Permittivity tensor in single crystals for $\gtrsim 1$ GeV γ rays

V. A. Maisheev, V. L. Mikhalev, and A. M. Frolov

Institute of High Energy Physics
(Submitted 8 November 1991)
Zh. Eksp. Teor. Fiz. **101**, 1376–1394 (April 1992)

The permittivity tensor for $\gtrsim 1$ GeV γ rays in the region of single-crystal orientations corresponding to coherent pair production is determined in a model in which the single crystal is treated as a region with an electric field generated by atoms arranged in an ordered fashion in space. The polarization characteristics of normal electromagnetic waves propagating in single crystals are found using the permittivity tensor. In particular, it is shown that there exist solutions of the dispersion equation for elliptically polarized waves. Other electrodynamic processes in single crystals, including Cherenkov radiation from charged particles, are also studied.

According to theoretical and experimental investigations (see Refs. 1 and 2 and the literature cited there) electrodynamic processes in single crystals differ significantly from the same processes in an amorphous medium. This is because a single crystal is a medium that is filled with an electric field that is generated by an ordered arrangement of atoms. It is with such a field that elementary particles interact. For electrodynamic processes the most important properties of single-crystal fields are their magnitude, periodicity, and anisotropy.

In some sense the electric field of a single crystal is similar to that of a dielectric in which are present virtual electron-positron pairs, which are analogous to the bound charge in a dielectric medium. The polarization of these pairs alters the external field and ultimately causes Maxwell's equations to be nonlinear for the fields of the single crystal with an electromagnetic wave propagating in them. Of course, a single crystal contains carriers of electric charge (electrons, ions, etc.), but their direct presence is significant only if the frequencies of the electromagnetic radiation passing through the single crystal are low, while at high frequencies the fields formed by these charges play the main role. These considerations give a graphic picture of the process; a more accurate description of the vacuum polarization can be found in the literature.[3]

The interaction of an electromagnetic wave with the fields of single crystals can be described by introducing the permittivity tensor, as done for a constant uniform field.[4,5] The field of the wave satisfies the standard equations of linear electrodynamics in a medium, in the present case an electromagnetic vacuum, while such a tensor makes it possible to describe, from a unified standpoint, a number of processes occurring in single crystals, such as dichroism accompanying the passage of γ rays, Cherenkov radiation from charged particles, and others. There is no doubt that the solution of these problems is of interest in practical applications for obtaining polarized beams of high-energy electrons and γ-rays.[6–12]

The present paper is concerned with determining the permittivity tensor in single crystals oriented in the region of coherent pair production[13–15] for $\gtrsim 1$ GeV γ-rays, and examining some electrodynamic processes using this tensor.

ELECTRIC FIELDS OF SINGLE CRYSTALS

The three-dimensional potential of a single crystal (averaged over thermal fluctuations of the lattice) has the form[2]

$$\varphi(\mathbf{r})=\frac{4\pi eZ}{\Delta}\sum_{\mathbf{g}} U(\mathbf{g})e^{-i\mathbf{gr}}, \tag{1}$$

where e and Z is the electron charge and the atomic number of the material of the single crystal, Δ is the cell volume, $\mathbf{g}$ is a reciprocal lattice vector,[13] $\mathbf{r}$ is the radius vector drawn from the origin of the unit cell

$$U(\mathbf{g})=S(\mathbf{g})\frac{1-F(g)}{g^2}\exp(-{}^1/_2Ag^2), \tag{2}$$

$S(\mathbf{g})$ is the structure factor, $F(g)$ is the form factor of an atom in the single crystal, and A is the mean-square amplitude of thermal vibrations of the atoms. Thus the potential of the single crystal is determined, if we know the quantities $U(\mathbf{g})$, which can be measured (strictly speaking the quantity $|U(g)|^2$ is measured) from the diffraction of x-rays or electrons.

A γ-ray can propagate in a single crystal at an arbitrary angle to the crystallographic axes of the crystal. It is of interest to find the intensity of the electric field $\vec{\mathscr{E}}$ in a Cartesian coordinate system one axis of which is oriented along the direction of motion of the particle. In this coordinate system we obtain for the components of the vector $\vec{\mathscr{E}}$ from Eq. (1)

$$\mathscr{E}_k=\frac{4\pi eZ}{\Delta}i\sum_{n_1,n_2,n_3} U(n_1,n_2,n_3)\,(n_jG_j\alpha_{kj})\exp(-ix_in_jG_j\alpha_{ij}),$$
$$k=1,2,3, \tag{3}$$
$$n_k=0,\pm1,\pm2,\pm3,\ldots,$$

where α_{ij} are the elements of the matrix of the transformation from the system of crystallographic axes into a new coordinate system (x_1,x_2,x_3), the vector $\mathbf{g}$ is expanded in terms of the basis vectors of the reciprocal lattice

$$\mathbf{g}=n_1\mathbf{G}_1+n_2\mathbf{G}_2+n_3\mathbf{G}_3$$

and $U(n_1,n_2,n_3) = U(\mathbf{g})$. In this notation any of the three axes can be taken as the direction of motion. In the case that the matrix α_{ij} is unitary the formula (3) describes the field in the system of crystallographic axes.

It is well known that the properties of the field of an ultrarelativistic uniformly moving particle are close to those of the field of a light wave.[4] We show that the field of the single crystal also has the same properties. For this we trans-

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form into a coordinate system moving relative to the single crystal with velocity v (along the x_1 axis) close to the velocity of light c. In this system we have for the intensities of the electric field $\vec{\mathscr{E}}'$ and magnetic field $\mathbf{H}'$

$$\mathscr{E}_1' = \frac{4\pi eZ}{\Delta} i \sum_{n_1,n_2,n_3} U(n_1,n_2,n_3)\,(n_jG_j\alpha_{1j}) \times \exp\{-i[x_2'(n_jG_j\alpha_{2j})+x_3'(n_jG_j\alpha_{3j})] \times\exp\{-i[\gamma(x_1'-vt')(n_jG_j\alpha_{1j})]\},$$

$$\mathscr{E}_2' = \frac{4\pi eZ}{\Delta} \gamma i \sum_{n_1,n_2,n_3} U(n_1,n_2,n_3)\,(n_jG_j\alpha_{2j}) \times \exp\{-i[x_2'(n_jG_j\alpha_{2j})+x_3'(n_jG_j\alpha_{3j})]\} \times\exp\{-i[\gamma(x_1'-vt')(n_jG_j\alpha_{1j})]\},$$

$$\mathscr{E}_3' = \frac{4\pi eZ}{\Delta} \gamma i \sum_{n_1,n_2,n_3} U(n_1,n_2,n_3)\,(n_jG_j\alpha_{3j}) \times \exp\{-i[x_2'(n_jG_j\alpha_{2j})+x_3'(n_jG_j\alpha_{3j})] \times\exp\{-i[\gamma(x_1'-vt')(n_jG_j\alpha_{1j})]\},$$

$$H_1'=0, \quad H_2' = -\frac{v}{c}\mathscr{E}_3', \quad H_3' = \frac{v}{c}\mathscr{E}_2',$$

where $\gamma = (1-v^2/c^2)^{-1/2}$ and the variables x_k, t and x'_k, t' are related by the Lorentz transformations (t is the time). It is obvious that the longitudinal component of the field satisfies $\mathscr{E}_1' \ll \mathscr{E}_2' \approx H_3'$, $\mathscr{E}_3' \approx -H_2'$ if the factor γ is sufficiently large, and the formula (4) can be regarded as an expansion in plane waves, where the quantity $|\gamma v(n_jG_j\alpha_{ij})|$ plays the role of frequency. In order to find the number of equivalent photons[4,13] corresponding to the field of the particle we find the total electromagnetic energy flux along the direction $\mathbf{v}$:

$$J = \frac{c}{4\pi}\int \mathscr{E}'^2 dx_2\,dx_3\,dt' = \frac{4\pi e^2Z^2\gamma V}{\Delta}\sum_{n_1,n_2,n_3}\Phi(n_1,n_2,n_3) \times[(n_jG_j\alpha_{2j})^2+(n_jG_j\alpha_{3j})^2], \tag{5}$$

where

$$\Phi(n_1,n_2,n_3) = |U(n_1,n_2,n_3)|^2 = |S(\mathbf{g})|^2[1-F(g)]^2 e^{-Ag^2}/g^4,$$

