Captured-ion acceleration in the front of a magnetosonic shock wave with isomagnetic discontinuity

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A complete acceleration cycle of ions trapped in an isomagnetic discontinuity of a steady-state magnetosonic shock wave is investigated in detail. The conditions under which the ions are captured are determined. The total number of trapped particles, the number of those passing through and reflected once from the discontinuity, and the number of particles accelerated in a specified energy interval are determined. The energy drawn by the accelerated ions from the wave is estimated. All the quantities of interest are determined as functions of the electric field in the discontinuity, of the amplitude of the potential, of the temperature, and of the masses of the ions incident on the discontinuity. The potential and magnetic-field perturbations produced by the accelerated ions contained in the front are estimated.

1. INTRODUCTION

The resonance mechanism of particle acceleration is being quite diligently investigated of late, although its idea was advanced long ago.[1] Its gist is that a positive-potential wave moving across a magnetic field in collisionless plasma traps some of the plasma ions and accelerates them effectively along its front. A typical example of a structure with a potential discontinuity that moves across a magnetic field is a perpendicular magnetosonic shock wave (MSSW). Such shock waves have been investigated in sufficient detail in a laboratory plasma as well as in outer space (see the reviews, Refs. 2 and 3). Interest in MSSW has increased because they are among the main sources of high-energy particles and of heating in both laboratory and outer-space plasma. In solar-terrestrial physics, in particular, effects due to MSSW are most frequently discussed in connection with plasma processes in the sun's chromosphere, corona, and coronal loops, in solar wind, in the earth's magnetosphere, etc. One of the first attempts at a sufficiently profound investigation of using this mechanism to accelerate protons in MSSW excited during the evolution of solar flares was undertaken in Ref. 4. These ideas were further developed in other studies.[5–10] Resonant acceleration of ions was recently observed for the first time in laboratory experiments both in MSSW propagation and in a neutral current layer.[11–14]

It is common knowledge that the maximum velocity of ions resonantly propagating in an MSSW front is given by $w_m = c\,E/B$, where E is the maximum value of the electric field in the front, B is the magnetic field at the point where the electric field is a maximum, and c is the speed of light. E is usually estimated from the equation $E = U_m/d$, where U_m is the amplitude of the potential and d is the spatial dimension of the potential discontinuity, so that $w_m = cU_m/dB$. According to the theory[1,4,5] U_m does not differ greatly from K_0/e, where K_0 is the energy of the ion incident on the discontinuity at the shock-wave velocity and e is the ion charge. In experiments, as a rule, the amplitude of the potential is smaller than K_0/e.[2,3,15,16]

The width d of the potential discontinuity in a shock wave depends strongly both on the Alfven Mach number M and on the angle θ between the electromagnetic-field vector and the normal to the front. The dependence of d on the angle θ was investigated in detail in Ref. 7, where it was shown that $d \approx c/\omega_{pe}$ in the angle interval $\theta_c < \theta < \pi/2$, where $\theta_c = \operatorname{arctg}[m/m_e)^{1/2}]$; next, when θ decreases from θ_c to zero the width d of the front increases and reaches at $\theta = \pi/4$ the value $d \simeq c/\omega_{pi}$ (here ω_{pe} and ω_{pi} are respectively the plasma electron and plasma ion frequencies, while m and m_e are the masses of the ion and electron).

The (theoretically) minimum possible value of d, which determines the maximum E and is used as a rule by many workers for estimates, is the characteristic width of the magnetosonic soliton $(M < 2, \theta = \pi/2)$: $d = c/\omega_{pe}$ In Refs. 1 and 4–10 there were obtained, just for the minima of $d = c/\omega_{pe}$ and $U_m = K_0/e$, estimates of the maximum possible energies of the accelerated ions for a perpendicular MSSW.

As shown in Ref. 7, the maximum energy of the resonantly accelerated ions for a quasiperpendicular MSSW is m/m_e times larger than for a quasiparallel MSSW. We therefore confine ourselves here specially to the case of a strictly perpendicular MSSW, since it is the most important one for resonance acceleration of ions.

Laboratory investigations as well as satellite measurements of a near-terrestrial shock wave, show[2,3] that for a perpendicular MSSW with $M < 3$ the magnetic field in the front increases monotonically, and the growth scales range from 10 c/ω_{pe} to c/ω_{pi}, i.e., the real scale d is much larger (and the maximum energy is much lower) than for solitons. In the case $M > 3$ the shape of the front becomes more complicated: a "pedestal" of scale $\sim c/\omega_{pi}$ appears ahead of the region of the main growth of the magnetic field, a magnetic-field spike is observed directly behind the front, and intense oscillations develop in the front.

What is most important, and is of particular interest to us, is that a strong discontinuity of the potential is formed at $M > 3$ in the region of the growing magnetic field, and within this discontinuity the magnetic field is practically constant.[2,3,11,17,18] The maximum velocity w_m, and hence the maximum energy, calculated for parameters typical of the

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region of isomagnetic discontinuity of an MSSW potential, can be substantially larger than in a magnetosonic soliton. Moreover, we shall show below that in this case the ions can be theoretically accelerated to unlimited energy. It is therefore most important to investigate in detail the laws governing the trapping and acceleration of ions in an isomagnetic MSSW discontinuity at $M > 3$. This problem is in general extremely vital for MSSW, inasmuch as notwithstanding the numerous investigations, when it comes to resonant acceleration of ions in MSSW the dependences of the final ion energy on the plasma characteristics and on the wave parameters are not yet fully clear, and the number of particles reaching maximum velocity, the energy drawn from the wave by the accelerated particles, and many other questions are stil unanswered.

The present paper reports an analytic as well as computer-aided investigation, in the single-particle approximation, of a complete acceleration cycle of ions trapped in an isomagnetic discontinuities of a steady-state MSSW in a collision-free plasma. The conditions under which the particles are trapped are obtained. The total number of trapped particles, the number of particles passing once and reflected once from the discontinuity, and the number of particles accelerated in a specified energy interval are determined for plasma ions having a Maxwellian distribution function and impinging on an isomagnetic discontinuity. The energy drawn by the trapped ions from the MSSW is also estimated. All the parameters of interest are determined as functions of the electric field in the isomagnetic discontinuity, of the amplitude of the potential, and of the temperature of the ions incident on the discontinuity. The distribution function of the accelerated ions is determined in analytic form. The perturbations of the potential and of the magnetic field by the presence of trapped ions are estimated, thereby casting light on the influence of the accelerated particles on the macroscopic structure of the shock-wave front.

2. FORMULATION OF PROBLEM AND INITIAL EQUATIONS

For a perpendicular MSSW with an isomagnetic discontinuity, we confine ourselves to that region of its front where the potential discontinuity is localized. We assume this region to be a plane layer with x coordinates rigorously confined between $x = 0$ and $x = d$ (Fig. 1). The unperturbed plasma directly ahead of the discontinuity consists of electrons and ions with Maxwellian distributions in velocity (the unperturbed plasma is taken to be the one directly ahead of the discontinuity). A potential discontinuity in the form of a plane wave moves in the plasma strictly perpendicular to the magnetic field, with a constant velocity S and in the negative x direction. The magnetic field vector with modulus B is directed along z. In the wave front, whose width is d, the potential $P(x)$ is assumed to increase linearly from zero to a value U_m beyond which, behind the front, it remains constant (Fig. 1). (For brevity, the front is taken here and in Secs. 3–6 to mean the region of the isomagnetic discontinuity of the potential). The amplitude U_m of the potential is smaller than or equal to the energy of the main-plasma ions incident on the front (in the reference frame of the wave): $U_m < K_0/e$, where $K_0 = mS^2/2$. It is required to find the law of motion of the ions entering from the unperturbed plasma into the shock-wave potential discontinuity. The main simplification is that the electric and magnetic field are assumed to be independent of the coordinates within the confines of the potential discontinuity.

It is convenient to analyze the particle motion in the considered steady-state wave in a reference frame connected with the moving front. In the nonrelativistic approximation, a transition to the wave system will not change the magnetic-field vector components and the components $E_x = E(x) = -dP/dx$, but will lead to the appearance of a field component $E_y = SB/c$ in all of space.

