Magnetoelastic solitary waves and supersonic domain-wall dynamics

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The dynamics of domain walls in weak ferromagnets (of the $YFeO_3$ type) is investigated, taking into account the magnetoelastic interaction with the acoustic subsystem of the crystal. Special attention is devoted to the range of domain-wall velocities close to the velocity of sound, where the magnetoelastic interaction has a resonance character. An equation describing the dynamics of a domain wall and other nonlinear waves in a weak ferromagnet is derived, taking into account dissipation and interaction with the acoustic subsystem of the crystal. This equation reduces to the well-known sine-Gordon equation, if the dissipation constant and magnetostriction are made to approach zero. A method of successive approximations is proposed for determining the structure and the velocity of a domain wall as a function of the external magnetic field. It is shown that the velocity of a domain wall as a function of the magnetic field exhibits an anomaly near some branch of elastic waves. The main feature of this anomaly is that a section with negative mobility, where the stationary motion of a planar domain wall is unstable, is present. It is shown that the magnitude of these magnetoelastic anomalies depends strongly on the direction of motion of the domain wall relative to the crystallographic axes. The nonstationary dynamics of a domain wall and the accompanying deformation of the wall are investigated on the basis of a "condensed" description of the domain wall with the help of the coordinates of the center of gravity of the wall. It is shown that as the domain wall decelerates a localized elastic deformation (elastic soliton), propagating in the form of a wave packet with the velocity of longitudinal or transverse sound, separates from the wall. The character of the spatial distribution of the deformation and its evolution in time are studied. On the whole, they agree with recent experimental data on Brillouin scattering of light by a moving domain wall in $YFeO_3$.

1. INTRODUCTION

In the study of solitary waves—moving domain walls (kinks) and solitons—the question of the interaction of the waves with other degrees of freedom of the system is fundamental. This question is especially important and interesting in the case when this interaction is resonant. This situation is encountered in the case when a domain wall (DW) moves with velocity v, close to the velocity s_i of some branch of elastic waves, in a magnetically ordered medium. It is natural to expect that under resonance conditions (for $v \sim s_i$) the magnetoelastic (ME) interaction can radically change the character of the domain wall motion.

Indeed, significant anomalies in the dynamics of domain walls in yttrium orthoferrite were observed in Refs. 1–4. Yttrium orthoferrite is an antiferromagnet with orthorhombic symmetry (space group D_{2h}^{16}) and it exhibits weak ferromagnetism. The maximum velocity of domain walls in this material is equal[5-8] to $c = 2\times 10^6$ cm/s, i.e., the resonance conditions $v \sim s_i < c$ are obviously satisfied in it. This situation can also be realized in other weak ferromagnets of this class ($RFeO_3$) as well as in orthorhombic weak ferromagnets (for example, in iron borate $FeBO_3$, hematite α-Fe_2O_3, and other materials). In all these materials the maximum velocity of domain walls is virtually identical to the phase velocity of spin waves (in the nonlinear section of the spectrum) and exceeds the velocity of sound in them.

In the first experimental work on the dynamics of domain walls near the velocity of sound in weak ferromagnets, characteristic steps (or shelves) were observed on the curves of the velocity of a DW versus the magnetic field $v(H)$ at values of the velocity close to the velocities of longitudinal and transverse sound.

These anomalies originate with the strong increase in the amplitude of the wave of deformation, accompanying the moving DW, as the velocity of the DW approaches the velocity of sound.[9-13] A theory of these anomalies was proposed, under somewhat different assumptions, in Refs. 9 and 10. In Ref. 10 it was predicted that the planar front of a DW will be unstable near the velocity of sound. Different manifestations of this instability, including the formation of a very interesting dissipative structure on the front of the DW, were investigated in detail in Refs. 14–16.[1)]

Many interesting experimental results concerning the behavior of DWs in the region $v \gg s$ were obtained by M. V. Chetkin and his colleagues with the help of double-frame high-speed photography (see the review Ref. 5 and the references cited there).

Brillouin scattering of light by a moving DW was recently investigated in Refs. 19–21. These experiments make it possible to trace directly the amplitude of the deformation wave (by acoustic phonons), accompanying the DW, and to investigate very fine details of the interaction of a DW and the elastic subsystem, in particular, the behavior of a solitary elastic wave under unsteady DW conditions, detachment of the wave from the DW, phonon damping, etc.

All this made it necessary to elaborate the theory. The problem of magnetoelastic interaction at $v \sim s$ has still not been completely solved, in spite of much theoretical work that has been performed.[3,9,10,12] In Refs. 3, 10, and 12 a model system consisting of a ferromagnet with one sublattice was

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investigated. In Ref. 9 a two-sublattice antiferromagnet was studied. But the domain wall was assumed to have the same structure as a DW at rest. These and other model assumptions not only limit the possibility of making a quantitative comparison between the theory and experiment, but they leave some fundamental questions open [for example, the question of the dependence of the magnetoelastic anomalies on the direction of motion of DWs in a crystal and the real structure of the wall (curvature)].

Some aspects of the nonstationary motion of DWs near the velocity of sound were discussed in Refs. 14 and 16 under the assumption that the domain wall and the elastic wave move together as a unit, i.e., leaving open the question of the possible decoupling of these two waves.

In the present paper we give a theoretical description of the phenomenon, discussed above, on the basis of a quite realistic Lagrangian of the nonlinear magnetoelastic field of a two-sublattice antiferromagnet exhibiting weak ferromagnetism and a phenomenological Rayleigh dissipative function. In analyzing the nonstationary processes we focus attention on the decoupling of the magnetic and elastic degrees of freedom and we discuss some new possibilities, opened up here, for experimental investigations.

2. LAGRANGIAN AND THE RAYLEIGH DISSIPATIVE FUNCTION

Consider a weak orthorhombic ferromagnet of the $YFeO_3$ type. We describe it in a two-lattice approximation with the help of the dimensionless ferromagnetism vector (**m**) and antiferromagnetism vector (**l**). We characterize the elastic deformation of the crystal by the displacement vector **u**. Minimizing the thermodynamic potential of the system $\Phi(\mathbf{m},\mathbf{l},\mathbf{u})$ with respect to m and using the facts that

$$\mathbf{ml}=0,\ l^2=1-m^2\approx 1,$$

we represent Φ as follows (see, for example, Ref. 22):

$$\Phi=\Phi_M+\Phi_E+\Phi_{ME}, \tag{1}$$

$$\Phi_M={}^1\!/_2A(\nabla\mathbf{l})^2-{}^1\!/_2\chi_\perp[H^2-(\mathbf{Hl})^2]-m_z^0H_zl_x-m_x^0H_xl_z+{}^1\!/_2K_{ac}l_z^2+{}^1\!/_2K_{ab}l_y^2, \tag{2a}$$

$$\Phi_E={}^1\!/_2c_1\varepsilon_{xx}^2+{}^1\!/_2c_2\varepsilon_{yy}^2+{}^1\!/_2c_3\varepsilon_{zz}^2+c_4\varepsilon_{xx}\varepsilon_{yy}+c_5\varepsilon_{xx}\varepsilon_{zz}+c_6\varepsilon_{zz}\varepsilon_{yy}+2c_7\varepsilon_{xy}^2+2c_8\varepsilon_{xz}^2+2c_9\varepsilon_{yz}^2, \tag{2b}$$

$$\Phi_{ME}=\delta_{xx}\varepsilon_{xx}+\delta_{yy}\varepsilon_{yy}+\delta_{zz}\varepsilon_{zz}+2\delta_{xy}\varepsilon_{xy}+2\delta_{xz}\varepsilon_{xz}+2\delta_{yz}\varepsilon_{yz}, \tag{2c}$$

where

$$\delta_{xx}=\delta_1(l_x^2-l_z^2)+\delta_2(l_y^2-l_z^2),\ \delta_{yy}=\delta_3(l_x^2-l_z^2)+\delta_4(l_y^2-l_z^2),$$
$$\delta_{zz}=\delta_5(l_x^2-l_z^2)+\delta_6(l_y^2-l_z^2),\ \delta_{xy}=\delta_7l_xl_y,$$
$$\delta_{xz}=\delta_8l_xl_z,\ \delta_{yz}=\delta_9l_yl_z,$$
$$\varepsilon_{ik}={}^1\!/_2(\partial u_i/\partial x_k+\partial u_k/\partial x_i), \tag{3}$$

A is the inhomogeneous-exchange constant, $\chi_\perp=M_0/2H_E$ is the transverse susceptibility, m_z^0 and m_x^0 are components of the weak-ferromagnetic moment, K_{ac} and K_{ab} are anisotropy constants, and c_i and δ_i are, respectively, the elastic and magnetoelastic constants.

