Minority-carrier spectrum in a system of quantum dots in a strong magnetic field

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Holes injected into a lattice of electronic quantum dots move in the potential valleys between the dots. The Coulomb interaction of the holes with localized electrons results in renormalization of the hole cyclotron frequency by a polaron mechanism. This effect is calculated without the assumption that the interaction is weak.

Progress in submicron technology has made it possible to develop, in the last few years, regular periodic structures of artificial atoms—quantum dots, in which the parameters of the confining potential and the number of electrons per "atom" can be changed in a controlled manner within very wide limits. Such objects are usually prepared from a structure containing a two-dimensional electron gas. Many interesting effects, well known in solid-state physics, can be modeled by a system of quantum dots under essentially completely controllable conditions.

In the present paper we investigate a polaron-type effect, governed by the Coulomb interaction between the electrons of quantum dots and nonequilibrium holes, which are injected into the sample or are photoproduced in the sample. In a sufficiently strong magnetic field (the cylotron radius of the hole is shorter than the period of the lattice of quantum dots) the situation at hand can be reduced to an exactly solvable model. The observable effects are the dependence of the hole cyclotron resonance frequency or the interband-luminescence frequency on the number of electrons in the quantum dots.

We consider a lattice of quantum dots, in which localization of electrons is achieved by creating a periodically modulated electrostatic potential. Such a system was realized, for example, in Ref. 1. A hole, evidently, feels the effect of the same potential, but with the opposite sign, so that the regions occupied by the electrons in quantum dots are potential barriers for holes and the intervals between the dots are valleys, where the potential energy of the holes has minima. We denote the latter potential energy as $V(\mathbf{r})$, where $\mathbf{r}$ is the coordinate of the hole, $V(\mathbf{r}+\mathbf{a}_n)=V(\mathbf{r})$, and $\mathbf{a}_n$ is the set of lattice vectors of the quantum dots.

We use a parabola with frequency Ω to approximate the potential restricting the motion of electrons at a quantum dot. The theoretical and experimental justifications for this approximation can be found in the review by A. Kumar[2] and in references cited there. We also assume that the period of the lattice of quantum dots is large compared with the electronic radius of the dots. Then a hole is located primarily at large distances from the electrons at the quantum dots, and its Coulomb field within each dot, in leading order, can be regarded as uniform. It is known[3] that such a field, in the case of a harmonic confining potential of the quantum dot, excites only the degree of freedom that corresponds to the motion of the center of mass of the system of electrons. Designating the coordinate of the center of mass of a quantum dot, located at a lattice site $\mathbf{a}_n$, by $\mathbf{u}_n$ we can write the interaction energy as

$$H_{int}=\sum_{\mathbf{a}_n}\mathbf{u}_n\mathbf{F}(\mathbf{r}-\mathbf{a}_n), \tag{1}$$

where the force $\mathbf{F}(\mathbf{r}-\mathbf{a}_n)$ at distances $|\mathbf{r}-\mathbf{a}_n|$ which are larger than the electronic radius of a quantum dot is asymptotically equal to

$$N\tilde{e}^2(\mathbf{r}-\mathbf{a}_n)/|\mathbf{r}-\mathbf{a}_n|^3.$$

Here N is the number of electrons at a quantum dot and e^2 is the square of the effective charge (taking into account dielectric constants of the media in contact with one another). The potential energy associated with the oscillations of electrons at the dots is equal to

$$\sum_{\mathbf{a}_n}Nm_e\Omega^2\mathbf{u}_n^2/2,$$

and for the complete Hamiltonian of the system under study in a magnetic field we have

$$\hat{\mathcal{H}}=\sum_n\left\{\frac{1}{2Nm_e}\left(\mathbf{p}_h-\frac{Ne}{2c}[\mathbf{B}(\mathbf{a}_n+\mathbf{u}_n)]\right)^2+\frac{1}{2}Nm_e\Omega^2u_n^2\right\}+\frac{1}{2m_h}\left(\hat{\mathbf{p}}_h-\frac{e}{2c}[\mathbf{Br}]\right)^2+V(\mathbf{r})+H_{int}, \tag{2}$$

where m_e and m_h are the electron and hole effective masses, $\mathbf{P}_h$ is the hole momentum, and $\mathbf{P}_n$ is the momentum associated with the coordinate $\mathbf{u}_n$.

