Domain-wall dynamics and magnetization relaxation in magnetic materials with a magnetic aftereffect

B. A. Ivanov and S. N. Lyakhimets

Institute of Metal Physics, Academy of Sciences of the Ukraine

M. Kisielewski and A. Maziewski

Branch of Warsaw University, Bialystok, Poland

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Experiments show that the magnetization relaxation which occurs during the pulsed magnetization reversal of magnetic films with a magnetic aftereffect (iron garnet films with a cobalt impurity) is of a two-step nature. The first step is a rapid jump in the magnetization. This jump is quite sensitive to the pulse length. The second step is an anomalously slow, nonexponential relaxation (with a time scale $\tau_r \sim 40$ s). The magnetic anisotropy induced in the material is responsible for the magnetic aftereffect. However, the relaxation time τ of this anisotropy is much shorter than the magnetization relaxation time: $\tau_r/\tau \sim 10^3$. A theory is derived for the dynamics of domain walls in a medium with a magnetic memory. This theory gives a systematic description of both steps of the relaxation. A factor of importance for the first step is that the domain walls are not 180° walls. The dynamics of the walls is described by an integral equation whose time-dependent kernel (a memory function) determines the time delay in the magnetic aftereffect. An analytic solution found for this equation describes nonexponential relaxation of the magnetization with a time scale $\tau_r \sim \tau(d/2\Delta)$, where d is the period of the domain structure, and Δ is the width of a domain wall. Comparison reveals that the theoretical curves for the jump in the magnetization in the first step and for the magnetization relaxation in the second step agree well with the experimental curves. A new method is proposed for measuring the properties of an induced magnetic anisotropy.

1. INTRODUCTION

Some problems of importance in physics today involve the existence of nonlinear solitary waves, i.e., solitons. Problems involving the motion of solitons have been analyzed and have been used to describe the properties of real condensed media for many important physical entities.[1] Prominent among these problems are those concerning the dynamics of domains and domain walls in magnetic materials[2,3] which can be viewed in terms of magnetic solitons.[4] For none of these problems, to the best of our knowledge, has there been a detailed study of the motion of solitons (e.g., domain walls) in media with a memory.

In the physics of real magnetic materials and in related applications, on the other hand, it is important to reach an understanding of the magnetic losses which occur in magnetically soft materials. These losses are determined to a large extent by the magnetic viscosity and by the magnetic aftereffect.[5,6] While the eddy-current loss—another important relaxation mechanism—has been studied thoroughly, this cannot be said of the manifestations of a magnetic aftereffect. This problem requires further study at the level of the microscopic mechanisms and also at the macroscopic level, corresponding to a description of the changes which occur in domain structures and in the dynamics of domain walls.[1)] The magnetic aftereffect is intimately related to the induced magnetic anisotropy.[2)] We will be discussing this question below. One reason for this unsatisfactory state of the question, in our opinion, is the complexity of the actual domain structure of bulk magnetic materials such as alloys of the Permalloy type, silicon iron, etc. This structure is characterized by domains which are not in a 180° neighborhood, an irregular arrangement of domains, etc. Another important factor is that it is difficult to directly observe domains in these magnetic materials.

Epitaxial films of an yttrium iron garnet with a partial substitution of cobalt for iron were used as models in Refs. 7 and 8 for studying many pertinent aspects of this problem. Many of the distinguishing features mentioned above are seen in these magnetic materials. There is a strong magnetic aftereffect. There are also domain walls which are not in a 180° neighborhood. This model material has an advantage over materials of the Permalloy type in that it has a high magnetooptic figure of merit, a domain structure which runs completely through the material, and a domain structure which is fairly regular. It thus becomes possible to carry out a detailed study of the magnetization reversal by magnetooptic methods, both quasistatic and dynamic. Preliminary studies of the magnetic reversal of these magnetic materials with the help of pulsed fields[7] have demonstrated several aspects of the dynamics of the domain walls which, in our opinion, cannot be explained by the existing theories. We are thinking primarily of the slow relaxation of the magnetization, with a time scale ranging up to minutes.

In this paper we are reporting an experimental and theoretical study of relaxation in the magnetization reversal of these materials. Using specific analysis of the dynamics of domain walls in this system, we point out some new general features of the motion of nonlinear perturbations (solitons) in physical media with a memory effect.

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2. TEST SAMPLES AND EXPERIMENTAL PROCEDURE

The test films (with the composition $Y_2Ca_1Fe_{3.9}Co_{0.1}Ge_1O_{12}$) were grown by liquid-phase epitaxy from a $PbO–B_2O_3$ molten solution on a substrate of gadolinium-garnet cut perpendicular to the [100] axis. The growth procedure is described in Ref. 9. The films have a saturation magnetization $4\pi M_S = 80$ G (here and below, all properties are given for room temperature). A study of the ferromagnetic resonance showed that the energies of the natural cubic anisotropy and of the effective uniaxial anisotropy (consisting of the growth anisotropy and the shape anisotropy) are comparable and are characterized by the respective constants $K_1 = -10^4$ erg/cm^3 and $K_u = -2.5\cdot 10^3$ erg/cm^3 (Ref. 8). This anisotropy is responsible for the non-180° domains: In the domains, the magnetization vector makes an angle $\theta_* \sim 60°$ with the normal to the film, and the angle through which the magnetization rotates in a wall, $\pi - 2\theta_*$, is approximately 60° (Ref. 8). The domain structure of the demagnetized samples is labyrinthine, with slight variations in the relative proportions of the various phases due to a small deviation of the uniaxial-anisotropy axis from the normal to the film.[8]

