Passage of light through a system of two bistable thin films

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We give a theoretical discussion of the passage of light through a system of two bistable thin films of two-level atoms (centers). We show for the limiting case of an inertialess medium that, depending on the external parameters, the light dynamics of the system admit the occurrence of both regular and chaotic regimes. If the distance between the films is an integer multiple of the wavelength the system of the two films behaves as a separate bistable element. Self-oscillations arise if an odd number of half-wavelengths can be fitted between the films and absorption in the medium is inappreciable. We prove that it is possible for the system to operate as a multivibrator with different regimes.

INTRODUCTION

Optical bistability is nowadays a well studied property of the nonlinear interaction between light and matter.[1,2] The practical application of bistability is first and foremost connected with the construction of an optical computer where the role of the elementary cell is intended for the bistable element while different elements are coupled by a beam of light. With this goal in mind the problem of the passage of light through a chain of several bistable elements is of immediate importance.

The problem of the propagation of light through a chain of coupled nonlinear elements can be considered from the point of view of an approach which has been called synergetic.[3,4] Such an approach assumes that a set of several interacting subsystems (in the present case bistable elements) as a combined system demonstrates a new aspect which is not present in an elementary subsystem. In Ref. 5 the collective dynamics of coupled nonlinear elements was considered (using the Ikeda model[6]); the idea was expressed there that it is possible to use the collective properties of a chain to store and process information. The present paper can be considered as a first step in the study of the collective properties of a set of optically bistable thin films.

As a basic model we choose in the present paper a thin film of two-level atoms (centers). This model has recently attracted the attention of many researchers.[7–15] For our purposes the following facts were the main ones for choosing the model.

1. The small film thickness (much smaller than the wavelength of the incident radiation) makes it possible to avoid solving complicated partial differential equations and the problem reduces to a study of a dynamic system.

2. The equations for the film turn out to be identical to the equations for the case of an optical bistable nonlinear interferometer in the mean field approximation.[1] This fact indicates the possibility of applying the results of the present paper to other bistable elements.

3. Because it is thin the film is sensitive to the phase properties of the light. This makes it possible to use in addition to characteristics such as the intensity yet another control parameter of a coherent light beam—the phase of the field.

The paper is organized as follows. In Sec. 1 we write down the basic equations of the model and then consider the nonlinear mapping to which the problem reduces. We then give an analysis of the stability of the stationary states and determine the possibility for the occurrence of complex regimes (including a dynamic chaos regime). In the Conclusion we analyze the main properties of the system.

1. EQUATIONS OF THE MODEL

We consider the following construction of thin films where the films are positioned at a distance l from one another (Fig. 1) in a medium with a complex refractive index $n_c = n - i\varkappa$. The thickness L is assumed to be much smaller than the wavelength of the incident field.

We write the linearly polarized light fields in the form

$$\mathscr{E}_j^{(+)}(z,\ t)=E_j^{(+)}(z,\ t)\exp\{i[\omega t-k(z-z_j)]\}+\text{c.c.},\qquad(1)$$

$$\mathscr{E}_j^{(-)}(z,\ t)=E_j^{(-)}(z,\ t)\exp\{i[\omega t+k(z-z_j)]\}+\text{c.c.},\qquad(2)$$

where $j = 0, 1, 2$; $E_j^{(+)}$ and $E_j^{(-)}$ are the slowly varying field amplitudes and $k = n_c\omega/c$ is the wavenumber. The "+" superscript corresponds to the field propagating in the positive z direction and the "−" superscript to propagation in the negative z direction. It is clear that the field $\mathscr{E}_{j+1}$ differs from $\mathscr{E}_j$ by the magnitude of the secondary field $\mathscr{P}_j$ induced by the atomic polarization of the jth film. One can show by solving the Maxwell equations[10,11] that the following equations hold for the field amplitudes:

$$E_{j+1}^{(+)}(z_j,t)=E_j^{(+)}(z_j,t)-2i\pi\omega nLP_j(t)/c,\qquad(3)$$

$$E_j^{(-)}(z_j,t)=E_{j+1}^{(-)}(z_j,t)-2i\pi n\omega LP_j(t)/c,\qquad(4)$$

where $P_j(t) = P(z_j,t)$ is the slowly varying part of the polarization produced by the atoms of the jth film. The complete expression for the atomic polarization has the form

