Dynamics of *W*-atoms under multifrequency excitation

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We have studied the effects of light pressure on a *W*- (a double Λ-) system when there are four traveling optical waves. We find that light-pressure effects are completely determined by the relative phase Φ of the exciting fields. The temporal evolution of the velocity distribution has been derived for various values of Φ. We calculate the dependence of the transverse temperature T of a beam of *W*-atoms on the phase Φ, and show that under certain conditions T can reach $\sim 3\cdot 10^{-6}$ K.

It is well known[1-3] that when quantum systems with closed-loop transitions coupled to a field are excited, a variety of effects associated with the sensitivity of the system to the relative phase of the applied fields come into play. Examples include Λ- and *V*-systems whose interaction loops are closed by a radio-frequency field.[2]

At the same time, any atomic system can in principle also be closed[1)] by optical fields—via additional levels, for example. The most interesting system with an interaction loop closed strictly by optical fields is the *W*-system (Fig. 1), which has been closely scrutinized of late with regard to the feasibility of making a laser with no population inversion.[3] It has in fact been pointed out[3] that a *W*-system will also be sensitive to the relative phase of the incident fields. Naturally, this sensitivity is of great interest in various fields of physical inquiry, both theoretical and applied.

In the present paper, we study the dynamics of a beam of *W*-atoms subject to strong light pressure. We find that the light-pressure effects are completely determined by the relative phase of the exciting fields. For example, for certain values of the phase, one observes light pressure typical of a two-level atom, while for other phases the force acting on a *W*-atom shows a narrow dip near zero velocity (a "velocity well"), resulting in a highly distorted velocity distribution. It is then possible to have an ensemble of *W*-atoms for which the temperature is much lower than the Doppler limit $T_0 = \hbar\gamma/k_B \sim 10^{-4}$ K (γ is the atomic transition's natural halfwidth and k_B is Boltzmann's constant). In the work reported here we have detected the direct effects of the relative phase on the temperature of cold atoms, and have shown that for deep cooling to take place ($T_0 \ll \hbar\gamma/k_B$), the phase relationship of the optical fields involved in the interaction must be taken into consideration.

Consider the *W*-system of atomic levels depicted in Fig. 1. We shall assume that transitions $|m\rangle$–$|n\rangle$ ($m = 1,2$; $n = 3,4$), which comprise a closed interaction loop, are dipole allowed, and that transitions $|1\rangle$–$|2\rangle$ and $|3\rangle$–$|4\rangle$ are dipole forbidden. We denote the probability of spontaneous downward transitions in the $|m\rangle$–$|n\rangle$ channel by γ_{nm}, and the rate of the cross-relaxation transition $|1\rangle$–$|2\rangle$ by Γ.

The electromagnetic field that the *W*-system interacts with is a sum of four waves propagating in the z direction:

$$\mathbf{E} = \sum_{\substack{m=1,2\\ n=3,4}} \mathbf{e}_{nm} E^0_{nm} \cos(\omega_{nm} t + (-1)^m k_{nm} z + \chi_{nm}). \qquad (1)$$

Here ω_{nm} and χ_{nm} are the frequencies and phases of the optical fields, k_{nm} is the moduli of the wave vector ($k_{nm} = \omega_{nm}/c$), and $\mathbf{e}_{nm}$ is a polarization unit vectors.

We now find the force F_z exerted by light pressure on a *W*-atom in the field (1). For strong optical transitions ($\hbar^2 k^2/2m \ll \hbar\gamma$), and at times $t \gg \gamma^{-1}$, it has been shown[4] that

$$F_z = 2\hbar \sum_{\substack{n=3,4\\ m=1,2}} (-1)^m k_{nm} \operatorname{Re}(i V_{nm} \rho_{mn}), \qquad (2)$$

where ρ_{mn} is an off-diagonal element of the atomic density matrix, and $V_{nm} = \langle n|\hat{dE}|m\rangle = V^0_{nm}\exp(i\varphi_{nm})$ is the matrix element of the dipole interaction operator ($\varphi_{nm} = \chi_{nm} + \vartheta_{nm}$, where ϑ_{nm} is the phase of the dipole moment matrix element).

