Nonuniform stationary coherent states (the CTP effect) for the $j_g = 3/2 \to j_e = 1/2$ and $j_g = 2 \to j_e = 1$ in a resonant interaction with a nonuniformly polarized field

A. V. Taĭchenachev, A. M. Tumaĭkin, V. I. Yudin, and M. A. Ol'shanyi[1)]

Novosibirsk State University

(Submitted 30 September 1991)
Zh. Eksp. Teor. Fiz. **101**, 1787–1796 (June 1992)

For the optical transitions $j_g = 3/2 \to j_e = 1/2, j_g = 2 \to j_e = 1$, classes of resonant field configurations are identified in which there are nonuniform stationary coherent states that do not interact with radiation: $\widehat{V}(\mathbf{r})|\psi(\mathbf{r})\rangle = 0, \hat{p}^2|\psi(\mathbf{r})\rangle = \varepsilon|\psi(\mathbf{r})\rangle$. These are the states responsible for the effect of coherent trapping of populations (CTP), which makes it possible to produce 1-D, 2-D, and 3-D superdeep cooling of the atoms.

1. INTRODUCTION

Aspect *et al.*[1] reported the detection of 1-D resonance cooling of atoms below the recoil energy, $k_B T < (\hbar k)^2/2M$, in oppositely directed (along the z axis) orthogonally polarized waves (the $\sigma_+ - \sigma_-$ field configuration) in the $j_g = 1 \to j_e = 1$ transition (the $2^3s_1 \to 2^3p_1$ transition in a beam of metastable ^{4}He atoms). A theoretical analysis of this effect was proposed in Ref. 2. According to Ref. 2, as a result of radiation relaxation processes, the atoms accumulate in a state that does not interact with the electromagnetic field, i.e., coherent trapping of populations takes place. In the $\sigma_+ - \sigma_-$ configuration, which is a special case of a field with nonuniform polarization, the atomic state which does not react with the field proves to be selective in velocities—the distribution over the z component of momentum consists of two δ functions located at the points $\pm \hbar k$. It was shown in Ref. 2 that the "temperature" T, defined as the square of the halfwidth of the peaks in the momentum distribution, tends to zero in inverse proportion to the time t of interaction of the atoms with the field, $T \propto t^{-1}$. The possibility of 1-D superdeep cooling by means of the $j_g = 3/2 \to j_e = 1/2$ and $j_g = 2 \to j_e = 1$ transitions in a $\sigma_+ - \sigma_-$ field was discussed in Ref. 3. Various schemes of 2-D superdeep cooling by means of the $j_g = 1 \to j_e = 1$ transition were proposed in Refs. 2 and 3. Recently, on the basis of an analysis of the problem in the coordinate representation, Ol'shanyi and Minegin[4] obtained a field configuration, viz., six counterpropoagating (along the coordinate axes) orthogonally polarized waves (a superposition of three $\sigma_+ - \sigma_-$ configurations), that induced 3-D superdeep cooling on the $j_g = 1 \to j_e = 0$, and $j_g = 1 \to j_e = 1$ transitions.

On the other hand, it was shown in Ref. 5 that for atoms with angular momenta $j_g = j \to j_e = j - 1$ and $j_g = j' \to j_e = j'$ (j' being an integer), in a resonance field with an arbitrary uniform elliptical polarization, there exist stationary coherent states (SCS) that do not interact with the field. The SCS constitute a coherent superposition of the wave functions of the Zeeman sublevels of the ground state and hence, have a zero natural width.

In the present work, for the transitions $j_g = 3/2 \to j_e = 1/2$ and $j_g = 2 \to j_e = 1$, we have found nonuniform stationary coherent states which extend SCS to the case of a field with nonuniform polarization (i.e., changing over distances on the order of a wavelength λ). Below, to avoid introducing new abbreviations, we shall also use SCS in the case of a nonuniformly polarized field. These SCS do not interact with the field, have a zero natural width, and are the eigenfunctions of the kinetic energy operator of the atom. With the exception of a few cases (see Sec. 5 below), the momentum distribution in SCS describes the localization of atoms on a sphere in momentum space: $p^2 = (\hbar k)^2$ ($k = 2\pi/\lambda$, where $\mathbf{k}$ is the wave vector of the field).