In the expression for Φ_M we neglected the anisotropy of inhomogeneous exchange and the fourth-order anisotropy constants, which are not significant in the phenomena we are studying. Small terms associated with the antisymmetric components of the distortion tensor of the crystal are dropped in Φ_E and Φ_{ME}, and terms of the form $\varepsilon_{ik}m_pl_q$, which are small compared with the $\varepsilon_{ik}l_pl_q$ terms and have virtually no effect on the final results, are neglected in the magnetoelastic energy. In what follows we assume that in the homogeneous equilibrium state of the crystal the antiferromagnetism vector **l** is oriented parallel to the a axis (and **m**||**c** axis). We assume that the external field is oriented along the c axis:

$$\mathbf{H}=(0,0,H).$$

In order to describe the dynamics of the system we employ the following Lagrangian density L and Rayleigh dissipative function R, which depend only on the antiferromagnetism vector **l** and the displacement vector **u**:[7,23-27]

$$L={}^1\!/_2\rho\dot{\mathbf{u}}^2+{}^1\!/_2(\chi_\perp/\gamma_2)\dot{\mathbf{l}}^2-(\chi_\perp/\gamma)\mathbf{H}[\mathbf{l}\dot{\mathbf{l}}]-\Phi, \tag{4}$$

$$R={}^1\!/_2\alpha(M_0/\gamma)\dot{\mathbf{l}}^2+{}^1\!/_2\sum\eta_{iklm}\dot{\varepsilon}_{ik}\dot{\varepsilon}_{lm}, \tag{5}$$

where $\gamma=2\mu_B/\hbar$ is the gyromagnetic ratio for Fe^{3+} ions, α is the damping constant for the magnetic subsystem, η_{iklm} is the viscosity tensor of the elastic subsystem and has the same symmetry properties as the tensor of the elastic moduli in Eq. (2b), and ρ is the density of the crystal.

It is easy to derive from Eqs. (4)–(6) the corresponding equations of motion for the angles θ and φ, determining the orientation of **l**, where $l_x=\sin\theta\cos\varphi$, $l_y=\sin\theta\sin\varphi$, and $l_z=\cos\theta$, and the displacement vector **u**:

$$\ddot{\theta}-\dot{\varphi}^2\sin\theta\cos\theta+\gamma H\dot{\varphi}\sin 2\theta+(\gamma\omega_E/M_0)\delta\Phi/\delta\theta+\alpha\omega_E\dot{\theta}=0, \tag{6}$$

$$\ddot{\varphi}\sin^2\theta+\dot{\varphi}\dot{\theta}\sin 2\theta-\gamma H\dot{\theta}\sin 2\theta+(\gamma\omega_E/M_0)\delta\Phi/\delta\varphi+\alpha\omega_E\dot{\varphi}\sin^2\theta=0, \tag{7}$$

$$\rho\ddot{\mathbf{u}}=-\partial\Phi/\partial\mathbf{u}-\partial R/\partial\dot{\mathbf{u}}, \tag{8}$$

where $\omega_E=2\gamma H_E=\gamma M_0/\chi_\perp$.

In what follows we confine our attention to the analysis of the basic equations and formulas for the particular case of the motion of a domain wall along a definite crystallographic axis. We discuss the results for the general case without detailed derivation.

Of the two possible types of DWs realized in orthoferrites, depending on the ratio of the anisotropy constants K_{ac} and K_{ab} and characterized by the rotation of the vector **l** either in the ac plane (ac-type DW, $K_{ac}<K_{ab}$) or in the ab plane (ab-type DW, $K_{ac}>K_{ab}$), we confine our attention to ac-type DW, which is the most typical type.

Consider a planar ac-type DW, oriented perpendicular to the a axis (x axis). For $H=0$, when the DW is at rest, Eqs. (6)–(7) have the exact solution $\varphi=0$ and $\theta=\theta(x)$, corresponding to rotation of **l** and **m** in the ac plane. During the motion of the DW in the presence of an external field and dissipation, this plane, strictly speaking, will no longer be the plane of rotation of **l**. As shown in Ref. 13, however, for the typical conditions of observation of DWs ($v\approx s_{t,l}$ and $H\leqslant 200$ Oe), the deviation of **l** from the ac plane does not exceed 0.1° and can be neglected. As a result, the equations (6)–(8) for the angle θ and the displacement vector **u** assume the form

$$A(\theta''-\ddot{\theta}/c^2)=-K_{ac}\cos\theta\sin\theta-m_z^0H\cos\theta+\delta_lu_l'\sin 2\theta+\delta_tu_t'\cos 2\theta+(\alpha M_0/\gamma)\dot{\theta}, \tag{9}$$

$$\rho\ddot{u}_l-c_lu_l''-\eta_l\dot{u}_l''=\delta_l\theta'\sin 2\theta, \tag{10a}$$

$$\rho\ddot{u}_t-c_tu_t''-\eta_t\dot{u}_t''=\delta_t\theta'\cos 2\theta, \tag{10b}$$

where

$$u_l=u_x,\ u_t=u_z\ ,$$

the second component of the transverse deformation satisfies $u_y=0$ in this geometry, and

$$\theta'=\partial\theta/\partial x,\ \theta''=\partial^2\theta/\partial x^2,$$

$$\delta_l=2\delta_1+\delta_2,\ \delta_t=\delta_8,\ c_l=c_1,\ c_t=c_8,\ \eta_l=\eta_1,\ \eta_t=\eta_8.$$

The quantity

$$c=\gamma(A/\chi_\perp)^{1/2}$$

is the maximum velocity of a DW, equal to the spin-wave velocity.[7]

3. NONLINEAR EIGENVALUE PROBLEM: FORBIDDEN VELOCITY BAND AND BRANCHING OF THE SOLUTIONS; MECHANISMS OF MAGNETOELASTIC ANOMALIES

We seek the solution of Eqs. (9)–(10) in the form of a solitary wave, describing the stationary motion of the DW, i.e.,

$$\theta=\theta(x-vt),$$

$$u_{l,t}=u_{l,t}(x-vt).$$

Substituting these functions into Eqs. (9)–(10) we obtain a nonlinear eigenvalue problem, in which the velocity v is the eigenvalue and the magnetic field is an external parameter.

The idealized model of an antiferromagnet is described by the sine-Gordon equation. It presupposes that the following conditions hold: the magnetic anisotropy has the simplest quadratic form (which is satisfied in $YFeO_3$ to a good degree of accuracy); there is no dissipation, external magnetic field, magnetoelastic interaction, etc. In this case the moving domain wall is a soliton (kink), whose velocity can assume arbitrary values in the interval $(-c,c)$, i.e.,

$$-c<v<c.$$

This degeneracy reflects the existence of a definite dynamical symmetry in the problem: in the present case, with respect to the Lorentz transformation with the limiting velocity c.

The dependence $v(H)$ of the domain-wall velocity on the external field is determined when the interaction with an external field, dissipation, and in our case magnetoelastic interaction are taken into account simultaneously in the equations of motion. Taking these perturbations into account removes the degeneracy, as a result of which to each value of the field H there corresponds one or several values of the velocity, i.e., branching of the solutions is possible.

Before analyzing Eqs. (9)–(10) we present some qualitative considerations, which explain the mechanism by which the magnetoelastic interactions affect the structure and character of the motion of the domain wall.

As we have already mentioned above, magnetoelastic anomalies appear in the motion of domain walls primarily as a result of the strong increase in the magnitude of the elastic deformation accompanying the wall as the sound velocity is approached. In the absence of dissipation in the elastic subsystem, according to Eqs. (10), the deformations behave as follows in the case of steady motion of the DW:

$$u_t'=-\varepsilon_t^0\cos\theta\sin\theta/(1-v^2/s_t^2),\quad u_l'=\varepsilon_l^0\cos^2\theta/(1-v^2/s_l^2),\tag{11}$$

where

$$\varepsilon_{t,l}^0=(u_{t,l}')_0=\delta_{t,l}/c_{t,l},$$

i.e., as the sound velocity is approached they increase without bound. Dissipation limits the increase in the deformation.

a) Dynamic renormalization of the thermodynamic potential; forbidden velocity band. A consequence of the strong increase in the amplitude of the elastic wave accompanying the DW is significant renormalization of the magnetic-anisotropy energy in the ac plane:

$$\tilde{\Phi}_A=\Phi_A+\Phi_{\rm ME}+\Phi_{\rm E}={}^1/_2\tilde{K}_{ac}\sin^2\theta+{}^1/_4\tilde{K}_2\sin^4\theta,\tag{12}$$

where

$$\tilde{K}_{ac}=K_{ac}-K^t_{\rm ME}/(1-v^2/s_t^2),\tag{12a}$$

$$\tilde{K}_2=K_2+2K^t_{\rm ME}/(1-v^2/s_t^2)-2K^l_{\rm ME}/(1-v^2/s_l^2),$$

$$2K^{t,l}_{\rm ME}=\delta^2_{t,l}/c_{t,l}.\tag{12b}$$

Near s_t and s_l the increase in the magnetoelastic energy is so significant that the total anisotropy energy can change sign. The change in the anisotropy alters the structure and size of the domain wall, which in turn can affect the dynamical behavior of the wall. The differential equations describing the DW can, in this case, have bifurcation points, i.e., points at which the phase portrait of the equations changes qualitatively. Then, in some interval of velocities near s_t or s_l

$$\Delta v\approx s_{t,l}K^{t,l}_{\rm ME}/K_{ac},$$

as the analysis[10] for a nondissipative system shows, there are no solutions at all of the solitary-wave type with prescribed equilibrium position at infinity. Thus a gap appears in the spectrum of admissible velocities of solitary waves of the DW type.[2)] The appearance of such forbidden bands is a direct consequence of the existence of resonances in the magnetoelastic interaction.[10,29] This is probably a general property of the dynamics of solitons interacting with the resonant subsystems. This mechanism for the magnetoelastic effect on the dynamics of DWs can also operate with weak dissipation in the elastic subsystem, when the amplitude of the magnetostriction deformations with $v\sim s_{t,l}$ is quite large.