The magnetic aftereffect is manifested in an interesting way: a shape memory of the domain structure.[7] Specifically, in the case of magnetization to saturation by a pulsed field with a pulse length up to a few seconds, the shape of the labyrinth is restored after the field is turned off and after the subsequent relaxation.

To study the magnetization relaxation, we studied the reconstruction of the domain structure after the application of saturating field pulses. The pulse length was varied from 10 ms to a few seconds; the pulse height was ≈ 70 Oe, sufficient to saturate the film. Specifically, we measured the time evolution of the average magnetization of the film after the application of the pulse. The magnetization was determined from the Faraday effect as linearly polarized light passed through the film. The experimental magnetooptic apparatus was controlled by a PC, with a special built-in card. This apparatus also made it possible to automatically determine the external conditions, e.g., the magnitude and time evolution of the applied magnetic field. This apparatus enabled us to record and to store in the computer memory, for averaging and processing, the quantities which determine the magnetization of the film, etc. The optical layout of the apparatus is similar to that described in Ref. 10.

This is an appropriate procedure for these particular samples, since small rotations of the polarization of the light (a fraction of a degree) lead to a significant scatter in the experimental points, because the films are thin. The time resolution was better than 1 ms.

It turned out that the experimental time dependence of the Faraday rotation angle φ after the field pulse is determined to a large extent by the length of the pulse (Fig. 1). We can see several main stages on this time dependence. For clarity we will be discussing the characteristic behavior in terms of the normal component of the magnetization, expressed in units of the saturation magnetization, $M_S\cos\theta_*$, instead of in terms of the angle φ.

In the first stage, just after the pulse ends, the magnetization decreases very rapidly (in practice, over a time shorter than the time resolution of the apparatus). The characteristic velocity of the domain walls in this stage was determined in Ref. 11 by high-speed photography. It was found to range up to 200 m/s. After this stage, the magnetization assumes a certain value M_0 which is different from both M_S and the remanent magnetization M_r. Interestingly, this value is determined by the pulse length D. Figure 2 shows M_0 versus D. The characteristic value of D at which the sharp change in M_0 occurs turned out to be on the order of 0.1 s. The subsequent stages of the demagnetization are far slower, with time scales on the order of tens of seconds. An attempt to describe this relaxation by a law $M(t) - M_r \propto \exp(-t/\tau_r)$ showed that the typical relaxation time τ_r in the initial stage was hundreds of times that in the final stage. This nonexponential behavior is not unique. Models with several relaxation times are presently used to describe experimental results on ferrites, alloys of the Permalloy type, and other materials.[6] However, these models are unsuccessful in describing the present experiments, primarily the $M_0(D)$ dependence. We show below that a comprehensive description of the entire set of experimental data can be generated on the basis of the theory presented below, which has only one relaxation time, τ (which characterizes the magnetic viscosity of the material), but which takes the dynamics of the non-180° domain walls into account systematically, with allowance for the magnetic aftereffect.

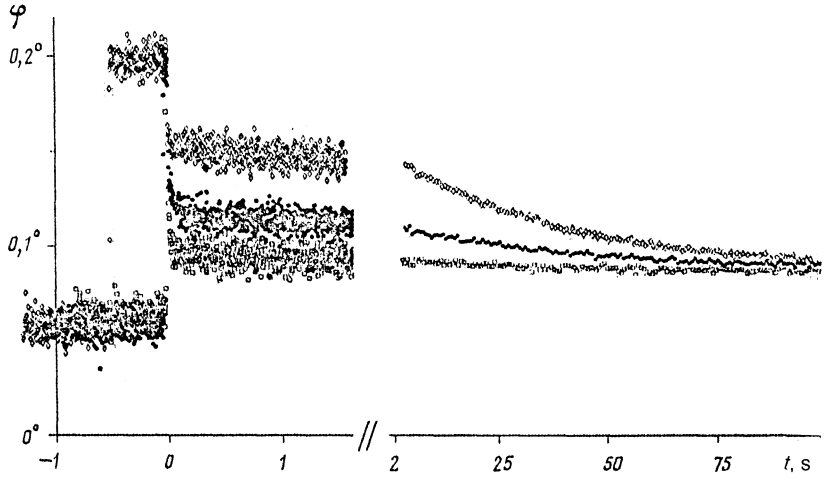

FIG. 1. Time evolution of the Faraday rotation angle φ for three lengths of the field pulse, D. ◇—$D = 0.5$ s; ●—$D = 0.4$ s; □—$D = 0.02$ s. Here $t = 0$ is the time at which the pulse ends. For the lower group of points, the value of the angle φ at $t < 0$ (before the application of the field pulse) is $\varphi \approx 0.051°$ and corresponds to a demagnetized state; for the upper group of points, the value is $\varphi \approx 0.21°$ and corresponds to a saturation state.

