Absorption of weak electromagnetic radiation by a semiconductor in the field of a strong electromagnetic wave in a quantizing magnetic field

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The two-band model of a semiconductor, described by an equation of the Dirac type, is used to calculate the absorption coefficient of weak electromagnetic radiation by a semiconductor placed in a constant magnetic field and acted upon by a strong resonant laser field. The dependences of the absorption coefficient on temperature and frequency are determined for a weak as well as a strong electromagnetic field. It is shown that the absorption spectrum has a number of singularities whose positions depend substantially on the magnetic field and on the laser-emission intensity. The results are compared with those obtained earlier on the basis of a simpler model of the semiconductor.

A number of recent papers[1-6] are devoted to a theoretical investigation of optical phenomena brought about by restructuring of the energy spectrum of a semiconductor by strong laser emission that causes resonant single-photon transitions of an electron between the edges of the valence and conduction bands. The used semiconductor model[7,8] is described by an equation formally analogous to the Dirac equation. The solutions of the two-band equation determine the states of the electrons and holes in the narrow-band semiconductors PbS, PbSe, and PbTe. Account is taken of the spin properties of the carriers, and in the presence of an electric field also of the spin-orbit interaction.

It is shown in Refs. 1 and 2 that allowance for the spin properties leads to a result that is substantially new compared with an earlier physical one,[9] a dependence of the quasienergy spectrum of the electrons and holes of the semiconductors on the type of polarization of the strong laser field. According to Refs. 3–5, the dependence of the quasienergy spectrum of the carriers on the type of polarization of the exciting field makes it relatively easy to control the optical characteristic of the semiconductor.

Another possibility of controlled variation of the optical characteristics of semiconductor materials acted upon by intense laser radiation is by using additional static fields. It is shown in Ref. 6 that the static magnetic field **H** that causes a radical restructuring of the carrier quasienergy spectrum produces substantial changes in the semiconductor recombination-radiation spectrum. The presence of an additional parameter—the magnetic-field strength—makes control of these variations possible.

The present paper is a continuation of an examination, initiated in Ref. 6, of the optical effects due to the joint action of strong laser radiation and a static magnetic field. It is devoted to a theoretical investigation of the absorption of weak electromagnetic radiation by a semiconductor placed in a constant magnetic field

$$\mathbf{H}=(0,\ 0,\ H) \tag{1}$$

and acted upon by an alternating electric field

$$\mathbf{E}(t)=(0,\ 0,\ E_0\sin\omega_0 t), \tag{2}$$

having a frequency ω_0 close to the band gap ε_g of the semiconductor (a system of units with $\hbar=1$ is used),

$$\omega_0=\varepsilon_g+\Delta.\quad |\Delta|\ll\varepsilon_g.$$

It is assumed that the magnetic field is strong enough, such that

$$\omega_H\gg\tau^{-1},$$

where ω_H is the cyclotron frequency of the electron (hole) and τ is the relaxation time of the electron subsystem. The alternating electric field is assumed to be strong in the sense of satisfaction of the inequality[1]

$$\rho\omega_0\gg\tau^{-1},$$

where $\rho=2e_0E_0s\omega_0^{-2}\ll 1$, $s=-(\varepsilon_g/2m_0)^{1/2}$, and e_0 and m_0 are, respectively, the absolute value of the charge and the effective mass of the electron. We describe the weak electromagnetic field with the aid of the potential $\mathbf{A}=(\mathbf{r},t)=\mathbf{a}e^{-i\omega t}+\mathbf{a}^*e^{i\omega t}$.

We describe the absorption coefficient $K(\omega)$ with the aid of the approximate solutions obtained in Ref. 10 (see also Ref. 6) for the case of an external field constituting a superposition of fields (1) and (2). The use of the semiconductor model proposed in Refs. 7 and 8 in the problem on hand is of interest in view of the satisfactory agreement between the theory based on the use of a Dirac-type two-band equation and the experimental data on multiphoton magneto-absorption in the narrow-band semiconductors PbS, PbSe, and PbTe (Ref. 11).

