf{k}\,d\mathbf{p}_1|V_0(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^6|\nabla\varepsilon(\mathbf{p}_1)|}\times\left(\frac{\max(\varepsilon_0,T,\Delta)}{E_0}\right)^g\left[\operatorname{th}\frac{\tilde\omega(\mathbf{k})-\zeta}{2T}-\operatorname{th}\frac{\tilde\omega(\mathbf{k}+\mathbf{p}_1-\mathbf{p})-\zeta}{2T}\right] \tag{8}$$
$$\times[i(\varepsilon_k-\varepsilon_m)-\tilde\omega(\mathbf{k})+\tilde\omega(\mathbf{k}+\mathbf{p}_1-\mathbf{p})]^{-1}\int_{-\infty}^{\infty}\frac{\Delta(\mathbf{p}_1,\varepsilon_m)\,d\xi}{\varepsilon_m^2+\xi^2+\Delta^2(\mathbf{p}_1,\varepsilon_m)}.$$

Here $f_1(\mathbf{p},\varepsilon_k)$ is the part of $\Sigma_1(\mathbf{p},\varepsilon_k)$ which is odd in terms of ε_k, and

$$\Delta(\mathbf{p},\varepsilon_k)=\Sigma_2(\mathbf{p},\varepsilon_k)[1+if_1(\mathbf{p},\varepsilon_k)/\varepsilon_k]^{-1}. \tag{9}$$

Here $M(\mathbf{p}-\mathbf{p}_1)$ is the screened matrix element of the electron-phonon interaction, and $\omega_0(\mathbf{p}-\mathbf{p}_1)$ is the phonon frequency. The integration over $\mathbf{p}_1$ is over the Fermi surface, and that over $\mathbf{k}$ is over the Brillouin zone.

To write the Éliashberg equations in this form is to assume that the representation of Bloch functions holds for defectons. In other words, the defecton relaxation time τ must satisfy the inequality $\tau\tilde\varepsilon_0\gg1$.

As was shown in Ref. 4, we have

$$\tau^{-1}=\frac{2\pi gT}{1+\exp(\Delta/T)}, \tag{10}$$

and this equality holds in a normal metal if $T\ll\tilde\varepsilon_0/2g$ or $T\ll1$–10 K. No systematic description of the defecton subsystem has been worked out for higher temperatures, at which the condition $\tau\tilde\varepsilon_0<1$ holds, but coherent tunneling remains the primary mechanism for the diffusion of defectons. As the temperature is raised further, incoherent hops from one interstitial site to another assume a major role, and the defectons can be described in the coordinate representation.

At temperatures $T\gtrsim10$ K, however, the inequality $\tau\tilde\varepsilon_0\gg1$ may be satisfied in a superconductor with $T_c\gg T$; here T_c is the temperature of the superconducting transition caused by the electron-phonon interaction.

By analogy with Ref. 4, we then find the following expression for the electron-phonon interaction from Eqs. (7) and (8) in the weak-coupling approximation, in the first nonvanishing order in $\tilde\varepsilon_0$ ($\tilde\varepsilon_0\ll T\ll\Delta$):

$$\Delta(\varepsilon_k)=\pi T\sum_{\varepsilon_m}\int\frac{d\mathbf{p}_1\,d\mathbf{p}|M(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^3|\nabla\varepsilon(\mathbf{p})||\nabla\varepsilon(\mathbf{p}_1)|}\times\frac{\omega_0^2(\mathbf{p}-\mathbf{p}_1)}{\omega_0^2(\mathbf{p}-\mathbf{p}_1)+(\varepsilon_k-\varepsilon_m)^2}\times\frac{\Delta(\varepsilon_m)}{[\varepsilon_m^2+\Delta^2(\varepsilon_m)]^{1/2}}\left[\int\frac{d\mathbf{p}}{|\nabla\varepsilon(\mathbf{p})|}\right]^{-1}$$
$$+x\int\frac{d\mathbf{p}_1\,d\mathbf{p}\,d\mathbf{k}|V_0(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^6|\nabla\varepsilon(\mathbf{p})|\,|\nabla\varepsilon(\mathbf{p}_1)|}\times\left(\frac{\Delta_0}{E_0}\right)^g[\tilde\omega(\mathbf{k})-\tilde\omega(\mathbf{k}+\mathbf{p}_1-\mathbf{p})]^2\left\{\frac{1}{\varepsilon_k^2+\Delta_0^2}+\frac{2\varepsilon_k^2-\Delta_0^2}{2\varepsilon_k(\varepsilon_k^2+\Delta_0^2)^{3/2}}\right.$$
$$\left.\times\ln\frac{(\varepsilon_k^2+\Delta_0^2)^{1/2}+\varepsilon_k}{(\varepsilon_k^2+\Delta_0^2)^{1/2}-\varepsilon_k}\right\}\left[\int\frac{d\mathbf{p}}{|\nabla\varepsilon(\mathbf{p})|}\right]^{-1}. \tag{11}$$