and V is the volume of the single crystal. Hence we find the number of pseudophotons (determined by the indices n_1, n_2, n_3) per atom of the single crystal:

$$N_\gamma(n_1,n_2,n_3) = \frac{4\pi e^2Z^2}{\Delta N_s c}\frac{\Phi(g)(g^2-g_\parallel^2)}{\hbar|g_\parallel|}, \tag{6}$$

where N_S is the number of atoms per unit cell, $g_\parallel = n_jG_j\alpha_{1j}$ is the projection of the vector $\mathbf{g}$ on the direction of motion, and $\hbar$ is Planck's constant. The number density of pseudophotons in momentum space $\mathbf{q}$ is given by (per atom of the single crystal)

$$\frac{dN_\gamma}{d\mathbf{q}} = \frac{4\pi e^2Z^2}{\Delta N_s c}\sum_{\mathbf{g}}\frac{\Phi(q)(q^2-q_\parallel^2)}{\hbar|q_\parallel|}\delta(\mathbf{q}-\mathbf{g}), \tag{7}$$

and the two sets of indices n_1, n_2, n_3 and $-n_1, -n_2, -n_3$ correspond to the same energy of a pseudophoton $|\hbar(n_jG_j\alpha_{1j})\gamma c|$. The total number of pseudophotons is determined by the sum over all reciprocal lattice vectors g in the relation (6). The main difference between the pseudophoton spectra of an atom and a single crystal is that in the first case the spectrum is continuous while in the second case the spectrum is discrete. The existence of thermal oscillations of the atoms of the single crystal radically changes the spectral density of pseudophotons for large g. Likewise, the spectrum lacks the divergence, characteristic for an individual atom, in the limit $q_\parallel \to 0$ [the term with $n_1, n_2, n_3 = 0$ is eliminated from the spectrum (6) or (7), since this term corresponds to an additive constant in Eq. (1)]. Pseudophotons in single crystals have also been studied in Refs. 2 and 13.

Knowing the cross section $\sigma_{\gamma\gamma}$ of the process[4] $\gamma\gamma \to e^+e^-$, we can calculate the coherent part of the pair production cross section in single crystals from the formula

$$\sigma_p = \sum_{n_1,n_2,n_3}\sigma_{\gamma\gamma}(w)N_\gamma(n_1,n_2,n_3), \tag{8}$$

where

$$w = \left(1-\frac{m^2c^4}{\omega_1\omega_2}\right)^{1/2} = \left(1-\frac{2m^2c^4}{E_\gamma\hbar|n_jG_j\alpha_{1j}|}\right)^{1/2},$$

ω_1 and ω_2 are the energy of the γ-ray and the pseudophoton, respectively ($\omega_1\omega_2$ is a relativistic invariant). Hence it is obvious that only the lattice sites n_1, n_2, n_3 for which $\omega_1\omega_2 > m^2c^4$ holds contribute to the cross section. The quantity σ_p, calculated from the formula (8), is equal to the Born cross section, i.e., for a single crystal the logarithmic indefiniteness characteristic for the pseudophoton method is absent.

The cross section σ_p depends on the linear polarization of the γ-ray[6]

$$\sigma_p = \mathscr{A} + \mathscr{B}(\mathbf{e}\boldsymbol{\tau})^2, \tag{9}$$

where the unit vectors $\mathbf{e}$ and $\boldsymbol{\tau}$ determine the polarization plane of the γ-ray beam and some definite plane relative to the crystallographic axes (the wave vector of the γ-ray lies in both planes). The formula (9) essentially determines a symmetric tensor of rank two, whose components we denote as $\sigma_{kl}(k,l=1,2)$.

The case when the γ-ray propagates in the single crystal at a small angle $\theta \ll 1$ relative to the crystallographic axis, determined, for example, by the factor $\mathbf{G}_1$, is of practical interest. The angle α, chosen as the plane angle between the plane containing the momentum of the γ-ray and the vector $\mathbf{G}_1$ and the plane containing the vectors $\mathbf{G}_1$ and $\mathbf{G}_2$, determines the azimuthal direction of motion relative to the axis $\mathbf{G}_1$. There is no loss of generality in this approach, since the choice of crystallographic axes is not unique and any axis can be chosen as the $\mathbf{G}_1$ axis. In this case

$$g_\parallel = G_1n_1+\theta(G_2n_2\cos\alpha+G_3n_3\sin\alpha), \tag{10}$$

and it is obvious from a comparison of Eqs. (7) and (8) that the terms of the reciprocal lattice with $n_1 = 0$ make the main contribution to the cross section. This fact is well known in the theory[6,13] and the γ-ray energy is $\gtrsim 1$ GeV when this approximation holds. This means that the coherent pair production occurs, in practice, in the field

$$\mathscr{E}_1 \approx 0,$$

$$\mathscr{E}_2 \approx \frac{4\pi eZi}{\Delta}\sum_{n_2,n_3}U(0,n_2,n_3)\,(n_2G_2)\exp[-i(x_2n_2G_2+x_3n_3G_3)], \tag{11}$$

$$\mathscr{E}_3 \approx \frac{4\pi eZi}{\Delta}\sum_{n_2,n_3}U(0,n_2,n_3)\,(n_3G_3)\exp[-i(x_2n_2G_2+x_3n_3G_3)].$$

This field is obtained by averaging (3) over the longitudinal coordinate with a unitary matrix (α_{ij}). An important case is realized when the angle is sufficiently large (but $\theta \ll 1$). Then only one transverse coordinate dominates

$$\mathscr{E}_p = \frac{4\pi eZ}{\Delta} \sum_{n_p} U(n_p)\,(n_p G_p) \exp(-ix_p n_p G_p),\ n_p = \pm 1, 2, \ldots, \tag{12}$$

where from the set of numbers n_2 and n_3 the particular numbers n_2 and n_3 with the same ratio n_2/n_3 are selected, $n_p^2 G_p^2 = n_2^2 G_2^2 + n_3^2 G_3^2$, and x_p is a variable transverse coordinate. In this case the field of the signal crystal reduces to the effect of the field of one plane.

The conditions which the angles θ and α must satisfy will be presented below.

As is well known, the theory of coherent pair production is valid only for orientation angles

$$\theta_p > \frac{V_0}{mc^2} = \varphi_B$$

(see Refs. 1 and 2), where θ_p is the entry angle of the γ-ray relative to the axis or plane with the potential V_0. For $\theta_p < \varphi_B$ the character of pair production is close to that of the same process in a constant field.[2,9,10] The strength of the interaction of the γ-ray with the electric field is characterized by the parameter[2] $\chi = \gamma \mathscr{E}/\mathscr{E}_0$, where $\mathscr{E} = m^2c^3/e\hbar$. We obtain the following estimate:

$$|\chi| \approx \gamma \frac{V_0}{1/2 d} \frac{e\hbar}{m^2c^3} = \left(\frac{eV_0}{mc^2}\right) \frac{2}{\pi} \left(\frac{G_p \hbar}{mc}\right) \frac{\gamma}{2} = \frac{2}{\pi} \frac{\varphi_B}{\varphi_1} \approx \frac{\varphi_B}{\varphi_1}, \tag{13}$$

where d is the interplanar spacing and φ_1 is the plane angle at which the first harmonic comes into play [see Eq. (8)]. Thus for coherent pair production, when the effect is most fully manifested (near the maximum of the first harmonic), small values of the parameter $|\chi| \lesssim 1$ are typical.

We note that the pair production process in single crystals also occurs on fluctuations of the electric field. In Refs. 13–15 it is shown that this process can be taken into account by adding to the cross section σ_p a term that is virtually identical to the cross section in the case of an isolated atom, i.e., the pair production processes in the potential (1) and on fluctuations of the potential (1) are virtually independent of one another.

