Non-distorting action of inhomogeneous medium of a nematic liquid crystal on an ordinary wave (theory and experiment)

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It is predicted and experimentally confirmed that an ordinary wave can pass through a thick (≈ 5 mm) cells with a nematic liquid crystal with practically no distortion. The physical idea is the following: 1) the polarization of ordinary and extraordinary waves follows the local orientation of the optical axis; 2) the phase velocity of an ordinary wave is independent of the director orientation. The topology of polarization around a disclination line is investigated.

1. INTRODUCTION

In most liquid-crystal displays the polarization plane of an *o*- or *e*-wave is made to follow the local orientation of the director in twisted cells with nematic liquid crystals (NLC). It is known at the same time that an NLC in a thick cell is not transparent. The opacity has the following two causes.

First, a rather large fraction of the light scattering is due to small-scale director fluctuations at thermodynamic equilibrium (the extinction coefficient is in this case $R\sim 1$–10 cm^{-1}). This form of scattering can be compensated for by external E or H fields, but these fields must be excessively strong.

The second cause may be the inhomogeneity of a large-scale director (including disclinations) in thick ($L \gtrsim 1$ mm) cells. Refraction of light by these smooth inhomogeneities is usually a source of those parts of the image which have not yet been "sprayed" into a relatively large angle by the thermal fluctuations. This is in fact the case for an *e*-wave, since the refractive index $n_e(\theta)$ depends strongly on the angle θ between the director $\mathbf{n}$ and the wave vector $\mathbf{k}$

$$\theta=\theta(\mathbf{r})=\arccos\frac{\mathbf{kn}}{k},\qquad n_e(\theta)=\frac{n_\perp n_\parallel}{(n_\perp^2\sin^2\theta+n_\parallel^2\cos^2\theta)^{1/2}}. \tag{1}$$

Here $n_\parallel$ and $n_\perp$ are the refractive indices of the NLC for light polarized along and across the director, respectively. At $\lambda=2\pi c/\omega=0.63\ \mu$m (He–Ne laser) $n_\parallel-n_\perp\approx 0.1$–$0.2$, $\delta\theta\approx 1$ rad, and $L\approx 1$ mm the change of the *e*-wave phase

$$\gamma_e(x,y)=\frac{\omega}{c}\int_0^L n_e[\theta(x,y,z)]dz \tag{2}$$

has inhomogeneities $\delta\gamma_e\sim 2\pi(n_\parallel-n_\perp)L/\lambda\sim 10^3$ rad.

The deflection angles α_x and α_y of a light beam passing through a cell can be estimated by calculating the direction of the normal to the wave front. The surface of the latter in air is given by

$$(\omega/c)z+\gamma_e(x,y)=\text{const},$$

so that for a deflection angle $\alpha_x\approx\partial z/\partial x$ we have the estimate $\alpha_x\sim(c/\omega)\partial\gamma_e/\partial x\sim c\delta\gamma_e/\omega\Delta x\sim 0.1$ rad if we assume the scale Δx of the transverse inhomogeneity to be $\Delta x\sim 0.1$ cm.

The main idea of the present paper is based on the fact that the refractive index $n_o\equiv n_\perp$ of an *o*-wave is independent of θ. The adiabatic passage of an *o*-wave through the inhomogeneities of an NLC is therefore not accompanied by phase distortions,

$$\gamma_o(x,y,z)=\omega z n_\perp/c.$$

In Sec. 2 we calculate the correction to the phase velocity of an *o*-wave in a twisted NLC cell to which an external magnetic field is applied. The latter makes it possible, in principle, to measure the number of half-turns of the director. In Sec. 3 we describe the results of experimental investigations of increased transparency of NLC to an *o*-wave and to the feature of passage of light around a disclination. In Sec. 4 we discuss possible applications of the effect.