We shall consider the motion of the trapped ions in the single-particle approximation, assuming that the number of these particles is small. If the ion incidentally incident on the unperturbed plasma encounters a potential barrier significantly higher than its kinetic energy, it will certainly be reflected from the front. After reflection, the ion landing in the region ahead of the front will be returned to the front by the fields B_z and E_y, will be reflected again, and thus oscillate about an equilibrium position determined in this case by the coordinate $x = 0$ until it lands for some reason behind the front. During the entire oscillation time the ion is trapped by the wave of the potential and is accelerated by the field E_y along the front of this wave. The ion trapping time and the number of times that the trapped particle crosses the yz plane are larger the larger the ratio of the wave potential U_m to the initial ion energy. This, in general, is the qualitative picture of the motion and acceleration of the particles.

We proceed now to a quantitative description. Given the electric fields and under the assumptions made, the equation of motion of a typical ion in the xy plane is

$$dv/dt = eE(x)/m + fw(t), \tag{1}$$

$$dw/dt = f[S - v(t)], \tag{2}$$

where $v = v(t) = dx/dt$, $w = w(t) = dy/dt$ are the x and y components of the ion velocity, $f = eB/mc$ is the gyrofrequency, and

$$E(x) = \begin{cases} -E, & 0 \leqslant x \leqslant d, \\ 0, & x < 0,\ x > d. \end{cases}$$

For the ion kinetic energy $K(t) = m[v^2 + w^2]/2$ we have the equation

$$dK/dt = eE(x)v(t) + eE_y w(t). \tag{3}$$

Integrating (2) and (3) once, we get

$$w(t) = f[St - x(t)] + w_0, \tag{4}$$

$$K(t) = m(v_0^2 + w_0^2)/2 + eE(x)x(t) + eE_y y(t). \tag{5}$$

Account is taken in (4) and (5) of the initial conditions

$$x(0) = y(0) = 0,\ v(0) = v_0,\ w(0) = w_0, \tag{6}$$

where v_0 is the absolute value, since only ions with $v > 0$ land in the front. The quantity w_0 can be positive or negative.

We shall track the motion of the particle until it reaches the point $x = d$ (Fig. 1). From the system (1) and (2) we obtain after the nth crossing of the yz plane by the particle (we call such a crossing a collision) solutions in the form

$$v(t) = P_n \cos(F_n(t)) + Q_n \sin(F_n(t)) + S, \tag{7}$$

$$w(t) = Q_n \cos(F_n(t)) - P_n \sin(F_n(t)) + SD, \tag{8}$$

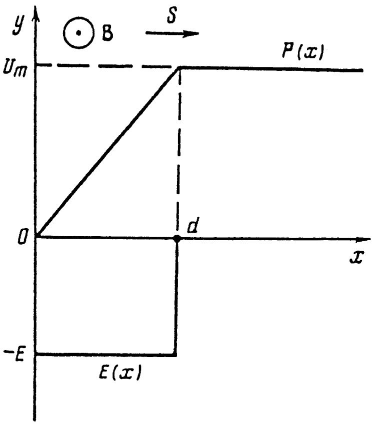

FIG. 1. Schematic pattern of field distribution in an isomagnetic discontinuity of a shock wave.

$$x(t)f=P_n\sin(F_n(t))-Q_n[\cos(F_n(t))-1]+SF_n(t). \quad (9)$$

$$y(t)f=Q_n\sin(F_n(t))+P_n[\cos(F_n(t))-1]+SDF_n(t), \quad (10)$$

where

$F_n(t)=f(t-t_n)$, $P_n=v(t_n)-S$, $Q_n=w(t_n)-SD$, $D=cE/BS$,

t_n is the instant of the nth collision: $t_n \leqslant t \leqslant t_{n+1}$,

$$SD=\begin{cases} cE/B, & 0\leqslant x\leqslant d,\\ 0, & x<0,\ x>d\end{cases}$$

and is determined from the equation

$$x(t_n)=0. \quad (11)$$

For the instant of the nth collision we obtain from (4) $w(t_n)=Sft_n+w_0$. During the time of ion motion $t_n^{\rm col}=t_{n+1}-t_n$ between two successive collisions the y-component of the velocity changes by an amount $dw=Sft_n^{\rm col}$, i.e., the total change of the y-component is proportional to the time of motion. The change of the velocity x-component during the same time can be obtained from the equation[4]

$$v[t_{n+1}]=-v(t_n)+2S-SF_n(t_n^{\rm col})\operatorname{ctg}(F_n(t_n^{\rm col})). \quad (12)$$

Analysis of (12) leads to the conclusion that during the time of particle motion from one collision to another the x-component of its velocity reverses sign, and the modulus increases when the ion moves ahead of the front ($x<0$) and decreases when it moves in the front (i.e., at $0\leqslant x\leqslant d$). The increase or decrease depends on the time of motion in accordance with (12).

From Eqs. (7)–(10) one can obtain "local" energy conservation laws that hold for the particles only in the intervals between successive collisions. For ions moving ahead of the front ($x<0$) or behind it ($x>d$) we obtain a conservation law in the form

$$[v(t)-S]^2+w^2(t)=[v(t_n)-S]^2+w^2(t_n). \quad (13)$$

The analogous relations for ions in the interval $0\leqslant x\leqslant d$ are

$$[v(t)-S]^2+[w(t)-SD]^2=[v(t_n)-S]^2+[w(t_n)-SD]^2. \quad (14)$$

In (13) and (14) $t_n\leqslant t\leqslant t_{n+1}$.

To obtain the most complete picture of the motion of particles trapped in the front we need information on the constants of time at which the particle is farthest from the yz plane. These instants t_m are determined from the condition $v(t_m)=0$ from which, using (7) and (11), we can obtain the explicit expression

$$t_m-t_n=\frac{2}{f}\operatorname{arctg}\frac{-Q_n-(Q_n^2+P_n^2-S^2)^{1/2}}{S-P_n} \quad (15)$$

for ions moving in the space $0\leqslant x\leqslant d(v(t_n)>0)$ and

$$t_m-t_n=\frac{2}{f}\operatorname{arctg}\frac{-w(t_n)+[w^2(t_n)+P_n^2-S^2]^{1/2}}{S-P_n}, \quad (16)$$

for ions moving ahead of the front ($v(t_n)<0$). In Eqs. (15) and (16), t_m-t_n is the time of ion motion from the instant of its nth crossing of the yz plane to the point of its maximum distance from this plane, $t_n\leqslant t_m\leqslant t_{n+1}$.

Obviously, at $x>0$ the quantity $x(t_m)$ determines the depth of penetration of the particles into the front, and hence the potential $U_m=Ex(t_m)$ surmounted by the particle. Using (9) we obtain for U_m

$$U_m=(E/f)\,[P_n\sin(F_n(t_m))-Q_n\cos(F_n(t_m))+Q_n+SF_n(t_m)]. \quad (17)$$

Another quantity conserved in our problem, apart from $w(t)$ and $K(t)$ obtained from the conservation of the generalized momentum and of the energy [Eqs. (4), (5)], is the adiabatic invariant I, the calculation details for which are given in the Appendix. We present here the final expression:

$$I=-\frac{Dv}{S}+\frac{v^2+w^2}{S^2}\arcsin\frac{v}{(v^2+w^2)^{1/2}}+\frac{(SD-w)^2+v^2}{S^2}\arcsin\frac{v}{[(SD-w)^2+v^2]^{1/2}}, \quad (18)$$

where v and w are taken at $x=0$.

Equations (4)–(18) determine completely the character of the trapped-particle motion.