2. STATEMENT OF THE PROBLEM

We consider atoms with angular momenta j_g in the ground state and j_e in the excited state, which interact resonantly with a monochromatic electromagnetic field

$$\mathbf{E}(\mathbf{r}, t) = e^{-i\omega t}\mathbf{e}(\mathbf{r}) + \text{c.c.} \tag{1}$$

When the scattered field is neglected, the complex vector amplitude $\mathbf{e}(\mathbf{r})$ satisfies the free wave equation and transversality condition:

$$(\Delta + k^2)\mathbf{e}(\mathbf{r}) = 0, \quad k = \omega/c, \qquad (\nabla\mathbf{e}(\mathbf{r})) = 0. \tag{2}$$

Here the field polarization, which is determined by the unit vector $\mathbf{e}(\mathbf{r})/(|\mathbf{e}(\mathbf{r})|^2)^{1/2}$, can change from one point to another in an arbitrary fashion (provided Eqs. (2) are satisfied), i.e., the configuration of the field is not yet fixed.

The matrix elements of the operator describing the interaction between the atoms and the field (1) in the resonance approximation are

$$\langle j_e\mu|\widehat{V}(\mathbf{r}, t)|j_g m\rangle = -e^{-i\omega t}\langle j_e\|\mathbf{d}\|j_g\rangle \sum_{q=\pm 1, 0} (-1)^{j_e-\mu}\begin{pmatrix} j_e & 1 & j_g \\ -\mu & q & m \end{pmatrix} e_q(\mathbf{r}), \tag{3}$$

where the indices m and μ label the Zeeman sublevels of the ground level and excited level, respectively; $\langle j_e\|\mathbf{d}\|j_g\rangle$ is the reduced matrix element of the dipole moment; the angular-momentum selection rules are contained in the $3jm$ symbols $\begin{pmatrix} j_e & 1 & j_g \\ -\mu & q & m \end{pmatrix}$; and $e_q(\mathbf{r})$ are the circular components of the vector $\mathbf{e}(\mathbf{r})$. The time dependence $\sim e^{-i\omega t}$ is not essential in the subsequent discussions and will not be written out explicitly.

The statement of the problem of finding stationary states that do not interact with the field is clearest in the coordinate representation. The unknown states $|\psi(\mathbf{r})\rangle$ must meet three requirements:

1) they must be stable in relation to radiation relaxation, and therefore, $|\psi(\mathbf{r})\rangle$ is sought in the form of a superposition of the wave functions of the Zeeman sublevels of the ground state:

 0038-5646/92/060952-05$05.00

$$|\psi(\mathbf{r})\rangle=\sum_m b_m(\mathbf{r})|j_g m\rangle, \quad m=-j_g,\ldots,j_g; \tag{4}$$

2) they must not interact with the field (1), i.e.,

$$\hat{V}(\mathbf{r})|\psi(\mathbf{r})\rangle=0, \tag{5}$$

which for amplitude $b_m(\mathbf{r})$ can be written in the form[5]

$$\sum_{m,q}(-1)^{j_e-\mu}\begin{pmatrix} j_e & 1 & j_g \\ -\mu & q & m \end{pmatrix} e_q(\mathbf{r})b_m(\mathbf{r})=0; \tag{5a}$$

3) they must be the eigenstates for the kinetic energy operator

$$\frac{\hat{p}^2}{2M}|\psi(\mathbf{r})\rangle=\varepsilon|\psi(\mathbf{r})\rangle, \quad \hat{\mathbf{p}}=-i\hbar\nabla, \tag{6}$$

which for the states (4) means

$$\left(\Delta+\frac{2M\varepsilon}{\hbar^2}\right)b_m(\mathbf{r})=0. \tag{6a}$$

Such states are not destroyed as a result of the free motion of the atom and are strictly stationary when radiative processes, including all the recoil effects (except for the processes of interatomic exchange of photons, which are not considered here), are taken into account.

When the field polarization is constant in space $e_q(\mathbf{r})=f(\mathbf{r})\varepsilon_q$, $\varepsilon_q=$ const, the problems of distribution of atoms over the internal and translational degrees of freedom under conditions of coherent trapping of populations separate:

$$b_m(\mathbf{r})=\varphi(\mathbf{r})\beta_m, \quad \beta_m=\text{const}.$$

As was shown in Ref. 6, for the transitions $j_g=j\to j_e=j-1$ and $j_g=j'\to j_e=j'$ (j' being an integer), the solution of the problem exists for an arbitrary elliptical polarization ε_q: the amplitudes β_m are independent of the light intensity and are completely determined by its polarization, and $\varphi(r)$ is an arbitrary eigenfunction of the kinetic energy operator. Hence, the atomic density matrix in SCS is in the form of the direct product $\rho=\rho_{\text{trans}}\rho_{\text{int}}$, where the distribution over the internal degrees of freedom ρ_{int} is given by the polarization of the field $\langle j_g m|\rho_{\text{int}}|j_g m'\rangle\propto\beta^*_m\beta_{m'}$ and is independent of the space variables, and the distribution over the translational degrees of freedom ρ_{trans} is determined by the initial conditions.