b) Branching of solutions. A second consequence of the increase in elastic deformations as $v\to s$ is that the dissipation in the elastic subsystem increases. The dissipative function of the elastic subsystem

$$R={}^1/_2\eta_l(\dot{u}_l')^2+{}^1/_2\eta_t(\dot{u}_t')^2\tag{13}$$

in this case has a maximum at the points $v=s_t$ and $v=s_l$. The increase in dissipation near the sound velocities results in an increase of the magnetic field, creating pressure on the DW, as a result of which on the $v(H)$ curve near $v=s_t$ and $v=s_l$ sections appear where the velocity remains practically constant as the field increases (see Fig. 1). Under certain conditions this mechanism results in a multivalued magnetic-field dependence $v(H)$ which contains a section where the differential mobility

$$\mu=dv/dH$$

is negative. We show below that this mechanism is primarily

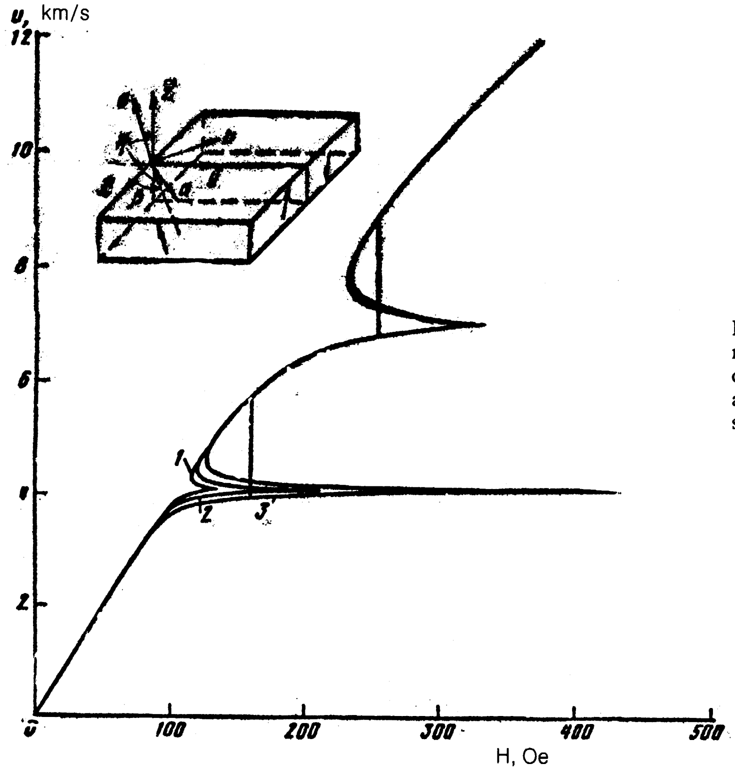

FIG. 1. Domain-wall velocity versus the external magnetic field: *1*) the normal to the plane of the DW is parallel to the a-axis; *2, 3*) the normal of the plane of the DW is inclined to the a-axis in the ac-plane by $\eta = 4°$ and 8°, respectively. The vertical lines depict transitions, which correspond to Maxwell's rule, between stable branches of the curve *3*.

responsible for magnetoelastic anomalies in the dynamics of domain walls in orthoferrites.

4. PERTURBATION THEORY FOR SOLITONS: DETERMINATION OF $v(H)$

We assume that the damping in the elastic subsystem is strong enough that the renormalization of the anisotropy constants is insignificant in the entire range of velocities of the DW (strongly dissipative approximation). In this case the deformations can be assumed to be small and the equations of motion can be solved by the method of successive approximations. The small parameters will then be the quantities

$$\delta_{t,l}|u'_{t,l}|_{max}/K_{ac} \sim \alpha M_0 v/\gamma K_{ac}\Delta_0 \sim H m_z^0/K_{ac} \ll 1,$$

where

$$\Delta_0 = (A/K_{ac})^{1/2},$$

for which we introduce the single designation ν. We discuss below in greater detail the applicability of this approximation. First we examine the case of steady motion of a DW.

We seek the solution of Eqs. (9)–(10) in the form of a solitary wave, which describes a domain wall moving with constant velocity, i.e.,

$$\theta = \theta(x - vt), \quad u_{x,z} = u_{x,z}(x - vt).$$

In the zeroth-order approximation in the small parameters ν the dependence $\theta(x - vt)$ is determined by the equation

$$\hat{L}(\theta_0) \equiv A(1 - v^2/c^2)\theta_0'' + K_{ac}\cos\theta_0 \sin\theta_0 = 0, \quad (14)$$

where we have introduced the nonlinear operator $\hat{L}(\theta_0)$. Under the boundary conditions

$$\theta_0(x\to\infty) = -\pi/2, \; \theta_0(x\to-\infty) = \pi/2, \; \theta_0'(x\to\pm\infty) = 0, \quad (14a)$$

the solution of Eq. (14) is

$$\theta_0' = -\Delta^{-1}\cos\theta_0 = -1/\Delta \,\mathrm{ch}\,\xi, \quad \sin\theta_0 = \mathrm{th}\,\xi, \quad (15)$$

where

$$\xi = (x - vt)/\Delta, \; \Delta(v) = \Delta_0(1 - v^2/c^2)^{1/2}, \; \Delta_0 = (A/K_{ac})^{1/2}.$$

Substituting Eq. (15) into Eqs. (9)–(10) we obtain to first order in the small parameter ν

$$\hat{L}(\theta) = f(\theta_0, x), \quad (16)$$

where

$$f(\theta_0, x) = -m_z^0 H\cos\theta_0 - (\alpha M_0 v/\gamma)\theta_0' + \delta_l u_l' \sin 2\theta_0 + \delta_t u_t' \cos 2\theta_0. \quad (17)$$

The deformations in Eq. (17) are determined by Eqs. (10), in whose right-hand side $\theta_0(x - vt)$ from Eq. (15) must be substituted. The solutions of these equations with the boundary conditions

$$u_{t,l}(x\to\pm\infty) = u'_{t,l}(x\to\pm\infty) = u''_{t,l}(x\to\pm\infty) = 0 \quad (\mathrm{O}')$$

have the form

$$u_{t,l} = (\delta_{t,l}/2\pi)\int_{-\infty}^{\infty} dq\,\varphi_{t,l}(q, v)\exp[iq(x - vt)]/ q^2(s_{t,l} - v^2 - ivq\bar{\eta}_{t,l}), \quad (18)$$

where

$$\varphi_t(q, v) = \pi q^2\Delta^2/\mathrm{ch}(\pi q\Delta/2), \; \varphi_l(q, v) = \pi q^2\Delta^2/\mathrm{sh}(\pi q\Delta/2), \quad (19)$$
$$s_{t,l} = (c_{t,l}/\rho)^{1/2}$$

are the velocities of transverse and longitudinal sound, respectively, and

$$\tilde\delta_t=\delta_t/\rho,\ \tilde\delta_l=i\delta_l/\rho,\ \tilde\eta_{t,l}=\eta_{t,l}/\rho.$$

Expanding $\theta(x-vt)$ in Eq. (16) in powers of the small parameter ν

$$\theta=\theta_0+\theta_1+\dots, \tag{20}$$

where

$$|\theta_1|\ll|\theta_0|,$$

we find that the first-order correction θ_1 satisfies the equation

$$\hat L_1\theta_1\equiv\partial\hat L(\theta_0)/\partial\theta_0\theta_1=f(\theta_0,x). \tag{21}$$

The nonlinear equations (14)

$$\hat L(\theta_0)=0$$

with the boundary conditions (14a) has a continuous velocity spectrum, i.e., its solution θ_0 depends on the velocity v as a parameter, varying continuously in the interval $-c<v<c$, as follows from Eq. (15). When a magnetic field is switched on in the presence of dissipation, a definite value of the velocity, depending on the magnitude of the field and the dissipation parameters, is selected.[3)] This value of the velocity can be found with the help of the well-known method for eliminating secular terms in the higher-order equations. According to this method, the following condition must be satisfied in order for the inhomogeneous equation (21) to be consistent:

$$\int_{-\infty}^{\infty}\chi(x)f(\theta_0,x)dx=0, \tag{22}$$

where $\chi(x)$ is the solution of the homogeneous equation

$$\hat L_1\chi=0.$$

It is easy to verify that

$$\chi=\partial\theta_0/\partial x.$$

Using the formulas (15), (17), and (18) we can obtain from Eq. (22), after some transformations, the following equation for the function $v(H)$:

$$H=v/\mu_0(1-v^2/c^2)^{1/2}-F_{\rm ME}(v)/2m_z^0, \tag{23}$$

where

$$\mu_0=\Delta_0 m_z^0/\alpha M_0$$

is the mobility of the domain wall in the absence of magnetoelastic interaction,

$$F_{\rm ME}(v)=F^t_{\rm ME}(v)+F^l_{\rm ME}(v), \tag{24}$$

$$F^{t,l}_{\rm ME}(v)=-(v\delta^2_{t,l}\tilde\eta_{t,l}/\pi\rho)\int_0^\infty dq\varphi^2_{t,l}(q,v)/[(s^2_{t,l}-v^2)^2+(vq\tilde\eta_{t,l})^2], \tag{25}$$

and $\varphi_{t,l}(q,v)$ are determined by Eqs. (19).