3. DYNAMICS OF PARTICLES IN FRONT. TRAPPING CONDITIONS

In the case in question, no forces whatever act on the particle along the z axis (along the magnetic-field direction). We confine ourselves therefore to an analysis of the ion motion in the xy plane. The unperturbed-plasma particles, moving in two dimensions, land on the wave front with arbitrary velocity components of v and w. We examine in detail the behavior of a particle arriving at the point $x=0$ (see Fig. 1). Obviously, the only particles landing in the front are those having positive velocity x-components at $x=0$ of a selected coordinate frame moving together with the shock wave: $v > 0$. Analysis of the solutions shows that all the particles landing for the first time in the front can be arbitrarily divided into two groups. The first consists of passing particles that land directly behind the wave front (in the region $x > d$). The second, trapped group is returned by the electromagnetic field to the region ahead of the front and lands again at the point $x = 0$ after moving on closed trajectories. The greater part of the ions lands again in the front after the first reflection and joins the transmitted ones. Some particles are multiply reflected, and it is they which are subject to effective resonant acceleration by the field E_y.

To determine which of the particles is trapped and which passes through, it suffices to analyze, with the aid of relations (7)–(10), the behavior of a particle in the region $0 \leqslant x \leqslant d$. If the x-component of the velocity vanishes in this region, the particle is trapped, otherwise it remains passing. The problem reduces thus to a determination of the region of existence of solutions for Eq. (15), which requires in fact an investigation of the function $t_m = t_m(v_0, w_0)$ on the v_0, w_0 plane (recall that v_0 and w_0 are the initial particle-velocity components at the instant $t_1 = 0$). Analysis shows that the boundary between the trapped and passing components on this plane is determined either by the vanishing of the radicand of Eq. (15):

$$(w_0 - SD)^2 + (v_0 - S)^2 = S^2, \tag{19}$$

or by the relation

$$x(t_m) = d. \tag{20}$$

Actually, (19) is reached only as a limit, since the radicand in (15) cannot be less than zero. To understand the meaning of the condition (19), we turn to the local conservation law (14). It is easily seen that (19) is obtainable from (14) by putting

$$v(t) = v(t_m) = 0, \quad w(t) = w(t_m) = SD, \quad v(t_n) = v_0, \quad w(t_n) = w_0.$$

Condition (19) means thus that the particle had reached a speed SD at its stopping point, and an elementary analysis of the ion motion in the front shows that the particle become passing as soon as this value is reached by the y-component of the particle velocity at $v > 0$.

We arrive thus at an important but physically simple conclusion that a particle becomes passing in two cases: 1) when the y-component of the velocity at the stopping point is equal to or larger than SD; 2) the stopping point is behind the coordinate $x > d$. Otherwise the particle is trapped.

Curves *1* and *2* on the $v_0 w_0$ plane (Fig. 2) are plots of (19) and (20), respectively. The region bounded on one side by curves *1* and *2* and on the other by the w_0 axis is the region of the values of v_0 and w_0 at which the particles are captured by the wave. Particles with v_0 and w_0 outside this region are passing. As seen from Fig. 2, the inequality $v_0 \ll SD - w_0$ is satisfied in the greater part of the values of v_0 and w_0 belonging to curve *2*. So long as this inequality holds, we have for the time of motion up to the stopping point the approximate equation $ft_m = v_0/(SD - w_0) \ll 1$. Substituting hence the value of t_m in Eq. (20) we obtain its approximate analog

$$mv_0^2/2 = eU_m(1 - w_0/SD). \tag{21}$$

It is easy to verify that relation (21) can be obtained from the equation of motion (1) of the particle in the region $0 < x < d$ by assuming the velocity w_0 to be constant, i.e., in the case when the velocity increment dw during time of motion of the particle from $x = 0$ to $x = d$ is substantially smaller than w_0, viz., $dw = Sft_m \ll w_0$.

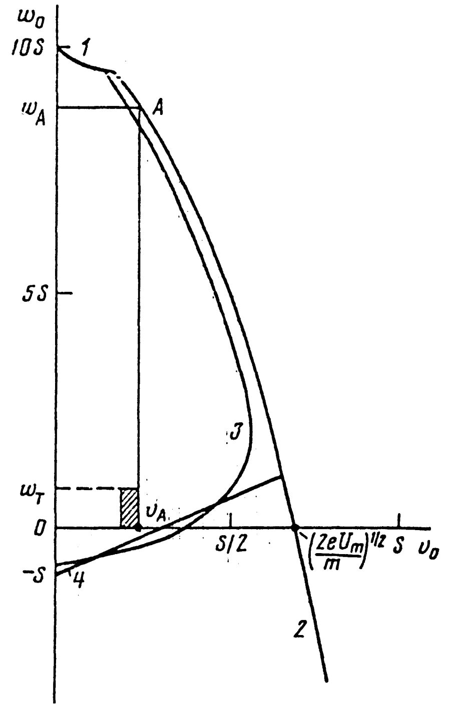

FIG. 2. Boundaries, on the $v_0 w_0$ plane, between the passing, trapped, and once-reflected particles ($U = 0$, $D = 10$).

4. COMPUTER PROCEDURE. DIMENSIONLESS PARAMETERS OF THE PROBLEM

The computation procedure that yields, given the initial conditions, the final values of the accelerated-particle parameters of interest take the following form. We assume that the ion initially approaches the front from an unperturbed plasma and crosses the yz plane for the first time at the parameters given by the conditions (6) (collision $n = 1$, $t_1 = 0$). Next, using the corresponding equations, we can calculate any quantity at any instant of time up to the next collision. First of interest is whether the particle is stopped at $x > 0$. If the particle is not stopped, i.e., if the radicand in (15) is negative, or if the stopping point is outside the front, i.e., at $x > d$, the computation is discontinued and the particle is assumed to be passing. If the particle is trapped, we use Eq. (11) to calculate the time of motion $t_n^{\rm col}$ between collisions. Knowing $t_n^{\rm col}$, we determine for the next collision the velocity components v and w as well as the path of the particle along the y axis between collisions. Using the obtained v, w, and y as initial values, we repeat the computation and verification by the procedure described above.

The main difficulty in the above procedure is the determination of the roots of Eq. (11), which calls for a computer. A good approximation of $t_n^{\rm col}$ is $2(t_m - t_n)$, where $t_m - t_n$ is the time of particle motion after the nth collision to the turning point [Eq. (15)]. The value of $2(t_m - t_n)$

obtained using (15) was the first approximation in the calculation of the root of Eq. (11). A special program for finding this root, consuming the bulk of the computer time, was accurate enough, as verified continuously by computing the quantity

$$R=2[K(t)-K(0)-eEx(t)-eE_y(t)]/mS^2,$$

obtained from the energy-conservation law (5). In each variant of the computation, the deviation of R from the initial value did not exceed 10^{-6} (an ES-1061 computer was used).

For convenience, all the equations were written in dimensionless form. The time was normalized to $1/f$, the coordinate to S/f, the velocity to S, and the energy and temperature to $mS^2/2$.

The dimensionless parameters of the problem were

$$D=cE/BS=E/E_y,\quad U=2eU_m/mS^2,\quad C=S/(2T_i/m)^{1/2},$$

where T_i is the ion temperature. The parameter D is connected with the electric field E, and the parameter U with the potential discontinuity in the wave. Obviously, the three quantities E, U_m, and d are related; $E=U_m/d$, so that two of them, E and U_m, could be specified. We point out that the parameter C is expressed in terms of the known plasma parameter $\beta_i = 8\pi nT_i/B^2$, viz.

$$C=M\beta_i^{-1/2},\qquad M=S(4\pi nm)^{1/2}/B$$

(n is the plasma density). Using parameters customarily encountered in MSSW propagation under laboratory and outer-space conditions,[2,3,13,19] we obtain the ranges of the dimensionless quantities of interest:

$$D=1-1000,\quad U=0,1-1,\quad C=0,1-10.$$

One may question here the choice of the maximum of D and the minimum of C. The value $C=0.1$ was chosen to allow for the fact that in an MSSW with $M>3$ the ions in the pedestal of the shock wave can be heated, so that their temperature directly ahead of the isomagnetic discontinuity can be quite high. The choice of the maximum $D=1000$ is justified in Sec. 7.