PERMITTIVITY TENSOR IN SINGLE CRYSTALS

We write the equations of the electromagnetic field in a medium in the following form:[16,17]

$$\begin{aligned} \operatorname{rot} \mathbf{B} &= \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t}, \quad \operatorname{div} \mathbf{D} = 0, \\ \operatorname{rot} \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad \operatorname{div} \mathbf{B} = 0, \end{aligned} \tag{14}$$

where $\mathbf{E}$ is the intensity of the electric field and $\mathbf{D}$ and $\mathbf{B}$ are the electric and magnetic induction vectors. All properties of the medium—the electric field of the single crystal (11) is the medium—are reflected in the relation between the vectors $\mathbf{B}$, $\mathbf{E}$, and $\mathbf{D}$. We represent the relation between $\mathbf{D}$ and $\mathbf{E}$ in the form

$$D_i(\omega) = \varepsilon_{ij} E_j(\omega),\ i, j = 1, 2, 3, \tag{15}$$

where $\varepsilon_{ij} = \varepsilon_{ij}' + i\varepsilon_{ij}''$ is the complex permittivity tensor and ω is the frequency of the γ-ray.

In order to determine the tensor ε_{ij} in the case of a monochromatic field

$$\mathbf{E} = {}^1/_2 \{\mathbf{E}_0 \exp[i(\mathbf{kr} - \omega t)] + \mathbf{E}_0^* \exp[-i(\mathbf{kr} - \omega t)]\}, \tag{16}$$

where $\mathbf{k}$ is the wave vector of the γ-ray, we find the average energy lost by the electromagnetic wave per unit volume and per unit time[16,17]

$$\tilde{q} = \frac{1}{4\pi V} \int_V \mathbf{E} \frac{\partial \mathbf{D}}{\partial t} dV = \frac{\omega}{8\pi} \varepsilon_{ij}'' E_{0j} E_{0i}^*. \tag{17}$$

The mechanism by which the wave loses energy is electron-positron pair production in the field of the single crystal. The process is determined primarily by the transverse part of the permittivity tensor, while the longitudinal components of the tensor are higher-order infinitesimals in the interaction constant α.[1,2] Taking this into account, and in the coordinate system one axis of which is oriented parallel to the wave vector of the γ-ray, we have

$$\tilde{q} = \frac{\omega}{8\pi} \varepsilon_{kl}''(\omega) E_{0l} E_{0k}^* = \frac{\omega}{c} \varepsilon_{kl}''(\omega) e_l e_k^* Q, \quad l, k = 1, 2, \tag{18}$$

where

$$e_l = \frac{E_{0l}}{(E_{01}E_{01}^* + E_{02}E_{02}^*)^{1/2}}, \quad Q = \frac{c}{8\pi} (E_{01}E_{01}^* + E_{02}E_{02}^*).$$

On the other hand, knowing the tensor σ_{kl} we can write

$$\tilde{q} = N\sigma_{kl} e_l e_k^* c n_\gamma E_\gamma = N\sigma_{kl} e_l e_k^* Q, \tag{19}$$

where cn_γ is the γ-ray flux density and e is the polarization vector of the γ-ray [the quantum analog of this vector in the formula (18)].

Comparing Eqs. (18) and (19) we find the imaginary part of the permittivity tensor

$$\varepsilon_{kl}'' = \frac{N\sigma_{kl} c}{\omega} = \frac{N\sigma_{kl}\hbar c}{E_\gamma}, \tag{20}$$

where E_γ is the energy of the γ-ray.

We determine the real components of the permittivity tensor with the help of the dispersion relations[17]

$$\varepsilon_{kl}' - \delta_{kl} = \frac{2}{\pi} \int \frac{x\varepsilon_{kl}''(x)\,dx}{x^2 - \omega^2}, \tag{21}$$

where δ_{kl} is the Kronecker δ-function and the integral is a principal value integral. The integrals arising here are calculated with the help of the theory of functions of a complex variable. Then in the system of coordinates such that one axis is oriented parallel to the direction of motion of the γ-ray and the other two axes lie in planes determined by the vectors $\mathbf{G}_1$, $\mathbf{G}_2$ and $\mathbf{G}_1$, $\mathbf{G}_3$, the tensor ε_{kl} is a sum over reciprocal lattice vectors $\mathbf{g} = n_2\mathbf{G}_2 + n_3\mathbf{G}_3$ ($n_1 = 0$ $\theta \ll 1$) and has the following components:

$$\varepsilon_{11}' = \frac{S'}{2} + \frac{BN\bar{\sigma}}{8\pi} \frac{\hbar}{mc} \sum_g \Phi(g)\,(g_2^2 - g_3^2) z_g^2 F_1'(z_g),$$

$$\varepsilon_{22}' = \frac{S'}{2} - \frac{BN\bar{\sigma}}{8\pi} \frac{\hbar}{mc} \sum_g \Phi(g)\,(g_2^2 - g_3^2) z_g^2 F_1'(z_g),$$

$$\varepsilon_{12}'=\varepsilon_{21}'=\frac{BN\bar{\sigma}}{8\pi}\frac{\hbar}{mc}\sum_{g}\Phi(g)(2g_2g_3)z_g^2F_1'(z_g), \quad (22)$$

$$S'=2+\frac{BN\bar{\sigma}}{\pi}\frac{\hbar}{mc}\sum_{g}\Phi(g)(g_2^2+g_3^2)z_g^2F_2'(z_g),$$

$$z_g=\frac{2mc^2}{E_T\theta(g_2\cos\alpha+g_3\sin\alpha)}=\frac{1}{n_2W_V+n_3W_H}.$$

The summation over g in Eq. (22) satisfies the condition $z_g>0$,

$$F_1'(z)=\begin{cases}\left[(1-z)^{1/2}+\frac{z}{2}\ln\frac{1+(1-z)^{1/2}}{1-(1-z)^{1/2}}\right]^2\\ +\left[(1+z)^{1/2}-\frac{z}{2}\ln\frac{(1+z)^{1/2}+1}{(1+z)^{1/2}-1}\right]^2-\frac{\pi^2z^2}{4}, & 0<z\leqslant 1,\\ -[(z-1)^{1/2}-z\operatorname{arcctg}(z-1)^{1/2}]^2\\ +\left[(1+z)^{1/2}-\frac{z}{2}\ln\frac{(1+z)^{1/2}+1}{(1+z)^{1/2}-1}\right]^2, & z>1,\end{cases} \quad (23)$$

$$F_2'(z)=\begin{cases}-3-\left(1+z-\frac{z^2}{2}\right)\frac{1}{4}\ln^2\frac{1+(1-z)^{1/2}}{1-(1-z)^{1/2}}\\ -\left(1-z-\frac{z^2}{2}\right)\frac{1}{4}\ln^2\frac{(1+z)^{1/2}+1}{(1+z)^{1/2}-1}\\ +\frac{(z+1)(1-z)^{1/2}}{2}\ln\frac{1+(1-z)^{1/2}}{1-(1-z)^{1/2}}\\ -\frac{(z-1)(1+z)^{1/2}}{2}\ln\frac{(1+z)^{1/2}+1}{(1+z)^{1/2}-1}\\ +\frac{\pi^2}{4}\left(1+z-\frac{z^2}{2}\right), & 0<z\leqslant 1,\\ -3+\left(1+z-\frac{z^2}{2}\right)\operatorname{arcctg}^2(z-1)^{1/2}\\ -\left(1-z-\frac{z^2}{2}\right)\frac{1}{4}\ln^2\frac{(1+z)^{1/2}+1}{(1+z)^{1/2}-1}\\ +(1+z)(1-z)^{1/2}\operatorname{arcctg}(z-1)^{1/2}\\ -\frac{(z-1)(1+z)^{1/2}}{2}\ln\frac{(1+z)^{1/2}+1}{(1+z)^{1/2}-1}, & z>1.\end{cases} \quad (24)$$

$$\varepsilon_{11}''=\frac{S''}{2}-\frac{BN\bar{\sigma}}{16}\frac{\hbar}{mc}\sum_{g}\Phi(g)(g_2^2-g_3^2)F_1''(z_g),$$

$$\varepsilon_{22}''=\frac{S''}{2}+\frac{BN\bar{\sigma}}{16}\frac{\hbar}{mc}\sum_{g}\Phi(g)(g_2^2-g_3^2)F_1''(z_g),$$

$$\varepsilon_{12}''=\varepsilon_{21}''=-\frac{BN\bar{\sigma}}{16}\frac{\hbar}{mc}\sum_{g}\Phi(g)(2g_2g_3)F_1''(z_g), \quad (25)$$

$$S''=\varepsilon_A+\frac{BN\bar{\sigma}}{2}\frac{\hbar}{mc}\sum_{g}\Phi(g)(g_2^2+g_3^2)F_2''(z_g).$$