The first term on the right-hand side of Eq. (23) gives the familiar magnetic-field dependence $v(H)$ in the absence of magnetoelastic interaction: According to this dependence the velocity of as $H\to\infty$ the DW saturates.[7,8] The second term is determined by the strong decelerating force $F_{ME}(v)$, arising as a result of the interaction of the DW with elastic deformations. To within a constant factor, this decelerating force is the rate of dissipation in the elastic system. The dependence $F_{ME}(v)$ can be approximated in the form (see also Ref. 5)

$$F^t_{\rm ME}(v)=-{}^{14}/_{15}K^t_{\rm ME}D_t/[(1-v^2/s_t^2)^2+{}^7/_5D_t^2], \tag{26a}$$

$$F^l_{\rm ME}(v)=-{}^{16}/_{15}K^l_{\rm ME}D_l/[(1-v^2/s_l^2)^2+{}^4/_5D_l^2]. \tag{26b}$$

where

$$D_{t,l}=v\tilde\eta_{t,l}/\Delta(v)s_{t,l}.$$

The expressions (26) are identical to Eqs. (25) with

$$v=s_{t,l}$$

as well as with

$$|1-v^2/s^2_{t,l}|\gg D^2_{t,l}.$$

As one can see from Eqs. (25)–(26), as $v\to s_t$ or s_l, when the deformations $u_{t,l}$ (18) and the associated rate of dissipation in the elastic subsystem increase, the corresponding quantities $|F_{ME}(v)|$ reach a maximum, and this can result in the appearance of sections with negative differential mobility of DWs in the magnetic-field dependence $v(H)$.

5. ORIENTATIONAL DEPENDENCE

In order to compare the theory with experiments performed with different directions of motion of the DW relative to the crystallographic axes, we used the scheme described above to calculate the elastic deformations and the magnetic-field dependence $v(H)$ for a planar ac-type DW, moving along some direction in the crystal given by the angles β and η (see inset in Fig. 1). In the coordinate system $(\tilde x,\tilde y,\tilde z)$ tied to the DW (the $\tilde x$-axis is perpendicular to the plane of the DW), the components of the displacement vector $\mathbf{u}$ are equal to

$$\tilde u_x\equiv u_l=(1/2\pi\rho)\int_{-\infty}^{\infty}dq[i\delta_{l_1}\varphi_l(q,v)+\delta_{l_2}\varphi_t(q,v)]\cdot\times\exp[iq(x-vt)]/q^2(s_l^2-v^2-ivq\tilde\eta_l), \tag{27a}$$

$$\tilde u_y\equiv u_{t_1}=(1/2\pi\rho)\int_{-\infty}^{\infty}dq[i\delta_{t_{11}}\varphi_l(q,v)+\delta_{t_{12}}\varphi_t(q,v)]\times\exp[iq(x-vt)]/q^2(s_t^2-v^2-ivq\tilde\eta_t), \tag{27b}$$

and analogously for $u_{t_2}\equiv u_z$ with the substitutions in Eq. (27b)

$$\delta_{t_{11}}\to\delta_{t_{21}},\quad \delta_{t_{12}}\to\delta_{t_{22}},$$

where

$$\delta_{l_1}=[(2\delta_1+\delta_2)\cos^2\beta+(2\delta_3+\delta_4)\sin^2\beta]\cos^2\eta+(2\delta_5+\delta_6)\sin^2\eta,$$

$$\delta_{l_2}=\delta_8\cos\beta\sin2\eta.$$

$$\delta_{t_{11}}={}^1/_2[(2\delta_1+\delta_2)-(2\delta_3+\delta_4)]\cos\eta\sin^2\beta.$$

$$\delta_{t_{12}} = -\delta_8 \sin\beta \sin\eta, \tag{28}$$

$$\delta_{t_{21}} = {}^1/_2 [(2\delta_5+\delta_6) - (2\delta_1+\delta_2)\cos^2\beta - (2\delta_3+\delta_4)\sin^2\beta] \sin 2\eta,$$

$$\delta_{t_{22}} = \delta_8 \cos\eta \cos\beta.$$

In deriving Eq. (27) we assume for simplicity that the elastic subsystem is isotropic. This presumes that the elastic constants are related by

$$c_1=c_2=c_3=c_l,\ c_4=c_5=c_6=c_l-2c_t,\ c_7=c_8=c_9=c_t,$$

and the damping constants η_i are related by analogous relations. We note that, as follows from Eq. (27b), the shear deformation $u_{t_{1,2}}$ is determined not only by the comparatively small "transverse" constant δ_8 but also by the large "longitudinal" constants $2\delta_1 + \delta_2$, etc., appearing in $\delta_{t_{11}}$ and $\delta_{t_{21}}$, and for this reason it can be comparable to u_l when the DW moves is an arbitrary direction.

The function $v(H)$ for the case at hand will be determined by the formulas (23) and (24), in which

$$F^t_{\rm ME} = -(v\tilde\eta_t/\pi\rho) \int_0^\infty dq [\varphi_t^2(q,v)(\delta^2_{t_{12}}+\delta^2_{t_{22}})$$

$$+\varphi_l^2(q,v)(\delta^2_{t_{11}}+\delta^2_{t_{21}})]/[(s_t^2-v^2)^2+(vq\tilde\eta_t)^2], \tag{29a}$$

$$F^l_{\rm ME} = -(v\tilde\eta_l/\pi\rho) \int_0^\infty dq [\varphi_t^2(q,v)\delta_{l_2}^2 + \varphi_l^2(q,v)\delta_{l_1}^2]/$$

$$[(s_l^2-v^2)^2+(vq\tilde\eta_l)^2], \tag{29b}$$

The maximum width of the shelves at $v = s_t$ and $v = s_l$ on $v(H)$ can be estimated using Eq. (29) as well as Eqs. (23) and (26):

$$\Delta H_t \approx {}^1/_3 [(\delta_{t_{12}}^2+\delta_{t_{22}}^2) + 2(\delta_{t_{11}}^2+\delta_{t_{21}}^2)] \Delta(s_t)/m_z^0 \rho s_t \tilde\eta_t, \tag{30a}$$

$$\Delta H_l \approx {}^1/_3 [\delta_{l_2}^2 + 2\delta_{l_1}^2] \Delta(s_l)/m_z^0 \rho s_l \tilde\eta_l, \tag{30b}$$

One can see from Eqs. (28)–(30) that for a DW moving along the crystallographic axes $a(\beta = \eta = 0)$, $b(\beta = \pi/2,\ \eta = 0)$, or $c(\beta = 0,\ \eta = \pi/2)$ the anomaly (shelf) in $v(H)$ at $v = s_l$ occurs in any case, while the anomaly at $v = s_t$ occurs only when the DW moves along the axes a or c. Figure 1 shows $v(H)$ evaluated numerically from the formulas (23), (28), and (29) for $YFeO_3$ with the following values of the parameters of this crystal:

$2\delta_1 + \delta_2 \approx 2\delta_3 + \delta_4 \approx -2.3\times10^7$ ergs/cm^3,
$2\delta_5 + \delta_6 \approx 6\times10^7$ ergs/cm^3 (Ref. 22),
$\delta_8 \approx 0.5\times10^7$ ergs/cm^3 (Ref. 33),
$s_t = 4.1\times10^5$ cm/s,
$s_l = 7\times10^5$ cm/s (Ref. 3),
$\mu_0 = 4\times10^3$ cm/s·Oe (Refs. 3 and 21),
$c = 2\times10^6$ cm/s (Ref. 2), $\Delta_0 = 1.1\times10^{-6}$ cm.