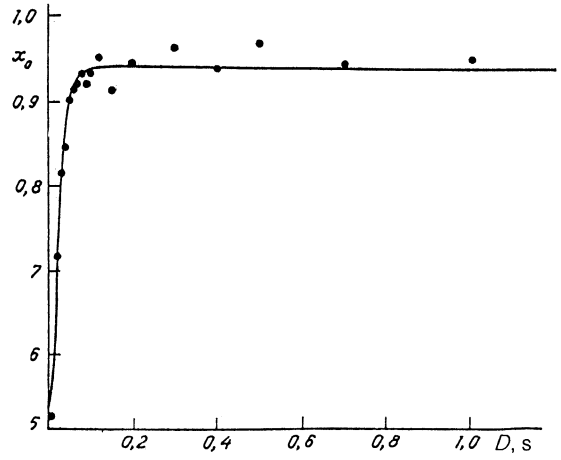

FIG. 2. The normalized magnetization M_0 established immediately after the end of the pulse, versus the length of the field pulse, D: $x_0(D) = M_0(D)/M_S$. The points show experimental data. The solid line was plotted from Eq. (15) with the parameter values $\varepsilon/k = 0.41$, $\tau = 0.016$ s, and $x_c = 0.53$.

3. THEORY

The magnetic energy, which determines the structure of the domain walls in the material of interest here, can be written as the sum of two terms:

$$W\{\mathbf{m}\} = W_0\{\mathbf{m}\} + W_1\{\mathbf{m}\}, \tag{1}$$

where $\mathbf{m}$ is a unit vector along the magnetization.

The first term incorporates the strongest interactions, which determine the equilibrium values of the magnetization in the domains and the structure of possible domain walls, i.e., the nonuniform-exchange energy and the energies of the uniaxial and cubic anisotropies:

$$W_0\{\mathbf{m}\} = \int d\mathbf{r}\{A(\nabla\mathbf{m})^2 + K_1(m_x^2m_y^2 + m_x^2m_z^2 + m_y^2m_z^2) + K_u m_z^2\}, \tag{2}$$

where K_1 and K_u are the constants of the cubic and uniaxial anisotropies, and A is the nonuniform-exchange constant. If only W_0 is taken into account, the position of the domain wall in the material is not fixed.

The second term in (1) represents the weaker interactions which determine the position of the domain wall. Foremost among them is the magnetic-dipole energy in the external magnetic field $\mathbf{H}$. This energy determines the period of the domain structure and also the energy (the induced magnetic anisotropy). As we will see below, the induced magnetic anisotropy in our case (with non-180° domain walls) is a source of magnetic viscosity. It determines the dynamic and relaxation properties of the domain walls. We choose $W_1\{\mathbf{m}\}$ in the form

$$W_1(\mathbf{m}) = \int d\mathbf{r}\left\{-\frac{M_s}{2}\mathbf{m}(\mathbf{H}_m + 2\mathbf{H}) + w^i\{\mathbf{m}\}\right\}. \tag{3}$$

Here $\mathbf{H}_m$ is the demagnetizing field, and $w^i(\mathbf{m})$ is the energy density of the induced magnetic anisotropy. In the dynamic regime this energy density depends not only on the orientation of $\mathbf{m}$ at the given time t but also on the orientation of $\mathbf{m}(t')$ at earlier times $t' < t$. In other words, $w^i\{\mathbf{m}\}$ is a functional of $\mathbf{m}(t)$. In the approximation of a single time constant τ characterizing the relaxation of the induced magnetic anisotropy, we can write the following expression for $w^i\{\mathbf{m}\}$ in the dynamic regime as follows:[5]

$$w^i\{\mathbf{m}, t\} = \int_0^t \frac{dt'}{\tau}\exp\left(\frac{t'-t}{\tau}\right) w_\infty{}^i(\mathbf{m}, \mathbf{m}(t')) + w^i(\mathbf{m}, 0)\exp\left(-\frac{t}{\tau}\right), \tag{4}$$

where the function $w^i_\infty(\mathbf{m},\mathbf{m}')$ determines the change in the energy of the material which occurs when the magnetization $\mathbf{m}$ is rotated rapidly after it has been directed along $\mathbf{m}'$ for a long time. In a cubic material $w^i_\infty(\mathbf{m},\mathbf{m}')$ is given by an expression which was first proposed by Néel:[5]

$$w_\infty{}^i(\mathbf{m}, \mathbf{m}') = -F(m_x^2m_x'^2 + m_y^2m_y'^2 + m_z^2m_z'^2 - G(m_xm_ym_x'm_y' + m_xm_zm_x'm_z' + m_ym_zm_y'm_z'), \tag{5}$$

where F and G are the constants of the induced magnetic anisotropy. The energy density in (1), with (2)–(5), determines the dynamics of the magnetization. This dynamics is described by the Landau–Lifshitz equation

$$\frac{\partial\mathbf{m}}{\partial t} = \frac{g}{M_s}\left[\mathbf{m}, \frac{\delta W}{\delta\mathbf{m}}\right] + \mathbf{R}, \tag{6}$$

where g is the gyromagnetic ratio, and $\mathbf{R}$ is a relaxation term describing the intrinsic magnetic relaxation. We wish to stress that $\mathbf{R}$ does not determine all relaxation processes. We know that retardation effects due to the induced magnetic anisotropy also result in a relaxation of the magnetization energy. In the approach of this paper, these effects are embodied in the dynamic part of Eq. (6).