Here $\Delta(\varepsilon_k)=\Delta_0+\Delta_1(\varepsilon_k)$, where Δ_0 is the gap due to the electron-phonon interaction, $\Delta_1(\varepsilon_k)$ is a small correction to Δ_0 for the electron-defecton interaction, and x is the concentration of defectons. The second term on the right side of (11) is the low-frequency defecton component $\Delta_1'(\varepsilon_k)$ of Δ. This component falls off at frequencies $|\varepsilon_k|>\Delta_0$. Substituting $\Delta_1'(\varepsilon_k)$ into the first term on the right side of Eq. (11), we find the frequency-independent defect component Δ_1'' of Δ:

$$\Delta_1''=\sum_{\varepsilon_k}\frac{\Delta_1'(\varepsilon_k)\varepsilon_k^2}{(\varepsilon_k^2+\Delta_0^2)^{3/2}}\Big/\sum_{\varepsilon_k}\frac{\Delta_0^2}{(\varepsilon_k^2+\Delta_0^2)^{3/2}}$$
$$=\frac{5x}{6T\Delta_0}\int\frac{d\mathbf{p}\,d\mathbf{p}_1\,d\mathbf{k}|V_0(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^6|\nabla\varepsilon(\mathbf{p})|\,|\nabla\varepsilon(\mathbf{p}_1)|}\left(\frac{\Delta_0}{E_0}\right)^g\times[\tilde\omega(\mathbf{k})-\tilde\omega(\mathbf{k}+\mathbf{p}_1-\mathbf{p})]^2\left[\int\frac{d\mathbf{p}}{|\nabla\varepsilon(\mathbf{p})|}\right]^{-1}. \tag{12}$$

The quantities Δ_1' and Δ_1'' are comparable in magnitude, and

$$\frac{\Delta_1'}{\Delta_0}\approx\frac{xg\tilde\varepsilon_0^2}{T\Delta_0}\left(\frac{E_0}{\Delta_0}\right)^{1-g}=\frac{xg\varepsilon_0^2}{T\Delta_0}\left(\frac{E_0}{\Delta_0}\right)^{1-g-2\beta}$$

For $\tilde\varepsilon_0\sim3$ K, $T\sim10$ K, $\Delta_0\sim50$ K, $x\sim10^{-2}$, $g\sim0.3$, and $E_0\sim5\cdot10^4$ K we find $\Delta_1'/\Delta_0\sim10^{-2}$.

We see thus that the IR renormalizations reduce the defecton component of the superconductivity by about an order of magnitude, due to both a narrowing of the defecton band and an attenuation of the effective electron-defecton interaction.

At $T\sim1$–10 K, however, the interaction of defectons with each other plays an important role. At a temperature

$$T_0=W_0/|\ln x| \tag{13}$$

a stratification into two phases, with high and low defecton concentrations, occurs. Alternatively, clusters of a finite number of particles form.[9] Here W_0 is the specific binding energy of the defects in a cluster or in the high-concentration phase. Since the probability for the tunneling of a cluster of two or more particles is negligible, only isolated defectons contribute substantially to the superconductivity. The concentration of these isolated defectons, x_{eff}, is[9]

$$x_{eff}=x\left[1+\gamma x\left(\frac{T}{W_0}\right)^2\exp(W_0/T)\right]^{-1},\quad\gamma\sim1. \tag{14}$$

It falls off exponentially at $T < T_0$. Since we would have $W_0 \sim 10^2$–10^3 K for hydrogen in a metal, it would appear difficult to observe a defecton superconductivity.