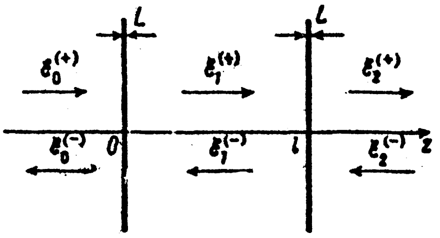

FIG. 1. Bistable films (thickness L) at a distance l from one another in a medium with a refractive index $n_c = n - i\varkappa$.

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$$\mathscr{P}_j^{(+)}(z,\ t)=P_j(t)\exp[i\omega t-ik(z-z_j)]+\text{c.c.},\tag{5}$$

$$\mathscr{P}_j^{(-)}(z,\ t)=P_j(t)\exp[i\omega t+ik(z-z_j)]+\text{c.c.}\tag{6}$$

In Eqs. (3)–(6) we have used the fact that the thickness L of the film is much smaller than the wavelength, so that the film is a "point source" relative to the z axis. The following equations follow from Eqs. (3) and (4):

$$E_{j+1}^{(+)}+E_{j+1}^{(-)}=E_j^{(+)}+E_j^{(-)},\tag{7}$$

which corresponds to the total field amplitude being constant to the right and to the left of each film.

The dynamics of the polarization for the films are described by the Bloch equations[16]

$$\frac{dR_j}{dt}=-\frac{R_j}{T_2}+i\frac{pW_jE_j}{\hbar},\tag{8}$$

$$\frac{dW_j}{dt}=-\frac{W_j+1}{T_1}+\frac{ip}{2\hbar}(E_j^*R_j-E_jP_j^*),\quad j=1,2,\tag{9}$$

where W_j is the difference in populations between the levels, R_j is the slowly varying part of the off-diagonal element of the density matrix of the two-level atom, p is the transition dipole moment, and T_1 and T_2 are the longitudinal and transverse relaxation times. We assumed for Eqs. (8) and (9) that the condition of exact resonance between the incident field and the atomic transition was satisfied. The polarization in the film is $P_j=pNR_j$, where N is the density of the two-level atoms. The quantity E_j in Eqs. (8) and (9) is the effective electric field acting on the atoms. The effective field is determined not only by the external incident field, but also by the internal polarization of the medium. It was shown in Refs. 10 and 11 that one can write the effective field in the form

$$E_j(t)=E_{j-1}^{(+)}(z_j,t)+E_j^{(-)}(z_j,t)-i\hbar R_j(t)/pT_R,\tag{10}$$

where the quantity $T_R=\hbar c/2\pi nNL\omega p^2$ is the characteristic parameter of the thin film and determines the superradiation processes in it.[11,14] The presence of the last term in Eq. (10) reflects the internal feedback between the atoms in the film. Thanks to this fact a mirrorless optical bistability is realized in the system. When substituting (10) into (8) and (9) for the stationary case ($dR_j/dt=0, dW_j/dt=0$) we are led to the relation

$$y=x\left(1+\frac{2C}{1+|x|^2}\right),$$
$$C=T_2/2T_R,\quad x=E_j,\quad y=E_{j-1}^{(+)}(z_j,t)+E_j^{(-)}(z_j,t),\tag{11}$$

which was first obtained for describing the optical bistability of nonlinear interferometers in the mean field approximation.[1] We draw attention in Eq. (11) to the fact that the phases of the incident (x) and the transmitted (y) fields are the same. This fact turns out to be very important for the interaction of light with films. The similarity of the thin film model and the nonlinear interferometer suggest that the results of the present paper may be used for other bistable devices.

One must note that in considering the problem of the effective electric field (10) in Refs. 8 and 12 the necessity was discussed of introducing yet another term, $\propto\frac{4}{3}\pi P$, which corresponds to the Lorentz correction.[17] The problem then becomes more complicated and we do not consider the Lorentz correction in our paper.