2. CORRECTIONS TO THE PHASE OF AN ADIABATICALLY PROPAGATING *o*-WAVE

The Maxwell equations for monochromatic radiation can be written in the form

$$\operatorname{rot}\operatorname{rot}(\hat\varepsilon^{-1}\mathbf{D})-\frac{\omega^2}{c^2}\mathbf{D}=0, \tag{3}$$

where $\mathbf{D}\exp(-i\omega t)+\text{c.c.}$ is the electric induction vector, $\hat\varepsilon$ is the dielectric-constant tensor, and $\hat\varepsilon\hat\varepsilon^{-1}=1$. The dielectric constant of an NLC with local orientation of the director $\mathbf{n}(\mathbf{r})$ ($|\mathbf{n}|\equiv 1$) at optical frequencies can be represented in the form

$$\varepsilon_{ik}=\varepsilon_\perp\delta_{ik}+(\varepsilon_\parallel-\varepsilon_\perp)n_in_j+i\mu_1e_{ijs}H_s+i\mu_2(e_{ips}n_pn_j+n_in_pe_{pjs})H_s. \tag{4}$$

Here $\varepsilon_\perp=n_\perp^2$, $\varepsilon_\parallel=n_\parallel^2$, $n_\perp$ and $n_\parallel$ are the corresponding refractive indices, μ_1 and μ_2 are constants indicative of the influence of a static magnetic field of Faraday type on the propagation of light, and e_{ijk} is the unit antisymmetric Levi–Civita tensor. In particular, were we to put $\varepsilon_\parallel\approx\varepsilon_\perp=n^2$ and $\mu_2=0$, the Faraday rotation could take the form

$$\frac{d\alpha}{dz}\left[\frac{\text{rad}}{\text{cm}}\right]=\frac{\omega}{c}\frac{\mu_1H}{2n}. \tag{5}$$

The field $\mathbf{D}(\mathbf{r})$ corresponding to an *o*-wave propagating in the z direction can be represented in the form

$$\mathbf{D}(\mathbf{r})=e^{ik_{o}z}[A(\mathbf{r})\mathbf{e}_o(\mathbf{r})+B(\mathbf{r})\mathbf{e}_e(\mathbf{r})+C(\mathbf{r})\mathbf{e}_z], \tag{6}$$

where

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$$k_o=\omega n/c,\ |\mathbf{e}_e|=|\mathbf{e}_o|=1,\ \mathbf{e}_e\sim\mathbf{n}(\mathbf{r})-\mathbf{e}_z(\mathbf{n}\mathbf{e}_z),\ \mathbf{e}_o=[\mathbf{e}_z\mathbf{e}_e].$$

For a homogeneous medium $[\mathbf{n}(\tau)=\mathbf{n}_0=\text{const}]$ without a magnetic field, the solution for the o-wave is $A=\text{const}$, $B=C=0$. If $\mathbf{n}(\mathbf{r})$ varies smoothly and $\mathbf{H}\neq 0$, the components B and C contain the terms $(n_\parallel-n_\perp)^{-1}d\mathbf{n}/dz$ and $(n_\parallel-n_\perp)^{-1}H$, while C contains $d\mathbf{n}/d\mathbf{r}_\perp$ and $\mathbf{H}$ without the small denominator $n_\parallel-n_\perp$ (see below), and $\mathbf{r}_\perp=(x,y)$. We shall therefore neglect C.

The coupled equations for the slowly varying amplitudes $A(\mathbf{r})$ and $B(\mathbf{r})$ can be obtained by substituting (6) in (3) and discarding the inessential terms

$$\frac{\partial A}{\partial z}=\left\{\frac{n_\perp^2}{n_e^2(\theta)}\frac{\partial\varphi}{\partial z}-\frac{1}{2}\left[\frac{\mu_1}{n_e^2(\theta)}+\frac{\eta_2\sin^2\theta}{n_\parallel^2}\right]\frac{\omega}{c}n_\perp H\right\}B(z),\tag{7}$$

$$\frac{n_\perp^2}{n_e^2(\theta)}\frac{\partial B}{\partial z}=-\frac{i}{2}\frac{\omega n_\perp}{c}\left[\frac{n_\perp^2}{n_e^2(\theta)}-1\right]B(z)$$
$$-\left\{\frac{\partial\varphi}{\partial z}-\frac{1}{2}\left[\frac{\mu_1}{n_e^2(\theta)}+\frac{\mu_2\sin^2\theta}{n_\parallel^2}\right]\frac{\omega}{c}n_\perp H\right\}A(z).\tag{8}$$