The summation over g in Eq. (25) satisfies the condition $0<z_g\leqslant 1$:

$$F_1''(z)=z^4\left[\ln\frac{1+(1-z)^{1/2}}{1-(1-z)^{1/2}}+\frac{2(1-z)^{1/2}}{z}\right],$$

$$F_2''(z)=z^2\left[\left(1+z-\frac{z^2}{2}\right)\ln\frac{1+(1-z)^{1/2}}{1-(1-z)^{1/2}}-(1-z)^{1/2}(1+z)\right], \quad (26)$$

$\lambda_c=\hbar/mc$ is the Compton wavelength of an electron,

$$B=\frac{16\pi^2}{N_S\Delta}, \quad \bar{\sigma}=\alpha Z^2r_e^2,$$

α is the fine-structure constant, and r_c is the classical electron radius. The term ε_A in the formula (25) takes into account the absorption of γ-rays on thermal vibrations of the lattice and is equal to

$$\varepsilon_A=\frac{\bar{\sigma}Nc\hbar}{E_T}\left(\frac{2}{3}\psi_1^{am}+\frac{1}{9}\psi_2^{am}\right), \quad (27)$$

where we have ψ_1^{am} and $\psi_2^{am}\approx$const and these quantities are determined in the theory.[13-15] Since $\varepsilon_A E_\gamma\approx$const, the corresponding terms in ε_{11}' and ε_{22}' are negligibly small. Here we employed the system of units in which the reciprocal lattice constant is measured in units of λ_c^{-1} and the direct lattice constant is measured in units of λ_c; this system is adopted in the theory of coherent radiation[13-15] and is convenient for performing specific calculations numerically. The relations (22), (25), (29), (31), (37), and (38) are written in this system of units.

It is obvious from the relations (22) and (25) that the components of the tensor ε_{kl} are quadratic functions of the electric field of the single crystal. This is a manifestation of the nonlinearity of Maxwell's equations for the system consisting of the field of the single crystal in which an electromagnetic wave propagates. In other words, the resulting field of such a system is not a superposition of fields of the single crystal and the wave. Although this nonlinearity is weak, it does result in easily observable physical processes, some of which are examined below.

In a number of problems in crystal optics it is more convenient to employ the tensor η_{kl}, which is the inverse of the tensor ε_{kl}. When $|\varepsilon_{kl}-\delta_{kl}|\ll 1$, these tensors are related by

$$\eta_{kl}+\varepsilon_{kl}=2\delta_{kl}. \quad (28)$$

One can see from the expressions (22)–(26) that the components of the tensor ε_{kl} are functions of the two universal parameters W_H and W_V (if the term ε_A is neglected); this makes it easier to study processes occurring with different energies and orientations. The dependence on the parameters W_H and W_V is essentially a manifestation of spatial dispersion, i.e., the dependence of the components of the tensor ε_{kl} on the wave vector of the γ-ray. Thus in classical crystal optics the tensor ε_{kl} for a monochromatic wave is often determined by only several numbers.[17]

For orientation angles close to $\varphi_B=V_0/mc^2$ (but larger than it), the components of the tensor can be easily modified by introducing an effective mass, as done in Ref. 2 for the cross sections for pair production in single crystals.

As we have already indicated, two types of orientations are important: axial and planar. Axial orientation occurs when we have $W_H\lesssim 1$ and $W_V\lesssim 1$; planar orientation is realized when we have $(W_H^2+W_V^2)^{1/2}\gg 1$, but $W_V/W_H\approx G_2n_2/G_3n_3$. In the planar case only terms with $n_2/n_3=$ const need be included in the expressions (22) and (25), since the contribution of the remaining terms is negligibly small.

For high γ-ray energies ($z\to 0$) and fixed angles θ and α the anisotropy of the properties of the single crystals is low

and the tensor ε_{kl} reduces to the following value of the real permittivity:

$$\varepsilon' = \frac{S'}{2} = 1 - \frac{16\pi N e^6 Z^2}{N_s \Delta m c^2 E_\gamma^2} \sum_g \frac{\Phi(g) g^2 \ln^2(2mc^2/E_\gamma g_\parallel) \vartheta(g_\parallel)}{g_\parallel^2}, \tag{29}$$

where ϑ is the Heaviside unit step function. It is helpful to compare this formula with the well-known relation for the permittivity at high energies[16,17]

$$\varepsilon'(\omega) = 1 - \frac{4\pi N_e e^2}{m\omega^2}, \tag{30}$$

where N_e is the number of electrons per unit volume. The formula (29) takes the form (30), if it is assumed that the number of virtual electron-positron pairs with effective mass $mg_\parallel$ per unit volume is equal to

$$N_p(g) = N N_\gamma \frac{e^2 \ln^2(2mc^2/E_\gamma g_\parallel)}{\pi \hbar c}, \tag{31}$$

where $N_\gamma(g)$ is the number of pseudophotons, determined by the formula (6), and the relation (29) can be written as

$$\varepsilon'(\omega) = 1 - \frac{4\pi e^2}{\omega^2} \sum_g \frac{N_p(g)}{m g_\parallel} \vartheta(g_\parallel). \tag{29a}$$

This result shows that an electromagnetic wave interacts with one component of the pair, since the pairs occupy a spatial region of characteristic size $\sim\lambda_c$, while the wavelength of a γ-ray is much smaller [hence the dependence $\varepsilon' \sim \omega^{-2}$ (Ref. 16)]. Under these conditions, however, a virtual pair is a quite rigid system (at sufficiently high frequencies the pair production cross section decreases), so that it interacts with the wave as a particle with effective mass mg and charge e.

In the general case the symmetric complex tensor ε_{kl} does not reduce to principal axes (i.e., there does not exist a coordinate system in which the tensors ε'_{kl} and ε''_{kl} are simultaneously diagonal). Thus the principal axes of the tensors ε'_{kl} and ε''_{kl} are not parallel and make with one another some angle β which depends on the parameters W_H and W_V. In the symmetry planes of the single crystal (in the coordinates W_H and W_V), however, we have $\beta = 0$ or 90° and the angle β approaches the same values away from the axis W_H, $W_V \gg 1$. Thus the tensor ε_{kl} cannot be reduced to principal axes only when the γ-rays make a small angle with the crystallographic axis also. The physical nature of this phenomenon is explained by the nonaxial nature of the electric field of the single crystal. Mathematically this results in the fact that for fixed angles of orientation, but different γ-ray energies the position of the principal axes of the tensor ε''_{kl} is different; but, since the relation (21) is satisfied, the principal axes of the tensors ε'_{kl} and ε''_{kl} become nonparallel.

Everything said above is also true for the tensor η_{kl}. Thus in the system of the principal axes of the tensor η'_{kl} the tensor η_{kl} has the form

$$\begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix} + i \begin{pmatrix} a & M \\ M & b \end{pmatrix}. \tag{32}$$

The manner in which the components of the tensor η_{kl} transform when the coordinate system is rotted around the wave vector of the γ-ray by an angle φ [relative to the system in which the tensor η_{kl} has the form (32)] is obvious.