Replacing the off-diagonal elements

$$\rho_{mn} \to \rho_{mn} \exp\{-i[\omega_{nm} t + \varphi_{nm} + (-1)^m k_{nm} z]\},$$
$$\rho_{12} \to \rho_{12} \exp\{-i[(\omega_{31} - \omega_{32})t + (\varphi_{31} - \varphi_{32}) - (k_{31} + k_{32})z]\},$$
$$\rho_{34} \to \rho_{34} \exp\{-i[(\omega_{42} - \omega_{32})t + (\varphi_{42} - \varphi_{32}) + (k_{42} - k_{32})z]\}$$
$$(m=1,\ 2;\quad n=3,\ 4)$$

we obtain the following set of equations for the elements $\rho_{mn}(\mathbf{r},\mathbf{p},t)$ of the *W*-atom's density matrix:

$$\dot\rho_{nn} = \gamma_{4n}\rho_{44} + \gamma_{3n}\rho_{33} + 2\operatorname{Re}(i\rho_{n3} V_{3n}^0) + 2\operatorname{Re}(i\rho_{n4} V_{4n}^0),$$
$$\dot\rho_{ss} = -(\gamma_{s1} + \gamma_{s2})\rho_{ss} - 2\operatorname{Re}(i\rho_{1s} V_{s1}^0) - 2\operatorname{Re}(i\rho_{2s} V_{s2}^0),$$
$$\dot\rho_{12} = -[i(\Omega_M - 2kv_z) + \Gamma]\rho_{12} + i\rho_{13} V_{32}^0 - iV_{13}^0\rho_{32}$$
$$+ (i\rho_{14} V_{42}^0 - iV_{14}^0 \rho_{42}) e^{-i\Phi},$$
$$\dot\rho_{ns} = -[i(\Omega_{s1} - \delta_{2n}\Omega_M + (-1)^n k v_z) + \gamma_{ns}]\rho_{ns} + iV_{ns}^0(\rho_{nn} - \rho_{ss})$$
$$+ i\rho_{nm} V_{ms}^0 \exp[i(2\delta_{n1} - 1)\delta_{s4}\Phi]$$
$$- iV_{nr}^0 \rho_{rs} \exp[i(2\delta_{s4} - 1)\delta_{1n}\Phi],$$
$$\dot\rho_{34} = -[i(\Omega_{41} - \Omega_{31}) + \gamma_{34}]\rho_{34} + i(\rho_{31} V_{14}^0 - V_{31}^0 \rho_{14}) e^{-i\Phi}$$
$$+ i\rho_{32} V_{24}^0 - iV_{32}^0 \rho_{24} \qquad (3)$$

($n,m = 1,2$; $n \neq m$; $s,r = 3,4$; $s \neq r$; and δ_{mn} is the Kronecker delta).

We use the following notation in (3): $\Phi = \varphi_{31} - \varphi_{32} + \varphi_{42} - \varphi_{41}$ is the total phase around the atomic loop; $\Omega_{nm} = \omega_{nm} - \omega^0_{nm}$ is the mistuning of an exciting field (ω^0_{nm} is the frequency of transition $|m\rangle$–$|n\rangle$);

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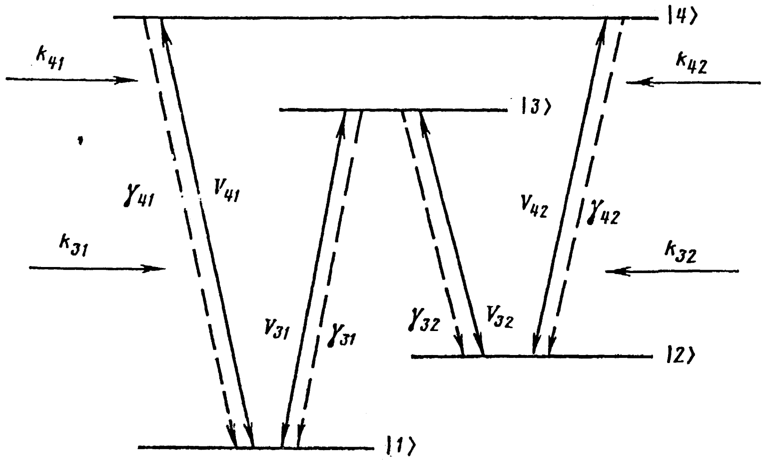