Thus the problem has taken on the typical polaron form, where collective oscillations of electrons in the quantum dots play the role of phonons.[1)]

The potential energy of the "hole + electrons in quantum dots" system consists of the last two terms in Eq. (2) and the term

$$\frac{1}{2}\sum Nm_e\Omega^2\mathbf{u}_n^2$$

The positions of the minima (and in general the extremal points) in the configuration space of the system are determined by the system of equations

$$Nm_e\Omega^2\mathbf{u}_n+\mathbf{F}(\mathbf{r}-\mathbf{a}_n)=0, \tag{3a}$$

$$\nabla_r\left[\sum_n\mathbf{u}_n\mathbf{F}(\mathbf{r}-\mathbf{a}_n)+V(\mathbf{r})\right]=0. \tag{3b}$$

Substituting $\mathbf{u}_n$ from Eq. (3a) into Eq. (3b), we verify that the coordinates of the hole which correspond to the extrema of the total potential energy are determined by the extrema of the function

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$$W(\mathbf{r})=V(\mathbf{r})-[(N\tilde{e}^2)^2/2Nm_e\Omega^2]\sum_n \mathbf{F}^2(\mathbf{r}-\mathbf{a}_n). \qquad (4)$$

Consider a square lattice of quantum dots, in which the coordinates of the sites are n_1L and n_2L, where L is the period and n_1 and n_2 are integers. Then, obviously, the minima of $V(\mathbf{r})$ are located at the points $(n_1+1/2)L$ and $(n_2+1/2)L$. The second term in Eq. (4) has a minimum near the lattice sites. Thus the situation is a multivalley one, in the sense that $W(\mathbf{r})$ has equivalent minima at four points in each unit cell which are located on the diagonals of a square.

The motion of a hole near a minimum in the presence of a magnetic field, oriented perpendicular to the plane of the structure, is localized in a region of size $(\hbar/m_h\omega)^{1/2}$, where ω is the hybrid frequency of the hole. The motion can also be finite near a maximum of the potential, if the magnetic field is sufficiently strong. In general, for a two-dimensional particle in a neighborhood of an extremum of the potential energy with the principal curvatures ν_1^2 and ν_2^2, the hybrid frequencies in a perpendicular magnetic field are equal to

$$\tilde{\omega}_{1,2}^2=\frac{\nu_1^2+\nu_2^2+\omega_c^2}{2}\pm\frac{1}{2}[(\nu_1^2+\nu_2^2+\omega_c^2)^2-4\nu_1^2\nu_2^2]^{1/2},$$

where ω_c is the cyclotron frequency. It is easy to verify that the for $\nu_1^2,\nu_2^2>0$ (minimum) $\tilde{\omega}_1^2,\omega_2^2>0$, i.e., the motion is finite; for $\nu_1^2,\nu_2^2<0$ (maximum) both hybrid frequencies ω_1 and ω_2 are real (i.e., the motion is finite), if $\omega_c>|\nu_1|+|\nu_2|$. Finally, in the case of a saddle point $\nu_1^2,\nu_2^2<0$ one of the hybrid frequencies is always imaginary, i.e., the motion is infinite no matter how strong the magnetic field is. We note in passing that the frequently used argument about the motion of a Larmor center along lines of constant level is not always applicable: Near a maximum of the potential all lines of constant level are closed, but the motion becomes finite only in a field exceeding some critical value.