Figure 1 illustrates the behavior of the quantities $S'/2 - 1$ (top) and $S''/2$ (bottom) in the (W_H, W_V) plane for a silicon single crystal near the ⟨001⟩ axis. These quantities represent the permittivity of the single crystal neglecting the anisotropy of the properties of the crystal. The picture is symmetric with respect to the diagonal (the (100) plane), i.e.,

$$S'(W_H, W_V) = S'(W_V, W_H), \quad S''(W_H, W_V) = S''(W_V, W_H)$$

(along the diagonal $\beta = 0$ or 90°). The figure was construct-

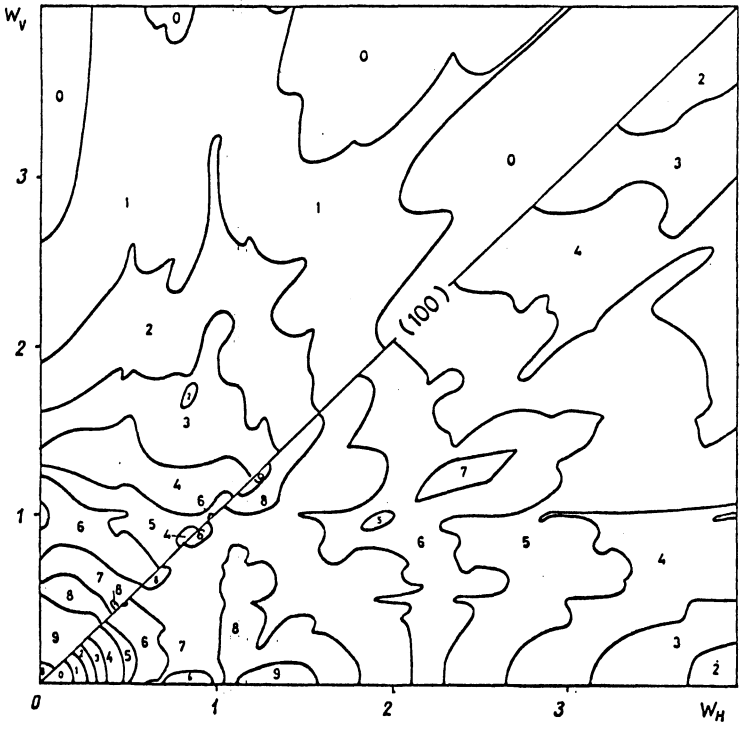

FIG. 1. Permittivity $\varepsilon' = 1 + S'/2$ and $\varepsilon'' = S''/2$ of a silicon single crystal near the ⟨001⟩ axis in the coordinates W_H and W_V [for W_H and $W_V = 0$ the γ-ray propagates along the ⟨001⟩ axis and for $W_H = 0$ the γ-ray propagates in the (110) plane]. Further explanations are given in the text.

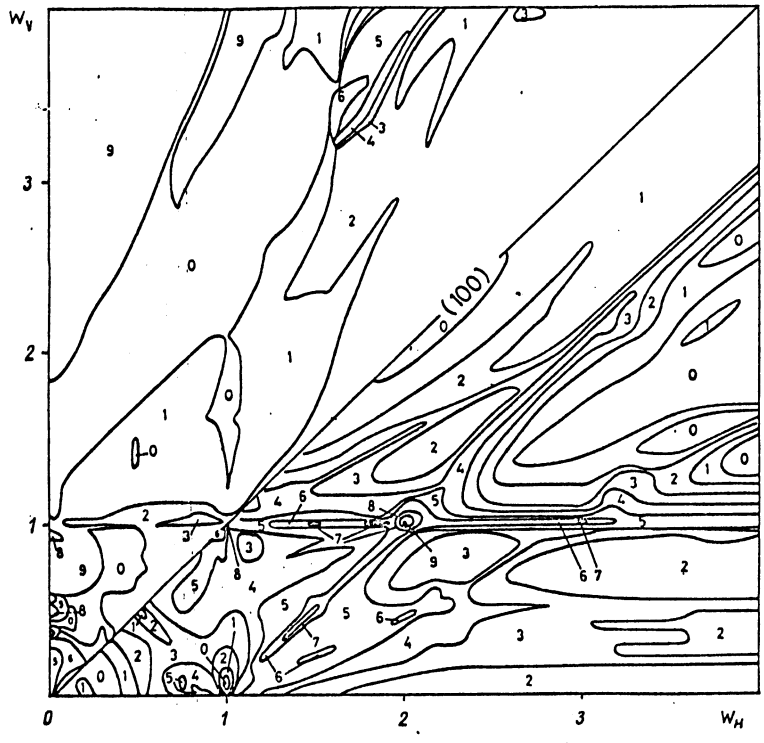

FIG. 2. Values of $|\varepsilon'_{11} - \varepsilon'_{22}|$ along the principal axes of the tensor ε'_{kl} (bottom) and the angle φ' between these axes and the $(\mathbf{G}_1,\mathbf{G}_2)$ plane (top). The choice of crystallographic axes corresponds to Fig. 1. Further explanations are given in the text.

ed according to the following principle: The maximum value of the quantity $S'/2-1$ or $S''/2$, equal to I, is divided into ten equal parts; the zero level corresponds to the values of $S'/2-1$ or $S''/2$ corresponds to the values of $S'/2-1$ or $S''/2$ from 0 to $0.1I$, and so on, $I_{S'/2-1} = 1.08\cdot 10^{-16}$, $I_{S''/2} = 0.67\cdot 10^{-16}$. In the calculations the Moliére form factor was employed.[5]

Figure 2 illustrates the values of $\Delta\varepsilon' = |\varepsilon'_{11} - \varepsilon'_{22}|$ in the principal axes of the tensor ε'_{kl} near the $\langle 001\rangle$ axis of silicon. The quantity $\Delta\varepsilon'$ is shown at the bottom of the figure and the angle φ', which the principal axes of the tensor ε'_{kl} make with the (G_1,G_2) plane, is shown at the top. The principal of construction is the same as in the case of Fig. 1. $I_{\varphi'} = 180°$, $I_{\Delta\varepsilon'} = 3.3\cdot 10^{-17}$. Similarly Fig. 3 illustrates the values of $\Delta\varepsilon'' = |\varepsilon''_{11} - \varepsilon''_{22}|$ in the principal axes of the tensor ε''_{kl} near the $\langle 001\rangle$ axis of silicon. The angle φ'', which the principal axes of the tensor ε''_{kl} make with the $(\mathbf{G}_1,\mathbf{G}_2)$ plane ($I_{\varphi'} = 180°$), is shown at the top and $\Delta\varepsilon''$

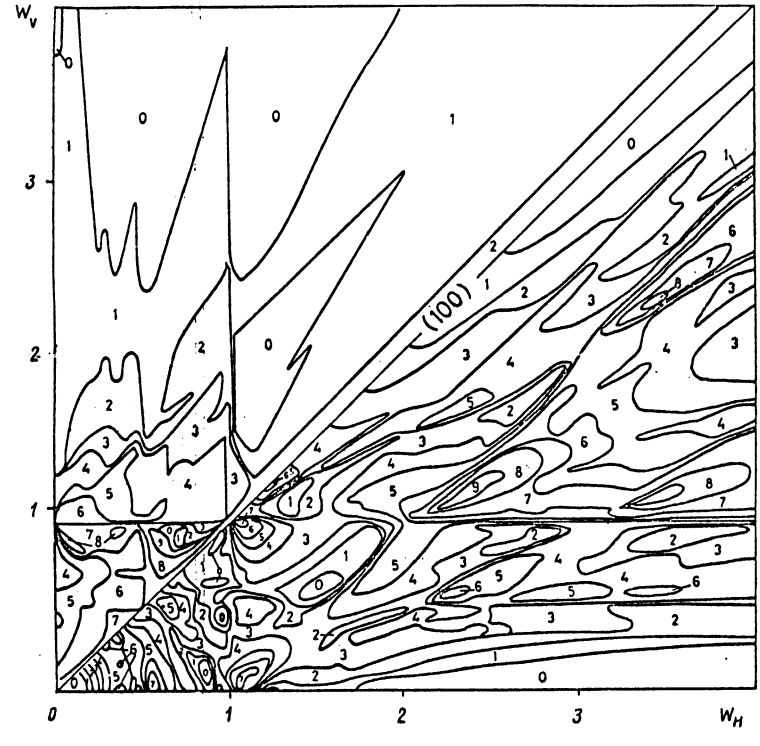

FIG. 3. Values of $|\varepsilon''_{11} - \varepsilon''_{22}|$ in the principal axes of the tensor ε''_{kl} (bottom) and the angle φ'' between these axes and the $(\mathbf{G}_1,\mathbf{G}_2)$ plane (top). The choice of the crystallographic axes corresponds to Fig. 1. Further explanations are given in the text.

($I_{\Delta\varepsilon''} = 1.94\cdot 10^{-17}$) is shown at the bottom. In Figs. 1 and 3 the term ε_A is neglected ($\varepsilon_A = 1.32\cdot 10^{-15}/E_\gamma$ GeV for silicon). We also note that $\beta = \varphi' - \varphi''$. Thus it is easy to see that the data in Figs. 1–3 completely determine the tensor ε_{kl} near the ⟨001⟩ axis of silicon.