FIG. 1. A W-system: a closed interaction loop comprising the transitions $|1\rangle-|3\rangle$, $|3\rangle-|2\rangle$, $|2\rangle-|4\rangle$, and $|4\rangle-|1\rangle$. Transitions $|1\rangle-|3\rangle$ and $|1\rangle-|4\rangle$ are excited by fields with frequencies ω_{31} and ω_{41} propagating in the z direction; transitions $|2\rangle-|3\rangle$ and $|2\rangle-|4\rangle$ are excited by fields with frequencies ω_{32} and ω_{42} propagating in the z direction.

$\Omega_m = \Omega_{31} - \Omega_{32}$. In deriving (3), we have assumed multiphoton resonance ($\omega_{31} - \omega_{32} = \omega_{41} - \omega_{42}$), and also that the wave vectors all have the same modulus, $k_{mn} = k$. Solving the system (3) in the stationary case, we obtain the off-diagonal elements ρ_{mn} of the density matrix, and thereby determine the force due to light pressure using (2).

In Fig. 2 we have plotted the light-pressure force F_z as a function of v_z, having solved (3) numerically for various values of Φ. Varying the total phase Φ around the atomic loop clearly produces a qualitative change in the force exerted on a W-atom by light pressure. With $\Phi = \pi(2n+1)$ ($n = 0,1,2,...$), for example, F_z has the familiar Lorentzian profile typical of two-level atoms.[4] As soon as the phase approaches $2\pi n$, however, a narrow dip in the force profile (a "velocity well") appears near zero velocity, leading to qualitatively different W-atom dynamics as compared with $\Phi = \pi(2n+1)$.

Note that physically, the appearance of a velocity well near $\Phi = 2\pi n$ can be interpreted in terms of coherent population capture in the W-system.[5] The mechanism whereby the phase dependence comes about is then completely analogous to the mechanism that prevails in a closed Λ system,[2] and is associated with the direct influence of the closure fields on the magnitude ρ_{12} of the low-frequency coherence.

We emphasize here that since these effects are coherent, phase fluctuations in the exciting fields can make them vanish. For the effects to be manifested in a stable fashion, it is necessary to employ cross-correlated fields[6] such as those obtained by using acoustooptic modulators.

The evolution of the velocity distribution of a beam of W-atoms undergoing transverse cooling (beam collimation) is illustrated in Fig. 3. It can be seen that for $\Phi = 0$, the distribution has a narrow peak approximately 12 cm/sec wide on the left-hand shoulder of the velocity well (the width of the original velocity distribution is $\Delta v_z \sim 1$ m/sec, corresponding to $T \sim T_0$, and can be derived by normal Doppler cooling[4]). At the same time, for $\Phi = \pi$, light pressure produces no additional deformation of the W-atom velocity distribution, merely displacing the distribution as a whole toward positive velocities without changing the original width.