By simple calculations similar to those in Ref. 5, we get

$$K(\omega)=\frac{m_0e_0^2s^2\omega_H}{4\pi c\omega\varepsilon^{1/2}}\sum_{l=0,1}\sum_{m,m'=0,1,2}|e^{(m-m')}|^2\Lambda_{m,m'}^{(l)}(\omega). \tag{3}$$

$$\begin{aligned}\Lambda_{m,m'}^{(0)}(\omega)={}&(m_0s)^{-2}\int dp_z[(2-\delta_{m,0})\delta_{m,m'}p_z^2\\&+{}^1/_4(\delta_{m,m'+1}+\delta_{m,m'-1})(p_m^2+p_{m'}^2)]\\&\times\{[f_m(p_z)-f_{m'}(p_z)][U_m(p_z)U_{m'}(p_z)\\&-V_m(p_z)V_{m'}(p_z)]^2[\delta(E_m(p_z)\\&-E_{m'}(p_z)-\omega)-\delta(E_m(p_z)-E_{m'}(p_z)+\omega)]\\&+[1-f_m(p_z)-f_{m'}(p_z)][U_m(p_z)\\&\times V_{m'}(p_z)+V_m(p_z)U_{m'}(p_z)]^2[\delta(E_m(p_z)+E_{m'}(p_z)-\omega_0-\omega)\\&-\delta(E_m(p_z)+E_m(p_z)-\omega_0+\omega)]\}\end{aligned} \tag{4}$$

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$$\Lambda^{(1)}_{m,m'}(\omega)=\int dp_z[(2-\delta_{m,0})\delta_{m,m'}+\delta_{m,m'+1}+\delta_{m,m'-1}]\{[f_m(p_z)$$
$$-f_{m'}(p_z)]U_m^2(p_z)V_{m'}^2(p_z)[\delta(E_m(p_z)-E_{m'}(p_z)+\omega_0-\omega)$$
$$-\delta(E_m(p_z)-E_{m'}(p_z)$$
$$-\omega_0+\omega)]+[1-f_m(p_z)-f_{m'}(p_z)]$$
$$\times[U_m^2(p_z)U_{m'}^2(p_z)\delta(E_m(p_z)+E_{m'}(p_z)-\omega)$$
$$-V_m^2(p_z)V_{m'}^2(p_z)\delta(E_m(p_z)+E_{m'}(p_z)+\omega-2\omega_0)]\}. \quad (5)$$

Here

$$e^{(0)}=e_z,\quad e^{(\pm1)}=e_x\pm ie_y,\quad \mathbf{e}=\frac{\mathbf{a}}{|\mathbf{a}|},$$
$$U_m(p_z)=\left\{\frac{1}{2}\left[1+\frac{|\delta_m(p_z)|}{\lambda_m(p_z)}\right]\right\}^{1/2},$$
$$V_m(p_z)=\operatorname{sign}\delta_m(p_z)\left\{\frac{1}{2}\left[1-\frac{|\delta_m(p_z)|}{\lambda_m(p_z)}\right]\right\}^{1/2},$$
$$\lambda_m(p_z)=\left[\delta_m^2(p_z)+\left(\frac{\rho}{2}\right)^2\right]^{1/2},\quad \delta_m(p_z)=\frac{2\varepsilon_m(p_z)}{\omega_0}-1,$$
$$\varepsilon_m(p_z)=s[(m_0s)^2+p_m^2+p_z^2]^{1/2},$$
$$p_z=\left(\frac{2e_0Hm}{c}\right)^{1/2}.$$

The quantity

$$E_m(p_z)=\frac{\omega_0}{2}[1+\lambda_m(p_z)\operatorname{sign}\delta_m(p_z)]$$

has the meaning of the electron (hole) quasienergy, and its dependence on the z-component of the momentum $\mathbf{p}$ is shown in the figures. The stationary distribution functions of the electrons (holes) take the form of the Fermi distributions[6]

$$f_m(p_z)=\{\exp[\beta(E_m(p_z)-\omega_0/2]+1\}^{-1}, \quad (6)$$

where $\beta=1/k_BT$, k_B is the Boltzmann constant and T is the absolute temperature of the heat bath. Expression (6) is valid under the conditions

$$\omega_H,\ \rho\omega_0<\Omega,\ (\Delta/\rho\omega_0)^2\ll\tau_R/\tau,$$

in which Ω is the photon frequency and τ_R is the lifetime of the electron–hole pair.