Since there are no data on the sound damping coefficients η_i for $YFeO_3$, we employed the values $\eta_l = 0.5$ ergs·s/cm^3 and $\eta_t = 0.1$ ergs·s/cm^3, which are of the same order of magnitude as for $ErFeO_3$ (Ref. 34).

As one can see from Fig. 1, when the DW moves strictly along the a-axis (curve *1*) distinct anomalies occur both at $v = s_l$ and $v = s_t$. We note that the anomaly at $v = s_t$, with all other conditions being equal, generally speaking, is appreciably smaller than the anomaly at $v = s_l$, since it is determined by the small value of the shear-deformation constant δ_8. The magnitude of this anomaly increases rapidly as the direction of motion of the DW deviates from the crystallographic axes. Thus when the normal to the DW deviates from the a axis toward the c axis by only 4°–8° the quantity ΔH_t increases by several times (see curves *2* and *3* in Fig. 1). This also refers to the DW moving along the b axis with the normal to the DW deviating from it by a small amount in the bc plane. The vertical lines in Fig. 1, schematically showing transitions between the stable branches of $v(H)$, are determined from Maxwell's generalized rule (Ref. 16; see Sec. 10 below).

When the DW moves in a direction, lying in the ab plane, at an arbitrary angle β between the normal to the DW and the a axis, the quantity $\Delta H_l(\beta)$ remains virtually constant, while $\Delta H_t(\beta)$ decreases as $\beta \to \pi/2$. The dependence $\Delta H_t(\beta)$ is illustrated in Fig. 2 for the cases when the normal to the DW lies in the ab plane ($\eta = 0$; curve *1*) and when the normal to the DW deviates from this plane by a small angle $\eta = 4°$ (curve *2*). We note that if with

$$2\delta_1+\delta_2 = 2\delta_3+\delta_4$$

the quantity $\Delta H_t(\beta)$ decreases monotonically (curves *1* and *2* in Fig. 2), then even a small difference in these quantities, resulting in $\delta_{t_{11}} \neq 0$ (28), can cause a peak to appear in $\Delta H_t(\beta)$.

For DWs of the intermediate type[2,5,11,19-21] (the plane of the plate is perpendicular to the optical axis of the crystal, which in $YFeO_3$ lies in the bc plane at an angle $\theta_0 = 52°$ to the c axis and the plane of the DW is perpendicular to the a axis), which are most often encountered in experiments, the magnetic-field dependence $v(H)$ in the case when the DW moves strictly along the a axis or in a direction deviating slightly from the plane of the plate is almost completely analogous to the corresponding curves shown in Fig. 1. However

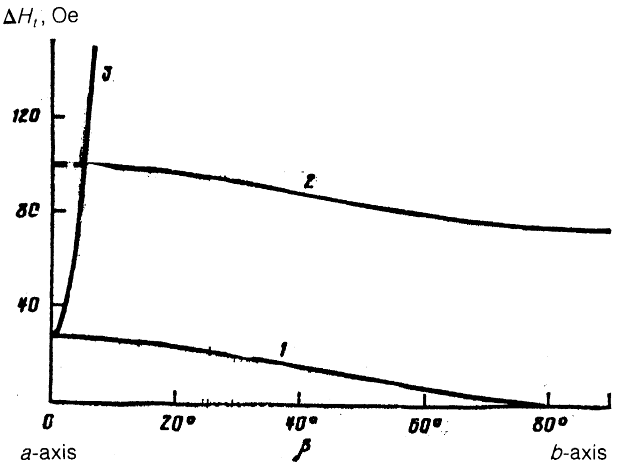

FIG. 2. Orientational dependences of the width of the magnetoelastic anomaly ΔH_t: *1, 2*) with a change in the direction of motion of the DW in the ab plane from the a axis (Néel DW) to the b axis (Bloch DW) (*1*—the normal to the surface of the DW lies in the ab plane; *2*—the normal to the DW surface is inclined to the ab plane by an angle $\eta = 4°$); *3*) for a DW of the intermediate type, the normal to whose surface deviates from the a axis in the plane of the plate.

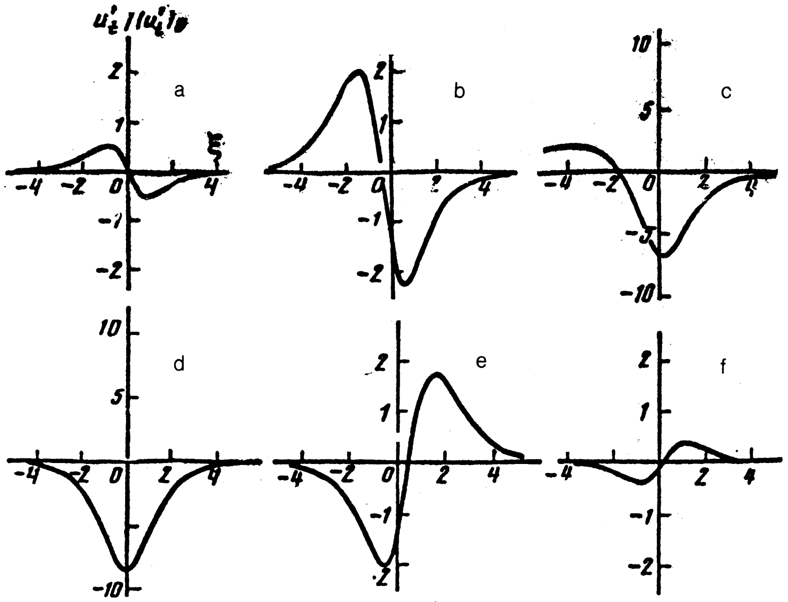

FIG. 3. Transverse deformation distribution for different stationary velocities of the DW: $v/s_t = 0$ (a), 0.9 (b), 0.99 (c), 1 (d), 1.1 (e), and 1.5 (f).

when the normal to the DW deviates by a small amount from the a-axis in the plane of the plate the magnetoelastic anomaly increases very strongly at the velocity of transverse sound (in contrast to the plate perpendicular to the c axis). Curve 3 in Fig. 2 illustrates for this case the increase in the width corresponding to the shelf ΔH_t.

6. STRUCTURE OF A SOLITARY DEFORMATION WAVE

We now examine the distribution of the deformations $u_l'(x)$ and $u_t'(x)$, accompanying the moving DW. We first analyze the case when the DW moves strictly along a crystallographic axis, for example, along the a axis. From the expressions (18), far from the sound velocities we have for $u_{t,l}'(x)$

$$u_t' \approx (\varepsilon_t^0/\mathrm{ch}\,\xi)\,[-\mathrm{th}\,\xi + (v/c)\,(2\,\mathrm{th}^2\,\xi - 1)/2q_t(1-v^2/s_t^2)^2], \qquad (31a)$$

$$u_l' \approx (\varepsilon_l^0/\mathrm{ch}^2\,\xi)\,[1-(v/c)\,\mathrm{th}\,\xi/q_l(1-v^2/s_l^2)^2], \qquad (31b)$$

Near $v = s_t$

$$u_t' \approx (\varepsilon_t^0/D_t)\exp(K_t\xi)\,[-1/\mathrm{ch}\,\xi + K_t(\xi/\mathrm{ch}\,\xi + \mathrm{arccosth}\,\xi - r_t)], \qquad (31c)$$

and for $v \approx s_l$

$$u_l' \approx -(\varepsilon_l^0/D_l)\exp(K_l\xi)\,[\mathrm{th}\,\xi - \mathrm{sign}\,K_l - K_l(\xi\,\mathrm{th}\,\xi - \ln 2\,\mathrm{ch}\,\xi)], \qquad (31d)$$

where

$$\xi = (x-vt)/\Delta, \quad K_{t,l}(v) = (1-v^2/s_{t,l}^2)/D_{t,l}(v),$$

$$r_t = (1-\mathrm{sign}\,K_t)\pi/2,$$

and $\varepsilon_{t,l}^0$ and $D_{t,l}$ are determined, respectively, in Eqs. (11) and (26).

For simplicity we assumed $v \ll c$ in Eqs. (31). The formulas (31) are in good agreement with the numerical results for the deformations computed according to (18) and presented in Figs. 3 and 4 for $\eta_l = \eta_t = 0.5$ ergs·s/cm^3.