PROPAGATION OF γ-RAYS IN SINGLE CRYSTALS

The main problem of crystal optics is to investigate the propagation of monochromatic plane waves, characterized by definite values of the frequency ω and wave vector $\mathbf{k}$, in single crystals. Such waves, satisfying a homogeneous wave equation, are called normal electromagnetic waves,[17] and they have the form

$$\mathbf{E}=\mathbf{E}_0 \exp -i(\mathbf{kr}-\omega t), \quad k=\frac{\omega}{c}\tilde{n}\mathbf{S},$$

where $\tilde{n}$ is the complex index of refraction and $\mathbf{S} = \mathbf{k}/|\mathbf{k}|$ is a real unit vector. The vectors $\mathbf{D}$ and $\mathbf{B}$ have the same form.

From Maxwell's equations (14) we obtain the wave equation[17]

$$\operatorname{rot}\operatorname{rot}\mathbf{E}+\frac{1}{c^2}\frac{\partial^2\mathbf{D}}{\partial t^2}=0. \tag{33}$$

Taking into account the relation between $\mathbf{D}$ and $\mathbf{E}$ in a system of coordinates in which the axis x_1 is oriented parallel to the wave vector, we obtain

$$\begin{aligned}\eta_{11}\frac{\partial^2 D_1}{\partial x_1^2}+\eta_{12}\frac{\partial^2 D_2}{\partial x_1^2}-\frac{1}{c^2}\frac{\partial^2 D_1}{\partial t^2}=0,\\ \eta_{21}\frac{\partial^2 D_1}{\partial x_1^2}+\eta_{22}\frac{\partial^2 D_2}{\partial x_1^2}-\frac{1}{c^2}\frac{\partial^2 D_2}{\partial t^2}=0.\end{aligned} \tag{34}$$

For a monochromatic plane wave it follows from these equations that

$$(\tilde{n}^{-2}\delta_{kl}-\eta_{kl})D_l=0. \tag{35}$$

From the condition that the two homogeneous equations be compatible we find the index of refraction of the γ-rays:

$$\tilde{n}^{-2}=S/2\pm(S^2/4-D_M)^{1/2}, \tag{36}$$

where S and D_M are, respectively, the trace and determinant of the matrix η_{kl}. Thus two waves with different indices of refraction $\tilde{n}_1$ and $\tilde{n}_2$, in particular, with different absorption cross sections, can propagate in the single crystal.

We first study the case when the γ-rays propagate near the surface. For these orientations the tensor η_{kl} can be reduced to principal axes and the solutions of the dispersion equation will be two linearly polarized waves (parallel and perpendicular to the field of the plane). The corresponding values of these indices of refraction $\tilde{n}_\parallel$ and $\tilde{n}_\perp$ are determined by the relations

$$\begin{aligned}\operatorname{Re}(\tilde{n}_\parallel-\tilde{n}_\perp)&=-\frac{BN\bar{\sigma}}{8\pi}\frac{\hbar}{mc}\sum_{n=1}^{\infty}\Phi(Gn)G^2n^2z_n^2F_1'(z_n),\\ \operatorname{Re}(\tilde{n}_\parallel+\tilde{n}_\perp)&=2+\frac{BN\bar{\sigma}}{2\pi}\frac{\hbar}{mc}\sum_{n=1}^{\infty}\Phi(Gn)G^2n^2z_n^2F_2'(z_n),\\ \operatorname{Im}(\tilde{n}_\parallel-\tilde{n}_\perp)&=\frac{BN\bar{\sigma}}{16}\frac{\hbar}{mc}\sum_{n=1}^{\infty}\Phi(Gn)G^2n^2F_1''(z_n)\vartheta(1-z_n),\end{aligned} \tag{37}$$

$$\operatorname{Im}(\tilde{n}_\parallel+\tilde{n}_\perp)=\varepsilon_A+\frac{BN\bar{\sigma}}{4}\frac{\hbar}{mc}\sum_{n=1}^{\infty}\Phi(Gn)G^2n^2F_2''(z_n)\vartheta(1-z_n),$$

where

$$z_n=\frac{2mc^2}{(E_\gamma Gn\theta_p)},$$

and θ_p is the angle of entry of a γ-ray relative to the plane. The asymptotic behavior of the refractive indices in the limit $z\to\infty$ (low energies or small angle of orientation θ_p) is of interest:

$$\operatorname{Re}(\tilde{n}_\parallel)=1+\frac{7}{45\pi}BN\bar{\sigma}\frac{\hbar}{mc}\sum_{n=1}^{\infty}\Phi(Gn)(Gn)^2=1+\frac{7\alpha}{90\pi}\frac{\langle\mathscr{E}\rangle^2}{\mathscr{E}_0^2}, \tag{38}$$

$$\operatorname{Re}(n_\perp)=1+\frac{4}{45\pi}BN\bar{\sigma}\frac{\hbar}{mc}\sum_{n=1}^{\infty}\Phi(Gn)(Gn)^2=1+\frac{4\alpha}{90\pi}\frac{\langle\mathscr{E}\rangle^2}{\mathscr{E}_0^2},$$

where $\langle\mathscr{E}\rangle^2$ is the mean-squared magnitude of the field of the plane in the single crystal (see Appendix).

These formulas are similar to the analogous relations in a uniform constant electromagnetic field.[4,5] The values of $\tilde{n}_\parallel$ and $\tilde{n}_\perp$ remain valid also for small orientation angles $\theta_p < \varphi_B$, so long as $\chi \lesssim 1$.[11] Figure 4 illustrates the behavior of the refractive indices for the (110) plane in silicon as a function of the parameter $W_H \neq E_\gamma G\theta_p/(2mc^2)$.

We now study the general case of the propagation of γ-rays in single crystals. From Eq. (35) we find the ratios of the components of the vector $\mathbf{D}$

$$\frac{D_1}{D_2}=\varkappa=\frac{\tilde{n}^{-2}-\eta_{11}}{\eta_{12}}=\frac{\eta_{12}}{\tilde{n}^{-2}-\eta_{22}}=\frac{|D_1|}{|D_2|}e^{i\delta}, \tag{39}$$

where δ is the phase shift between D_1 and D_2. By adjusting the angle φ the ratio $\varkappa$ can be reduced to zero or to the form $\varkappa = i\rho$ (since $|D_1||D_2|\sin\delta = b_1b_2$, where b_1 and b_2 are the semiaxes of the ellipse and $|\rho| = b_1/b_2$).[18] The first case corresponds to the propagation of a linearly polarized wave and the second case corresponds to an elliptically polarized wave; in addition, $\rho > 0$ ($\rho < 0$) corresponds to left(right)-hand polarization of the γ-ray.

The character of the polarization of the γ-rays is determined by the form of the tensor η_{kl}. If we have $M = 0$ or $A - B = 0$, then two linearly polarized waves will correspondingly propagate in the single crystal along the principal axes of the tensors η'_{kl} and η''_{kl}. It is the case $M = 0$ that is studied in the theory of coherent pair production.[6,7,15]

If $(A - B)\cdot M \neq 0$, then two elliptically polarized waves will propagate in the single crystal, and in addition their circular P_{circ} and linear P_l polarizations are found from the relations

$$\varkappa+\varkappa^*=0, \tag{40}$$

$$\frac{\varkappa-\varkappa^*}{2}=i\rho, \tag{41}$$

$$P_{\text{circ}}=\frac{2\rho}{1+\rho^2}, \tag{42}$$

$$P_l=(1-P_{\text{circ}}^2)^{1/2}. \tag{43}$$

From the relation (40) we determine the angle through which the polarization ellipse turns, and we substitute this