In the case of nonuniform polarization, we found the solution of the system (5), (6) for the transitions $j_g=1\to j_e=0$, $j_g=1\to j_e=1$, $j_g=2\to j_e=1$, $j_g=3/2\to j_e=1/2$, and this solution demonstrates a strong correlation of the internal and translational degrees of freedom of the atoms, which is determined by the configuration of the field $\mathbf{e}(\mathbf{r})$. We shall examine the SCS for specific types of transitions.

3. SCS FOR THE TRANSITIONS $j_g=3/2\to j_e=1/2$, $j_g=2\to j_e=1$

As was noted in the introduciton, we previously found SCS only for the transitions $j_g=1\to j_e=0$, $j_g=1\to j_e=1$,[4,5] and therefore, the problem of finding the SCS for other transitions is of definite physical interest. The present section gives the solution of this problem for the transitions $j_g=3/2\to j_e=1/2$ and $j_g=2\to j_e=1$.

(a) The transition $j_g=3/2\to j_e=1/2$

We write Eqs. (5a) for atoms with angular momenta $j_g=3/2$ in the ground state and $j_e=1/2$ in the excited state in the explicit form

$$\begin{gathered} 3^{1/2}e_{-1}(\mathbf{r})b_{3/2}(\mathbf{r})+2^{1/2}e_0(\mathbf{r})b_{1/2}(\mathbf{r})+e_{+1}(\mathbf{r})b_{-1/2}(\mathbf{r})=0, \\ e_{-1}(\mathbf{r})b_{1/2}(\mathbf{r})+2^{1/2}e_0(\mathbf{r})b_{-1/2}(\mathbf{r})+3^{1/2}e_{+1}(\mathbf{r})b_{-3/2}(\mathbf{r})=0. \end{gathered} \tag{7}$$

We recall that Eq. (7) follows from the fact that the probability amplitudes of atoms in the excited $|j_e\pm 1/2\rangle$ states under conditions of coherent trapping of population are zero.

The general solution of Eq. (7) may be represented in the form of a superposition of linearly independent solutions (the quantization axis is directed along the z axis):

$$|\psi(\mathbf{r})\rangle=\begin{bmatrix} b_{3/2} \\ b_{1/2} \\ b_{-1/2} \\ b_{-3/2} \end{bmatrix}=C_1(\mathbf{r})\begin{bmatrix} -e_{+1}(\mathbf{r})/3^{1/2} \\ 0 \\ e_{-1}(\mathbf{r}) \\ -\dfrac{2^{1/2}e_{-1}(\mathbf{r})e_0(\mathbf{r})}{3^{1/2}e_{+1}(\mathbf{r})} \end{bmatrix}+C_2(\mathbf{r})\begin{bmatrix} -\dfrac{2^{1/2}e_{+1}(\mathbf{r})e_0(\mathbf{r})}{3^{1/2}e_{-1}(\mathbf{r})} \\ e_{+1}(\mathbf{r}) \\ 0 \\ -e_{-1}(\mathbf{r})/3^{1/2} \end{bmatrix}. \tag{8}$$

In addition to Eq. (7), the SCS must satisfy Eq. (6a), which imposes certain conditions on the functions $C_1(\mathbf{r})$ and $C_2(\mathbf{r})$. The general solution (8) enable us to distinguish two classes of field configuration in which SCS exist.

I. $e_0(\mathbf{r})=0$, i.e., the local vector of field polarization lies in the xy plane, and $(\mathbf{e}(\mathbf{r})\mathbf{e}_z)=0$ at all points of space. In this case, the SCS on the transition $j_g=3/2\to j_e=1/2$ are a superposition of two orthogonal states

$$|\psi(\mathbf{r})\rangle=\frac{C_1}{V^{1/2}}\begin{bmatrix} -e_{+1}(\mathbf{r})/3^{1/2} \\ 0 \\ e_{-1}(\mathbf{r}) \\ 0 \end{bmatrix}+\frac{C_2}{V^{1/2}}\begin{bmatrix} 0 \\ e_{+1}(\mathbf{r}) \\ 0 \\ -e_{-1}(\mathbf{r})/3^{1/2} \end{bmatrix}, \tag{9}$$

where $C_{1,2}$ are constants related by the normalization condition

$$|C_1|^2\left\langle|e_{+1}(\mathbf{r})|^2\cdot\frac{1}{3}+|e_{-1}(\mathbf{r})|^2\right\rangle_V+|C_2|^2\left\langle|e_{-1}(\mathbf{r})|^2\cdot\frac{1}{3}+|e_{+1}(\mathbf{r})|^2\right\rangle_V=1.$$

Here the angle brackets denote averaging over the normalization volume $V:\langle\ldots\rangle_V=V^{-1}\int\ldots d^3r$.