We assume below that the motion of a hole is finite, and in addition the size of the region where the hole is localized is assumed to be small compared with the lattice period of the quantum dots. Then H_{int} can be expanded in powers of the small deviations of $\mathbf{r}$ from the extremum point $\mathbf{r}_0$, retaining only the term proportional to $(\mathbf{u}_n-\mathbf{u}_{n0})\cdot(\mathbf{r}-\mathbf{r}_0)$, where $\mathbf{u}_{n0}$ is found from Eq. (3a) at $\mathbf{r}=\mathbf{r}_0$. Then we arrive at the exactly diagonalizable Hamiltonian of coupled harmonic oscillators in a magnetic field. The closest analogy is the problem of local modes in crystal lattice dynamics. The levels corresponding to coupled oscillations of a hole and electrons at the dots split off the band of oscillations of quantum dots with frequency Ω.[2)] It is these levels that contain the desired polaron shift of the hole cyclotron frequency. The corresponding characteristic frequencies are found from the condition that the following system of equations have a solution:

$$(\Omega^2-\omega^2)A_{nx}=i\omega\omega_eA_{ny}+\frac{1}{m_e}[T_{xx}(\mathbf{a}_n)B_x+T_{xy}(\mathbf{a}_n)B_y],$$

$$(\Omega^2-\omega^2)A_{ny}=-i\omega\omega_eA_{nx}+\frac{1}{m_e}[T_{yx}(\mathbf{a}_n)B_x+T_{yy}(\mathbf{a}_n)B_y],$$

$$(\nu_{xx}^2-\omega^2)B_x=-(i\omega\omega_h+\nu_{xy}^2)B_y+\frac{1}{m_h}\Big[\sum_n T_{xx}(\mathbf{a}_n)A_{nx}+T_{yx}(\mathbf{a}_n)A_{ny}\Big],$$

$$(\nu_{yy}^2-\omega^2)B_y=(i\omega\omega_h-\nu_{xy}^2)B_x+\frac{1}{m_h}\Big[\sum_n T_{xy}(\mathbf{a}_n)A_{nx}+T_{yy}(\mathbf{a}_n)A_{ny}\Big]. \qquad (5)$$

Here the two-dimensional vectors $\mathbf{A}_n$ correspond to the amplitudes of the displacements of the center of mass of the quantum dot at the site $\mathbf{a}_n$, $\mathbf{B}$ is the same thing for a hole, ω_e and ω_h are the electron and hole cyclotron frequencies, and $\nu_{\alpha\beta}^2$ $(\alpha,\beta=x,y)$ are coefficients in the quadratic form arising when $V(\mathbf{r})$ is expanded around the extremum point $\mathbf{r}_0$, and

$$T_{\alpha\beta}(\mathbf{a}_n)\equiv\partial F_\alpha(\mathbf{r}_0-\mathbf{a}_n)/\partial r_\beta.$$

Finding A_{nx} and A_{ny} from the first two equations of Eqs. (5) and substituting into the last two equations of Eqs. (5), we obtain the dispersion equation for the frequencies which are split off:

$$(\omega^2-\omega_+^2)(\omega^2-\omega_-^2)(\omega^2-\nu_+^2)(\omega^2-\nu_-^2)-\gamma^4[4\omega_e\omega_h\omega^2(\omega^2-\omega_+^2)(\omega^2-\omega_-^2)+{}^5/_2(\omega^2-\Omega^2)(\omega^2-\omega_+^2)(\omega^2-\omega_-^2)\times(2\omega^2-\nu_{xx}^2+\nu_{yy}^2)]+\gamma^8[{}^{25}/_4(\omega^2-\Omega^2)^2-4\omega_e^2\omega^2]=0, \qquad (6)$$

where

$$\omega_\pm=(\Omega^2+\omega_e^2/4)^{1/2}\pm\omega_e/2,$$

$$\nu_\pm^2={}^1/_2(\nu_{xx}^2+\nu_{yy}^2+\omega_h^2)\pm[{}^1/_4(\nu_{xx}^2+\nu_{yy}^2+\omega_h^2)^2-(\nu_{xx}^2\nu_{yy}^2-\nu_{xy}^4)]^{1/2},$$

$$\gamma^4\equiv(N\tilde{e}^2)^2\lambda/m_em_h, \qquad \lambda\equiv\sum_n|\mathbf{r}_0-\mathbf{a}_n|^{-6}$$