We note the following features of $u_t'(\xi)$ and $u_l'(\xi)$. The

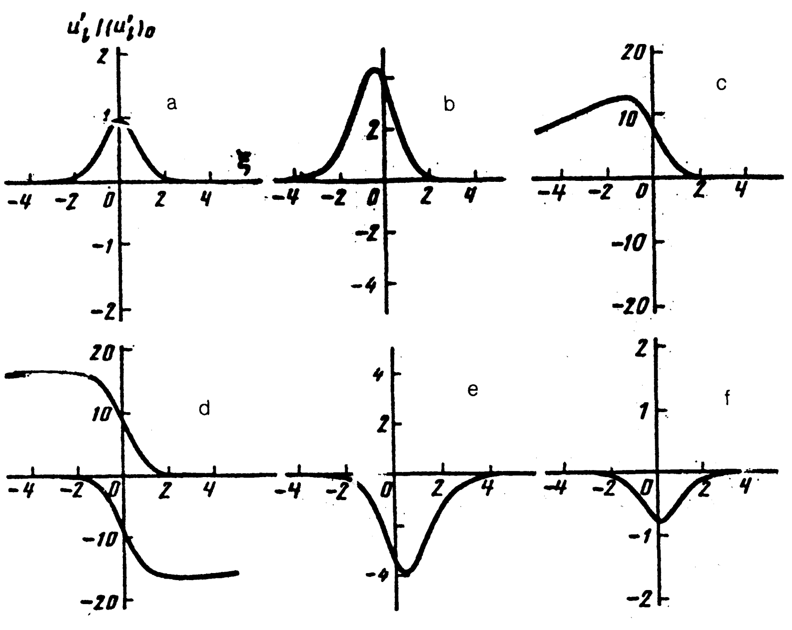

FIG. 4. Longitudinal deformation distribution for different stationary velocities of a domain wall: $v/s_l = 0$ (a), 0.9 (b), 0.99 (c), 0.999 (1d), 1.001 (2d), 1.1 (e), and 1.5 (f).

symmetry of the distribution of the deformations in a moving DW is lower than that of a DW at rest. At $v = s_t$ the deformation u_t' once again becomes symmetric with respect to the center of the DW, but it has a qualitatively different form than in the case $v = 0$ (see Figs. 3a and d). At the point $v = s_l$ the longitudinal deformation does not satisfy the boundary conditions (14a)—it is discontinuous. At this point there is no continuous solitary-wave solution of the with the boundary conditions (14a). For any small deviation of the velocity from s_l, however, there does exist a solution of the desired form. As the velocity passes through the sound velocity s_l the character of the deformation u_l' changes. For $v < s_l$ a slower drop in the deformation from the center of the DW to the periphery occurs behind the moving DW (Fig. 4c), while for $v > s_l$ the dropoff occurs in front of the moving DW (Fig. 4e). This jump in the distribution of the deformation u_l' accompanying the DW nonetheless does not result in a discontinuity in the field dependence of its velocity at the transition through the velocity of longitudinal sound, i.e., $H(v)$ does not have a jump at $v = s_l$. But a discontinuity appears in the derivative of this function, i.e., $H(v)$ is not smooth at $v = s_l$. The distribution of the deformation changes continuously as the velocity passes through the transverse sound velocity, and for this reason $H(v)$ is smooth at $v = s_t$.

In the case when the DW moves in an arbitrary direction the distribution of each component of the deformation $u_l'(\xi)$ and $u_{t_{1,2}}(\xi)$ will be, according to Eq. (27), a linear superposition of the longitudinal and shear deformations examined above.

We now refine the condition under which the strongly dissipative approximation, which we employed and according to which the renormalization of the anisotropy constants due to the magnetoelastic interaction should be small, is applicable. It follows from Eqs. (31c) and (31d) that the maximum deformation at $v \approx s_{t,l}$ is

$$|u_{t,l}'|\max \approx \varepsilon_{t,l}''/D_{t,l} = \delta_{t,l}\Delta_0/\eta_{t,l}s_{t,l}.$$

As a result, we obtain from the condition

$$\delta_{t,l}|u_{t,l}'|_{max} \ll K_{ac}$$

the criterion for applicability of the strongly dissipative approximation:

$$\eta_{t,l} \gg \eta_{t,l}^2 = \delta_{t,l}^2\Delta_0/K_{ac}s_{t,l}.$$

Assuming that

$$K_{ac} \approx 6\times 10^5 \text{ ergs/cm}^3,$$
$$\delta_l \approx (2\text{–}6)\times 10^7 \text{ ergs/cm}^3 \quad \text{(Ref. 22)},$$
$$\delta_8 \approx 0.5\times 10^7 \text{ ergs/cm}^3 \text{ (Ref. 33)},$$
$$s_t = 4.1\times 10^5 \text{ cm/s},$$
$$s_l = 7\times 10^5 \text{ cm/s} \quad \text{(Ref. 3)},$$
$$\Delta_0 = 1.1\times 10^{-6} \text{ cm},$$

we obtain $\eta_l^* = 10^{-2}$–10^{-3} ergs·s/cm^3 and $\eta_t^* = 10^{-4}$ ergs·s/cm^3. According to the data from Ref. 34 the sound damping constant in orthoferrites at room temperature is of the order of 3 ergs·s/cm^3, which indicates that the strongly dissipative approximation is applicable.

7. NONUNIFORM MOTION OF A DW; CONDENSED DESCRIPTION OF DW DYNAMICS

An equation describing the nonuniform motion of a DW can be derived by the approach developed above (elimination of secular terms). We employ the "condensed" description of a DW with the help of the coordinates of its center

$$x_0 = x_0(t,\ y,\ z).$$

In this case the distribution in the moving DW is found in the form

$$\theta(t,\ \mathbf{r}) = \theta[(x - x_0(t,\ y,\ z))/\Delta(t,\ y,\ z)] \equiv \theta(\xi). \tag{32}$$

Substituting Eq. (32) into Eq. (11) for θ and assuming once again that the magnetoelastic interaction is a small perturbation (strongly dissipative approximation), we seek the solution in the form of an expansion (20) in the small parameter ν (see above). The following equation of motion for the center of the DW can be derived from the condition that the secular terms (22) be eliminated in the equation for the higher-order approximation (θ_1):

$$\partial(m\dot{x}_0)/\partial t + m\dot{x}_0/\tau + \mathrm{div}_\perp(mc^2\,\mathrm{grad}_\perp\, x_0)$$
$$= 2m_z^0(H + H'x_0) + F_{\mathrm{ME}}, \tag{33}$$

where

$$m = m_0/(1 - \dot{x}_0^2/c^2)^{1/2}, \quad m_0 = 2\chi_\perp/\gamma^2\Delta_0$$

are the effective masses per unit area of the moving and stationary DWs, respectively;

$$\tau = 1/\alpha\omega_E$$

is the decay time of the magnons;

$$H' = dH/dx$$

is the gradient of the inhomogeneous magnetic field;

$$F_{\mathrm{ME}} = \int_{-\infty}^{\infty} (d\theta_0/d\xi)\,(\partial\Phi_{\mathrm{ME}}/\partial\theta_1)\,d\xi$$
$$= \int_{-\infty}^{\infty} d\xi\,(\partial\theta_0/\partial\xi)\,[(\delta_{l_1}\sin 2\theta_0 + \delta_{l_2}\cos 2\theta_0)u_l'$$
$$+ (\delta_{t_{11}}\sin 2\theta_0 + \delta_{t_{12}}\cos 2\theta_0)u_{t_1}' + (\delta_{t_{21}}\sin 2\theta_0 + \delta_{t_{22}}\cos 2\theta_0)u_{t_2}'], \tag{34}$$

is the magnetoelastic decelerating force acting on a DW moving in an arbitrary direction in the crystal (see above).[4)] The quantity $\theta_0(\xi)$ is determined by the formulas (15) and the deformations $u_{l,t}'(t,x)$ are determined from the corresponding Eqs. (10) and (8), describing the nonuniform motion of the elastic subsystem. For a DW moving along the a axis, the corresponding quantities u_l and $u_t = u_{t_2}$ ($u_{t_1} = 0$) are determined by the expressions

$$u_{t,l}(t,x) = u_{t,l}^{(0)}(t,x) + u_{t,l}^{(1)}(t,x), \tag{35}$$

$$u_{t,l}^{(0)}(t,x) = (\delta_{t,l}/2\pi)\int_{-\infty}^{\infty} dq\,\varphi_{t,l}(q,v_0)\,\chi_{t,l}(q,t,v_0)\exp(iqx - t\Gamma_{t,l}), \tag{36a}$$

$$u_{t,l}^{(1)}(t,x)=(\tilde{\delta}_{t,l}/2\pi)\int_{-\infty}^{\infty}dq\int_0^t dt'\varphi_{t,l}(q,\dot{x}_0')(q\tilde{s}_{t,l})^{-1}$$

$$\times\sin[(t-t')q\tilde{s}_{t,l}]\exp[iq(x-x_0')-(t-t')\Gamma_{t,l}], \qquad (36b)$$

where

$$\chi_{t,l}(q,t,v_0)=[\cos(qt\tilde{s}_{t,l})+\tilde{s}_{t,l}^{-1}(iv_0-q\tilde{\eta}_{t,l}/2)$$
$$\times\sin(qt\tilde{s}_{t,l})]/q^2(s_{t,l}^2-v^2-ivq\tilde{\eta}_{t,l}),$$
$$\tilde{s}_{t,l}=(s_{t,l}^2-q^2\tilde{\eta}_{t,l}^2/4)^{1/2},\quad \Gamma_{t,l}={}^1/{}_2q^2\tilde{\eta}_{t,l},$$
$$x_0'=x(t'),\quad \dot{x}_0'=\dot{x}_0(t'). \qquad (37)$$

In deriving Eqs. (35)–(37) we assume that initially (at $t=0$) the $u_{t,l}(0,x)$ are determined by the distribution (18) for uniform motion of a DW with the velocity $v=v_0$. The extension of the expressions for the deformations to arbitrary direction of motion of the DW does not present any difficulties and actually reduces to replacing in Eq. (36) the quantities

$$\tilde{\delta}_l\varphi_l\equiv i\delta_l\varphi_l/\rho$$

by

$$(i\delta_{l_1}\varphi_l+\delta_{l_2}\varphi_t)/\rho$$

etc., by analogy to the case of uniform motion [see Eqs. (18) and (20)].