We refine the values of the E_j fields acting on the atoms of the two films:

$$E_1(t)=E_{0i}(0,t)-i\frac{i\hbar R_1(t)}{pT_R}+\left[E_{2i}\left(l,t-\frac{nl}{c}\right)-\frac{i\hbar R_2(t-nl/c)}{pT_R}\right]e^{-ikl},\tag{12}$$

$$E_2(t)=E_{2i}(l,t)-\frac{i\hbar R_2(t)}{pT_R}+\left[E_{0i}\left(0,t-\frac{nl}{c}\right)-\frac{i\hbar R_1(t-nl/c)}{pT_R}\right]e^{-ikl},\tag{13}$$

where $E_{0i}(0,t)$ and $E_{2i}(l,t)$ are the amplitudes of the external fields which are incident, respectively, from the left on the first and from the right on the second film. The terms with retardation in Eqs. (12) and (13) indicate the effect the films have upon one another. The factor $\exp(-ikl)$ indicates that in our problem not only the amplitude, but also the phase relations between the incident fields are important. For instance, depending on the phases, the films may turn out to be either in the nodes or in the antinodes of the standing wave formed by the coherent light fields propagating in opposite directions to one another. We give the incident fields in the form

$$E_{0i}(0,t)=\begin{cases}0, & t<0\\ E_{0i}, & t\geqslant 0\end{cases},\tag{14}$$

$$E_{2i}(l,t)=\begin{cases}0, & t<0\\ E_{2i}, & t\geqslant 0\end{cases}\tag{15}$$

where E_{0i} and E_{2i} are real time-independent amplitudes. The final dynamics of the passage of light through films is described by Eqs. (8), (9), and (12)–(15).

2. CASE OF AN INERTIALESS MEDIUM

Equations (8), (9), (12), and (13) with the conditions (14) and (15) constitute a set of six nonlinear differential equations with retarded arguments. In view of the impossibility of finding a general solution of the set we consider some simplifications. We assume that the medium is inertialess, i.e., that the relaxation times T_1 and T_2 are sufficiently small:

$$T_1\ll nl/c,\quad T_2\ll nl/c,\tag{16}$$

so that after a feedback time nl/c relaxation processes are damped in the films. One can then neglect the derivatives in (8) and (9) and consider all changes occurring in the system on a time scale $\Delta t=nl/c$. Equations (8) and (9) are then reduced to the following mapping: for the first film

$$r_1(m+1)=\frac{-2iC\{a-ir_1(m+1)+[b-ir_2(m)]e^{-ikl}\}}{1+|a-ir_1(m+1)+[b-ir_2(m)]e^{-ikl}|^2},\tag{17}$$

and for the second film

$$r_2(m+1)=\frac{-2iC\{b-ir_2(m+1)+[a-ir_1(m)]e^{-ikl}\}}{1+|b-ir_2(m+1)+[a-ir_1(m)]e^{-ikl}|^2},\tag{18}$$

where we have introduced the notation

$$a=(T_1T_2)^{1/2}pE_{0i}/\hbar,\ b=(T_1T_2)^{1/2}pE_{2i}/\hbar,\ r_j=(T_1T_2)^{1/2}R_j/T_R,$$

while increasing m by unity corresponds to increasing the time t by $\Delta t = nl/c$. If we assume that when the fields are incident on the films the atoms are in the ground state, we can write the initial conditions in the form

$$r_j(0)=0,\quad j=1,\ 2. \tag{19}$$

If the incident fields a and b are equal to one another the problem becomes symmetric (see Fig. 1). The processes occurring in the two films turn out to be identical. Expressions (17) and (18) are the same. The dynamics of the system are described by the same mapping, (17) or (18). This case turns out to be equivalent to the problem, considered in Ref. 13, of the passage of light through a thin film of two-level atoms behind which there is a reflecting surface.

Note that $r_1(m+1)$ in the mapping (17) depends solely on $r_2(m)$, but is independent of $r_1(m)$; i.e., the state of the first film at the $m+1$st time depends solely on processes in the second film at the mth time. The same is true for the second film, described by the mapping (18). The change in a film thus affects the film itself through the other film after the lapse of a time interval $2\Delta t$ and the two-dimensional mapping reduces to two one-dimensional ones.