The solution of Eq. (9) is fairly obvious, since for this field configuration, from the general scheme of interaction we can distinguish two simple Λ systems which are not coupled by induced transitions, shown in Fig. 1b; for each of these systems there exists a solution of Eqs. (7) which is linear in the field amplitudes $e_{\pm 1}(\mathbf{r})$.

In general, this field configuration is a superposition of plane waves $\mathbf{e}(\mathbf{r})=\Sigma_l\mathbf{e}_l\exp(i\mathbf{k}_l\mathbf{r})$, whose linear polarizations $\mathbf{e}_l$ lie in the xy plane, and whose wave vectors $\mathbf{k}_l$ are directed in a fairly arbitrary manner relative to the xy plane, provided the orthogonality condition $\mathbf{k}_l\cdot\mathbf{e}_l=0$ is satisfied

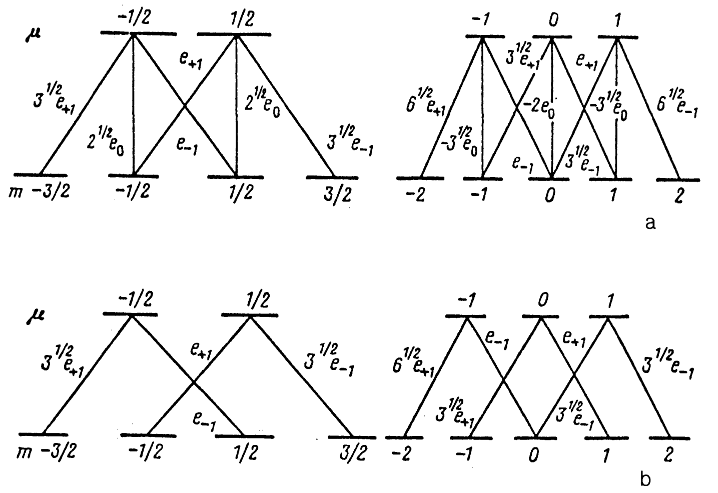

FIG. 1. a) General scheme of induced transitions for atoms with angular momenta $j_g = 3/2 \to j_e = 1/2$, $j_g = 2 \to j_e = 1$. The quantization axis is directed along the z axis. Relative amplitudes of the transitions are given. b) Same for $e_0 = 0$. Simple Λ systems are distinguished:

$\{|j_g, -3/2\rangle, \quad |j_g, 1/2\rangle, \quad |j_e, -1/2\rangle\}$,
$\{|j_g, -1/2\rangle, \quad |j_g, 3/2\rangle, \quad |j_e, 1/2\rangle\}$,
$\{|j_g, -1\rangle, \quad |j_g, +1\rangle, \quad |j_e, 0\rangle\}$.

(see Fig. 2). It is the arbitrariness of the directions of $\mathbf{k}_l$ that makes it possible to obtain 1-D, 2-D, or 3-D superdeep cooling in this field configuration. We note that in addition to the linearly polarized waves, the condition $e_0(\mathbf{r}) = 0$ is satisfied by two waves propagating oppositely along the z axis and having an arbitrary elliptical polarization (they are denoted by $\mathbf{E}_{(+z)}, \mathbf{E}_{(-z)}$ in Fig. 2). As an example, we can examine the following superposition of waves (see Fig. 3): a standing wave of linear polarization $\mathbf{e}_y$ propagates along the x axis, a standing wave of linear polarization $\mathbf{e}_x$ propagates along the y axis, and a $\sigma_+ - \sigma_-$ wave propagates along the z axis.

II. Another class of field configurations for which SCS exist on the transition $j_g = 3/2 \to j_e = 1/2$ is distinguished from the general solution (8) by the condition $e_{\pm 1}(\mathbf{r}) = f(\mathbf{r})\varepsilon_{\pm 1}$, where $\varepsilon_{+1}, \varepsilon_{-1}$ are arbitrary constants, i.e., the circular components of the field have the same spatial structure and ratio $e_{+1}(\mathbf{r})/e_{-1}(\mathbf{r}) = \varepsilon_{+1}/\varepsilon_{-1} = \text{const}$.