For our analysis it is convenient to go over to an equation for the position of the center of the domain wall. This reduced equation can be derived from (6) in the usual way, on the basis of soliton perturbation theory.[1] These equations are widely used for domain walls in magnetic materials. They are usually written in the form $\lambda\dot{x}(t) = F(x(t))$, where x is the coordinate of the domain wall, and the viscosity constant λ is determined by $\mathbf{R}$ (Ref. 2; the inertial effects determined by the term $m\dot{x}$ are usually small, and we will not discuss them here). The force $F(x)$ is given by the expression $F(x) = -\partial V(x)/\partial x$, where $V(x)$ is the energy in (1), expressed per unit area of the domain wall and calculated for the magnetization distribution corresponding to a planar domain wall centered at point x. In the usual case (without a magnetic aftereffect), the energy $V(x)$ and the force $F(x)$ are ordinary functions of x, and $F(x)$ is a potential force. When induced magnetic-anisotropy effects are taken into account, $F(x)$ depends not only on the value of x at the given instant but also on the history.

For our case, $V(x)$ is determined exclusively by W_1 and can be written

$$V(x) = V^i(x) + V^m(x) + V^z(x) = \int_{-\infty}^{+\infty} d\xi[w^i\{\mathbf{m}(\xi-x)\} + w^m\{\mathbf{m}(\xi-x)\} + w^z\{\mathbf{m}(\xi-x)\}], \tag{7}$$

where the coordinate ξ runs perpendicular to the plane of the domain wall.

The dipole energy $V^m(x)$ determines only the potential force which fixes the equilibrium position of the domain wall. We approximate it by the quadratic expression

$$V^m(x)=kx^2/2, \tag{7a}$$

where we put the origin for the x scale at the equilibrium position of the domain wall. The energy of the domain wall in the external field is simply

$$V^z(x)=-hx. \tag{7b}$$

In order to calculate the coefficients k and h, we need to specify the particular domain structure of the material.

Let us consider the particular domain structure in a YIG:Co film. This magnetic material has four easy axes because of the relations $K_1<0$ and $K_u>0$ (in the case $K_u=0$, these axes coincide with [111] axes). In spherical coordinates $\mathbf{m}=(\cos\psi\sin\theta, \sin\psi\sin\theta, \cos\theta)$ the equilibrium orientation of $\mathbf{m}$ in the domains is determined by the angles $\theta_*=\arcsin\sqrt{(2+q)/3}$, where $q=2K_u/|K_1|$ and $\varphi_*^n=\pi/4+n\pi/2$. The index $n=1,2,3,4$ specifies the easy axis. Accordingly, there are domain walls of four types in a YIG:Co film.[8,12] In these walls, $\mathbf{m}$ rotates from θ_* to $\pi-\theta_*$ in the plane defined by the angles φ_*^n. For the most favorable Bloch domain wall (a planar wall), we can use the standard minimization procedure with (2) to write an equation which determines the structure of the domain wall:

$$l_0(\partial\theta/\partial\xi)=(\cos^2\theta-\cos^2\theta_*), \tag{8}$$

where $l_0=2\sqrt{2A/3|K_1|}$. The structure of the domain wall is determined by elementary integration of (8):

$$\operatorname{tg}\theta=\operatorname{tg}\theta_*\operatorname{cth}(\xi/\Delta), \quad \Delta=l_0/\sin\theta_*\cos\theta_*, \tag{8'}$$

where Δ is the thickness of the domain wall.

We turn now to the contribution of the induced magnetic anisotropy, which determines the basic effects in our problem. The application of $V^i(x)$ gives rise not only to a potential force but also to reversible phenomena, in particular, additional viscosity.[1,6] For a specific calculation of $V^i(x,t)$, we need to substitute into (4) and (5) a specific rotation $\mathbf{m}(\theta(\xi-x))$ in domain wall (8), (8') and integrate over ξ. For $V^i(x,t)$ we then find

$$V^i(x,t)=\int_0^t\frac{dt'}{\tau}\exp\left(\frac{t'-t}{\tau}\right)\Xi(x,x(t'))+V^i(x,0)\exp\left(-\frac{t}{\tau}\right). \tag{9}$$