We have thus succeeded in the limit of an inertialess medium to reduce the problem to two nonlinear complex mappings given in implicit form. It is well known[18] that even the simplest logistic mappings have chaotic dynamics so that we have every reason to expect complex dynamic regimes in our system. To explore this problem we turn to finding stationary states and to studying their stability.

3. STATIONARY STATES AND STABILITY

We find the stationary states of the set (8) and (9) [or the fixed points of the mapping (17) and (18)]. We obtain a set of algebraic equations for the quantities r_{1s} and r_{2s} which characterize the stationary values of the polarizations in the films:

$$r_{1s}=\frac{-2iC[a-ir_{1s}+(b-ir_{2s})e^{-ikl}]}{1+|a-ir_{1s}+(b-ir_{2s})e^{-ikl}|^2}, \tag{20}$$

$$r_{2s}=\frac{-2iC[b-ir_{2s}+(a-ir_{1s})e^{-ikl}]}{1+|b-ir_{2s}+(a-ir_{1s})e^{-ikl}|^2}. \tag{21}$$

The stationary values of the populations are given by the expressions

$$W_{1s}=\frac{-1}{1+|a-ir_{1s}+(b-ir_{2s})e^{-ikl}|^2}, \tag{22}$$

$$W_{2s}=\frac{-1}{1+|b-ir_{2s}+(a-ir_{1s})e^{-ikl}|^2}. \tag{23}$$

Because they are complex the equations are still rather complicated. We consider two limiting cases, when the value of $\exp(-in\omega l/c)$ equals 1 or -1. The case $\exp(-in\omega l/c)=1$ corresponds to the case when between the films one can fit an integer number of wavelengths and the fields arrive at the films with the same phases. If $\exp(-in\omega l/c)=-1$ holds the distance between the films contains an odd number of half wavelengths and the waves "quench" one another. In those cases the equations simplify and it is possible to carry out an analytical study. We denote $\exp(-\varkappa\omega l/c)$ by s ($|s|<1$); i.e., we introduce a quantity indicating the attenuation of the light beam when it passes from one film to the other. We also introduce the notation $r_j = u_j + iv_j, j = 1,2$. Since the external fields are real quantities it follows from the equations that the quantity $u_j(t)$, which is initially zero, remains the same also in what follows. Equations (20) and (21) become real and take the form

$$v_{1s}=\frac{-2C[a+v_{1s}+(b+v_{2s})s]}{1+[a+v_{1s}+(b+v_{2s})s]^2}, \tag{24}$$

$$v_{2s}=\frac{-2C[b+v_{2s}+(a+v_{1s})s]}{1+[b+v_{2s}+(a+v_{1s})s]^2}. \tag{25}$$

In the $s=1$ case (medium without absorption) the right-hand sides of Eqs. (24) and (25) are the same. This means that the stationary states for v_1 and v_2 will be equal to one another: $v_{1s}=v_{2s}=v_s$. According to (24) and (25) one finds the quantity v_s from the standard expression (11) for the bistable characteristic where $x=a+b+2v_s$, $y=a+b+v_s$. The system of two bistable films thus repeats the properties of a separate bistable element. The system behaves also similarly when $\exp(-in\omega l/c)=-1$ with the difference that the stationary values for the polarizations satisfy the equation $v_{1s}=-v_{2s}=v_s$ while the quantity v_s can be found from Eq. (11) where $x=a-b+2v_s$, $y=a-b+v_s$. The phase difference of the incident waves leads to the fact that in the first case ($s=1$) the fields magnify one another while in the second case ($s=-1$) they weaken one another. According to (22) and (23), the stationary values of the populations are equal to one another both in the first and in the second case.