5. DETERMINATION OF THE NUMBER OF TRAPPED, PASSING, AND ONCE-REFLECTED PARTICLES

We assume here in all the quantitative results and estimates that all the unperturbed plasma ions have a Maxwellian distribution in velocity. Since we are considering particle motion in the xy plane, we write this distribution in two-dimensional form (in the reference frame of the wave)

$$g(v,w)=\frac{n_0m}{2\pi T_i}\exp\left[\frac{-m(v-S)^2}{2T_i}-\frac{mw^2}{2T_i}\right], \qquad (22)$$

where n_0 is the density of the unperturbed plasma. We neglect the influence exerted on the initial ion distribution function by the reflected particles that may appear quite far ahead of the front. This assumption is justified so long as the number of reflected particles is negligible.

The total number of trapped particles is given by

$$n_t=\int_{-\infty}^{SD}dw\int_0^a dv\, g(v,w),\qquad a=\left[\frac{2eU_m}{m}\left(1-\frac{w}{SD}\right)\right]^{1/2},$$

i.e., by the integral over the area bounded in Fig. 2 by curve *2* and by the w_0 axis. We assume here that the relation (20) represented by curve *2* is valid for all w all the way to $w=SD$, i.e., we have ignored relation (19); computations show that this hardly affects the end result.

Integrating, we obtain ultimately the relative number of trapped particles in the form

$$N_T=\frac{n_t}{n_0}=\frac{1}{4}[1+\mathrm{erf}(CD)]\,\mathrm{erf}\,C+\frac{1}{2\pi^{1/2}}\times\int_0^\infty \exp(-Y^2)\,\mathrm{erf}\left[C\left\{\left[U\left(1+\frac{Y}{CD}\right)\right]^{1/2}-1\right\}\right]dY-\frac{1}{2\pi^{1/2}}\times\int_0^{CD}\exp(-Y^2)\,\mathrm{erf}\left[C\left\{1-\left[U\left(1-\frac{Y}{CD}\right)\right]^{1/2}\right\}\right]dY. \qquad (23)$$

The equation for the number of passing particles is quite simple:

$$N_P=[1+\mathrm{erf}\,C]/2-N_T. \qquad (24)$$

In (23) and (24) we use

$$\mathrm{erf}\,x=\frac{2}{\pi^{1/2}}\int_0^x \exp(-t^2)\,dt.$$

It follows from the numerical computations that most trapped ions are once-reflected. The limiting values v_0 and w_0 separating in the trapped-ion region the singly and multiply reflected particles are represented by curve *3* of Fig. 2. This dependence can be approximated by the simple relation

$$v_0={}^1/_2(2eU_m/m)^{1/2}(1+w_0/S),$$

represented in Fig. 2 by the straight line. The number of once-reflected particles can thus be represented on the (v_0, w_0) plane with sufficient accuracy by the region bounded by curves *2* and *4* and by the w_0 axis (this region lies below curve *4* in Fig. 2). We have thus for the number of once-reflected particles

$$n_r=\int_{-\infty}^{S}dw\int_0^a dv\,g(v,w)-\int_{-S}^{S}dw\int_0^b dv\,g(v,w),$$

$$b=\left(\frac{2eU_m}{m}\right)^{1/2}\frac{1+w/S}{2},$$

or in dimensionless form

$$N_R=\frac{n_r}{n_0}=\frac{1}{4}[1-\mathrm{erf}\,C]\,\mathrm{erf}\,C+\frac{1}{2\pi^{1/2}}\int_0^\infty \exp(-Y^2)\,\mathrm{erf}\left[C\left\{\left[U\left(1+\frac{Y}{CD}\right)\right]^{1/2}-1\right\}\right]dY-\frac{1}{2\pi^{1/2}}\int_0^C \exp(-Y^2)\,\mathrm{erf}\left[C\left\{1-\left[U\left(1-\frac{Y}{CD}\right)\right]^{1/2}\right\}dY\right.-\frac{1}{2\pi^{1/2}}\int_{-C}^{C}\exp(-Y^2)\,\mathrm{erf}\left\{C\left[\frac{U^{1/2}}{2}\left(1+\frac{Y}{C}\right)-1\right]\right\}. \qquad (25)$$

6. CONDITIONS FOR ION ESCAPE FROM TRAP. ESTIMATE OF THE NUMBER OF ACCELERATED PARTICLES AND OF THE ENERGY THEY DRAW FROM THE WAVE. INFLUENCE OF RESONANT IONS OF THE MACROSCOPIC STRUCTURE OF THE FRONT

We consider "deeply" trapped particles, i.e., those acquiring an energy close to the limit after executing a large number of oscillations on the front. The trapping conditions for a consecutive crossing of the xy by an oscillating ion are the same as for the first collision. Ultimately, when the particle incident on the front has the parameters curve *1* or *2*, or else outside the trapping region (Fig. 2), the particle passes through. The kinetic energy of an ion in the trapping regime grows continuously according to (5). We shall use the particle energy acquired by the particle at the point $x = 0$.

Let an ion with some energy K have at $x = 0$ the same parameters as when it lands at a certain point A of curve *2* (Fig. 2). At this point the velocity components are $v = v_A$ and $w = w_A$ and hence the ion energy is $K \approx mw_A^2/2$. Knowing final values of the components v_A and w_A (see Fig. 2) and taking their relation (21) into account, we obtain the adiabatic invariant as a function of w_A, viz., $I_A = I(w_A)$. Corresponding to this value of the adiabatic invariant at the initial instant of time is a whole set of values v_0 and w_0 that can be found using relations (A1) and (A4). We can find similarly the set of initial values v_0 and w_0 for some energy $K + dK$. Knowing the ranges of v_0 and w_0 at which the ions ultimately acquire an energy specified in the interval dK, and knowing the initial particle distribution (22) in velocity, the number of these ions can be found. Computations by this scheme yield a rather cumbersome expression. All the computations, however, can be substantially simplified by using the adiabatic invariant given by Eq. (A4). The connection between the initial and final values of the resonant-ion velocity components can then be obtained in explicit form by using I_A from (A3) and taking (21) into account:

$$v_0^{*}=SU^{0,6}\left(1-\frac{w}{SD}\right)^{0,2}\left(\frac{\pi w}{S}\right)^{-0,4}. \qquad (26)$$

A computer analysis shows that this simplification underestimates somewhat (within order-of-magnitude limits) the sough values compared with the exact ones. We, however, are quite satisfied, in view of the exceedingly simple manner we have obtained a very important estimate, with acceptable accuracy, of the number of accelerated particles. The region of the initial velocities of the ions accelerated in a specified energy interval are represented in this approximation by a rectangle (cross-hatched in Fig. 2), two sides of which are parallel to w_0 and two other to v_0. We choose the maximum value of w_0 in this region to be the ion thermal velocity $w_T = (2T_i/m)^{1/2}$. The reasons are, on the one hand, that at large values of w_0 the number of particles is exponentially small; on the other, we have confined ourselves to these characteristic velocity values to exclude ions with high initial energies (we assume that $w_T \ll SD$) and to consider thus only those particles which have acquired energy directly by acceleration, and furthermore an energy significantly higher than initial. The total number of particles accelerated in some energy interval dK is thus dependent now only in the velocity interval dv_0:

$$dn_a=A\exp[-(v_0-S)^2/w_T^2]dv_0,$$

where

$$A=\frac{n_0}{\pi w_T}\int_0^{w_T}\exp\left(-\frac{w_0^2}{w_T^2}\right)dw_0\approx\frac{n_0}{2\pi^{1/2}w_T}.$$

Taking (26) into account, we can express dn_a in terms of w: $dn_a = f(w)dw$, or in terms of $K = mw^2/2$ viz., $dn_a = h(K)dK$, where

$$f(w)=\frac{dn_a}{dw}=AU^{0,4}\left(1-\frac{w}{SD}\right)$$
$$\times\exp\left[-C^2\left(1-\frac{v_0^{*}}{S}\right)^2\right]\Big/2.5\pi^{0,4}w^{1,4}\left(1-\frac{w}{SD}\right)^{0,8} \qquad (27)$$

plays the role of the particle distribution function in velocity [v_0^* is determined by Eq. (26)], while

$$h(K)=\frac{dn_a}{dK}=AU^{0,4}\left(1-\frac{(2K/m)^{1/2}}{SD}\right)$$
$$\times\exp\left[-C^2\left(1-\frac{v^{*}}{S}\right)^2\right]\Big/5\pi^{0,4}K^{1,2}\left[1-\frac{(2K/m)^{1/2}}{SD}\right]^{0,8},$$

where

$$\frac{v^{*}}{S}=U^{0,6}\left[1-\frac{(2K/m)^{1/2}}{SD}\right]^{0,2}\left[\frac{\pi(2K/m)^{1/2}}{S}\right]^{-0,4}$$

assumes the role of the distribution function in the energies of the accelerated ions. The function $h(K)$ is shown in Fig. 3 for one of the parameter sets.