The function $\Xi(x,x')$ is

$$\Xi(x,x')=\int_{-\infty}^{+\infty}d\xi w_\infty{}^i(\theta(\xi-x),\theta(\xi-x')), \tag{10}$$

where $w_\infty^i(\theta,\theta')=w_\infty^i(\mathbf{m}(\theta),\mathbf{m}(\theta'))$. If the domain wall spends a sufficiently long time ($t\gg\tau$) in its initial position $x(0)$, then we have $V^i(x,0)=\Xi(x,x(0))$ in (9). The function $\Xi(x,x')$ can be evaluated easily in the two limiting cases $|x-x'|\ll\Delta$ and $|x-x'|\gg\Delta$:

$$\Xi(x,x')=\begin{cases}\text{const}+{}^1/_2p(x-x')^2+o((x-x')^4), & |x-x'|\ll\Delta,\\ \text{const}+\varepsilon|x-x'|, & |x-x'|\gg\Delta.\end{cases} \tag{10a}$$

The constants p and ε in (10a) are given in general by

$$p=-\int_{-\infty}^{+\infty}d\xi\tilde{w}^i(\theta(\xi),\theta(\xi))\left(\frac{\partial\theta}{\partial\xi}\right)^2 \tag{10b}$$

$$\varepsilon=w_\infty{}^i(\theta_*,\pi-\theta_*)-w_\infty{}^i(\theta_*,\theta_*), \tag{10c}$$

where $\tilde{w}^i(\theta,\theta')=\partial^2\tilde{w}_\infty^i(\theta,\theta')/\partial\theta\partial\theta'$. It is important to note that the distinctive features of the magnetization reversal in YIG:Co are intimately related to the fact that the domain walls are not 180° walls and $\varepsilon\neq0$. It is this circumstance which determines the shape memory of the domain structure[7] and also the jump and then the slow relaxation of the magnetization. Using (5) and (8), we can calculate p and ε:

$$p=g(\theta_*)G/\Delta, \quad \varepsilon=(\sin^2(2\theta_*)/2)G. \tag{11}$$

The function $g(\theta_*)$ in (11) is

$$g(\theta_*)=(1/16\cos\theta_*\sin\theta_*)\{[\cos(2\theta_*)(\pi-2\theta_*)+\sin2\theta_*](6f+5)-\sin(2\theta_*)(2f-1)\}, \quad f=F/G.$$

A simple calculation (Ref. 8, for example) yields k and h also. In our model, they are

$$k=16\pi\eta(M_s\cos\theta_*)^2/d, \quad h=2HM_s\cos\theta_*, \tag{12}$$

where η is a numerical coefficient with a value $\eta\sim1$, d is the period of the domain structure, and the field $\mathbf{H}$ is directed along the normal to the film $H=|\mathbf{H}|$. These equations make it possible to write the equation which we are seeking, for the instantaneous coordinate of the domain wall. It turns out to be a rather complicated nonlinear integral equation:

$$\int_0^t\frac{dt'}{\tau}\exp\left(\frac{t'-t}{\tau}\right)\Omega(x,x(t'))+\exp\left(-\frac{t}{\tau}\right)\Omega(x,0)+kx+\lambda\dot{x}=h_c\operatorname{sign}(\dot{x})+h(t), \tag{13}$$

where $\Omega(x,x')=(\partial/\partial x)\Xi(x,x')$.

In writing (13) we have also taken account of the coercivity of the material, by means of the parameter $h_c=2MH_c\cos\theta_*$, where H_c is the coercivity field.

Let us examine the properties of the integral operator in (13) in various limiting cases. We first note that a detailed analysis can be carried out for extremely slow motions, with $\dot{x}(t)\to0$ (we will refine the inequality below). In this case we have $\Omega(x(t),x(t'))\approx p[x(t)-x(t')]\approx p\dot{x}(t)(t-t')+o((\dot{x}\tau/\Delta)^2,\ddot{x}\tau^2/\Delta)$. The integral in (13) can then be calculated. The integral operator is replaced by $p\tau(\partial/\partial t)$. It leads to a renormalization of the viscosity coefficient $\lambda:\lambda\to\lambda+p\tau$. This simplification is legitimate, however, only under the rather strong inequalities $\dot{x}\tau\ll\Delta$ and $\tau\ddot{x}\ll\dot{x}$. As we will see below in the exact formula, these inequalities may not hold.

The second limiting case is a rapid motion of the domain wall: $\dot{x}\tau \gg \Delta$. As was mentioned in Ref. 13, the dissipative part of the integral operator is small in this case. At $t \gg \tau$, this operator is approximately $\varepsilon \operatorname{sign}(\dot{x}) + o(\Delta/\dot{x}\tau)$, and it describes a potential force acting on the domain wall. This case is pertinent to a description of the motion of a domain wall under the influence of an external field and also just after a field pulse in the first step of the experiment, during the rapid motion.

The assertions made above concerning the particular features of the motion of solitons in memory media are fairly general. When the soliton moves slowly, so that the medium has time to relax in the region in which the soliton is localized, the effect of the medium on the soliton is of the nature of a viscous damping force. When the motion is sufficiently rapid, the soliton is moving through a medium which does not have time to adjust. In this case the force exerted by the medium is a potential force.