4. TWO-LEVEL SYSTEMS

If the crystal contains, along with quantum defects, immobile heavy impurities, and if the concentration of these impurities satisfies $c \gtrsim x$, then a defecton will be trapped by a heavy impurity as the temperature is lowered.[9] A situation may arise in which a trapped defecton is still able to tunnel between two interstitial positions which are equivalent in terms of energy. This equivalence is disrupted by other lattice defects. A situation of this sort has been observed in niobium single crystals during the trapping of hydrogen or deuterium by impurities of carbon, oxygen, or nitrogen.[2,3] The two-level systems (2LSs) which arise in the process also contribute to the superconductivity, since there can be an inelastic scattering of electrons by the 2LSs, accompanied by a transition of a 2LS from one state to the other.

We write the Hamiltonian of a 2LS as

$$\mathcal{H}_{2\mathrm{LS}} = -\frac{\varepsilon}{2} c_1^+ c_1 + \frac{\varepsilon}{2} c_2^+ c_2 + \frac{\varepsilon_0}{2} (c_1^+ c_2 + c_2^+ c_1), \quad (15)$$

where the operators c_j^+ and c_j perform a second quantization for a defecton in potential well j, ε is the asymmetry of the 2LS caused by other defects, and $\varepsilon_0/2$ is the matrix element for a tunneling between the minima of the 2LSs.

The interaction of a 2LS with electrons is described by the Hamiltonian

$$H_{e,2\mathrm{LS}} = \sum_{\mathbf{p},\mathbf{q},j} V_0(\mathbf{q}) a^+(\mathbf{p}-\mathbf{q}) a(\mathbf{p}) \exp(i\mathbf{q}\mathbf{R}_j) c_j^+ c_j, \quad (16)$$

where $\mathbf{R}_j$ are the coordinates of the minima of the 2LSs.

After diagonalizing Hamiltonian $\mathcal{H}_{2\mathrm{LS}}$, and converting to states with energies $\pm E/2$, where

$$E = (\varepsilon^2 + \varepsilon_0^2)^{1/2}, \quad (17)$$

the Hamiltonian $\mathcal{H}_{e,2\mathrm{LS}}$ becomes

$$\mathcal{H}_{e,2\mathrm{LS}} = \sum_{\mathbf{p},\mathbf{q}} V_0(\mathbf{q}) a^+(\mathbf{p}-\mathbf{q}) a(\mathbf{p}) \{\exp(i\mathbf{q}\mathbf{R}_1) \times [\alpha^2 \tilde{c}_1^+ \tilde{c}_1 + \nu^2 \tilde{c}_2^+ \tilde{c}_2 - \alpha\nu (\tilde{c}_1^+ \tilde{c}_2 + \tilde{c}_2^+ \tilde{c}_1)] + \exp(i\mathbf{q}\mathbf{R}_2) [\nu^2 \tilde{c}_1^+ \tilde{c}_1 + \alpha^2 \tilde{c}_2^+ \tilde{c}_2 + \alpha\nu (\tilde{c}_1^+ \tilde{c}_2 + \tilde{c}_2^+ \tilde{c}_1)]\}. \quad (18)$$

Here the operators $\tilde{c}_j^+, \tilde{c}_j$ correspond to new states of energy $(-1)^j E/2$, $\alpha = [\frac{1}{2}(1 + \varepsilon/E)]^{1/2}$, and $\nu = [\frac{1}{2}(1 - \varepsilon/E)]^{1/2}$.

We have not written the terms corresponding to scattering by a heavy impurity. This scattering of electrons makes the usual contribution to the residual resistance but does not contribute to the superconductivity.

Calculating the IR renormalizations with Hamiltonian (18) is an extremely tedious process, so we will restrict the discussion to the case $k_F|\mathbf{r}| \ll 1$, where $2\mathbf{r} = \mathbf{R}_1 - \mathbf{R}_2$, and k_F is the Fermi momentum of the electrons. In this case the renormalization is determined primarily by the terms of the Hamiltonian which are diagonal in the states of the 2LS. The behavior found in this manner remains qualitatively the same in the case $k_F|\mathbf{r}| \sim 1$.