It is assumed here that the director has a polar angle $\theta(\mathbf{r})$ and an azimuthal angle $\varphi(\mathbf{r})$:

$$\mathbf{n}(\mathbf{r})=\mathbf{e}_z\cos\theta+\mathbf{e}_x\sin\theta\cos\varphi+\mathbf{e}_y\sin\theta\sin\varphi.\tag{9}$$

Since we chose the principal exponential dependence $\exp(ik_oz)$, these equations are good when $|B|\ll|A|$. In first order approximation B-order follows quasistatically the initial amplitude A. This can be shown by neglecting in (8) the term $\partial B/\partial z$, so that

$$B\sim\frac{ic}{\omega(n_\perp-n_\parallel)}\left[\frac{\partial\varphi}{\partial z}-\frac{1}{2}\left(\mu_1+\mu_2\sin^2\theta\right)\frac{\omega}{cn}H\right].\tag{10}$$

The dimensionless small parameter that determines the accuracy of the adiabatic following is the relation of $\partial\varphi/\partial z$ to the mismatch $w[n_e(\theta)-n_o]/c$ between the wave vectors k_e and k_o. The e-wave admixture can be qualitatively explained as follows. The o-wave polarization "wants" to go from a certain cross section z_0 to the next cross section z_0+h with its initial vector $\mathbf{e}_0(z_o)$. In the cross section z_0+h, however, the new polarization vector

$$\mathbf{e}_o(z_0+h)\approx\mathbf{e}_o(z_0)+h\mathbf{e}_e(z_0)\partial\varphi/\partial z$$

contains part of the unit vector $\mathbf{e}_e$ of the extraordinary wave. The effective length h over which such a transformation can be regarded as coherent is the mismatch length $h=c/\omega(n_o-n_e)$ and $h\partial\varphi/\partial z$ gives a good estimate of the mixing B.

It is of interest to note that if a magnetic field is present, the Faraday effect can produce the rotation speed necessary to follow up the rotation rate of the local optical axis. This can be regarded as an explanation of the summary contributions from $\partial\varphi/\partial z$ and H, which can cancel each other at a suitable value of H.

The admixed e-wave influences in turn the initial o-wave, resulting in an additional phase shift for the latter. Namely, substitution of B [Eq. (10)] in (7) yields

$$\partial A/\partial z=i\delta k_o(z)A,\tag{11a}$$

$$\delta k(z)\approx\frac{c}{\omega[n_\perp-n_e(\theta)]}\left[\frac{\partial\varphi}{\partial z}-\frac{1}{2}\left(\frac{\mu_1}{n_e^2}+\frac{\mu_2\sin^2\theta}{n_\parallel^2}\right)\frac{\omega}{c}n_\perp H\right]^2.\tag{11b}$$

We consider first the case $H\equiv 0$. The additional phase shift for the o-wave is

$$\gamma_o(x,y)=\int_0^L\delta k(x,y,z)dz+m\pi.$$

The term $m\pi$ describes here the possible reversal of the sign of the o-wave, when the angle $\varphi(z)$ is equal to $m/2$ complete revolutions along the ray $x=x_0$, $y=y_0$, and $0\leqslant z\leqslant L$:

$$m=\frac{1}{\pi}\int_0^L\frac{\partial\varphi}{\partial z}dz.\tag{12}$$

The boundary between regions with different values of m corresponds to the projection of a disclination line in the NLC volume on the xy plane.

Assume the existence of a certain local region for which $\partial\varphi/\partial z$ is almost constant over its entire length l. The number of independent contributions to $\gamma_o(x,y)$ is then approximately $N\sim L/l$ and the total fluctuation is

$$\delta\gamma_o\sim N^{1/2}l|\delta k|\sim\frac{c}{\omega}\frac{(Ll)^{1/2}}{n_e-n_o}\left(\frac{\partial\varphi}{\partial z}\right)^2.\tag{13}$$

A numerical estimate yields the following. Let $l\sim 10^{-2}$ cm, $|\partial\varphi/\partial z|\sim 1$ rad$/10^{-2}$ cm $=10^{-2}$ cm^{-1}, $L\sim 10^{-1}$ cm, $N=10$, and $n_e-n_o\sim 0.15$. We have then $\delta\gamma_o\approx 6.7\cdot 10^{-3}$ rad, meaning that the phase distortions of the o-waves are very small.