The function $\Lambda^0_{mm'}(\omega)$ describes the absorption of a weak electromagnetic signal of frequency $\omega\ll\omega_0$, due to the electron intraband transitions and also to the transitions between the conduction (valence) band and the single-photon replica of the valence (conduction) band. The function $\Lambda^{(1)}_{mm'}(\omega)$ is responsible for the absorption in the spectral region $|\omega-\omega_0|\ll\omega_0$, due to quantum transitions of an electron between the valence and conduction bands, their single-photon replicas, and transitions between the conduction (valence) band and its one-photon replica.

All the above transitions occur both when the Landau-level number $m=0, 1, 2, \ldots$ remains unchanged and when it changes by unity. The first and second types of transition are due, respectively, to absorption of weak signals with polarization $\mathbf{e}\|\mathbf{H}$ and $\mathbf{e}\perp\mathbf{H}$.

In the calculation of the functions (4) and (5) we confine ourselves to the band-parabolicity approximation, as-

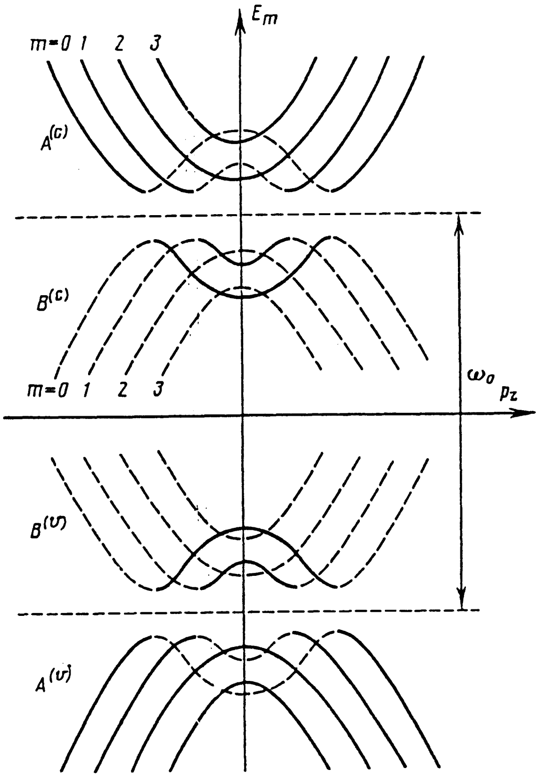

FIG. 1. Quasienergy spectrum of electron at $\Delta>0$. Solid lines—valence band and conduction band; dashed—single-photon replicas of these bands.

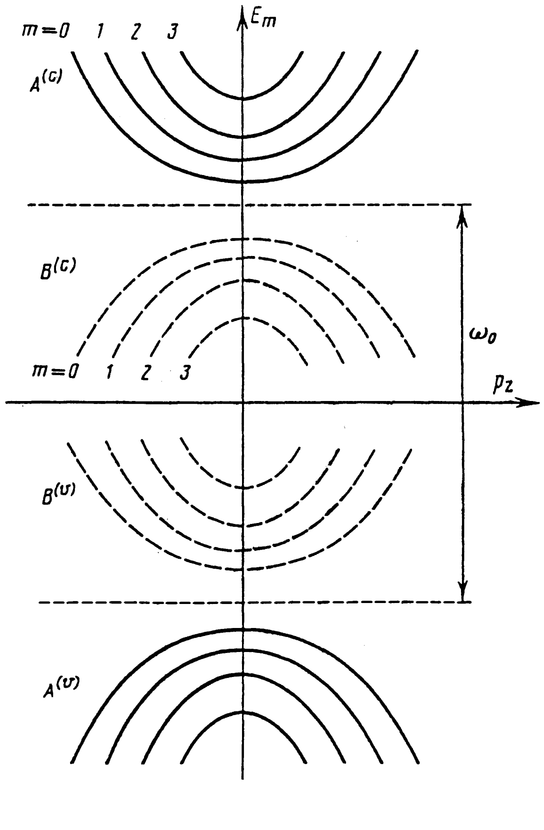

FIG. 2. Quasienergy spectrum of electron at $\Delta<0$.