As in the case of band motion, the renormalization of the vertex representing the electron-defecton interaction and the Green's function of the defecton are specified by the diagrams in Figs. 1 and 2, but now the dashed line should be understood as representing the seed Green's functions of a defecton:

$$\Psi_{j,j}^{(0)}(\varepsilon_k) = [i\varepsilon_k - (-1)^j E/2 + \xi]^{-1}, \quad j = 1, 2. \quad (19)$$

Summing the parquet diagrams, we find[5,6]

$$\Psi_{j,j} = \Psi_{j,j}^{(0)} [\max(T, \Delta)/E_0]^g, \quad (20)$$

$$V(\mathbf{q}) = V_0(\mathbf{q}) [E_0/\max(T, \Delta)]^g, \quad (21)$$

$$\tilde{\varepsilon}_0 = \varepsilon_0 [\max(\Delta, E, T)/E_0]^\beta. \quad (22)$$

In our approximation, we would have $\beta \sim g \cdot \sin^2(k_F r) \ll g$.

Since the defecton states are localized, the processes shown in Fig. 4 contribute along with the diagrams in Fig. 3 to the scattering.[12] These scattering processes, however, are elastic, and they do not influence the value of Δ. It changes only as a result of inelastic scattering of electrons, i.e., only as a result of the processes in Fig. 3 for which the defecton Green's functions correspond to different states. In this case the divergence of $V(\mathbf{q})$ is cut off at $\max(T, E, \Delta)$.

Replacing the integration in Eqs. (7) and (8) by a summation over the various 2LSs, and eliminating the elastic-scattering contribution, we find

$$f_1(\mathbf{p}, \varepsilon_k) = \pi T \sum_{\varepsilon_m} \int \frac{|M(\mathbf{p}-\mathbf{p}_1)|^2 d\mathbf{p}_1}{(2\pi)^3 |\nabla\varepsilon(\mathbf{p}_1)|} \frac{\omega_0^2(\mathbf{p}-\mathbf{p}_1)}{\omega_0^2(\mathbf{p}-\mathbf{p}_1) + (\varepsilon_k - \varepsilon_m)^2} \times \frac{i\varepsilon_m}{[\varepsilon_m^2 + \Delta^2(\mathbf{p}_1, \varepsilon_m)]^{1/2}} - T \sum_{\varepsilon_m, l} \int \frac{d\mathbf{p}_1 |V_0(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^3 |\nabla\varepsilon(\mathbf{p}_1)|} \left(\frac{\tilde{\varepsilon}_0}{E_l}\right)^2 \times \sin^2[(\mathbf{p}-\mathbf{p}_1)\mathbf{r}] \left(\frac{\max^2(T, \Delta)}{E_0 \max(T, E_l, \Delta)}\right)^g \times \frac{2E_l \operatorname{th}(E_l/2T)}{(\varepsilon_k - \varepsilon_m)^2 + E_l^2} \int_{-\infty}^{\infty} \frac{i\varepsilon_m \, d\xi}{\varepsilon_m^2 + \xi^2 + \Delta^2(\mathbf{p}_1, \varepsilon_m)} \quad (23)$$