Notice must be taken of an important physical aspect, namely, whether it is possible to measure the number m of half-revolutions of the o-polarization along the ray. Introduction of a magnetic field permits an affirmative answer to this question. In fact, if we measure the difference of the phase shifts for $\mathbf{H}=H_0\mathbf{e}_z$ and $\mathbf{H}=-H_0\mathbf{e}_z$,

$$\Delta\gamma=\gamma_o(x_0,y_0,H_0)-\gamma_o(x_0,y_0,-H_0)$$
$$=-2\int\frac{1}{[n_\perp-n_e(\theta)]}\frac{\partial\varphi}{\partial z}\left(\frac{\mu_1}{n_e^2}+\frac{\mu_2\sin^2\theta}{n_\parallel^2}\right)n_\perp H_0\,dz.\tag{14}$$

If $\theta(z)=\pi/2$ and $\varphi=\varphi(z)$, then the phase difference is

$$\Delta\gamma=2\frac{n_\perp}{n_\parallel^2(n_\perp-n_\parallel)}(\mu_1+\mu_2)H_0m.\tag{15}$$

Even in the case $\theta=\theta(z)$ the expression (14) shows that some information on $\partial\varphi/\partial z$ is contained in $\delta\gamma$.

Note that the phase shift $\Delta\gamma$ accumulates during successive back and forward passages of the light through the cell as in the time-asymmetric Faraday effect. An effect of this kind is discussed in Refs. 1 and 2.

Numerical estimates are not very promising for measurements using the Faraday effect, for $\theta(z)=\pi/2$ and $\partial\varphi/\partial z=\pi/L$ (i.e., for $m=1$) and $\mu_1\sim 10^{-10}$ rad/Oe (corresponding to a Verdet constant $r\approx 3.25\ 10^{-6}$ rad/cm$\cdot$Oe $\sim 10^{-2}$ min/Oe for $\lambda=6328$ Å) one can obtain $\Delta\gamma\approx 2\cdot 10^{-9}$ rad/Oe. This means that in an external magnetic field $H\sim 10^4$ Oe an o-wave passing through an NLC with disclinations will acquire a phase shift $\approx 2\cdot 10^{-5}$ rad per complete rotation of the polarization vector.

Thus, the presented theoretical treatment can be briefly expressed as follows: 1) it is possible to pass an image with the aid of an o-wave through relatively thick cells with NLC; 2) a possible phase jump π in the transmitted wave is possible; 3) by using a longitudinal magnetic field it is possible to

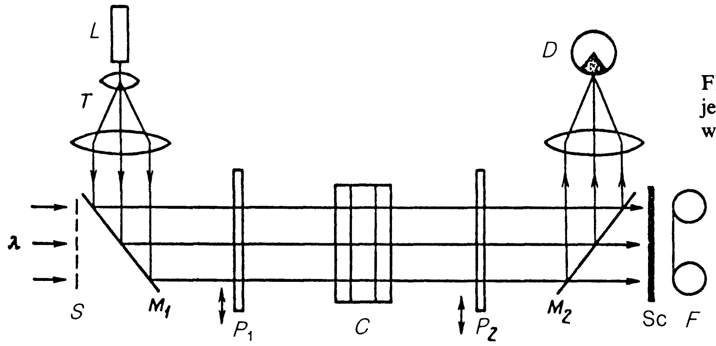

FIG. 1. Experimental setup: *L*-He-Ne laser, *T*—telescope, *S*—slide projector, *M* 1, *M* 2—semitransparent mirrors, *P* 1, *P* 2—polarizers, *C*—cell with NLC, Sc—screen, *D*—diode, *F*—photographic film.

determine the number of rotations of the polarization vector in π radians.