suming that $\omega_H \ll \varepsilon_g$. Integrating with respect to p_z with the aid of a delta function, we obtain

$$K(\omega)=\frac{m_0^2 e_0^2 s^3 \varkappa_H}{4\pi c\omega\varepsilon^{1/2}}$$

$$\times\sum_{j=\pm 1}\sum_{k,l=0,1}\sum_{m=0,1,2}(2-\delta_{k,0}\delta_{m,0})\,\mathrm{sh}\frac{\beta\xi_l\omega_0}{4}\left\{\mathrm{ch}\frac{\beta\xi_l\omega_0}{4}\right.$$

$$\left.+\mathrm{ch}\left[\frac{\beta k\varkappa_H\omega_0}{4}\left(\frac{\xi_l^2-k\varkappa_H^2-\rho^2}{\xi_l^2-k\varkappa_H^2}\right)^{1/2}\right]\right\}^{-1}$$

$$\frac{\theta[(\xi_l^2-k\varkappa_H^2-\rho^2)(\xi_l^2-k\varkappa_H^2)]\,\theta(D_{k,l,m,j})}{[(\xi_l^2-k\varkappa_H^2-\rho^2)(\xi_l^2-k\varkappa_H^2)]^{1/2}\,(D_{k,l,m,j})^{1/2}}\lambda_{k,l,m,j}, \tag{7}$$

$$\lambda_{k,0,m,j}=\frac{\rho^2\xi_0^2}{|\xi_0^2-k\varkappa_H^2|}\left[|e_z|^2\delta_{k,0}D_{k,0,m,j}\right.$$

$$\left.+(|e_x|^2+|e_y|^2)\frac{\delta_{k,1}(2m+1)\varkappa_H}{2}\right], \tag{8}$$

$$\lambda_{k,1,m,j}=\frac{1}{4}(|\xi_1^2-k\varkappa_H^2-\rho^2|^{1/2}+j|\xi_1^2-k\varkappa_H^2|^{1/2})^2\cdot$$

$$\times[|e_z|^2\delta_{k,0}+(|e_x|^2+|e_y|^2)\delta_{k,1}],$$

$$D_{k,l,m,j}=2\frac{\Delta}{\omega_0}-2(2m+k)\varkappa_H+j\xi_l\left(\frac{\xi_l^2-k\varkappa_H^2-\rho^2}{\xi_l^2-k\varkappa_H^2}\right)^{1/2}$$

$$\xi_l=\frac{2(\omega-l\omega_0)}{\omega_0},\quad \varkappa_H=\frac{2\omega_H}{\varepsilon_g}\ll 1. \tag{9}$$

Let us use (7)–(9) to study the features of the absorption of a weak signal of frequency $\omega \ll \omega_0$. We consider first a physical situation in which $\mathbf{e}\|\mathbf{H}$. In this case, if $0 < -\Delta < \Omega$, the semiconductor is transparent at $\omega < \Omega_0$, where $\Omega_m = [(2\omega_H m - \Delta)^2 + (\rho\omega_0/2)^2]^{1/2}$, $m = 0,1,2,\ldots$.

The absorption in the region $\omega > \Omega_0$ is due to quantum transitions of electrons between the quasienergy bands $A^{(i)}$ and $B^{(i)}$ $(i = c,v)$ and decreases as the temperature rises. The absorption coefficient has in this region a square-root singularity at the frequency $\omega = \Omega_m$: $K(\omega) \sim (\omega - \Omega_m)^{1/2}$. For $\Delta > 0$, a gap appears in the quasienergy spectrum of the semiconductor electrons. At the gap boundaries the spectrum has an additional van Hove singularity.[12] The transparency region is determined in this case by the inequality $\omega < \rho\omega_0/2$, and the absorption coefficient has in addition to the indicated square-root singularities a peak $K(\omega) \sim (\omega - \rho\omega_0/2)^{-1/2}$ at the boundary frequency $\omega = \rho\omega_0/2$.