We can now find the total number, of interest to us, of the particles accelerated from minimum velocity value, which we choose to be $SD/3$, to a maximum equal to SD:

$$n_a=\int_{SD/3}^{SD}f(w)dw, \qquad (28)$$

where $f(w)$ is defined by (27).

Knowing $f(w)$ we can determine the momentum-flux density, of interest to us, carried by the accelerated particles:

$$W_r=m\int_{SD/3}^{SD}f(w)w^2\,dw.$$

This quantity determines which fraction of the plasma momentum flux density n_0mS^2 incident on the shock-wave

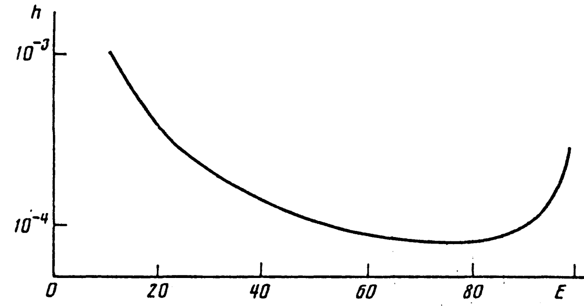

FIG. 3. Distribution function in energy ($E = 2K/mS^2$) of the accelerated ions ($D = 10$, $U = 1$, $T_I = 2T_i/mS^2 = 0.25$).

front is carried away by the accelerated ions, i.e., in fact the total energy extracted from the wave by these ions. This estimate was obtained under the assumption that the shock wave itself is not attenuated, i.e., its energy lost to particle acceleration is replenished by the work of external forces (a moving piston etc.).

We proceed now to discuss the influence of the trapped ion on the macroscopic structure of the shock front. This influence is due mainly to the presence of a group of charged particles in the front during the acceleration, each of which is continuously accelerated along the y axis. This influence is substantial in view of the particle accumulation in the front.[10] To understand the cause of the accumulation, we turn to the law of conservation of the flux of particles moving through the front. It is easy to verify that the flux of particles dragged out of the unperturbed plasma into the acceleration regime,

$$J=\int_0^{v_m} v\,dv\int_0^{w_T} dw g(v,w),$$

is approximately equal to $v_a v_m$, where $v_m \ll S$ is the initial velocity connected by relation (26) with the minimum value of the final particle velocity $w = SD/3$, and n_a is the number of accelerated particles (per unit volume). The captured particles, oscillating in the front, slowly "climb up" to the "hump" of the potential, as is clearly seen from the plotted time dependence of the potential surmounted by the ion in the course of its acceleration (Fig. 4). The speed of this peculiar drift of the particles can be estimated at $v_d \approx d/t_y$ (t_y is the acceleration time). From the flux-conservation law $J \approx n_a v_m \approx n_f v_d$, where n_f is the density of the particles accelerated in the front, it follows that $n_f/n_a \approx D^{1.6}U^{0.6}$. For typical MSSW parameters we usually have $D^{1.6}U^{0.6} \gg 1$ and $n_f/n_a \gg 1$, i.e., the density of the trapped ions in the front can be substantially larger than the density of that particle fraction accelerated from the unperturbed plasma.

To estimate the perturbations of the electric and magnetic fields by the trapped particles, we assume the density n_f of the accelerated ions is constant in the front, and that their space charge is not compensated for by electrons. The perturbed potential is then

$$U_f \approx e n_f d \approx e d U^{0.6} D^{1.6}\int_{SD/3}^{SD} f(w)\,dw.$$

The perturbed magnetic field is $B_f \approx j_f d/c$, where j_f is the accelerated-particles current density:

$$j_f = eU^{0.6}D^{1.6}\int_{SD/3}^{SD} f(w)\,w\,dw.$$

The value of $f(w)$ in the expressions for U_f and j_f is determined by Eq. (27).

7. CALCULATION RESULTS AND THEIR DISCUSSION

Figure 4 shows the time dependences of the energy $K(t)$, of the velocity $v(t)$, and of the potential $P(t)$ surmounted by the trapped ions in the course of the acceleration. The velocity and energy values are taken at the instants when the oscillating ion crosses the yz plane. What is remarkable here is the time behavior of the x-component of the velocity $v(t)$ in the case when the final energy is close to its limit. We see that $v(t)$ increases during the initial stage, reaches next a maximum at the instant $t = D/2f$ a maximum, and then decreases to the initial value. According to (4), the y-component of the velocity increases linearly with time. The kinetic energy K, which consists of two components $mv^2/2$ and $mw^2/2$, and increases like t^2 mainly as a result of the growth of the component $mw^2/2$.

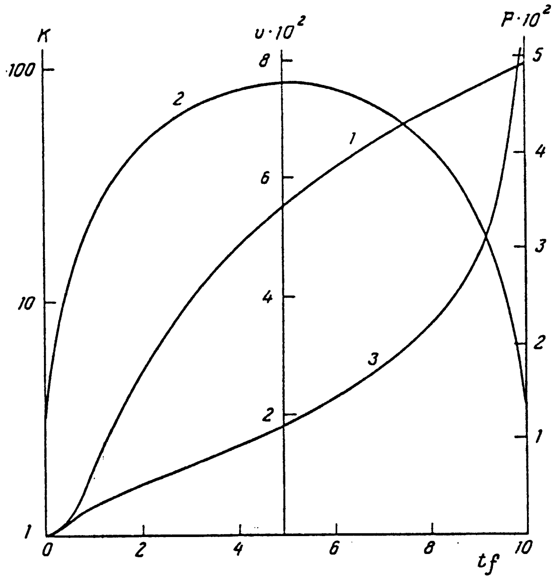

FIG. 4. Time dependences of the energy K (curve 1, of the x-component of the velocity v (2), and of the potential P (3) surmounted by the ion in the course of acceleration ($D = 10$, $U = 1$, $w_0 = 0$, $v_0 = 0.02$).

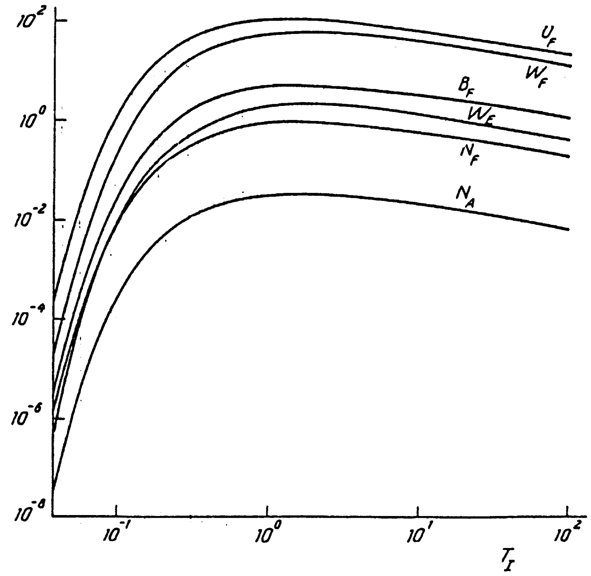

FIG. 5. Dependences of the density (N_F) of the accelerated ions located in the front, trapped in the acceleration regime (N_A), from an unperturbed plasma, of the energy density (W_E) drawn from the wave by the accelerated plasma, of the potential discontinuity (W_F) of the ions in the potential discontinuity, of the perturbed values U_F of the potential, and of the perturbed magnetic field B_F on the temperature $T_1 = 2T_i/mS^2$ ($D = 10$, $U = 0.6$, $M = 5$).