We will model the magnetization reversal in terms of a single effective domain wall. This is of course an approximation, but it leads to a rather good description of the basic effect, the motion of the domain wall under the influence of the restoring force when there is a pronounced magnetic viscosity. The model of the motion of the domain wall is as follows: We assume that, over the duration of the field pulse, the domain wall moves a large distance away from its equilibrium position $x = 0$ and then returns to this state. By taking this approach, we can avoid discussing the processes by which the domain wall is nucleated after the field is turned off. This approach is sufficient for our problem. Specifically, the slow motion in which we are interested here occurs from the position of the domain wall, $x = x_0$, which is smaller than the period of the domain structure, d. According to experiments, the nucleation of the domain wall is a very rapid event and can be classified as a fast step, which we would not have to discuss in detail for our purposes here.

Skipping over the details, we write an expression for the relief which is set up by the induced magnetic anisotropy and which determines the motion of the domain wall just after the field pulse is turned off, provided that at this instant the coordinate of the domain wall is $x = r$, where $r \gg d$:

$$\frac{\partial V^i}{\partial x} = \Omega(x, r) + [\Omega(x, 0) - \Omega(x, r)]\, e^{-D/\tau}. \qquad (14)$$

We can use this equation to study the motion of a domain wall during the fast step of the return to the equilibrium state. Specifically, it follows from Eq. (13), with (14), that if the pulse is long enough that the relief set up by the induced magnetic anisotropy does not have time to relax (see Fig. 3 and the associated caption) then the domain wall does not reach the equilibrium position $x = 0$. It instead stops at the point $x_0(D)$. An expression for $x_0(D)$ can be found from the equation $kx = h_c + \partial V^i/\partial x$, where $(\partial V^i/\partial x)$ is given by (14). Taking account of the asymptotic expressions for $\Omega(x,r)$, we can write explicit expressions for $x_0(D)$ in various limiting cases. We restrict the discussion to the most pertinent case, $x_0(D) \gg \Delta$ (a smaller value of x_0 could not be determined experimentally). In this case we have

$$x_0(D) = \begin{cases} h_c/k, & D < \tau/\ln 2, \\ h_c/k + (\varepsilon/k)(1 - 2e^{-D/\tau}), & D > \tau/\ln 2. \end{cases} \qquad (15)$$

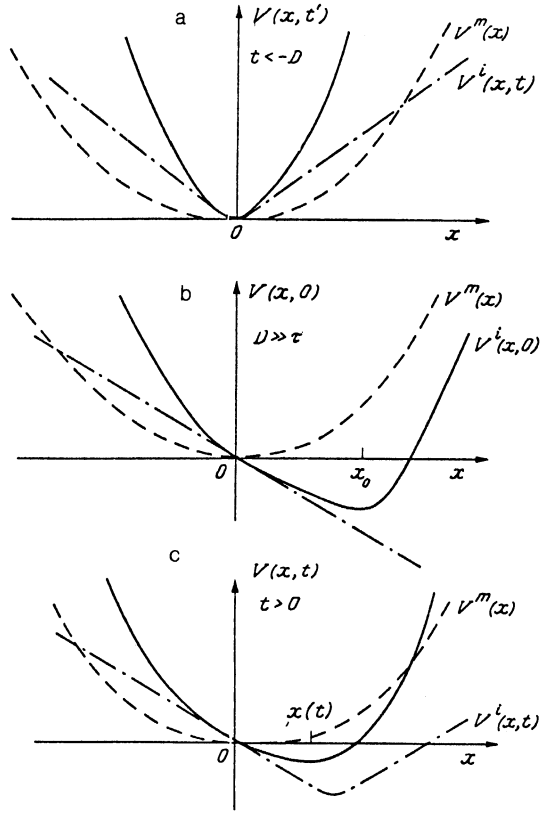

FIG. 3. Time evolution of the potential relief in which the domain wall moves in a self-consistent fashion: $V(x,t) = V^i(x,t) + V^m(x)$. The solid line shows the overall potential relief $V(x,t)$, which is the sum of the magnetostatic energy $V^m(x)$ (shown by the dashed line) and the energy of the induced magnetic anisotropy, $V^i(x,t)$ (the dot-dashed line). a—The relief before the application of the field pulse, at $t < -D$; b—the relief immediately after the field pulse, at $t = 0$, with $D \gg \tau$[$x_0 = x_0(D)$ is the initial position of the domain wall after the jump]; c—at an intermediate time, at which there is a constant, self-consistent adjustment of the energy of the induced magnetic anisotropy toward the instantaneous position of the domain wall, $x(t)$, as this wall moves toward its equilibrium position.

When effects of order Δ/x_0 are taken into account, this dependence is described by a smooth function. However, the actual corrections of this type to (15) are unimportant in an analysis of experimental data. An important point is that in the case $D \gg \tau$ (the case of most interest), the value of $x_0(D)$ remains nonzero as $h_c \to 0$ and is determined not by the coercivity but by the substation modification of the induced magnetic anisotropy. Experimental and theoretical plots of $x_0(D)$ are shown in Fig. 2.