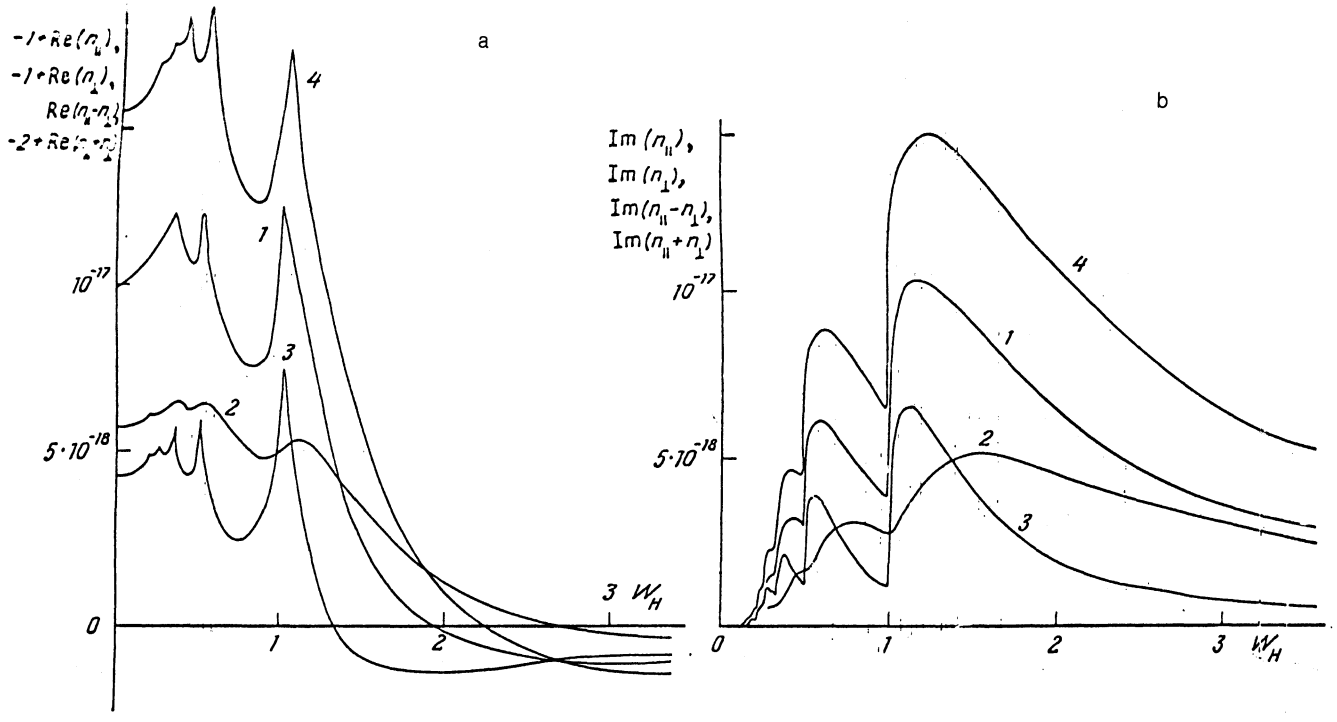

FIG. 4. Real (a) and imaginary (b) parts of the refractive indices for the (110) plane of silicon as a function of the parameter $W_H = GE_\gamma\theta_p/(2mc^2)$. The numbers on the curves correspond to the quantities shown on the left-hand side of the figure in the order top to bottom.

angle into Eq. (41) in order to determine ρ. From Eqs. (36) and (39) we obtain

$$\varkappa_{(1)}\varkappa_{(2)}=-\rho_{(1)}\rho_{(2)}=-1, \qquad \mathbf{D}_{(1)}\mathbf{D}_{(2)}=0, \tag{44}$$

where the indices in parentheses refer to waves with refractive indices $\tilde{n}_1$ and $\tilde{n}_2$. Hence it follows that the polarization ellipses of two waves have the same ratio of the axes, but the ellipses are turned relative to one another by 90°, and in addition the direction of rotation is the same in them. Some computational results for a silicon single crystal and the polarization characteristics of γ-rays propagating in it near the ⟨001⟩ axis are illustrated in Fig. 5.

The case when in the matrix (32) $a = b$ is especially simple. Then

$$|P_{circ}|=2\left|\frac{M}{B-A}\right|, \quad 2|M|\leqslant|B-A|, \qquad |P_{circ}|=\frac{1}{2}\left|\frac{B-A}{M}\right|, \quad 2|M|\geqslant|B-A|. \tag{45}$$

Although the two waves have the same absorption cross sections, the real parts of the refractive indices $\tilde{n}_1$ and $\tilde{n}_2$ are different ($\mathrm{Re}\tilde{n}_1 = \mathrm{Re}\tilde{n}_2$ only if $2|M| = |B - A|$).

The case studied here, of elliptical polarization of normal electromagnetic waves near the so-called singular axis, described in the literature,[16,17] where, strictly speaking, the condition $B - A = 2M \neq 0$ holds for the components of the tensor (32), i.e., γ-rays are certainly absorbed in the medium, is interesting in that $\tilde{n}_1 = \tilde{n}_2$, as a result of which there appear nontrivial solutions for the normal waves.[17] In addition, conditions at the vacuum-medium boundary must play a definite role in the formation of normal elliptical waves. Indeed, normal elliptical waves are solutions of the dispersion equation in an infinite medium. In order to resolve the question of the propagation of waves near a boundary (and therefore describe the establishment of polarization) it is necessary to study the system of equations (34) with bound-

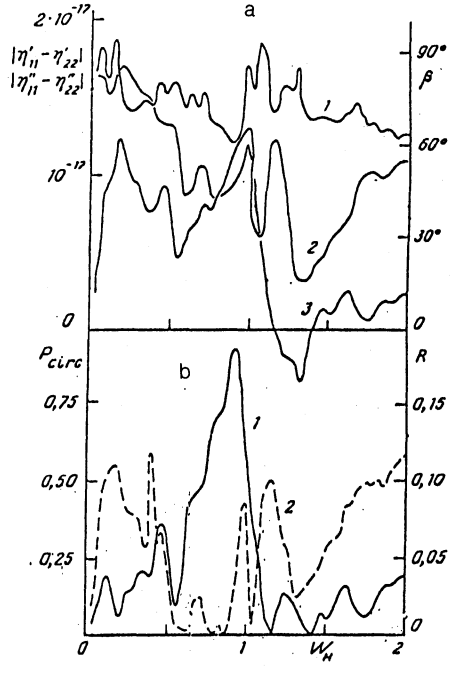

FIG. 5. Computed properties of a silicon single crystal and γ-rays propagating in it as a function of the orientation parameter W_H for $W_V = 1.16$ (the choice of crystallographic axes corresponds to Fig. 1). a) Values of $|\eta'_{11} - \eta'_{22}|$ and $|\eta''_{11} - \eta''_{22}|$ in the principal axes of the tensors η'_{kl} and η''_{kl} (curves 1 and 2), respectively; β is the angle between the principal axes of these tensors (curve 3); b) absolute magnitudes of the circular polarization P_{circ} of γ-rays (solid line) and the asymmetry parameter $R = \mathrm{Im}(n_2 - n_1)/\mathrm{Im}(n_1 + n_2)$. Here R does not include the absorption on thermal vibrations of the lattice.

ary conditions for **E** and **D**. However, this problem is made more complicated by the fact that near a boundary the expressions for the tensor ε_{kl} or η_{kl} are not valid, since the γ-ray absorption cross sections employed in the calculations are valid in a medium only at thicknesses greater than the pair production length l_γ. It can only be asserted that elliptical polarization of normal waves results from the interaction of γ-rays with the field of the single crystal at a distance $> l_\gamma$.

We note that for these orientations $\tilde{n}_\parallel$ and $\tilde{n}_\perp$ are different and the general case of propagation of γ-rays in single crystals was studied.

Although the present work concerns the interaction of γ-rays with single crystals, the relations obtained for the tensor ε_{kl} and the refractive indices will also be valid for the interaction of a γ-ray with the field of a linearly polarized wave, for example, when an intense laser beam encounters a beam of γ-rays (possibilities for performing such experiments on electron accelerators are already being studied). According to the formula (16), in order to describe such a process the number NN_γ of pseudophotons with energy $\hbar c\gamma g_\parallel$ per unit volume must be replaced in the relations obtained by the average number of photons per unit volume of the electromagnetic wave.