Unlike all other dependences on the dimensionless parameters, the temperature dependence of the ions (the parameter C) turned out to be nonmonotonic, see Fig. 5. This figure, as well as Figs. 7 and 9 below, is plotted on the dimensionless variables

$$W_F = W_f/n_0 mS^2, \quad W_E = W_e/n_0 mS^2, \quad N_F = n_f/n_0,$$

$$F_F = B_f/B, \quad U_F = U_f/U_m.$$

Figure 5 shows for practically all the quantities a well pronounced maximum at $T_i = (1\text{–}2)mS^2/2$. The temperature dependence to the left of the maximum (low-temperature region) is stronger than to the right (high temperatures). It turned out that at $U = 0.6$ the total number of trapped particles depends weakly on the temperature, and the relative number of passing ions ranges from 0.5 to 1.0 for the considered range of T_i. Worthy of attention here are the results of Ref. 10, where the values obtained for the number N_A of the accelerated ions are substantially higher than ours. The reason for this difference is that the author of Ref. 10 included in error with the accelerated ones all the trapped particles, including the singly reflected ones. That is to say, he actually used the value N_T in lieu of N_A, yet it follows from all our calculations that $N_T \gg N_A$ in all cases.

The dependences on the MSSW potential amplitude (Figs. 6 and 7) turned out to be quite strong: doubling U_m changes all the cited quantities by an order of magnitude or more. The dependences here are monotonic, and all the quantities but the number of the passing ions increase as a rule with increase of the potential. The increase of the number of trapped, once-reflected, and accelerated particles can be easily understood by examining Fig. 2. The increase of the perturbed magnetic field and of the energy density of the accelerated ions with increase of U_m is due to the increase of the number of accelerated particles as well as of their velocities, which increase with U_m on the average because of the increase of the limiting velocity. The latter in turn increases in proportion to the electric field in the field, which grows in turn together with U_m at a fixed width of the front.

Strange as it may seem, it is possible in the discussion of the dependence of the potential to touch upon the problem of acceleration of impurity ions having different masses in the front of the shock waves. The ion component of a real plasma usually consists of ions having different masses and charges, and constituting with respect to principal ions, the majority of the positive charges, while the remaining ions are regarded as impurities. In the general case these impurity ions can also be trapped and accelerated, and we consider therefore the problem of acceleration of singly charged ions whose masses are either larger or smaller than those of the main ions. We assume that the macroscopic structure of the shock wave is made up entirely of the principal ions and electrons. The amplitude of the potential is regarded as equal to the energy of the main ions, $U_m = mS^2/2e$, incident on the wave with velocity S. We denote the ratio of the masses of the impurity and main ions by k. For the ions of the light (heavy) impurities incident on the wave front at the same velocity S, the kinetic energy will therefore be k times smaller (larger) than U_m. It follows from this reasoning that the laws of motion of the impurity ions can be described by our derived Eqs. (4)–(18), in which the dimensionless variables must be renormalized and U_m must be decreased by a factor k for heavy impurities and increased by a factor k for the light ones. At equal final velocity, the energies of the principal and impurity ions will differ by a factor k, and all

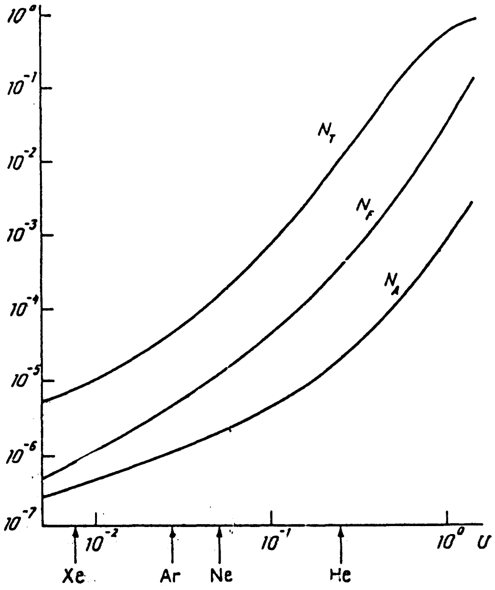

FIG. 6. Dependences of the density of the trapped ions (N_T), of the density of the ions in the front (N_F), and of the density of the ions accelerated from the unperturbed plasma (N_A) on the discontinuity of the potential U. The arrows correspond to the density of the heavy-impurity ions in the case when the shock wave behind the discontinuity of the potential $U = 1$ propagates in a hydrogen plasma. Each dimensionless-density value marked by an arrow is normalized to density of the corresponding impurity in the unperturbed plasma ($D = 10$, $T_i = 0.1$).

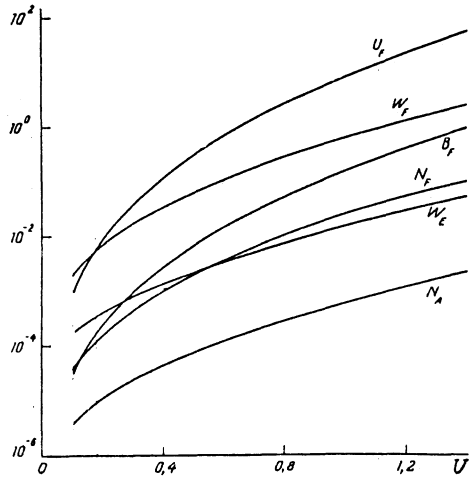

FIG. 7. Density of the accelerated ions in the front (N_F), density of the ions trapped in the acceleration regime from the unperturbed plasma (N_A), density of the energy drawn from by the accelerated ions from the wave (W_E), energy density of the ions in the potential discontinuity (W_F), and perturbed values of the potential U_F and of the magnetic field B_F, as functions of the discontinuity of the potential U.

the times and distances along the y axis will change by k times.

We consider first the case when the impurity-ion mass is lower than that of the principal one, $k < 1$. The possibility of accelerating the light-impurity ions was first pointed out in a preprint.[20] Following this preprint, the impurity ions will be regarded as cold. It is readily seen that in this approximation all the impurity ions are trapped and accelerated. It seems at first glance that their limiting velocity is in this case equal to SD, but this is not so. It follows from the calculation results illustrated in Fig. 8 that the final velocity of the trapped particles is approximately equal to SD so long as $D < 1/k$. For $D > 1/k$ the final velocity becomes smaller than SD and with increase of D it tends to a certain limiting value much smaller than SD. This is easily seen from an analysis of the computation results. The point is that if $D \gg 1/k$ the x-component of the velocity v is so accelerated that the "longitudinal" energy of the impurity ions becomes comparable with eU_m, permitting the particle to leave the trapping regime by overcoming the potential barrier and going behind the front before the y component of the velocity reaches the limit SD. For $D < 1/k$ the maximum value of U_m is such that the "longitudinal" energy is always smaller than eU_m.

Let us dwell in greater detail on the results for larger values of the parameter D. Note that going over to infinite values of D corresponds to an MSSW model[2] in which the potential jump is treated as an elastically reflecting piston. Taking into account the estimates obtained in Ref. 2, as well as Eq. (A3) in the Appendix, we obtain the connection between $w(t)$ and $v(t)$ in the form $w(t) = \text{const}\cdot[v(t)/v(0)]^3$. In our case $v(0) = S$, and for large D this equation permits an order-of-magnitude estimate of the asymptotic particle energy and acceleration time: $E_\infty \propto k^{-3}$, $t_\infty \propto k^{-3/2}$. These dependences are confirmed by computations (Fig. 8).