We now estimate the temperature of the W-atoms near zero velocity. To do so, we use (3) to find the momentum diffusion coefficient D_{zz} for $v_z = 0$,

$$D_{zz} = \hbar^2 k^2 [(\gamma_{31}+\gamma_{32})\rho_{33} + (\gamma_{41}+\gamma_{42})\rho_{44}].$$

The temperature of the cold atoms is then[4]

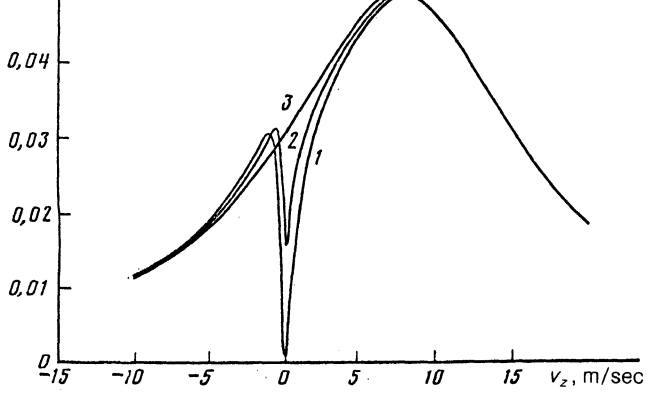

FIG. 2. The force F_z due to light pressure as a function of velocity v_z for $V^0_{mn} = 0.3$ (henceforth in units with $\gamma = 10^7$ Hz), $\gamma_{n1} = 1$, $\gamma_{n2} = 0.2$ ($m = 1,2$; $n = 3,4$), $\Gamma = 0.001$, $\Omega_{31} = \Omega_{41} = 2$, and $\Omega_M = 0$, and *1*) $\Phi = 0$; *2*) $\Phi = \pi/2$; *3*) $\Phi = \pi$.

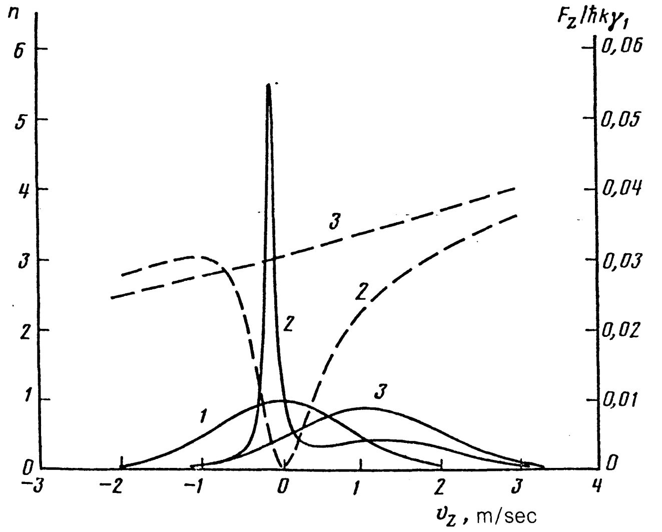

FIG. 3. Evolution of the velocity distribution of a W-atom under the influence of a force F_z for a time $\tau = 2\cdot 10^{-5}$ sec with $V^0_{mn} = 0.3$, $\gamma_{n1} = 1$, $\gamma_{n2} = 0.2$ ($m = 1,2$; $n = 3,4$), $\Gamma = 0.001$, $\Omega_{31} = \Omega_{41} = 2$, $\Omega_M = 0$, and *1*) $\tau = 0$; *2*) $\tau = 2\cdot 10^{-5}$ sec and $\Phi = 0$; *3*) $\tau = 2\cdot 10^{-5}$ sec and $\Phi = \pi$. The dashed curves show F_z for $\Phi = 0$ (*2*) and $\Phi = \pi$ (*3*).

$$T = \frac{D_{zz}}{\beta k_B}, \text{ where } \beta = -\left.\frac{\partial F_z}{\partial v_z}\right|_{v_z=0}$$

is the coefficient of dynamic friction near zero velocity.

In Fig. 4 we have plotted the transverse temperature T as a function of the overall mistuning $\Omega = \Omega_{nm}$ ($m = 1,2$; $n = 3,4$) of the exciting fields for $\Phi = 0$. We see that there is a range of Ω for which T is much lower than the Doppler limit T_0. For the parameters of curve *1* in Fig. 4, we reach $T_{\min} \approx 3.6\ \mu$K, which is only three times the temperature corresponding to the atom's recoil energy, $T_R = \hbar^2 k^2 / 2Mk_B \sim 1.2\ \mu$K (for the D_1 line of sodium).