Substituting the corresponding displacements (35)–(36) into Eq. (34), we obtain after some transformations the following expression for the magnetoelastic decelerating force:

$$F_{\rm ME}^{t,l}=(i\delta_{t,l}^2/2\pi\rho)\Big[\int_0^t dt'\int_{-\infty}^{\infty}dq\varphi_{t,l}(q,\dot{x}_0)\varphi_{t,l}(q,\dot{x}_0')(q\tilde{s}_{t,l})^{-1}\cdot$$
$$\times\sin[(t-t')q\tilde{s}_{t,l}]\exp[iq(x-x_0')-(t-t')\Gamma_{t,l}]$$
$$+\int_{-\infty}^{\infty}dq\varphi_{t,l}(q,\dot{x}_0)\varphi_{t,l}(q,v_0)\chi_{t,l}(q,t,v_0)\exp(iqx_0-t\Gamma_{t,l})\Big]. \qquad (38)$$

At $t=0$ these quantities transform into the expressions (25). An important feature of the expression (38) is that for nonuniform motion of the DW the magnetoelastic decelerating force depends on the previous history of the motion.

8. STABILITY OF A PLANAR DW

In the case of stationary motion of a DW, for $x_0(t)=v_0t$, Eq. (33) gives the previously obtained [see Eqs. (23), (25), and (29)] field dependence of the velocity of the DW. We now investigate the stability of this solution. For simplicity we assume $|\dot{x}|/c\ll1$. Let $x(t,y,z)=v_0t+x_1(t,y,z)$. Then the linearized equation for x_1 has the form

$$m_0\ddot{x}_1+(m_0/\tau)\dot{x}_1-m_0c^2\Delta_\perp x_1=\int_0^\infty f_{\rm ME}(\tau)[x_1(t)-x_1(t-\tau)]d\tau, \qquad (39)$$

where

$$\Delta_\perp x=\partial^2x/\partial y^2+\partial^2x/\partial z^2,$$
$$F_{\rm ME}(\tau)=f_{\rm ME}^t(\tau)+f_{\rm ME}^l(\tau), \qquad (40)$$

$$f_{\rm ME}^{t,l}(\tau)=-(\delta_{t,l}^2/2\pi\rho)\int_0^\infty dq\varphi_{t,l}^2(q,v_0)\tilde{s}_{t,l}^{-1}\sin(q\tau\tilde{s}_{t,l})$$
$$\times\exp[\tau(iqv_0-\Gamma_{t,l})]. \qquad (41)$$

Assuming $x_1\sim\exp(-i\omega t+i\mathbf{k}_\perp\mathbf{r}_\perp)$, we obtain the following equation for the characteristic frequencies:

$$\omega^2+i\omega\Gamma(\omega)-c^2k_\perp^2=0, \qquad (42)$$

where

$$\Gamma(\omega)=\tau^{-1}-\Gamma_{\rm ME}(w),\ \Gamma_{\rm ME}(w)=\Gamma_{\rm ME}^t(w)+\Gamma_{\rm ME}^l(w), \qquad (43)$$

$$\Gamma_{\rm ME}^{t,l}(\omega)=(\delta_{t,l}^2/2\pi\rho m_0)\int_{-\infty}^{\infty}dq\varphi_{t,l}^2(q,v_0)[W_{t,l}(q,v_0)$$
$$-W_{t,l}(q,v_0+\omega/q)]/i\omega, \qquad (44)$$
$$W_{t,l}(q,v_0)=1/(s_{t,l}^2-v_0^2-ivq\tilde{\eta}_{t,l}).$$

For small frequencies ($\omega\to0$) we have

$$\Gamma_{\rm ME}^{t,l}(\omega\to0)=m_0^{-1}\partial F_{\rm ME}^{t,l}/\partial v_0, \qquad (45)$$

where $F_{\rm ME}^{t,l}(v)$ is determined by the expression (25).

It follows from Eq. (42) that for $(1/\tau-\Gamma_{\rm ME})<0$ the uniform motion of a flat DW will be absolutely unstable. In the limit $\omega\to0$, as follows from Eqs. (45) and (25), this condition is satisfied near the sound velocities $s_{t,l}$ on the sections of the curve $v(H)$ which correspond to negative differential mobility of the DW. In these velocity ranges the DW should be expected to deviate from a planar shape.

9. EVOLUTION OF THE DEFORMATION WAVE DURING DECELERATION OF A DW

When the DW moves with constant velocity the elastic deformation of the crystal is localized on the wall. When the motion of the DW is nonuniform, for example, deceleration, under certain conditions the elastic deformation localized on the DW can become "detached" and propagate freely in the crystal in the form of a wave packet of elastic waves. This follows directly from the expressions (35)–(36) for the elastic displacements $u_{t,l}(t,x)$, in which the center of the moving DW $x_0(t)$, generally speaking, may not coincide with the region in which the distribution $u_{t,l}(t,x)$ is localized.[5]

This detachment of the elastic deformation accompanying a DW moving with a velocity close to the sound velocity (s_t) has actually been observed experimentally by means of Brillouin light scattering.[20–21] This happened as the DW decelerated and was observed only at low temperatures ($T\sim4.2$ K), when the sound damping decreases significantly. This was manifested in the Brillouin scattering spectra as a series of lines, one of which corresponded to scattering by the DW moving with velocity $v<s_t$ and the others corresponded to an elastic wave propagating with velocity $v=s_t$ ($v=s_l$). In addition, Demokritov *et al.*[20,21] were also able to determine the distribution of the deformation de-

tached from the DW in the elastic wave propagating with the velocity of transverse sound $v = s_t$: It had a sharp leading edge and a very flat (extended) trailing edge.

We now analyze on the basis of the expressions (35)–(36) for the elastic displacements the evolution of the deformation wave arising when the DW decelerates rapidly, for example, as a result of a decrease in the strength of the driving field. We assume that prior to deceleration the flat DW moved along the a axis of the crystal with velocity

$$v_0 \sim s_{t,l},$$

and after deceleration it moved with velocity v:

$$x_0(t) = v(t - t_0).$$

We are not interested in the short time interval during which the deceleration occurs. We assume that initially (at $t = 0$) the velocity of the DW drops rapidly to the value v. Substituting

$$x_0 = vt$$

into Eq. (35), we obtain

$$u_{t,l} = (\tilde{\delta}_{t,l}/2\pi) \int_{-\infty}^{\infty} dq \exp(iqx) [\varphi_{t,l}(q, v) \exp(-iqvt) / q^2(s_{t,l}^2 - v^2 - ivq\tilde{\eta}_{t,l}) + \exp(-t\Gamma_{t,l}) [\varphi_{t,l}(q, v_0)\chi_{t,l}(q, t, v_0) - \varphi_{t,l}(q, v)\chi_{t,l}(q, t, v)]], \quad (46)$$

where $\varphi_{t,l}$ and $\chi_{t,l}$ are determined by the expressions (19) and (37), respectively. The first term in Eq. (46) is the elastic deformation localized on the DW and propagating together with the wall with velocity v; the second term is the free elastic deformation, detached from the DW and propagating in the form of a wave packet with velocity $(s_{t,l})$.[6]

Generally speaking, this wave packet is damped and spreads out, since the damping rate

$$\Gamma_{t,l} = {}^1/_2 q^2 \tilde{\eta}_{t,l}$$

depends on the wave vector q.