The laws governing the resonant approximation of the impurities of the heavy ions are considered in an interpretation[21] of the energy spectra of ions with different masses and charges, measured with the "Voyager" spacecraft. The number of accelerated particles was however not estimated there. Assuming the temperatures of the principal and impurity ions to be the same, we can obtain this estimate from relations (4)–(18). Consider, for example, a hydrogen plasma in which an MSSW having a potential $U = 1(U = 2eU_m/mS^2$ where m is the hydrogen mass) propagates. The arrows in Fig. 6 show for this case the potentials that will be "seen" by singly charged ion of various impurity gases, as well as the densities of the impurity ions that are trapped, accelerated, and located in the front. The dimesionless density corresponding to any one arrow on these figures is normalized each to the density of the corresponding impurity in the unperturbed plasma. Obviously, the maximum velocity of the accelerated ions does not depend on their mass and is equal to SD, while the maximum energy of the heavy ions is k times larger than the main ones. All the characteristic times are increased by a factor k, as is therefore the distance traversed by an accelerated ion along the y axis. In a real plasma with resonantly accelerated ions it is precisely this distance (along y) which limits in fact their maximum energy. It is just this factor which accounts fully in Ref. 21 for the maximum values of energy of ions having different masses and accelerated by an interplanetary shock wave.

The dependences of the quantities of interest to us on the electric field in the front of a shock wave are shown in Fig. 9. It can be seen that the three quantities N_F, B_F, and W_E depend weakly on E. As E increases, the energy density

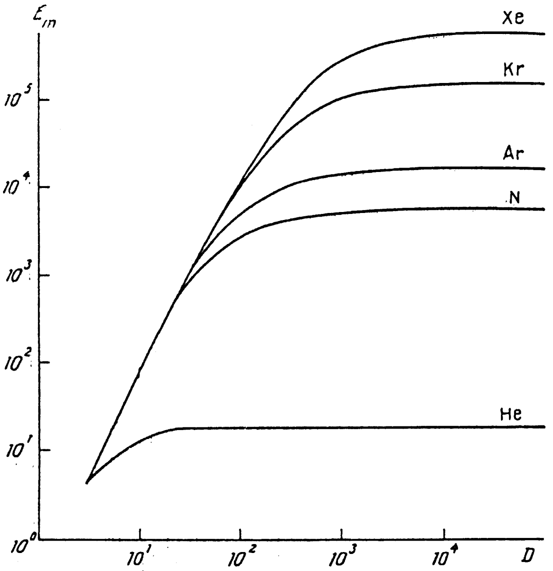

FIG. 8. Maximum energy of the impurity hydrogen ions accelerated in the front of a shock wave propagating in a plasma and having majority ions heavier than the impurity ions as a function of the electric field in the discontinuity ($D = cE/BS$). The amplitude of the shock wave is $U_m = mS^2/2e$, where m is the mass of the majority ion. The majority ions considered are those of xenon, krypton, argon, nitrogen, and helium.

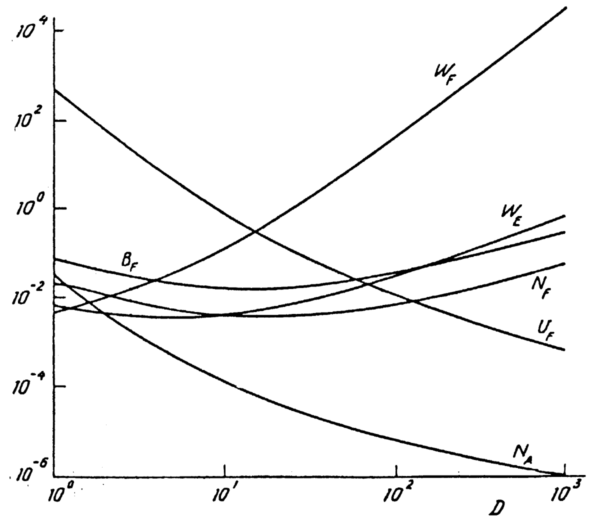

FIG. 9. Densities of the accelerated ions in the front (N_F), of the ions trapped into acceleration from an unperturbed plasma (N_A), of the energy drawn by the accelerated ions from the wave (W_E), of the energy of the ions in the potential discontinuity (W_F), and of the perturbed potential U_F and perturbed magnetic field B_F as functions of the electric field in the discontinuity ($D = cE/BS$).

W_F of the ions trapped in the front increases. This is not surprising, since when the trapped-ion density N_F in the front is increased the average ion energy increases in proportion to E. It must be noted here that under conditions when the accelerated-ion energy can become transformed into heat, the effective temperature of the heated ions is determined precisely by the value of W_F.

The magnetic field B_f perturbed by the current of accelerated ions varies in the same manner as the density n_f of the particles in the front. This can be verified by the relation that holds for a given wave amplitude: $B_F \approx en_f Ed/B \approx en_f U_m/B$. The perturbed potential U_f decreases substantially when E is increased. It has turned out that the relative number of trapped and once-reflected ions are independent of E at the given parameters U and T_1.

Naturally, the computations have confirmed a conclusion, drawn by many workers, that the maximum energy K_m of the accelerated ions is proportional to E^2 (or D^2. Most papers dealing with the acceleration of ions trapped in an MSSW front contain an estimate that can be expressed, in terms of the parameter D introduced by us, in the form

$$K_m=0.5mV_A{}^2D^2$$

(V_A is the Alfven velocity), where the maximum value of D^2 is equal to the ratio of the ion and electron masses: $D^2 = m/m_e$ (for hydrogen ions, in particular, $D^2 \approx 1800$). It is readily seen that the parameter D^2 is equal to m/m_e for a subcritical MSSW ($M<3$), in which the width of the discontinuity is taken to have the minimum value $d = c/\omega_{pe}$ and the maximum value $U_m = MmV_A^2/2e$ is used for the amplitude of the potential.

In the general case we can write for the maximum energy in a cold plasma

$$K_m=0{,}5mV_A{}^2UM^2D^2,$$

where the dimensionless potential U is a function of the Mach number M. On the basis of Refs. 4, 5, and 8, where solutions for a soliton are used, one can conclude that $UM^2 \approx M$, so that the equation for K_m becomes quite simple:

$$K_m=0{,}5mV_A{}^2MD^2. \tag{29}$$

The use of (29) for a supercritical MSSW at $M>3$ is questionable since, on the one hand, the soliton solution is not valid in this case, and on the other the front in a supercritical MSSW broadens substantially, D becomes small (for $d = c/\omega_{pi}$ we have $D \approx 1$), and consequently the idea of acceleration becomes meaningless. It appears, however, that it makes sense to use this equation for $M<3$ in an MSSW with an isomagnetic discontinuity, for the parameters of which the value of D may turn out to be large enough.

Let us choose 20 Debye radii as the typical dimension of the isomagnetic discontinuity of an MSSW potential.[2,3] This changes the maximum possible value of $D = (m/m_e)^{1/2}$ for the parameters of the front of a magnetosonic soliton by a factor $0.05c/v_{T_e}$ that depends on the temperature T_e of the electrons in the plasma ($v_{T_e} = (2T_e/m_e)^{1/2}$). Evidently, for $T_e < 1000$ eV the above factor is larger than unity, in particular 30 for $T_e = 1$ eV and 3 for $T_e = 100$ eV (the interval 1 eV $< T_c <$ 100 eV is typical of interplanetary and laboratory plasma). Thus, for an MSSW with an isomagnetic discontinuity in a hydrogen plasma with $T_c = 1$ eV it is in fact possible to have $D = 1000$ (as against $D \approx 40$ for a soliton). Furthermore, it follows from experiment[2,3,16] and computation[22] that as M increases the quantity $U = 2eU_m/mS^2$ decreases insignificantly, and that for the product of U and M^2 one can assume the relation $UM^2 \approx M$ which is valid for a subcritical MSSW.

We can thus summarize the foregoing as follows: Eq. (29) for K_m is valid at $M>3$ for an MSSW with an isomagnetic discontinuity in the front, i.e., for energy limit increases approximately in proportion to $M\cdot D^2$, while the factor D^2 can be of the order of m/m_e or even larger by one to three orders.