$$\Delta(\mathbf{p}, \varepsilon_k) = \frac{\pi T}{1 + i f_1(\mathbf{p}, \varepsilon_k)/\varepsilon_k} \times \sum_{\varepsilon_m} \int \frac{|M(\mathbf{p}-\mathbf{p}_1)|^2 d\mathbf{p}_1}{(2\pi)^3 |\nabla\varepsilon(\mathbf{p}_1)|} \frac{\omega_0^2(\mathbf{p}-\mathbf{p}_1)}{\omega_0^2(\mathbf{p}-\mathbf{p}_1) + (\varepsilon_k - \varepsilon_m)^2} \times \frac{\Delta(\mathbf{p}_1, \varepsilon_m)}{[\varepsilon_m^2 + \Delta^2(\mathbf{p}_1, \varepsilon_m)]^{1/2}} + \frac{T}{1 + i f_1(\mathbf{p}, \varepsilon_k)/\varepsilon_k} \times \sum_{\varepsilon_m, l} \int \frac{d\mathbf{p}_1 |V_0(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^3 |\nabla\varepsilon(\mathbf{p}_1)|} \times \left(\frac{\tilde{\varepsilon}_0}{E_l}\right)^2 \sin^2[(\mathbf{p}-\mathbf{p}_1)\mathbf{r}] \left(\frac{\max^2(T, \Delta)}{E_0 \max(T, E_l, \Delta)}\right)^g \times \frac{2E_l \operatorname{th}(E_l/2T)}{(\varepsilon_k - \varepsilon_m)^2 + E_l^2} \int_{-\infty}^{\infty} \frac{\Delta(\mathbf{p}_1, \varepsilon_m) \, d\xi}{\varepsilon_m^2 + \xi^2 + \Delta^2(\mathbf{p}_1, \varepsilon_m)}. \quad (24)$$

As was shown in Ref. 7, the distribution of ε_l values for the 2LSs in a crystal which results from the interaction with other point defects is Lorentzian with a width $\delta = cW_0$,

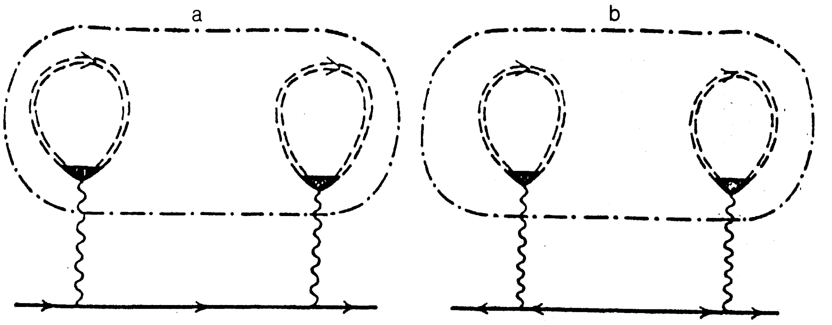

FIG. 4. Additional contribution of two-level systems to the eigenenergy parts (a) $\Sigma_1(\mathbf{p},\varepsilon_k)$ and (b) $\Sigma_2^+(\mathbf{p},\varepsilon_k)$ of the electron Green's functions. The dot-dashed oval shows that the defect lines which it envelops correspond to the same two-level system.

where c is the concentration of defects, and $W_0 \sim 10^2$–10^3 K. We can ignore the scatter in the values of ε_0 for the various 2LSs at $c \ll 1$.

Equations like (23) and (24) were derived in Ref. 13, but they ignored the asymmetry of the 2LSs, the IR renormalization, and the interference factor $\sin[(\mathbf{p}-\mathbf{p}_1)\mathbf{r}]$.

The contribution of 2LSs to superconductivity was also studied in Ref. 14. The interference factor was taken into account there, but the asymmetry of the 2LSs was not, and the IR renormalization was carried out only for the quantity ε_0, i.e., only partially. The replacement of E_0 by the frequency of the local defecton vibrations ω_d in (22) occurs because of the choice of a different initial approximation for ε_0 (Ref. 12).

We first assume that the defecton mechanism for superconductivity is the primary mechanism, and the electron-phonon interaction can be ignored. In the case $T_c < \tilde{\varepsilon}_0$ we then find the following equation for T_c from (23) and (24):

$$1+\sum_l \int \frac{2\,d\mathbf{p}_1\,d\mathbf{p}\,|V_0(\mathbf{p}-\mathbf{p}_1)|^2 \tilde{\varepsilon}_0^2 \sin^2[(\mathbf{p}-\mathbf{p}_1)\mathbf{r}]}{(2\pi)^3|\nabla\varepsilon(\mathbf{p})|\,|\nabla\varepsilon(\mathbf{p}_1)|E_l^3} \times\left(\frac{T_c^2}{E_0E_l}\right)^g\left[\int\frac{d\mathbf{p}}{|\nabla\varepsilon(\mathbf{p})|}\right]^{-1}$$
$$=\sum_l\int\frac{2\,d\mathbf{p}_1\,d\mathbf{p}\,|V_0(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^3|\nabla\varepsilon(\mathbf{p})|\,|\nabla\varepsilon(\mathbf{p}_1)|} \times\frac{\tilde{\varepsilon}_0^2\sin^2[(\mathbf{p}-\mathbf{p}_1)\mathbf{r}]}{E_l^3}\left(\frac{T_c^2}{E_0E_l}\right)^g \ln\frac{E_l}{T_c}\left[\int\frac{d\mathbf{p}}{|\nabla\varepsilon(\mathbf{p})|}\right]^{-1}. \tag{25}$$