3. EXPERIMENTAL METHODS

The experimental setup is shown in Fig. 1. Light from a slide projector or from a laser passed through a polarizer capable of altering the input polarization, and was next incident on a cell with an NLC. When a slide projector was used, the image of the slide was focused on a screen or photographic film. When a laser was used the divergence of its emission was corrected with a lens or a telescope. The cell was made up of two glass plates 4 mm thick separated by a teflon liner of thickness 0.4 to 5 mm and filled with 5 CB liquid crystal. A planar orientation of the NLC was reached by the standard method of coating the inner walls of the cell with a polymer followed by buffing with rough calico. The director orientation inside the volume, however, was not regular, especially in cells thicker than 1 mm.

In the absence of polarizers, the cell with the NLC was quite opaque and no image whatever could be seen through such a cell with the unaided eye. But when a polarizer was added to the system, and furthermore in such a way that either the incoming or the outgoing wave (or both simultaneously) had ordinary polarization, the image was transmitted through the cell without any distortion. The corresponding pictures of the original slide and if its images in *o*- and *e*-waves are shown in Fig. 2.

The cell measured 20×30 mm^2, and in our experiments almost all of it was illuminated. It can be noted that an *o*-wave permits information to pass through the cell with almost no distortion, even if the thickness is 5 mm. At the same time, an *e*-wave is strongly distorted even at ≈ 3 mm thick-

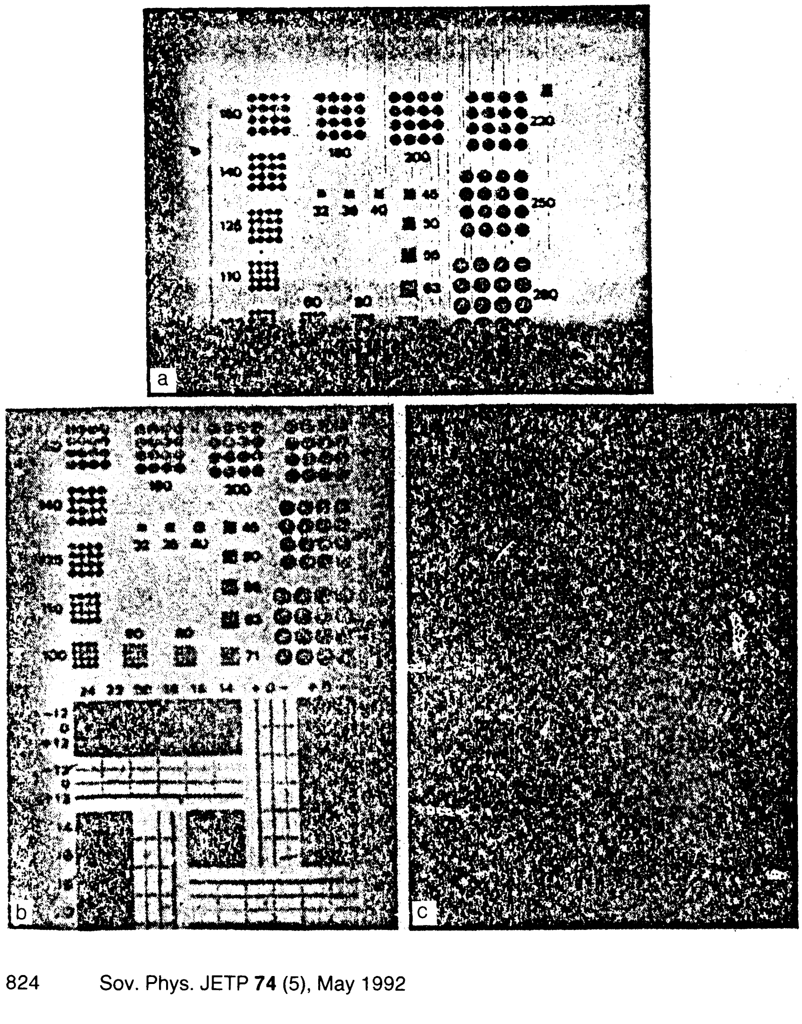

FIG. 2. Object slide (a) and its image on a screen behind an NLC cell 3 mm thick for an *o*-wave (b) and for an *e*-wave (c).