In this case, the NSCS are

$$|\psi(\mathbf{r})\rangle = \frac{C_1}{V^{1/2}} \begin{bmatrix} -e_{+1}(\mathbf{r})/3^{1/2} \\ 0 \\ e_{-1}(\mathbf{r}) \\ -\dfrac{\varepsilon_{-1}}{\varepsilon_{+1}} \dfrac{e_0(\mathbf{r})\cdot 2^{1/2}}{3^{1/2}} \end{bmatrix} + \frac{C_2}{V^{1/2}} \begin{bmatrix} -\dfrac{\varepsilon_{+1}}{\varepsilon_{-1}} \dfrac{e_0(\mathbf{r})\cdot 2^{1/2}}{3^{1/2}} \\ e_{+1}(\mathbf{r}) \\ 0 \\ \dfrac{-e_{-1}(\mathbf{r})}{3^{1/2}} \end{bmatrix}; \qquad (10)$$

C_1 and C_2 being constants related by the normalization condition $\int \langle \psi(\mathbf{r}) | \psi(\mathbf{r}) \rangle d^3 r = 1$. In general, this configuration is a superposition

$$\mathbf{e}(\mathbf{r}) = \sum_l \mathbf{E}_l \mathbf{e}_z \exp(i\mathbf{k}_l \mathbf{r}) + \boldsymbol{\varepsilon}(E_+ \exp(ikz) + E_- \exp(-ikz)), \qquad (11)$$

consisting of waves polarized linearly along $\mathbf{e}_z$, with amplitudes $\mathbf{E}_l$, whose wave vectors $\mathbf{k}_l$ are arbitrarily directed in the xy plane, and two waves counterpropagating along the z axis with amplitudes $E_\pm$, having the same elliptical polarization ε (see Fig. 4). It should be emphasized that the special case in which the waves propagating along the z axis have circular polarization, i.e., $\boldsymbol{\varepsilon} = \mathbf{e}_{\pm 1}$, is not suited for superdeep cooling, since it involves the existence of a state $|j_g, \mp 3/2\rangle$ (denoted by * in Fig. 5) that does not interact with the field when the atom has an arbitrary velocity. A simple example of a configuration inducing 3-D superdeep cooling is shown schematically in Fig. 6. Interestingly, for the configurations of II, in contrast to case I with an arbitrary elliptical polarization ε, it is impossible, by selecting the quantization axis, to reduce the scheme of the interaction between the Zeeman sublevels and the field to simple Λ systems. For both of the above classes of field configurations I and II, SCS (9) and (10) are the eigenfunctions of the kinetic energy operator of the atom [see Eq. (6)]. The kinetic energy eigenvalue is determined by the field wavelength

$$\varepsilon = \frac{(\hbar k)^2}{2M} = \left(\frac{2\pi\hbar}{\lambda}\right)^2 \frac{1}{2M}, \qquad (12)$$

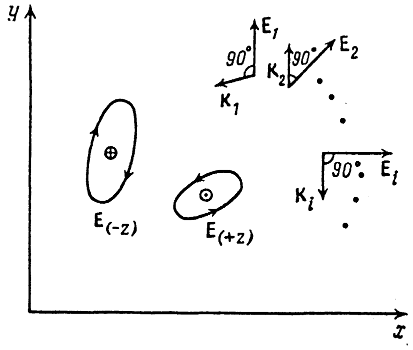

FIG. 2.

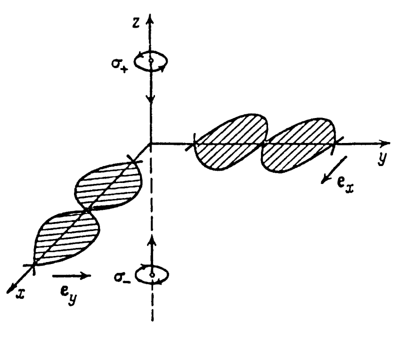

FIG. 3.

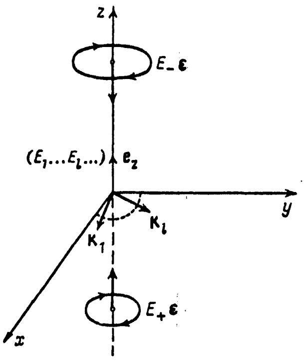

FIG. 4.

which means that the atoms are localized in momentum space on a sphere: $p^2 = (\hbar k)^2$. For the specific field configurations shown in Figs. 3 and 6, we have localization of the atoms at the points $\mathbf{p} = \pm \hbar k \mathbf{e}_i \ (i = x,y,z)$.