It is easy to see that the superconductivity is dominated by the 2LSs with $E_l \sim \varepsilon_0$, i.e., with $\varepsilon_l \lesssim \varepsilon_0$. We introduce the quantity

$$\lambda_d=\sum_l\int\Big\{2\,d\mathbf{p}\,d\mathbf{p}_1|V_0(\mathbf{p}-\mathbf{p}_1)|^2\tilde{\varepsilon}_0^2 \times\sin^2[(\mathbf{p}-\mathbf{p}_1)\mathbf{r}]\tilde{\varepsilon}_0^{2g}/(2\pi)^3|\nabla\varepsilon(\mathbf{p})|\,| \times|\nabla\varepsilon(\mathbf{p}_1)|E_l^3\left[\int\frac{d\mathbf{p}}{|\nabla\varepsilon(\mathbf{p})|}\right](E_0E_l)^g\Big\}. \tag{26}$$

If $g \ll \lambda_d$, then for T_c we find the following estimate:

$$T_c=\tilde{\varepsilon}_0\exp\left(-\frac{1+\lambda_d}{\lambda_d}\right). \tag{27}$$

In order of magnitude we have

$$\lambda_d=\frac{c_{2LS}\,g E_0}{\max(\tilde{\varepsilon}_0,\delta)}\left(\frac{\varepsilon_0}{E_0}\right)^g. \tag{28}$$

where c_{2LS} is the concentration of 2LSs. For $c_{2LS} \sim 10^{-2}$, and for the typical parameter values given above, we find $\lambda_d \sim 1$.

It is easy to see that a further increase in the concentration of 2LSs will not result in an increase in λ_d or T_c. The reason is that while the asymmetry of the 2LSs stems from their interaction with each other and from the condition $\delta > \tilde{\varepsilon}_0$, the ratio c_{2LS}/δ is equal to W_0^{-1} and is independent of c_{2LS}. Consequently, the increase in the asymmetry of the 2LSs with increasing concentration of these systems causes λ_d to approach a constant value.

For hydrogen in a metal, the typical values of T_c are 1–10 K. For heavier impurities (deuterium, tritium, and helium), they are several orders of magnitude lower.

Let us now find the corrections Δ_1' and Δ_1'' to Δ_0 for the electron-defecton interaction in the case in which the superconductivity results from a phonon mechanism. We assume $\delta \ll \Delta_0$. From (23) and (24) we then find

$$\Delta(\varepsilon_k)=\pi T\sum_{\varepsilon_m}\int\frac{|M(\mathbf{p}-\mathbf{p}_1)|^2\,d\mathbf{p}_1\,d\mathbf{p}}{(2\pi)^3|\nabla\varepsilon(\mathbf{p})|\,|\nabla\varepsilon(\mathbf{p}_1)|} \times\frac{\omega_0^2(\mathbf{p}-\mathbf{p}_1)}{\omega_0^2(\mathbf{p}-\mathbf{p}_1)+(\varepsilon_k-\varepsilon_m)^2} \times\frac{\Delta(\varepsilon_m)}{[\varepsilon_m^2+\Delta^2(\varepsilon_m)]^{1/2}}\left[\int\frac{d\mathbf{p}}{|\nabla\varepsilon(\mathbf{p})|}\right]^{-1}$$
$$+\sum_l\int\frac{d\mathbf{p}\,d\mathbf{p}_1|V_0(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^3|\nabla\varepsilon(\mathbf{p})|\,|\nabla\varepsilon(\mathbf{p}_1)|} \times\left(\frac{\tilde{\varepsilon}_0}{E_l}\right)^2\sin^2[(\mathbf{p}-\mathbf{p}_1)\mathbf{r}]\left(\frac{\Delta_0}{E_0}\right)^g\Delta_0 \times\left\{\ln\left[\frac{(\varepsilon_k^2+\Delta_0^2)^{1/2}+\varepsilon_k}{(\varepsilon_k^2+\Delta_0^2)^{1/2}-\varepsilon_k}\right]\Big/\varepsilon_k(\varepsilon_k^2+\Delta_0^2)^{1/2}\right\}E_l \times\operatorname{th}\frac{E_l}{2T}\left[\int\frac{d\mathbf{p}}{|\nabla\varepsilon(\mathbf{p})|}\right]^{-1}. \tag{29}$$