According to the ideas developed above and also according to the experiments, after the domain wall quickly reaches the point $x = x_0(D)$ it begins to move, considerably more slowly, toward the original equilibrium state ($x = 0$ and $h_c \to 0$). This step of the evolution should be described by means of the exact integral equation. We solve this equation under the assumption $|x(t) - x(t')| < \Delta$ for $t - t' < \tau$, and we verify this assumption below.

Under this condition, and with allowance for the circumstance that the integral is dominated by the region $t - t' \leqslant \tau$, by virtue of the exponential function $\exp[(t' - t)/\tau]$, we can approximate the kernel of the integral operator by the expression $\Omega(x,x') \approx p(x - x')$. We omit the dissipative term with $\lambda\dot{x}$ from (13), since the effec-

tive dissipation associated with the induced magnetic anisotropy is usually much greater than the intrinsic magnetic dissipation in the iron garnets. Equation (13) then becomes linear in x:

$$x(t)-x_c=(x_0(D)-x_c)e^{-t/\tau} - \frac{p}{k}\int_0^t \frac{dt'}{\tau}(x(t)-x(t'))\exp\left(\frac{t'-t}{\tau}\right), \quad (16)$$

where $x_c = h_c/k$, and we measure the time t from the end of the pulse.

Multiplying (16) by $e^{t/\tau}$, and differentiating it with respect to the time, we can go over from an integral equation to the differential equation

$$\dot{x}\left(1+\frac{p}{k}(1-e^{-t/\tau})\right)+\frac{x^{\bullet\bullet}}{\tau}=\frac{x_c}{\tau}. \quad (17)$$

This equation can be solved easily. As a result we find an equation for $x(t)$:

$$\frac{x(t)-x_c}{x_0(D)-x_c}=\frac{1}{[(\varkappa+1)e^{t/\tau}-\varkappa]^{1/(1+\varkappa)}}. \quad (18)$$

where $\varkappa = p/k$. This parameter plays an important role in the analysis below. The value of $\varkappa$ is determined by the relation between the dipole energy and the induced magnetic anisotropy. In other words, it depends on the properties of the material ($2\pi M_S^2, F$, and G) and also the type of domain structure. To estimate $\varkappa$ it is sufficient to assume $k \approx 2\pi M_S^2/d$, where d is the period of the domain structure, and $p \approx \max(F,G)/\Delta$. We then find $\varkappa \approx (\max(F,G)/2\pi M_S^2)(d/\Delta) \gg 1$. From (11) and (15) we also find the simple estimate $\varkappa \cong x_0(D=\infty)/\Delta \gg 1$. We turn not to an analysis of (18). For $t \gg \tau$, $x(t)$ tends toward x_c in accordance with $x(t)-x_c \approx (x_0(D)-x_c)\exp(-t/\tau_r)$, where $\tau_r = \tau(1+\varkappa) \approx \tau\varkappa$. The parameter τ_r, which characterizes the relaxation of the coordinate of the domain wall, x, and therefore the relaxation of the magnetization of the film after the field pulse in the last stage of the relaxation, is considerably longer than the relaxation time of the induced magnetic anisotropy, τ. We will write out some specific estimates below. If we instead take $t \ll \tau$, we find that the parameter $\varkappa$ does not appear in the result at all if $\varkappa \gg 1$, and the characteristic relaxation time is determined by τ.

Equation (18) thus determines the complicated nonexponential relaxation of the magnetization. This relaxation is described by a broad spectrum of relaxation times from τ to $\varkappa\tau$, as in an analysis of the experimental dependence $M(t)$. Before we go into a detailed description of the experimental data, we wish to verify our assumption $|x(t)-x(t-\tau)| \ll \Delta$, which we used in writing (16). It follows from (18) that with $\varkappa \gg 1$ this assumption is clearly valid at $t \gg \tau$. It turns out, however, that the initial velocity of the domain wall, $\dot{x}(0)$, is determined by the first term on the right side of (16) and is completely independent of the particular form of the kernel $\Omega(x,x')$ in the second integral term. The same value of the initial velocity, $\dot{x}(0) = (x_0(D)-x_c)(-1/\tau)$, follows from (18). It can thus be asserted that Eq. (18) is valid not only for $t \gg \tau$ but also for $0 < t \ll \tau$. In other words, this equation can be used throughout the stage of the slow motion of the domain wall after the initial jump.

4. DISCUSSION OF RESULTS

The theory gives a description of the $x(t)$ dependence in the slow stage of the motion as a general formula for various values of D and for a wide interval of t values. We would like to compare this dependence with the experimental demagnetization curve. To describe the motion of the domain wall, we convert the experimental dependence $M(t)$ into an effective displacement of the domain wall. This displacement is described by the dimensionless coordinate x, with the value $x = 1$ associated with the state of the film magnetized to saturation. With the equilibrium initial value $x = 0$ we associate the state of the film corresponding to an equilibrium domain structure (in this state the magnetization is nonzero, since the volumes of the domains are unequal because of the inclination of the axis; Sec. 2). Figure 4 shows curves of $x(t)$. In a similar way, we plot the dependence $x_0(D)$ corresponding to $M_0(D)$ (Fig. 2).