(b) The transition $j_g = 2 \to j_e = 1$

For the transition $j_g = 2 \to j_e = 1$, the system of equations (5a) has the form

$$
\begin{aligned}
&6^{1/2}e_{-1}(\mathbf{r})b_{+2}(\mathbf{r})-3^{1/2}e_0(\mathbf{r})b_{+1}(\mathbf{r})+e_{+1}(\mathbf{r})b_0(\mathbf{r})=0,\\
&3^{1/2}e_{-1}(\mathbf{r})b_{+1}(\mathbf{r})-2e_0(\mathbf{r})b_0(\mathbf{r})+3^{1/2}e_{+1}(\mathbf{r})b_{-1}(\mathbf{r})=0,\\
&e_{-1}(\mathbf{r})b_0(\mathbf{r})-3^{1/2}e_0(\mathbf{r})b_{-1}(\mathbf{r})+6^{1/2}e_{+1}(\mathbf{r})b_{-2}(\mathbf{r})=0.
\end{aligned} \tag{13}
$$

The general solution of Eq. (16) is a superposition of linearly independent solutions

$$
|\psi(\mathbf{r})\rangle = C_1(\mathbf{r})\begin{bmatrix} \dfrac{1}{2^{1/2}}\dfrac{e_{+1}(\mathbf{r})e_0(\mathbf{r})}{e_{-1}(\mathbf{r})} \\ e_{+1}(\mathbf{r}) \\ 0 \\ -e_{-1}(\mathbf{r}) \\ -\dfrac{1}{2^{1/2}}\dfrac{e_{-1}(\mathbf{r})e_0(\mathbf{r})}{e_{+1}(\mathbf{r})} \end{bmatrix}
+ C_2(\mathbf{r})\begin{bmatrix} \dfrac{e_0^2(\mathbf{r})}{e_{-1}^2(\mathbf{r})} - \dfrac{e_{+1}(\mathbf{r})}{e_{-1}(\mathbf{r})} \\ \dfrac{2^{1/2}e_0(\mathbf{r})}{e_{-1}(\mathbf{r})} \\ 6^{1/2} \\ \dfrac{2^{1/2}e_0(\mathbf{r})}{e_{+1}(\mathbf{r})} \\ \dfrac{e_0^2(\mathbf{r})}{e_{+1}^2(\mathbf{r})} - \dfrac{e_{-1}(\mathbf{r})}{e_{+1}(\mathbf{r})} \end{bmatrix} \tag{14}
$$

As is evident from Eq. (14), the SCS on the transition $j_g = 2 \to j_e = 1$ exist for the same two classes of fields as on the transition $j_g = 3/2 \to j_e = 1/2$:

I) in the case $e_0(r) = 0$ (Fig. 2),

$$
|\psi(\mathbf{r})\rangle = \frac{C_1}{V^{1/2}}\begin{bmatrix} 0 \\ e_{+1}(\mathbf{r}) \\ 0 \\ -e_{-1}(\mathbf{r}) \\ 0 \end{bmatrix}, \tag{15}
$$

II) in the case $e_{\pm 1}(\mathbf{r}) = f(\mathbf{r})\varepsilon_{\pm 1}$ ($\varepsilon_q = \text{const}$), which is described by the superposition (11) (Fig. 5):

$$
|\psi(\mathbf{r})\rangle = \frac{C_1}{V^{1/2}}\begin{bmatrix} \dfrac{1}{2^{1/2}}\dfrac{\varepsilon_{+1}}{\varepsilon_{-1}}e_0(\mathbf{r}) \\ e_{+1}(\mathbf{r}) \\ 0 \\ -e_{-1}(\mathbf{r}) \\ -\dfrac{1}{2^{1/2}}\dfrac{\varepsilon_{-1}}{\varepsilon_{+1}}e_0(\mathbf{r}) \end{bmatrix}. \tag{16}
$$

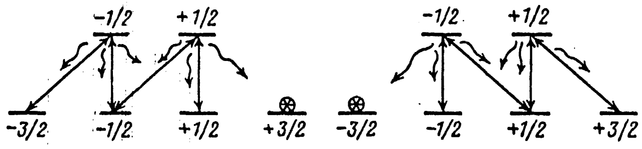

FIG. 5.

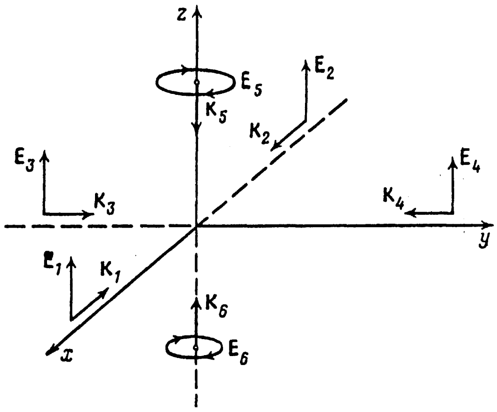

FIG. 6.