We turn to a study of the stability of the stationary states. First of all, we find the condition for stability of the original set (8) to (10). To do this we consider the dynamics of small perturbations

$$\Delta r_j(t)=r_j(t)-r_{js},$$
$$\Delta W_j(t)=W_j(t)-W_{js}.$$

After substituting the perturbations into the equations and linearizing in the standard way[19] we obtain a set of algebraic equations to find the growth rate σ characterizing the growth of the perturbations with time [$\Delta r_j(t)\propto\exp(\sigma t)$, $\Delta W_j(t)\propto\exp(\sigma t)$]. The condition for the compatibility of the algebraic system is the vanishing of the determinant

$$\begin{vmatrix} -\sigma\tau-\delta+\dfrac{\tau W_{1s}}{T_R} & f & W_{1s}\dfrac{\tau e^{-\sigma\Delta t}}{T_R}s & 0 \\ -v_{1s}-f & -\sigma\tau-1/\delta & -v_{1s}e^{-\sigma\Delta t}s & 0 \\ W_{2s}\dfrac{\tau e^{-\sigma\Delta t}}{T_R}s & 0 & -\sigma\tau-\delta+\dfrac{\tau W_{2s}}{T_R} & g \\ -v_{2s}e^{-\sigma\Delta t}s & 0 & -v_{2s}-g & -\sigma\tau-1/\delta \end{vmatrix}, \tag{26}$$

where we have $f=a+v_{1s}+(b+v_{2s})s$, $g=b+v_{2s}+(a+v_{1s})s$, $\tau=(T_1T_2)^{1/2}$, and $\delta=(T_1/T_2)^{1/2}$. The determinant (26) is written down for the $\exp(-nl\omega/c)=\pm 1$ case. The vanishing of the determinant can be somewhat simplified and be reduced to the form

$$\begin{vmatrix} -\sigma\tau-\delta+\dfrac{\tau W_{1s}}{T_R} & f \\ -v_{1s} & -f-\sigma\tau-1/\delta \end{vmatrix} \begin{vmatrix} -\sigma\tau-\delta+\dfrac{\tau W_{2s}}{T_R} & g \\ -v_{2s} & -g-\sigma\tau-1/\delta \end{vmatrix}$$

$$=\begin{vmatrix} f & W_{1s}\tau/T_R \\ -\sigma\tau-1/\delta & -v_{1s} \end{vmatrix}$$

$$\times\begin{vmatrix} g & W_{1s}\tau/T_R \\ -\sigma\tau-1/\delta & -v_{2s} \end{vmatrix} s^2e^{-2\sigma\Delta t}. \qquad (27)$$

We have thus obtained a transcendental equation for finding the parameter σ. We note that each of the determinants on the left-hand side of Eq. (27) corresponds to the stability condition of a separate bistable film. By putting $\sigma=0$ in (26) and (27) we get a relation between the control parameters a, b, s, and C of the system which gives us the boundary of the stability region of the stationary regime.

In the case of an inertialess medium [for the mappings (17) and (18)] the stability condition corresponds to the multiplier Λ being equal to unity.[3,4] One can then write condition (27) as follows

$$|\Lambda|=|\lambda_1\lambda_2|=1, \qquad (28)$$

where

$$\lambda_1=\frac{2C+2v_{1s}f}{1+2C+2v_{1s}f+f^2}s,\quad \lambda_2=\frac{2C+2v_{2s}g}{1+2C+2v_{2s}g+g^2}s.$$

The quantities λ_1 and λ_2 show the growth of the perturbations for the mappings (17) and (18), respectively. If a and b are the same the quantities λ_1 and λ_2 are equal to one another and are the same as the expression for the multiplier found in Ref. 13 in the study of the stability of the light dynamics of a thin film with an extra reflecting surface.

We note a simple method for studying the stability analytically. Instead of solving the algebraic nonlinear set of equations determining v_{1s} and v_{2s} it is much simpler to consider the inverse problem, i.e., to take as the known quantities $a+v_{1s}$ and $b+v_{2s}$. Afterwards one can easily calculate a, b, v_{1s}, and v_{2s} from Eqs. (24) and (25) and determine the stability of the system from (28) for some a, b, s, and C.