(we used the asymptotic expression for $\mathbf{F}(\mathbf{r}-\mathbf{a}_n)$ under the assumption that the extremum points $\mathbf{r}_0$ are located far from the lattice sites of the quantum dots). The desired roots, which correspond to the hole cyclotron resonance, are those solutions of Eq. (6) which pass into $\nu_\pm$ as $\gamma\to0$. Because of the complexity of the general formulas, we examine the simplest limiting case: $\nu_{\alpha\beta}^2\ll\omega_h^2$ (hole on an almost flat section of the potential relief) and $\gamma^2\ll\Omega^2$, ω_h^2, which corresponds to a weak-coupling polaron. We obtain

$$\omega_{res}^2=\omega_h^2+\gamma^4\frac{5(\omega_h^2-\Omega^2)+4\omega_e\omega_h}{(\omega_h^2-\omega_+^2)(\omega_h^2-\omega_-^2)}. \qquad (7)$$

For $\omega_h>\Omega$ (strong magnetic field) for a heavy hole the inequalities $\omega_+>\omega_h>\omega_-$ are satisfied. Then

$$\Delta\omega_h^2=\omega_{res}^2-\omega_h^2<0,$$

i.e., the change in the hole cyclotron mass Δm_h is positive (normal polaron effect). The frequency shift is proportional to γ^4, i.e., the square of the number of electrons at a dot. An anomalous polaron shift $\Delta m_h<0$ is obtained in the situation $m_h<m_e$ (light holes in Ge, Si) for sufficiently strong magnetic fields, when the inequalities $\omega_h>\omega_+>\omega_-$ are satisfied. Pseudocrossing of terms occurs near $\omega_+=\omega_h$:

$$\omega_{res}^{2}=\frac{\omega_h^2+\omega_+^2}{2}\pm\frac{1}{2}[(\omega_h^2-\omega_+^2)^2+\text{const}\,\gamma^4]^{1/2}, \qquad (8)$$

which should be exhibited in an experiment as splitting of the cyclotron peak of a light hole.

In conclusion we note that our harmonic-oscillator model becomes unstable when the value of γ is too high. This, of course, is a consequence of the expansion of H_{int} in powers of $\mathbf{u}_n$ and $\mathbf{r}-\mathbf{r}_0$. This instability disappears when the anharmonicity of the vibrations of the quantum dots and the fact that H_{int} decreases with increasing $|\mathbf{r}-\mathbf{r}_0|$ are taken into account. The threshold of instability is obtained from Eq. (6), if we set there $\omega=0$ (appearance of a soft mode). Then we obtain $\Omega^2 \min(\nu_1^2,\nu_2^2)>5\gamma^4/2$, where ν_1^2 and ν_2^2 are the principal values of the tensor $\nu_{\alpha\beta}^2$. For GaAs with $\hbar\Omega=2$ meV, $\hbar\nu_{1,2}\sim 0.2$ meV the region of stability corresponds to $N<30$, which is easily achievable for systems described in existing experiments.[4,5]

[1)] In the Hamiltonian (2) the dipole-dipole interaction of quantum dots with one another, which decreases with distance more rapidly than the charge-dipole interaction H_{int}, was omitted. Taking this interaction into account would result in smearing of the band of vibrational levels $\hbar\Omega(n+1/2)$, which correspond to "phonons" of the system of quantum dots. For the problem under study such smearing is not of fundamental significance.

[2)] In the absence of interaction between dots the width of this zone is equal to zero.

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