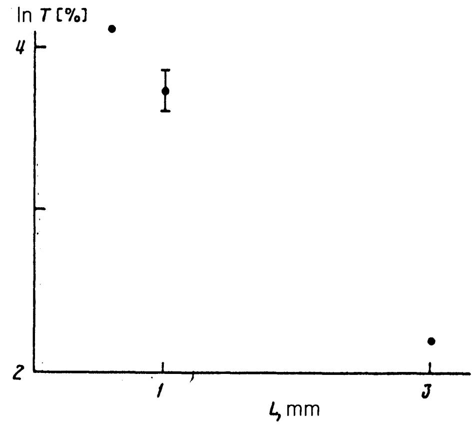

FIG. 3. Dependence of the transmission T (logarithmic scale) of a cell with NLC on its L thickness.

ness (see Fig. 2c). At thicknesses $\gtrsim 3$ mm it is altogether impossible to transmit any image with the aid of an e-wave.

The next run of experiments was performed with an He–Ne-laser source. A collimated beam of ≈ 2 mm diameter was directed to the cell with both o- and e-polarization. The light passed by the cell consisted of two parts. One corresponded to diffuse scattering of the incident wave over a wide angle (this was apparently scattering by thermodynamic vibrational fluctuations of the director orientation, called molecular scattering). At cell thicknesses $\geqslant 3$ mm this part of the radiation was almost completely depolarized, regardless of the type of incident wave (o- or e-). The second part was the image. This information-carrying part of the beam was considerably attenuated by molecular scattering (see above). Its degree of polarization, however, remained quite high for both o- and e-waves. This means that the assumption that the polarization of the radiation passing through the cell follows adiabatically the orientation of the director is well satisfied in our experimental situation.

The incident e-wave was strongly distorted, apparently owing to static inhomogeneities of the director inside the cell. We measured the radiation fraction carried by an o-wave through an iris 2 mm in diameter, mounted 10 cm behind the cell. The transmission T was 0.57, 0.37, and 0.08 for cells 0.6, 1, and 3 mm thick. Allowing for Fresnel loss to reflections, we have $T_{0.6} = 0.62$, $T_1 = 0.42$, $T_3 = 0.09$. Figure 3 shows the dependence of $\ln T$ on the cell thickness L. The (intensity) extinction coefficient R was ≈ 8 cm^{-1}, close to the theoretical estimates.[3,4]

It should also be noted that the use of Co–Sm magnets made it possible to induce strong cell inhomogeneities and thereby imitate an astigmatic lens for the e-wave. The focal lengths of such a "magnetic" lens ≈ 3 mm thick were $F \sim 5$–15 cm, depending on the magnetic-field "exposure" time, and the lens itself relaxes ≈ 5 minutes after the lens was moved away. All these manipulations with a magnetic field did not affect the o-wave passing through the cell.

Illumination with a wide laser beam made possible observation of disclinations inside the cell. The largest number was observed immediately after filling the cell with the liquid crystal, but after some time (from several hours to several days, depending on the cell thickness) their number decreased to a minimum or altogether to zero. For a cell 3 mm thick, the transverse distance between disclinations was ≈ 1 cm and the disclination shape was irregular. Figure 4a shows typical disclinations in an NLC (photographed for an o-wave). Figure 4b shows the transmitted o-polarized light at a distance 200 mm from the cell. The theory predicts a π phase shift in an o-wave passing from opposite sides of a disclination.

We have verified this fact by three different methods, two linear-optical and one nonlinear-optical.

1. Comparing the theoretical expression

$$E(z,x)=E_0 e^{ikz}\left(\frac{2}{i\pi}\right)^{1/2}\int_0^t e^{i\tau^2}\,d\tau,\qquad t=x\left(\frac{k}{2z}\right)^{1/2}$$

for Fresnel diffraction of a field $E(z=0,x) = E_0 x/|x|$ of wavelength $\lambda = 2\pi/k$ at a distance z with the experimental intensity distribution near a disclination. The agreement was fully acceptable.

2. Using the standard interferometry method to record the phase at an o-wave at its exit from the cell. The interference pattern in Fig. 5 shows clearly a π shift between adjacent parts of the wave front.