If $\mathbf{e}\perp\mathbf{H}$, there is no absorption in the regions $0 < \omega < (\Omega_1 - \Omega_0)/2$ and $\omega_H < \omega < (\Omega_1 + \Omega_0)/2$ at $\omega < \omega_H$. Absorption takes place in the region $(\Omega_1 - \Omega_0)/2 \leqslant \omega < \omega_H$ as a result of internal transitions of the electrons. As the absolute temperature is lowered to zero, the absorption vanishes. Absorption at $\omega \geqslant (\Omega_1 + \Omega_0)/2$ is accompanied by electron transitions between the quasienergy levels $A^{(i)}$ and $B^{(i)}$ $(i = c,v)$ and decreases as the temperature rises. These two absorption regions contain the frequencies $\omega = |\Omega_{m+1} \pm \Omega_m|/2$, at which the absorption coefficient has peaks. At $\Delta > \omega_H$ the transparency region defined by the condition $\omega_H < \omega < [\omega_H^2 + (\rho\omega_0/2)^2]^{1/2}$ splits into two absorption regions $0 < \omega < \omega_H$ and $\omega \geqslant [\omega_H^2 + (\rho\omega_0/2)^2]^{1/2}$. Absorption peaks exist both at $\omega = |\Omega_{m+1} \pm \Omega_m|/2$ and $\omega = [\omega_H^2 + (\rho\omega_0/2)^2]^{1/2}$. The latter is connected with the Van Hove singularities located near the quasienergy gap. As the temperature is lowered, the absorption in the region $0 < \omega < \omega_H$ decreases to zero, while in the region $\omega \geqslant [\omega_H^2 + (\rho\omega_0/2)^2]^{1/2}$ it increases.

The most interesting singularities are observed for weak-signal absorption in the spectral region $|\omega - \omega_0| \ll \omega_0$. Let us examine each of the cases $\mathbf{e}\|\mathbf{H}$ and $\mathbf{e}\perp\mathbf{H}$ separately.

At $\mathbf{e}\|\mathbf{H}$ we have weak-signal absorption and amplification regions separated by transparency regions. The function $K(\omega)$ has peaks in each region (absorption and amplification). Absorption and amplification result from electron transitions from the quasienergy band $A^{(v)}$ into the band $A^{(c)}$, and from $B^{(c)}$ into $B^{(v)}$, respectively. As the temperature is raised the absolute value of the absorption coefficient decreases. If $0 < -\Delta < \Omega$, the location of the amplification (absorption) region is determined by the condition $\omega \leqslant \omega_0 - \Omega_0$ $(\omega \geqslant \omega_0 + \Omega_0)$. The absorption-coefficient peaks occur at the frequencies $\omega = \omega_0 \pm \Omega_m$. For $\Delta > 0$ the semiconductor is transparent in the region $\omega_0 - \rho\omega_0/2 < \omega < \omega_0 + \rho\omega_0/2$.

A weak signal is amplified at $\omega < \omega_0 - \rho\omega_0/2$ and is absorbed at $\omega = \omega_0 + \rho\omega_0/2$. The frequencies $\omega = \omega_0 \pm \rho\omega_0/2, \omega_0 \pm \Omega_m$ determine the positions of the absorption-coefficient peaks.