COHERENT SCATTERING OF A γ-RAY IN THE FIELD OF THE SINGLE CRYSTAL

As is well known, the optical theorem[4] relates the amplitude of elastic scattering of a γ-ray through zero angle with the total cross section for the production of an electron-positron pair. Thus the cross section for scattering $d\sigma/d\Omega$ through zero angle in the field of the single crystal can be expressed in terms of the components of the tensor ε_{kl}. For planar orientation of the single crystal we obtain

$$\frac{d\sigma}{d\Omega}(\theta_1=0)=\frac{E_1^4[(\varepsilon'-1)^2+\varepsilon''^2]}{16\pi^2c^4\hbar^4N^2}, \tag{46}$$

where ε' and ε'' are the values of ε_{11} and ε_{22} (along the principal axes of the tensor ε_{kl}), depending on the polarization of the γ-ray (parallel or perpendicular to the plane), and $d\Omega$ is an element of solid angle. It is obvious that the dependence on the polarization of the γ-rays is also manifested in elastic scattering of γ-rays in single crystals. In addition, it is evident that the expression (46) contains interference between the scattering of γ-rays in the field of the single crystal (11) and the scattering by separate atoms. The cross section for the scattering of normal elliptic waves by zero angle can also be obtained similarly.

ROTATION OF THE POLARIZATION PLANE OF γ-RAYS IN SINGLE CRYSTALS

If in the matrix (32) we have $A = B$, then after a linearly polarized γ-ray traverses some distance in the single crystal the polarization plane of the γ-ray will, generally speaking, change.[12] The mechanism of this phenomenon is quite simple and is explained by the different rate of absorption of normal waves. Indeed, a completely polarized wave can be decomposed into two normal waves in orthogonal directions with different absorption cross sections $\sigma_\parallel$ and $\sigma_\perp$. Hence we find the angle of rotation Ω of the polarization plane

$$\frac{\operatorname{tg}\Omega(L)}{\operatorname{tg}\Omega(0)}=\exp\frac{(\sigma_\parallel-\sigma_\perp)L}{2}, \tag{47}$$

where $\Omega(0)$ is the initial polarization angle ($\Omega = 0$ along the plane with the cross section $\sigma_\parallel$). As one can see from Fig. 4, the condition $A = B$ is even satisfied in the planar case, though for one angle of entry of the γ-ray relative to the plane. Specific calculations show that near the axis there exist quite large regions of orientation when $A \approx B$ and where this effect should occur [for example, for the $\langle 001\rangle$ axis in silicon the center of such a region lies at $W_H \approx 2$ and $W_V \approx 4$ (see Fig. 2)].

CHERENKOV RADIATION IN SINGLE CRYSTALS

As is well known, a charged particle can emit Cherenkov radiation only in a medium whose permittivity $\varepsilon' > 1$. Calculations of the permittivity for high energy γ-rays show that $\varepsilon' > 1$ in single crystals near axes and planes, but because the difference $\varepsilon' - 1$ is small Cherenkov emission is possible only for very high-energy particles [we have $\gamma = (1 - v^2/c^2)^{-1/2} > 10^8$ near the $\langle 001\rangle$ axis in silicon]. In the subsequent calculations we neglect the anisotropy of the single crystal (it is small for the axial case) and we characterize its properties by the single quantity $\varepsilon' = 1 + S'/2$. For ultrarelativistic particles ($\gamma \gg 1$) and $\varepsilon' - 1 \ll 1$ the Frank-Tamm formula can be written in the form

$$\frac{d^2n_1}{dE_1dL}=\frac{e^2}{\hbar^2c^2}(\varepsilon'-1-\gamma^{-2})\theta(\varepsilon'-1-\gamma^{-2}), \tag{48}$$

where n_γ is the number of γ-rays with energy E_γ emitted over the thickness of the single crystal L. The results of calculations performed using this formula are presented in Fig. 6. The choice of angle θ is based on the consideration that it must be greater than the angle $\varphi_B = V_0/mc^2 \approx 0.15$ mrad (silicon, $\langle 001\rangle$ axis), and the choice of α permits extending somewhat the high-energy part of the spectrum, though the dependence on α is weak at the beginning and in the middle of the spectrum. Increasing the angle θ results in a shift of the spectrum toward low energies. The quantity dn_γ/dL for γ-rays with energy 0–1000 GeV is equal to 0.02 photons/cm and 0.01 photon/cm, respectively, for $\gamma = \infty$ and $\gamma = 2\cdot 10^8$. We note that Cherenkov emission will also exist for $\theta < \varphi_B$. Indeed, as has already been indicated, the asymptotic behavior of ε' in the limit $\theta \to 0$ is identical to that of ε' for small values of E_γ. Thus a high-energy particle will radiate for $\theta < \varphi_B$ as long as $\chi(E_\gamma) \lesssim 1$ holds. The use of single crystals

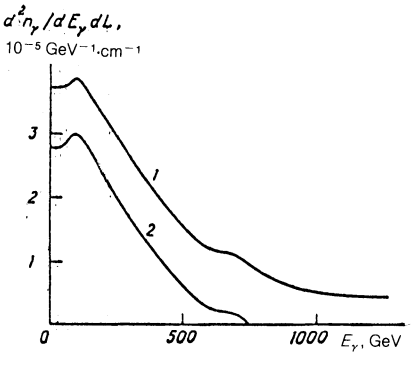

FIG. 6. Energy spectrum of Cherenkov γ-rays (for a 1 cm path) from charged particles with the relativistic factor $\gamma = (1 - v^2/c^2)^{-1/2} = \infty$ (1) and $2.2\cdot 10^8$ (2) and moving near the $\langle 001\rangle$ axis of silicon. The angles of orientation of the single crystal are $\theta = 0.25$ mrad and $\alpha = \tan^{-1} 1/4$.

consisting of materials with a high atomic number makes it possible to reduce somewhat the threshold energy of charged particles.

In the present paper we have derived the permittivity tensor for γ-rays $\gtrsim 1$ GeV on the basis of a representation of a single crystal as a region of electric field of atoms arranged in an ordered fashion in space. With the help of this tensor we found and studied the polarization characteristics of normal electromagnetic waves propagating in single crystals, and in particular we showed that there exist solutions of the dispersion equation for elliptically polarized waves. We also studied some other electrodynamic processes occurring in single crystals.

The existence of refractive indices of γ-rays different from unity in single crystals was associated to the nonlinearity of Maxwell's equations, and it is certainly of interest to observe such refractive indices experimentally. The existence of electron-photon beams with energies of hundreds of GeV on modern proton accelerators provides the necessary prerequisites for such experiments. Calculations show that it is quite possible to measure the difference of the real parts of the refractive indices[7,9,11,12] in such beams. Assessments of the possibility of measuring the absolute values of these indices are more pessimistic, but nonetheless Cherenkov emission from ultrahigh energy charged particles makes it possible, in principle, to determine these quantities experimentally.

From the standpoint of practical applications, the results obtained in this work can be used for generating high-energy polarized electron-photon beams and for determining their polarization characteristics.

APPENDIX

Some useful relations for the interplanar fields in a single crystal

$$\varphi(x)=\frac{8\pi eZ}{\Delta}\sum_{n=1}^{\infty}(-1)^n U(Gn)\cos(nGx),$$

$$\mathscr{E}(x)=\frac{8\pi eZ}{\Delta}\sum_{n=1}^{\infty}(-1)^n U(Gn)\,Gn\sin(nGx),$$

$$V_0=\varphi\left(\frac{d}{2}\right)-\varphi(0)=\frac{16\pi eZ}{\Delta}\sum_{n=1}^{\infty}U(2Gn),$$

$$\langle\varphi\rangle^2=\frac{1}{d}\int_{-d/2}^{d/2}\varphi^2(x)\,dx=\frac{32\pi^2e^2Z^2}{\Delta^2}\sum_{n=1}^{\infty}\Phi(Gn),$$

$$\langle\mathscr{E}\rangle^2=\frac{1}{d}\int_{-d/2}^{d/2}\mathscr{E}^2(x)\,dx=\frac{32\pi^2e^2Z^2}{\Delta^2}\sum_{n=1}^{\infty}\Phi(Gn)\,(Gn)^2,$$

where φ and $\mathscr{E}$ are the average potential and average electric field, respectively, of a plane of a single crystal, $G=2\pi/d$, d is the interplanar distance, and the value of the transverse coordinate $x=0$ corresponds to the midpoint between the planes. The rest of the notation is explained in the text. In order to compare the formula (3) with $\langle\mathscr{E}\rangle^2$ it is necessary to take into account the fact that $\Delta=N_S/N$. It is especially desirable to write the first three formulas in this form when $S(\mathbf{g})$ is a real number.

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Translated by M. E. Alferieff