3. Using the second harmonic (SH, $\lambda = 0.53\,\mu$m) of the radiation from a pulsed YAG:Nd laser ($\lambda = 1.06\,\mu$m). The setup consisted of two KTP crystals with the liquid-crystal cell between them (thanks are due to Yu. E. Kapitskiĭ and A. N. Chudinov for affording the possibility of performing these experiments with the setup they created for other purposes). The amplitude of the exit SH radiation was coherent with the sum of the SH amplitudes generated separately in the first and second crystals. By varying the distance between the KTP crystals we could vary the relative phase of these contributions: the dispersion of the refractive index of air produces a phase shift

$$\gamma_{2\omega}-2\gamma_\omega=2\omega d[n(2\omega)-n(\omega)]/c$$

at a distance d, and the use of a photodiode yielded the corre-

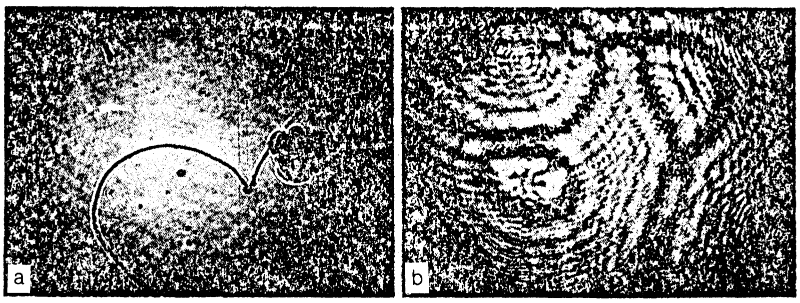

FIG. 4. Typical disclination in a thick cell with NLC illuminated by an o-wave; a—image of disclination in the cell plane; b—image of disclination in a plane 20 cm away from the cell.

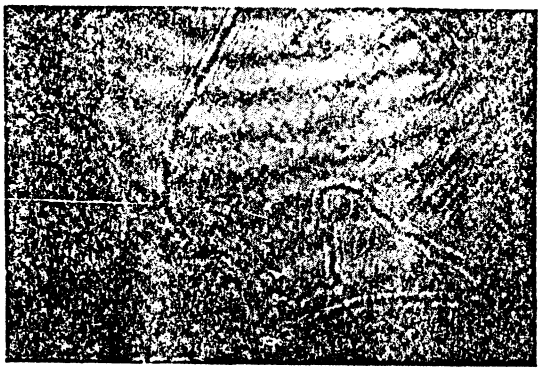

FIG. 5. Typical interference pattern of a π phase shift relative to opposite sides of a disclination.

sponding interference dependence

$$I_{2\omega}(d) \sim A + B\cos[\gamma_0 + (\gamma_{2\omega} - 2\gamma_\omega)].$$

We measured the change of γ_0 at the instant of a transverse displacement of the laser beam over the cell with a crossing of a disclination line. The measured value was $\Delta\gamma_0 = 2.8 \pm 0.4$ rad, in good agreement with the theoretical estimate $\Delta\gamma_0 = \pi$ (note that passage of an o-wave from opposite sides of a disclination reverses the signs of the term $\mathbf{E}_\omega$ and $\mathbf{E}_{2\omega}$, but leaves unchanged the sign of the term $\mathbf{E}_\omega \mathbf{E}_\omega$ responsible for the SH generation in the second KTP crystal).

DISCUSSION OF RESULTS

We touch here upon only one but very important aspect of our investigations, the difference between an undistorted image carried through a thick liquid crystal by an o-wave and by a wave repeatedly reflected in the same liquid crystal, having the same polarization, and incident at the same solid angle as the image wave. The difference is that an undistorted image, albeit very strongly distorted by the molecular scattering, preserves its time coherence with the beam incident on the cell, whereas the molecularly scattered part of the radiation loses this coherence, undergoing a spectral shift $\Omega/2\pi \sim 10$–100 Hz. This makes it possible to record the undistorted part of the e-wave by amplifying it, for example, in a photorefracting crystal pumped by the same laser beam as used to illuminate the cell, since the crystal response time $\tau \sim 1$–10 s does not permit the molecularly scattered wave to become amplified in view of its large frequency shift. Thus, the existence of undistorted transmission of an o-wave through thick cells with NLC gives grounds for hoping to use it in research as well as in technology.

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