4. INSTABILITY AND CHAOS

We show that the light dynamics of coupled thin films described by the mappings (17) and (18) admits the presence of complex regimes, including chaotic ones. It makes sense to consider initially the symmetric case of equal amplitudes a and b. We show in Fig. 2 the bifurcation diagram for $s=-0.8$ and $C=4$. Along the vertical axis we plot the field amplitude after the first film, $e_1^{(+)}(z=0)=a+v_{1s}$ which is the same as $e_1^{(-)}(z=l)=b+v_{2s}$. The first bifurcation of the loss of stability by the stationary regime occurs for $a=b\approx 9.85$ (it is not shown in the figure). For larger values of a self-oscillation regimes appear. The transition to chaos occurs according to Feigenbaum's period doubling scenario.[18] The chaos region is interrupted by regularity "windows" where periodic regimes, $3\Delta t$, $5\Delta t$,... exist, and also their sequence with a doubled period. We show in Fig. 2b the results of calculating the Lyapunov exponent[3,4]

$$\sigma_L=\lim_{N\to\infty}\frac{1}{N}\sum_{m=1}^{N}\ln\left|\frac{\Delta v_{m+1}}{\Delta v_m}\right|. \qquad (29)$$

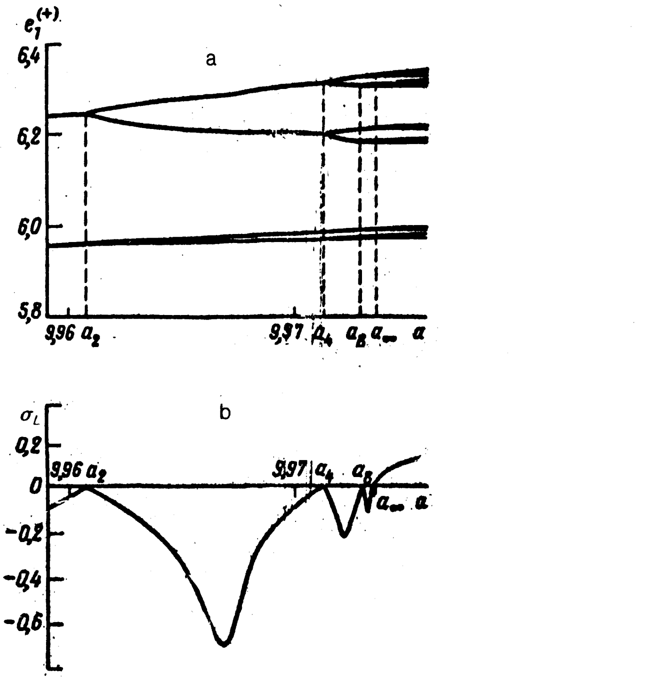

FIG. 2. a: Bifurcation diagram for the case in which the incident waves have equal amplitudes $a=b$, for $s=-0.8$, $C=4$. Along the ordinate is plotted the amplitude of the field transmitted through the film, $e_1^{(+)}(z=0)=e_1^{(-)}(z=l)$. b: Lyupanov exponent as function of the amplitude a.

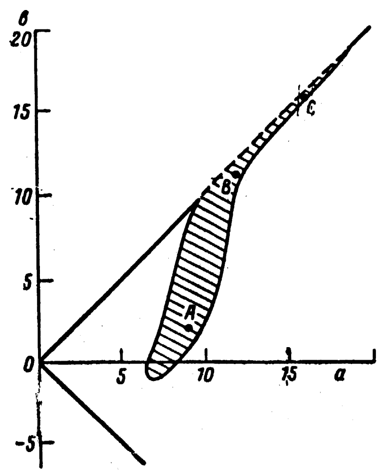

FIG. 3. The region where pulsation regimes exist is hatched in the plane of the control parameters a and b. The dashed section on the diagonal corresponds to the equal amplitudes case, shown in Fig. 2. The points A, B, and C correspond to different types of pulsating regimes.