In the case when $\mathbf{e}\perp\Omega$ and $\Delta < \omega_H$ there are three transparency regions: $\omega_0 - |\Omega_0 + \Omega_1|/2 < \omega < \omega_0 - \omega_H$, $\omega_0 + \omega_H < \omega < \omega_0 + |\Omega_0 + \Omega_1|/2$ and $\omega_0 - |\Omega_0 - \Omega_1|/2 < \omega < \omega_0 + |\Omega_0 - \Omega_1|/2$. A weak signal is amplified at $\omega < \omega_0 - |\Omega_0 + \Omega_1/2$ and $\omega_0 - \omega_H < \omega < \omega_0 - |\Omega_0 - \Omega_1|/2$ and absorbed at $\omega_0 + |\Omega_0 - \Omega_1|/2 < \omega < \omega_0 + \omega_H$ and $\omega > \omega_0 + |\Omega_0 + \Omega_1|/2$. Absorption and amplification peaks are located at $\omega = \omega_0 - |\Omega_m \pm \Omega_{m+1}|/2$ and $\omega = \omega_0 + |\Omega_m \pm \Omega_{m+1}|/2$, respectively. The amplification in the region $\omega_0 - \omega_H < \omega < \omega_0 - |\Omega_0 - \Omega_1|/2$ is accompanied by two types of electron quantum transitions, between the quasienergy bands $A^{(v)}$ and $B^{(c)}$, and also between the $B^{(v)}$ and $A^{(c)}$ bands. Electronic transitions of the same type cause also the absorption in the region $\omega_0 + |\Omega_0 - \Omega_1|/2 < \omega < \omega_0 + \omega_H$. In this case, however, the transitions are between Landau-level pairs of the indicated bands different from those in the case of amplification. As the temperature is lowered to zero, the amplification and absorption vanish from these regions. Electron transitions between the bands $B^{(c)}$ and $B^{(v)}$ lead to amplification in the region $\omega < \omega_0 - |\Omega_0 + \Omega_1|/2$ while transitions between $A^{(v)}$ and $A^{(c)}$ lead to absorption at $\omega > \omega_0 + |\Omega_0 + \Omega_1/2$. When the temperature is lowered to absolute value of the absorption coefficients increases only somewhat in these regions.

If $\mathbf{e}\perp\mathbf{H}$ but $\Delta > \omega_H$, the absorption coefficient

$$K(\omega)\begin{cases} >0 & \text{for}\quad \omega_0<\omega<\omega_0+\omega_H \text{ and } \omega>\omega_0 \\ & \quad +[\omega_H^2+(\rho\omega_0/2)^2]^{1/2}, \\ <0 & \text{for}\quad \omega_0-\omega_H<\omega<\omega_0 \text{ and } \omega<\omega_0 \\ & \quad -[\omega_H^2+(\rho\omega_0/2)^2]^{1/2}. \end{cases}$$

At the point $\omega = \omega_0$ it reverses sign, and at

$$\omega_0 - [\omega_H^2 + (\rho\omega_0/2)^2]^{1/2} < \omega < \omega_0 - \omega_H$$

and

$$\omega_0 + \omega_H < \omega < \omega_0 + [\omega_H^2 + (\rho\omega_0/2)^2]^{1/2}$$

it is equal to zero. The absorption and amplification peaks are produced at frequencies,

$$\omega = \omega_0 + |\Omega_m \pm \Omega_{m+1}|/2, \quad \omega = \omega_0 + [\omega_H^2 + (\rho\omega_0/2)^2]^{1/2}$$

and

$$\omega = \omega_0 - |\Omega_m \pm \Omega_{m+1}|/2, \quad \omega_0 + [\omega_H^2 + (\rho\omega_0/2)^2]^{1/2}$$

respectively. The amplification and absorption in the region $\omega_0 - \omega_H < \omega < \omega_0 + \omega_H$ vanish as $T \to 0$ and are due to electron transitions between the $A^{(v)}$ and $B^{(c)}$ and also the $B^{(v)}$ and $A^{(c)}$ quasienergy bands. The absorption (amplification) in the region

$$\omega \geqslant \omega_0 + [\omega_H^2 + (\rho\omega_0/2)^2]^{1/2} \quad (\omega \leqslant \omega_0 - [\omega_H^2 + (\rho\omega_0/2)^2]^{1/2})$$

is due to electron transitions between the bands $A^{(v)}$ and $A^{(c)}$ ($B^{(c)}$ and $B^{(v)}$). The absolute value of the absorption coefficient increases at $|\omega - \omega_0| \geqslant [\omega_H^2 + (\rho\omega_0/2)^2]^{1/2}$ when the temperature is lowered.