Once again we stress that in closed systems, the dependence on the phase relationships among the exciting fields is critical. In Fig. 5, we show T as a function of the total phase around the atomic loop. The indication is that for $\Phi = 2\pi n$ ($n = 0,1,2,...$), the temperature reaches a minimum that may be much lower than T_0. Likewise, near $\Phi = \pi(2n+1)$, there is an abrupt increase in the transverse temperature, due to the vanishing of β, the coefficient of dynamical friction (Fig. 2).

We have thus shown that, depending on the value of the phase Φ, a single interaction scheme of a W-atom and the field (1) can produce two fundamentally different physical outcomes. The first is encountered when $\Phi = 2\pi n$, whereupon there is a sharp deformation of the velocity distribution, and in principle, W-atom temperatures much lower than T_0 are attainable. In the second ($\Phi = \pi(2n+1)$), there is no such additional deformation of the velocity profile, and as before the temperature of the atomic beam cannot be lower than T_0.

In closing, we point out that Φ-dependence of the transverse temperature and of the shape of the velocity distribution can easily be investigated experimentally: on the one hand, techniques for observing light-pressure effects are well developed, and on the other, the required W-system can be set up in the D_1 line of sodium, to take one example. Four cross-correlated fields are required for such an experiment,[6]

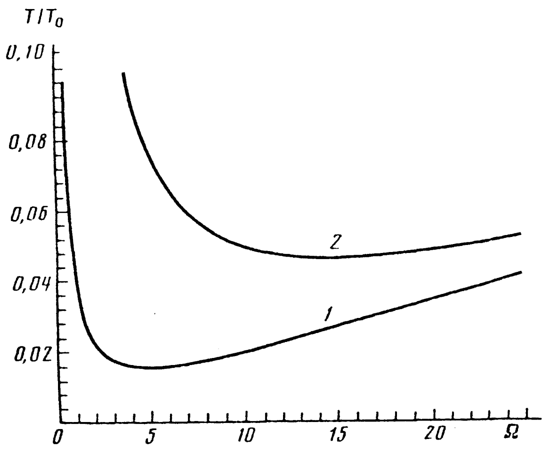

FIG. 4. Ω-dependence of the temperature T of a beam of W-atoms at $v_z = 0$ for $\gamma_{n1} = 1$, $\gamma_{n2} = 0.2$, $\Gamma = 0.001$, $\Omega_M = 0$, and $\Phi = 0$. The curves correspond to *1*) $V^0_{mn} = 0.1$ and *2*) $V^0_{mn} = 0.3$.

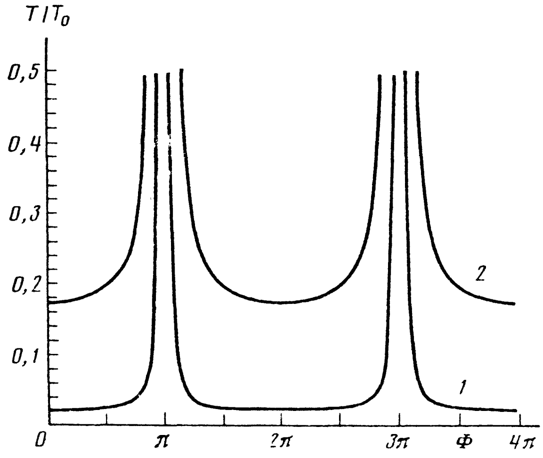

FIG. 5. Φ-dependence of the temperature T of a beam of W-atoms at $v_z = 0$ for $\Omega = 2$; other parameters are the same as in Fig. 4. The curves correspond to *1*) $V^0_{mn} = 0.1$ and *2*) $V^0_{mn} = 0.3$.

and these can be obtained using two acoustooptic modulators.

[1)]Here *closure* implies a closed loop of transitions coupled to the field.

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