The second term on the right side of (29) is the low-frequency increment $\Delta_1'(\varepsilon_k)$ in Δ_0. The correction Δ_1'', which is independent of ε_k, is given by [according to (12)]

$$\Delta_1''=\sum_l\int\frac{d\mathbf{p}\,d\mathbf{p}_1|V_0(\mathbf{p}-\mathbf{p}_1)|^2}{(2\pi)^3|\nabla\varepsilon(\mathbf{p})|\,|\nabla\varepsilon(\mathbf{p}_1)|}\left(\frac{\tilde{\varepsilon}_0}{E_l}\right)^2\sin^2[(\mathbf{p}-\mathbf{p}_1)\mathbf{r}] \times\left(\frac{\Delta_0}{E_0}\right)^g\frac{E_l\operatorname{th}(E_l/2T)}{\Delta_0}\left[\int\frac{d\mathbf{p}}{|\nabla\varepsilon(\mathbf{p})|}\right]^{-1}. \tag{30}$$

The ratio Δ_1/Δ_0 is, in order of magnitude,

$$\frac{\Delta_1}{\Delta_0}=\sum_l \frac{g\,\mathrm{th}(E_l/2T)\,\bar{\varepsilon}_0^2}{\Delta_0 E_l}\left(\frac{E_0}{\Delta_0}\right)^{1-g}. \tag{31}$$

Taking an average over the Lorentzian distribution of ε_l, we find

$$\frac{\Delta_1}{\Delta_0}=\frac{c_{2LS}\,g\tilde{\varepsilon}_0^2}{\Delta_0}\left(\frac{E_0}{\Delta_0}\right)^{1-g}$$

$$\times\begin{cases} T^{-1}, & \Delta_0 \gg T \gg \delta,\ \tilde{\varepsilon}_0;\\ \delta^{-1}\ln\dfrac{\delta}{\max(T,\tilde{\varepsilon}_0)}, & \Delta_0\gg\delta\gg T,\ \tilde{\varepsilon}_0;\\ \tilde{\varepsilon}_0^{-1}, & \Delta_0\gg\tilde{\varepsilon}_0\gg\delta,\ T. \end{cases} \tag{32}$$

For $c_{2LS}\sim 10^{-2}$, $g\sim 0.3$, $\tilde{\varepsilon}_0\sim 3$ K, $\Delta_0\sim 30$ K, and $E_0\sim 5\cdot 10^4$ K we find a value $\Delta_1/\Delta_0\sim 10^{-2}$–$10^{-1}$. The contribution of the 2LSs to the superconductivity thus reaches a substantial value at low temperatures.

5. CONCLUSION

It has been shown that the IR renormalizations substantially reduce the contribution of quantum defects to superconductivity.

A clustering of defectons resulting in their "freezing" would prevent observation of a superconductivity due to free defectons.

On the other hand, the component of the superconductivity which stems from the two-level systems which form when quantum defects are trapped by heavy impurities can reach an easily discernible value.

In the absence of other pairing mechanisms, the interaction of electrons with the 2LSs formed by hydrogen and a heavy impurity could give rise to a superconductivity with $T_c\sim 1$–10 K. Beginning at $c_{2LS}\sim 10^{-2}$, an increase in the concentration of 2LSs does not cause a further increase in T_c, since the further increase in c_{2LS} is offset by a decrease in the contribution of an individual 2LS, because of an increase in its asymmetry.

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