We determine the parameters of the problem, i.e., τ, $\varkappa$, and ε/k, by the method of least squares. The results of this fit of the $x(t,D)$ curves (Fig. 4) reveal good agreement between theory and experiment. This fit yields the values $\tau = 23$ ms, $\varkappa = 1350$, $\varepsilon/k = 0.32$, and $x_c = 0.53$. These values agree well with the values found by a fit based on the $x_0(D)$ dependence (Fig. 2): $\tau = 16$ ms, $\varepsilon/k = 0.41$, and $x_c = 0.53$.

There is some discrepancy here, which we do not regard as important. It may stem from the simplifications which we made in some secondary aspects of the problem. First, there was the assumption that the restoring force is a linear function of the displacement of the domain wall. Second, there is

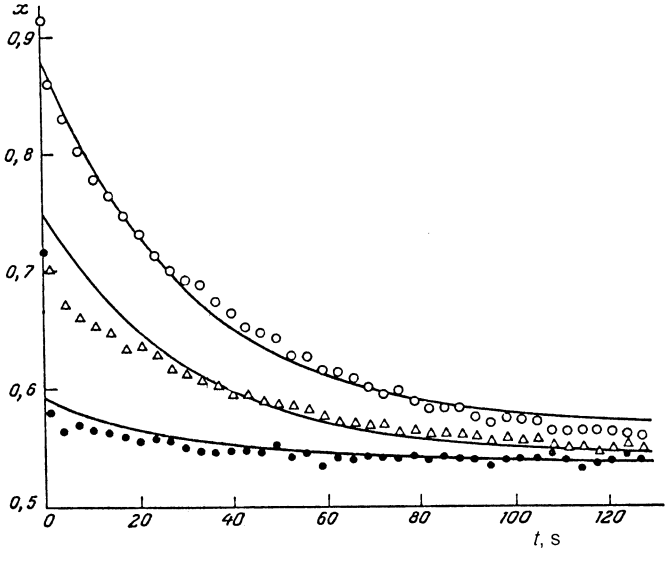

FIG. 4. Time evolution of the normalized magnetization, $x(t) = M(t)/M_S$, in the second, slow stage of the relaxation, for three values of the length D of the field pulse. The points are experimental: ○—$D = 0.5$ s; △—$D = 0.04$ s; ●—$D = 0.02$ s. The solid lines were calculated from (18) with the following values of the adjustable parameters: $\varepsilon/k = 0.32$, $\tau = 0.023$ s, $\varkappa = 1350$, and $x_c = 0.53$.

the approximation that there is only a single, effective domain wall (in a real material there would be a labyrinthine domain structure).

For an independent theoretical estimate of the parameter $\varkappa$, we use the formula $\varkappa = x_0(D=\infty)/\Delta$. Since $M_0(D) = 0.9M_S$ (Fig. 2), we assume $x_0(D=\infty) \approx d/2$, where d is again the period of the domain structure. According to Ref. 8, it is $d \simeq 20\,\mu$m. Taking $\Delta \simeq 10^{-6}$ cm, we find $\varkappa \simeq 10^3$, in agreement with the value found by the fitting procedure.

On the basis of the correspondence between the theoretical and experimental curves and also the correspondence between the parameter values found from the fit and the estimated values of these parameters, we can assume that the theory proposed here, which takes systematic account of the effects of the magnetic aftereffect in the nonlinear dynamics of a domain wall, is a good picture of the experimental situation for this magnetic material.

There are two points we would like to mention. The behavior which we have revealed here suggests a new method for measuring τ and also the field of the induced magnetic anisotropy, $H_{\text{IMA}} \simeq \max(F,G)/M_S$. We believe that this method can compete with the classical methods.[5,6] To measure τ it is sufficient to analyze the $x_0(D)$ dependence. Specifically, from the value $D = D_c$ corresponding to $x_0 = 0$ we immediately find the value $\tau = (\ln 2)D_c$. It is simple to find the field of the induced magnetic anisotropy from the asymptotic behavior of $x_0(D)$ as $D \to \infty$. Here it is sufficient to find ε from (10c) and (15); the coefficient k can either be calculated or found from independent quasistatic measurements. The error associated with the quadratic approximation of the restoring force can be reduced through an exact calculation of the restoring force and through a corresponding refinement of the expression for x_0.

The methods described here for measuring the parameters of the induced magnetic anisotropy are directly applicable only to non-180° domain walls. However, they can be modified to apply to magnetic materials with 180° domain walls. It would be sufficient, for example, to magnetize the film in an oblique field. After the pulse, the different domains are nonequivalent. As a result, we find the picture drawn above, with a jump of the domain wall to a position $x_0 \neq 0$, followed by a slow relaxation to the true equilibrium position.

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[1)] In addition to "magnetic aftereffect," the more precise terms "magnetic diffusion aftereffect" and "Richter aftereffect" are also used on occasion.

[2)] The magnetic anisotropy induced by magnetization is frequency referred to as the "field-induced magnetic anisotropy" in the literature. The first name is more general, because the induced-anisotropy effect also occurs in a zero field, in which case the role of the external field essentially reduces to one of fixing the orientation of the magnetization.

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