In contrast to the transition $j_g = 3/2 \to j_g = 1/2$, the function $C_2(\mathbf{r})$ in (14) is zero in both cases. The constants C_1 in Eqs. (15) and (16) are determined from the normalization condition. The SCS (15) and (16) describe the effects of localization of the atoms in the momentum space similarly to SCS on the transition $j_g = 3/2 \to j_e = 1/2$, which were discussed above.

We note in conclusion that the question of the existence of other field configurations for which SCS exist on the transitions $j_g = 3/2 \to j_e = 1/2$ and $j_g = 2 \to j_e = 1$ remains open.

4. EXPLICIT FORM OF SCS FOR THE $j_g = 1 \to j_e = 0$ AND $j_g = 1 \to j_e = 1$ TRANSITIONS

For reference purposes, we shall reproduce in our notation the results of Refs. 4 and 5, which obtained SCS on the transitions $j_g = 1 \to j_e = 0$ and $j_g = 1 \to j_e = 1$.

a) The $j_g = 1 \to j_e = 0$ transition

As above, the SCS are sought in the form of the superposition (4). In this case, the amplitudes $b_{\pm 1,0}(\mathbf{r})$ are circular components of the vector field $\mathbf{b}(\mathbf{r})$, and Eq. (5a) may be represented in the invarient form

$$
(\mathbf{e}(\mathbf{r})\mathbf{b}(\mathbf{r}))=0. \tag{17}
$$

The general solution (17) is written as a vector product

$$
\mathbf{b}(\mathbf{r}) = [\mathbf{A}(\mathbf{r})\mathbf{e}(\mathbf{r})], \tag{18}
$$

which can be represented explicitly as follows:

$$|\psi(\mathbf{r})\rangle = C_1(\mathbf{r}) \begin{bmatrix} e_0(\mathbf{r}) \\ e_{-1}(\mathbf{r}) \\ 0 \end{bmatrix} + C_2(\mathbf{r}) \begin{bmatrix} 0 \\ e_{+1}(\mathbf{r}) \\ e_0(\mathbf{r}) \end{bmatrix} + C_3(\mathbf{r}) \begin{bmatrix} e_{+1}(\mathbf{r}) \\ 0 \\ -e_{-1}(\mathbf{r}) \end{bmatrix}; \quad (18a)$$

(18a) satisfies Eq. (6a) if $C_i = \text{const}$, $i = 1, 2, 3$. Thus, the SCS on this transition exist for an arbitrary field configuration. The constants C_i are related by the normalization condition $\int \langle \psi(\mathbf{r}) | \psi(\mathbf{r}) \rangle d^3 r = 1$.

b) The $j_g = 1 \to j_e = 1$ transition

Since the angular momenta of the ground and excited states are equal to 1, Eq. (5a) reduces to the form

$$[\mathbf{e}(\mathbf{r})\mathbf{b}(\mathbf{r})] = 0. \quad (19)$$

The general solution of Eq. (20)

$$\mathbf{b}(\mathbf{r}) = C(\mathbf{r})\mathbf{e}(\mathbf{r}) \quad (20)$$

satisfies Eq. (6a) if $C = \text{const}$.

Thus, for an arbitrary field configuration on the $j_g = 1 \to j_e = 1$ transition there exist SCS which, with allowance made for the normalization condition, are written in the form

$$|\psi(\mathbf{r})\rangle = \frac{1}{[V\langle |e(\mathbf{r})|^2\rangle_V]^{1/2}} \begin{bmatrix} e_{+1}(\mathbf{r}) \\ e_0(\mathbf{r}) \\ e_{-1}(\mathbf{r}) \end{bmatrix}. \quad (21)$$

Some specific examples of 3-D superdeep cooling on the $j_g = 1 \to j_e = 0$ and $j_g = 1 \to j_e = 1$ transitions are given in Ref. 4.