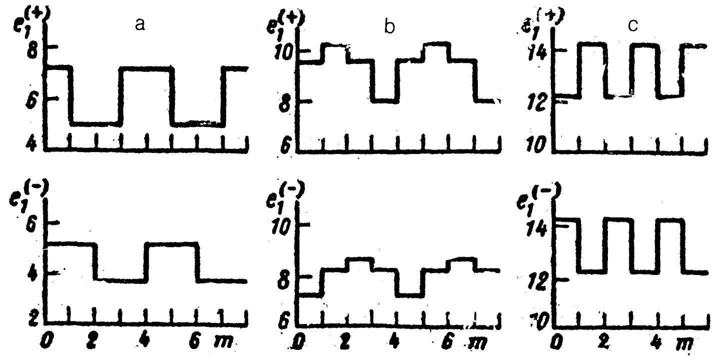

FIG. 4. Established regimes of the system (17) and (18) with the initial condition (19): a, b, and c correspond to the points A, B, and C in Fig. 3.

where the range of values of the parameter a corresponds to Fig. 2a. In the bifurcation points a_2, a_4, a_8 the Lyapunov exponent vanishes. The regular regimes are characterized by negative values of σ_L and when there is the transition to chaos for $a_\infty \approx 9.9736$ the exponent becomes positive. The chaos region occupies a small range of values of the parameter a and is shifted by $2\Delta t$ period oscillations which exist up to $a \approx 18.8$. Afterwards a stable stationary regime is again established which is determined by Eqs. (24) and (25).

A more complete picture describing the dynamics of the system is given by Fig. 3. We show here the plane of the control parameters a and b for $s = -0.8$ and $C = 4$ in which the hatched region corresponds to the loss of stability by the stationary regime. The curve bounding the region satisfies condition (28). Since the problem is symmetric in a and b it suffices to study the region of parameters bounded by the two diagonals (a fourth part of the plane). For the whole plane the picture turns out to be symmetric with respect to the diagonals. The dashed section on the diagonal indicates the instability section corresponding to Fig. 2. One must note that we have indicated in Fig. 3 the region where the instability leads to pulsations. Moreover, since the system is multistable there exist regions where the instability leads to another stationary regime. The intersection of the dashed region with the abscissa axis ($b = 0$) means that pulsations appear in the case where the field is only incident from one direction onto the film.

The calculation of the dynamics of the system using the mappings (17) and (18) with the initial condition (19) has shown that it is possible to establish different types of pulsating regimes which are shown in Fig. 4. The pulsating regimes are shown after having been already established after finishing a transitional process. Along the vertical axis in Fig. 4 we have plotted the amplitudes of the fields transmitted through the first film (upper series) and through the second film (lower series) at the same time. The point A in Fig. 3 corresponds to Fig. 4a, the point B to Fig. 4b, and the point C to Fig. 4c, and the A type of regimes occupy almost the whole dashed region. It is clear that A and B regimes correspond to a fourfold increase in period in contrast to the usual period doubling (C type). The B regimes occupy an intermediate parameter region between A and C. More complicated regimes, including chaotic ones, occupy a very small parameter region around the dashed section of the diagonal. Note also that the regimes shown in Fig. 4 were obtained using the initial condition (19). For instance, a B type regime is possible in the point A if we choose different initial conditions. Figure 4 thus demonstrates the possibility of a system of two bistable thin films as a multivibrator having a wealth of dynamic regimes.

We show in Fig. 5 the effect of the parameter s, which characterizes the absorption power of the medium between the films, on the development of the instability in the system. If the value of s is close to zero it means that the medium absorbs almost all the light transferred from one film to the other. There is no coupling between the films and no pulsations appear [this is indicated by Eq. (28) in which we have $|\Lambda| \propto s^2$]. The hatched region shown in Fig. 5 shows that for small negative s pulsations appear in a narrow range in the parameter a, but this range broadens when $|s|$ increases, shifting in the large a direction. Pulsations are thus most probable if the distance between the films contains an odd number of half wavelengths while the medium is not very absorptive.

We briefly discuss the role of the cooperative parameter C in the nonlinear dynamics. For small values of C the problem becomes nearly linear and there are neither bistability nor pulsations. When C increases the band in which pulsations exist occupies an ever more extensive region; for instance, the "tongue" shown in Fig. 5 broadens and shifts in the large a direction. For sufficiently large values of the parameter C the picture of the instability zones in the plane of the control parameters is rather complicated. An exhaustive study of all possible regimes becomes a very laborious problem.