According to (7) and (8) the absorption coefficient of a weak electromagnetic signal can be represented in the form

$$K(\omega) = K_\parallel(\omega) + K_\perp(\omega),$$

where

$$K_{\parallel,\perp}(\omega) \sim |\mathbf{e}_{\parallel,\perp}|^2,$$
$$\mathbf{e}_\perp = \mathbf{e} - \mathbf{e}_\parallel, \quad \mathbf{e}_\parallel = \mathbf{H}(\mathbf{eH})/H^2.$$

The absorption spectrum of a weak signal with arbitrary polarization **e** is therefore described by the curve obtained by simply adding the two relations described above, one for $\mathbf{e}_\perp = 0$ and the other for $\mathbf{e}_\parallel = 0$.

Comparing the foregoing results with those given in Ref. 9 (see also Ref. 5) we verify that a constant magnetic field influences considerably a weak-signal absorption signal in the presence of strong laser radiation. In the magnetic field the absorption becomes essentially anisotropic and has singularities that depend on the field.

The absorption of a weak electromagnetic signal by a semiconductor in a saturated state and in a quantizing magnetic field was investigated earlier in Ref. 13 on the basis of a simpler two-band model,[14] in which no account was taken of the spin properties of the carriers. It was assumed in Ref. 13 that $T = 0$. We have seen that entire weak-signal spectral absorption or amplification regions vanish in this case. Moreover, a case of interest in itself, when $\Delta < 0$, was not considered in Ref. 13. Assuming, following Ref. 13, that $T = 0$ and $\Delta > 0$, let us compare the results there and the analogous results in the present paper.

The main differences are the following:

1. The absorption in the low-frequency region of the spectrum ($\omega \ll \omega_0$), predicted in Ref. 13, preserves the anisotropy also as $H \to 0$, whereas the absorption coefficient (7) goes over as $H \to 0$ into the anisotropic expression given in Ref. 14.

2. According to (7), absorption of weak signal of frequency ω, located in the vicinity of the frequency ω_0 of strong laser radiation ($|\omega - \omega_0| \ll \omega_0$) is essentially anisotropic, with the number and dimensions of the transparency regions dependent both on the intensity of the latter and on the magnetic field intensity; the expression obtained in Ref. 13 for the absorption coefficient at frequencies $\omega \approx \omega_0$ is isotropic, and the dimension of the only transparency region is determined exclusively by the intensity of the strong laser radiation.

Note that if the summation over $m = 0, 1, 2, \ldots$ in (7) is replaced by summation over $m = \frac{1}{2}, 1 + \frac{1}{2}, 2 + \frac{1}{2}, \ldots$ and the factor $(2 - \delta_{K,0}\delta_{m,0})$ that takes into account the nondegeneracy of the lowest Landau level in spin is omitted, then we arrive, accurate to numerical factors, at the expressions of Ref. 13, which describes the absorption of a weak signal of frequency $\omega \ll \omega_0$ in the case $\mathbf{e} \perp \mathbf{H}$ and $\mathbf{e} \| \mathbf{H}$. To obtain a similar correspondence in the region of ω defined by the inequality $|\omega - \omega_0| \ll \omega_0$ we must put in (7) additionally $\mathbf{e}_\perp = 0$.

In the present paper, just as in Refs. 6 and 13 we use the homogeneous electric field approximation (2). The spatial homogeneity of this field calls, strictly speaking, for an additional investigation. As noted in Ref. 6, one can expect that the effects due to the finite value of to turn out to be negligibly small upon satisfaction of the inequality $\lambda \gg R$, in which $\lambda = 2\pi c/\omega_0\varepsilon^{1/2}$ is the laser emission wavelength and $R = (m_0\omega_H)^{1/2}$ is the characteristic dimension of the region of localization of the electron wave function in the xy plane.

We note in conclusion that allowance for the dissipative processes (e.g., interaction of the carriers with the crystal-lattice vibrations) will lead to a finite half-width of the quasienergy levels and to a smearing of the delta functions in (4) and (5). As a result, the peaks of the resonant absorption and amplification have finite height. For the semiconductor parameters $\varepsilon_g = 0.28$ eV, $m_0 = 0.1 m_e$, $\varepsilon = 16$, $\Gamma = 10^{11}\,\mathrm{s}^{-1}$ and for $E_0 = 10^5$ V/cm and $\omega_H = 0.1$ eV the value K at the resonance points reaches $5 \cdot 10^3\,\mathrm{cm}^{-1}$.

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