5. UNIQUENESS OF SCS

The SCS discussed in the preceding section are a special case of the general solutions (8), (14), (18a), (21), when we have $C_i(\mathbf{r}) = \text{const}$. However, for certain field configurations, by selecting the functions $C_i(\mathbf{r})$, one can obtain SCS that are different from those given above. For example, if there are three standing waves with complex amplitudes, which are directed along the coordinate axes

$$\mathbf{e}(\mathbf{r}) = \mathbf{E}_1 \cos(kx) + \mathbf{E}_2 \cos(ky) + \mathbf{E}_3 \cos(kz), \quad (22)$$

there exist SCS $|\tilde{\psi}(\mathbf{r})\rangle$ different from (9), (10), (16), (15), (18) or (21). We represent them in the form

$$|\tilde{\psi}(\mathbf{r})\rangle = \sin(kx)\sin(ky)\sin(kz)\,|\psi(\mathbf{r})\rangle, \quad (23)$$

where $|\psi(\mathbf{r})\rangle$ are any of the SCS (9), (10), (15), (16), (18a), (21), and each of the components of the vector $|\tilde{\psi}(\mathbf{r})\rangle$ has the form of a superposition with constant coefficients B_l^m ($l = 1,2,3$; $-j_g \leqslant m \leqslant j_g$):

$$\langle j_g m | \tilde{\psi}(\mathbf{r}) \rangle = B_1^m \sin(2kx)\sin(ky)\sin(kz) + B_2^m \sin(2ky)\sin(kx)\sin(kz) + B_3^m \sin(2kz)\sin(kx)\sin(ky). \quad (24)$$

It is easy to ascertain that the states (24) satisfy the equations (5): $\hat{V}(\mathbf{r})\,|\tilde{\psi}(\mathbf{r})\rangle = \sin(kx)\sin(ky)\sin(kz)\hat{V}(\mathbf{r})|\psi(\mathbf{r})\rangle = 0$ and (6):

$$\frac{\hat{p}^2}{2M}|\tilde{\psi}(\mathbf{r})\rangle = 6\frac{(\hbar k)^2}{2M}|\tilde{\psi}(\mathbf{r})\rangle. \quad (25)$$

As is evident from Eq. (25), the states $|\tilde{\psi}(\mathbf{r})>$ describe the localization of atoms in momentum space on a sphere: $p^2 = 6(\hbar k)^2$, in contrast to the SCS discussed above (9), (10), (15), (16), (18a), (21), where $p^2 = (\hbar k)^2$. Finding the various field configurations in which SCS of type (23) $|\tilde{\psi}(\mathbf{r})\rangle$ exist is a fairly intricate mathematical problem whose solution requires additional studies.

6. CONCLUDING REMARKS

For the remaining transitions of the class $j_g = j \to j_e = j$, and $j_g = j' \to j_e = j' - 1$ (j' being an integer), the solutions of Eqs. (5) depend nonlinearly on the field amplitudes $e_q(r)$ and evidently are not strictly stationary.

The SCS obtained in this work make it possible to use the $j_g = 3/2 \to j_e = 1/2$ and $j_g = 2 \to j_e = 1$ transitions not only for purposes of 3-D superdeep cooling, but also for creating light-induced space lattices of atomic multipole moments and for localizing atoms in caustics (singularities) of nonuniformly polarized fields by analogy with the effects discussed in Ref. 6.

[1)] M. A. Ol'shanyĭ—member of the staff of the Spectroscopy Institute, USSR Academy of Sciences, Troitsk, 142092.

[1]A. Aspect, E. Arimondo, R. Kaiser, N. Vansteenkiste, and C. Cohen-Tannoudji, Phys. Rev. Lett. **61**, 826 (1988).
[2]A. Aspect, E. Arimondo, R. Kaiser, N. Vansteenkiste, and C. Cohen-Tannoudji, J. Opt. Soc. Am. B **6**, 2112 (1989).
[3]F. Mauri, F. Popoff, and E. Arimondo, *Proceedings of International Workshop on Light-Induced Kinetic Effects on Atoms, Ions, and Molecules* (Marciana Marina, Italy, 1990), edited by L. Moi, S. Gozzini, C. Gabanini, E. Arimondo, and F. Strumia (ETS EDITPRICE PISA 1991), p. 89.
[4]M. A. Ol'shanii and V. G. Minogin, *Proceedings of International Workshop on Light Induced Kinetic Effects on Atoms, Ions, and Molecules* (Marciana Marina, Italy, 1990), edited by L. Moi, S. Gozzini, C. Gabanini, E. Arimondo, and F. Strumia (ETS EDITPRICE PISA 1991), p. 99.
[5]V. S. Smirnov, A. M. Tumaĭkin, and V. I. Yudin, Zh. Eksp. Teor. Fiz. **96**, 1613 (1989) [Sov. Phys. JETP **69**, 913 (1989)].
[6]A. V. Taĭchenachev, A. M. Tumaĭkin, V. I. Yudin, and M. A. Ol'shanyĭ, Pis'ma Zh. Eksp. Teor. Fiz. **53**, 336 (1991) [JETP Lett. **53**, 351 (1991)].

Translated by Adam Peiperl