CONCLUSION

The results obtained in the present paper show that the dynamics of the passage of light through a system of two bistable thin films is complicated. The system repeats the

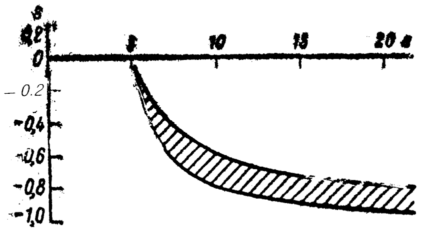

FIG. 5. Zone in the a, s plane where pulsations exist.

properties of a separate bistable element if one can fit an integer number of wavelengths between the films. If, however, the distance between the films contains an odd number of half-wavelengths, pulsations, including chaotic dynamics, may appear in the system if the absorption is weak. The system demonstrates multistability of different types of pulsation regimes.

If radiation with the same intensity is incident on the films from two sides the problem becomes symmetric and equivalent to the passage of light through a film with an additional reflecting surface.[13]

We note that the thin films of two-level atoms considered in our paper may turn out to be useful as a model for studying the nonlinear optical properties of thin semiconducting layers. Recent papers show that one can use the Bloch equations to describe both interband transitions[20,21] as well as the exciton mechanism for the interaction of light with semiconductors.[22] The short relaxation times for semiconducting materials produce a practical basis for the case of an inertialess medium considered in the present paper.

[1] L. A. Lugiato, *Progress in Optics* (ed. E. Wolf), North-Holland, Amsterdam (1984), Vol. 21, p. 71.
[2] H. Gibbs, *Optical Bistability* (Russian translation), Mir, Moscow (1988).
[3] H. Haken, *Advanced Synergetics: Instability Hierarchies of Self-Organizing Systems and Devices*, Springer-Verlag, New York (1983).
[4] A. Yu. Loskutov and A. S. Mikhaĭlov, *Introduction to Synergetics*, Nauka, Moscow (1990).
[5] K. Otsuka and K. Ikeda, Phys. Rev. **A39**, 5209 (1989).
[6] K. Ikeda, Opt. Commun. **30**, 257 (1979).
[7] O. N. Gadomskiĭ, Ukr. Fiz. Zh. **26**, 456 (1981).
[8] Y. Ben-Aryeh, C. M. Bowden, and J. C. Englund, Phys. Rev. **A34**, 3917 (1986).
[9] V. I. Rupasov and V. I. Yudson, Zh. Eksp. Teor. Fiz. **93**, 494 (1987) [Sov. Phys. JETP **66**, 282 (1987)].
[10] S. M. Zakharov and E. A. Manykin, Poverkhnost', No 2, p. 137 (1988).
[11] M. Benedikt and E. D. Trifonov, Phys. Rev. **A38**, 2854 (1988).
[12] M. Benedikt, V. A. Malyshev, E. D. Trifonov *et al.*, Phys. Rev. **A43**, 3845 (1991).
[13] A. M. Basharov, Zh. Eksp. Teor. Fiz. **94**, No 9, 12 (1988) [Sov. Phys. JETP **67**, 1741 (1988)].
[14] A. M. Samson, Yu. A. Logvin, and S. I. Turovets, Kvantovaya Elektron. (Moscow) **17**, 1223 (1990) [Sov. J. Quantum Electron. **20**, 1133 (1990)].
[15] Yu. A. Logvin, A. M. Samson, and S. I. Turovets, Kvantovaya Elektron. (Moscow) **17**, 1521 (1990) [Sov. J. Quantum Electron. **20**, 1425 (1990)].
[16] A. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms*, Wiley, New York (1975).
[17] M. Born and E. Wolf, *Principles of Optics*, 4th ed., Pergamon, Oxford (1970).
[18] M. Feigenbaum, Usp. Fiz. Nauk **141**, 343 (1983).
[19] L. E. El'sgol'ts and S. B. Norkin, *Introduction to the Theory of Differential Equations with a Fluctuating Argument*, Nauka, Moscow (1971).
[20] H. Haug, J. Lumin. **30**, 171 (1985).
[21] M. Lindberg and S. W. Koch, Phys. Rev. **B38**, 3342 (1988).
[22] O. N. Gadomskiĭ and R. A. Vlasov, *Optical Surface Echo Spectroscopy*, Nauka i Tekhnika, Moscow (1990).

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