It is noteworthy that increases of the parameter D and of the wave velocity S can lead to situations wherein the electric-field amplitude E in an MSSW isomagnetic discontinuity can exceed the magnetic field B. In this case the maximum velocity SD will tend to that of light and resonant acceleration of the ions trapped in the discontinuity to unlimited energy is theoretically possible.[23] For example, at $D = 1000$ the ratio $E/B = DS/c$ exceeds unity if $S > 3\cdot 10^7$ cm/s. These parameters are quite realistic for an MSSW with an isomagnetic discontinuity, and consequently a small group of trapped ions can be accelerated here to infinite energy.

Let us examine the conditions under which the results of the present paper are valid. One of the main assumptions stated in the formulation of the problem—constancy of the magnetic field in the potential discontinuity—is perfectly justified for an MSSW with an isomagnetic discontinuity. In our opinion, the assumption that the electric field is constant has no special effect on the end results.

Obviously, all the results are valid so long as the influence of the trapped ions on the structure of the front is small. The effectiveness of such an influence is manifest in the perturbations of the potential U_f and of the magnetic field B_f. If it is assumed that ratios B_f/B and U_f/U_m less than 10% are already negligible then as shown by the computations, the influence in question can be regarded as no longer significant for the following values of the parameters: $U<0.5$, $D>10$, $C>3$ ($T_i < 0.1\, mS^2/2$). Generally speaking, notwithstanding the smallness of the number of accelerated ions at certain parameters and the smallness of the perturbed B_f and U_f, the front still contains a noticeable energy density W_e drawn from the wave by the resonant particles, and a noticeable energy density W_f of these particles in the front. This is evidence that in a quantitative description of a shock wave the resonant ions must be taken into account in the relations at the discontinuity. Resonantly accelerated particles can in principle play in MSSW the role of collisionless dissipation, by analogy with reflected particles in a shock wave without a magnetic field.[1]

8. CONCLUSION AND PRINCIPAL DEDUCTIONS

We conclude with concise statements of our main results. Using a simple model to describe the structure of an isomagnetic discontinuity of an MSSW we obtained the conditions under which the particles are either trapped in the front, or go off immediately past the front, i.e., become passing. It has been made clear that the majority of the trapped particles undergo a single reflection. The total numbers of the passing, trapped, and singly reflected ions, as well as the numbers of ions accelerated in a specified energy interval, are determined for an unperturbed plasma with a Maxwellian ion distribution function. The energy drawn by the accelerated ions from the wave is estimated. For the number of accelerated and once-reflected ions, for the energy density of the accelerated ions located in the front, and for the perturbed magnetic field and the magnetic field produced by these particles we obtained the dependences on the potential discontinuity, on the electric field in the front, and on the perturbed-plasma ion temperature. We determined the distribution function of the accelerated ions. We touched upon the problem of resonant acceleration of singly charged ions of impurities with different masses in the front of an MSSW. The dependences of the maximum energy on the parameter D was obtained for light ion impurities having a zero temperature. For an unperturbed plasma containing only heavy impurity ions, we found which fraction of each impurity ions will be trapped and accelerated. We discussed the conditions under which our computations are valid and the pertinent range of variation of the problem parameters.

From among the main conclusion, we single out the following:

1. The high efficiency of resonant acceleration of ions in MSSW with isomagnetic discontinuities ($M > 3$) was demonstrated. It was found that in this case the maximum energy of the accelerated ions exceeds by 1–3 order the heretofore known possible energies obtained for a subcritical ($M < 3$) magnetosonic shock wave.[1,4–10] The excess was determined by the spatial scale of the potential discontinuity, by the electron temperature, and by the Mach number.

Generally speaking, it is quite feasible for an electric field in an isomagnetic MSSW discontinuity to exceed the magnetic field, so that trapped ions of unbounded energy can theoretically be obtained in a discontinuity under resonant acceleration.

2. All the computed quantities have maxima at an ion temperature $T_i \approx K_0 = mS^2/2$, while at $T_i \ll K_0$ they depend substantially on the variation of T_i and U_m.

3. The presence of accelerated ions in a wave front was shown to influence strongly the macroscopic structure of the front, thus attesting to the need for taking them into account in the derivation of the relations on the shock-wave discontinuity.

4. The results can be used as estimates for real magnetosonic waves with arbitrary Mach numbers and with quite complicated front structure, where a potential discontinuity as well as a magnetic-field discontinuity can exist.

APPENDIX

DETERMINATION OF THE ADIABATIC INVARIANT

To find the adiabatic invariant we turn to the energy conservation law (5). Substituting for $w(t)$ and $Y(t)$ in (5) their expressions in terms of $x(t)$ and t, obtained from Eq. (4), we obtain a conservation law in the form

$$mv^2/2+P(x,\ t)+Z(t)=mv_0^2/2,$$

where

$$P(x,t)=mf^2x^2/2+mf(SD-w_0-Sft)x,$$

$$Z(t)=mSf\int_0^t x(t)dt.$$

A part of the total energy

$$H(x,\ t)=mv^2/2+P(x,t)$$

is indicative of the motion of the particle along the x axis. This part of the total energy is not conserved. The potential part $P(x,t)$ of the energy $H(x,t)$ is useful since it is possible to estimate qualitatively from its form the motion of the trapped particle. A plot of the potential energy $P(x,t)$ has the appearance of a well whose form varies slowly with time. At $w_0 > 0$ a particle in this well oscillates about an equilibrium point with coordinate $x = 0$. During the initial state at $w_0 < 0$ both the well shape and the equilibrium-point coordinate change, i.e., the well-shaped deformation is more complicated in this case. For a trapped particle executing many oscillations about the equilibrium point $x = 0$ the energy $H(x,t)$ varies quite slowly with time compared with the period of the oscillations. This permits the use of the formalism of the adiabaticity of motion.[24] According to this theory the adiabatic invariant can be expressed in our case by a closed-contour integral $I = \oint x dv$. Substituting here the value of x obtained from the expression for $H(x,t)$ we obtain, putting $a(x,t) = [2H(x,t)/m]^{1/2}$,

$$I=-\frac{aD}{S}+\left(\frac{(D-w)^2}{S^2}+\frac{a^2}{S^2}\right)\arcsin\frac{a}{[(SD-w)^2+a^2]^{1/2}}+\frac{w^2+a^2}{S^2}\arcsin\frac{a}{[w^2+a^2]^{1/2}}.$$

We have assumed in this equation that $w = w_0 + Sft$, i.e., we take the values of I at the instants at which $x = 0$. For $t = 0$ we have $w = w_0 > 0$ and $a = v_0 > 0$.

We find now the approximate values of I in various limiting cases. At the initial instant when the inequalities $v_0 \ll w_0 < S$ hold we obtain

$$I\approx 0{,}5v_0^3/w_0S^2, \tag{A1}$$

and at $S > v_0 \gg w_0$ we have correspondingly

$$I\approx 0{,}5(v_0/S)^2. \tag{A2}$$

With the inequalities $SD - w \gg a$, and $w \gg a$ we have

$$I\approx 0{,}5a^3D/(SD-w)^2wS. \tag{A3}$$

The conservation of the adiabatic invariant I was constantly monitored in all the computations. In some cases I deviates from a constant at the very beginning of the computations. These cases were investigated in detail and have shown that if $w_0 < v_0$ the value of I at the initial instant exceeds the steady-state value for two or three oscillations. It was found that the initial excess of I depends only on the parameter v_0 and the excess varies like $v_0^{-1/2}$. The deviation of I from a constant is due to the fact that at the initial stage of the particle motion the period of its oscillations in the potential well is comparable with the characteristic change of its shape with time. After two or three oscillations the period decreases radically and the conditions for adiabaticity of the motion become much worse. Taking into account these fine points of the behavior of I, we monitored the initial value $I = I(v_0,w_0)$ at $w_0 < v_0$. For $w_0 < v_0 < S$, in particular, we used in lieu of (A1) the relation

$$I\approx 0{,}5(v_0/S)^{5/2}. \tag{A4}$$

Disregarding the initial stage of particle acceleration, a good approximation for I can be hereafter, for all v_0 and w_0, the value given by (A3).

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