According to the experiments of Refs. 20–21, phonons with $q \sim 2 \times 10^5\ \text{cm}^{-1}$, excited by the DW, have a lifetime of ~ 100 ns at $T = 4.2$ K. Hence it follows that

$$\eta_{t,l} = \tilde{\eta}_{t,l}\rho \sim 0.3 \times 10^{-2}\ \text{erg·s/cm}^3.$$

In order to determine the character of the evolution of the elastic deformation which becomes detached from the DW, we performed a numerical calculation of the distributions $u'_{l,t}(t,x)$, using the expression (46) (see Fig. 5). We assumed that the sound attenuation constant

$$a = \tilde{\eta}_{t,l}/2s_{t,l}\Delta_0$$

is $a \approx 10^{-3}$ (which corresponds to $\eta_t = 0.005$ g/cm·s and $\eta_l = 0.0084$ g/cm·s). All other parameters were assigned the same values as in the calculation of the field-dependence $v(H)$ (see above); the initial and final velocities of the DW were equal to

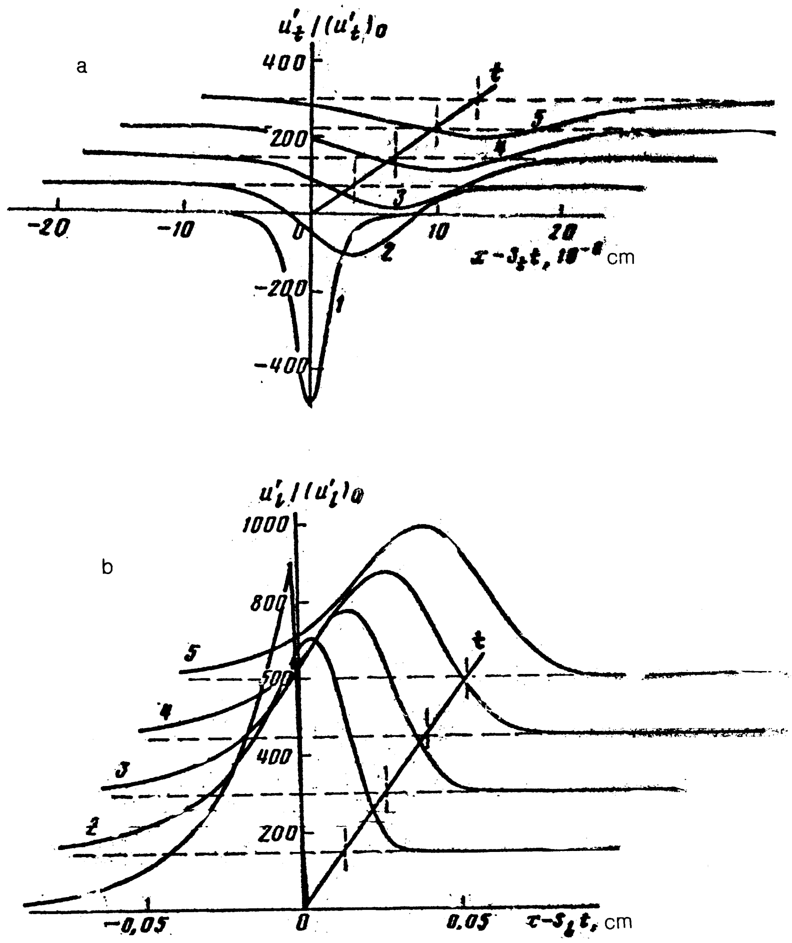

FIG. 5. Distribution of transverse (a) and longitudinal (b) deformations, detached from the domain wall, at different times relative to the time at which the wall stops ($t = 0$): a: $t = 0$ (*1*), 14 (*2*), 28 (*3*), 42 (*4*), 56 (*5*) ns; b: $t = 0$ (*1*), 0.04 (*2*), 0.08 (*3*), 0.12 (*4*), and 0.16 (*5*) s.

$v_0 = s_t, \; v = 0{,}9 s_t$

and

$1 - v_0/s_l = 0{,}5 \cdot 10^{-7}, \; v = 0{,}9 s_l$

in the calculation of the transverse and longitudinal deformations, respectively.

As one can see from Fig. 5, the character of the distribution of the transverse and longitudinal deformations in the corresponding wave packets and their evolution are qualitatively different. The transverse deformation (Fig. 5a) has the form of a narrow wave packet with a width of the order of Δ_0; this packet spreads out rapidly with time and decays (since it contains primarily harmonics with large values of q).

At the same time the distribution of the longitudinal deformation, characterized by a sharp leading edge and a very flat trailing edge, decays much more slowly than the transverse deformation and practically retains its shape. We note that the "tail" in the distribution of the longitudinal deformation, whose length, according to Eq. (31d), is equal to

$$L_0 = \Delta_0 D_l(s_l) / (1 - v_0^2/s_l^2) \approx \Delta_0 a / (1 - v_0/s_l),$$

lengthens as the velocity v_0 approaches s_l (see also Fig. 4).

The character of the distributions of the longitudinal and transverse deformations studied above occurs only when a planar DW moves strictly along definite crystallographic directions (in this case the a axis). When the direction of motion of the DW deviates from a crystallographic axis or as a result of inhomogeneous distortions of the profile of the wall (see below), the distribution of each component of the elastic deformation will consist of a linear superposition of these longitudinal and shear deformations with their characteristic singularities. Since the shear deformation is small in magnitude and decays rapidly, the shape of the distribution of the elastic deformation, propagating with the velocity of transverse sound, as in the case of the deformation propagating with the velocity of longitudinal sound, can also have a sharp leading edge and a flat (extended) trailing edge, as observed in the experiments of Refs. 20–21.

10. DISCUSSION AND CONCLUSIONS

Before discussing the experimental data and comparing theory with experiment, we point out the difficulties arising here. The magnetic-field dependence presented above for the stationary motion of DW $v(H)$ (Fig. 1), in particular, their S-shape and the existence of unstable sections with negative differential mobility indicate that here we are dealing with a strongly nonequilibrium system, which should exhibit critical behavior near the sound velocity with fluctuations significantly affecting the character of the observed processes. It can be asserted that fluctuation-induced first-order nonequilibrium phase transitions between two stable branches of the magnetic-field dependence $v(H)$ occur as the magnetic field is varied. Such critical behavior is manifested, in particular, in the wide spread of the experimentally observed $v(H)$ curves for different samples, methods of observation, and other experimental details (compare, for example, the curves $v(H)$ in Refs. 2, 3, 11, 21, and 37). The widths of the "shelves" vary, for example, from 30 to 500 Oe for DW moving along the a axis.

A fundamental manifestation of the critical behavior of a moving DW near the sound velocity is that in reality the transition of the system from one branch to another as the magnetic field increases or decreases occurs not at the endpoint of the stable section of a branch, but rather is determined by a rule analogous to Maxwell's rule for equilibrium phase transitions.[16] The size of the observed "shelf" in the curve $v(H)$ is significantly smaller than that given by the nonlinear problem for the eigenvalues $v(H)$. We note, however, that the measured field dependence $v(H)$ presented in Ref. 21 shows that different velocities coexist for the same value of the field; this is entirely typical for first-order phase transitions, because Maxwell's rule requires that the transition between stable branches must proceed quite slowly (adiabatically).

The second important factor, which must be taken into account in any analysis of experiments, is that under realistic conditions a moving DW is not planar. There can be different reasons for this. For example, the presence of some additional and entirely natural surface deceleration of the DW makes the DW convex, the more so the closer the velocity of the wall is to the sound velocity. It is then obvious that by virtue of the orientational dependence of the magnetoelastic stopping force (the Rayleigh dissipative function), an additional contribution to the magnetoelastic anomaly arises which is not small (compare curves *1* and *2*, *3* in Fig. 1). Curvature of the profile of the DW over the thickness of the plate can also arise as a result of a nonuniform planar field $H_x(z)$, necessarily present in many experimental situations where a gradient of the field $H_z(x)$ was employed.

We call attention to the natural and interesting possibility of spontaneous perturbation of the profile of a DW (possibly nonuniform). It is associated with the well-known concept of formation of dissipative structures in the thermodynamics of strongly nonequilibrium systems.

Indeed, according to this concept, an open system far from equilibrium tries to transform into a state or, in other words, rearrange itself so that the dissipation of negentropy flowing into the system would accelerate.[38] Applying this principle to our situation, we can expect that a moving planar DW restructures itself so that stronger dissipation mechanisms would be activated. In particular, such restructuring could be associated with bending of a DW across or along the plate. Such dissipative structures have been observed previously in $YFeO_3$ (Ref. 39) and they were investigated theoretically and experimentally in Refs. 14 and 40, but, it is true, in somewhat stronger fields, where they are easily observed visually.

Turning to the experimental data, presented, for example, in Refs. 2, 3, 11, 21, and 37, we note first of all that the orientational dependence of the velocity of a domain wall as a whole agrees poorly with the computed dependence [see Eqs. (23), (24), (28), and (29)]. Indeed, according to Eqs. (28) and (30), the length of a "shelf" with transverse sound for a Bloch wall moving along the b axis is close to zero[37] or appreciably smaller than for a Néel domain wall moving along the a axis.[3] The orientational dependence $v(H)$ accompanying a change in the direction of the velocity of the DW in the ab plane was investigated in Ref. 41; this dependence also agrees with the computed dependence (Fig. 2).

The second important question arises in connection with the anomalies in the velocity of longitudinal sound. According to the theory, for known values of the parameters the anomaly should be stronger than in the case of transverse sound (if the DW moves along rational axes of the crystal). On the other hand, significant anomalies for longitudinal sound which are appreciably stronger than the anomalies observed for transverse sound are not observed in experiments. This can be explained, as noted above, by a "jump" (nonequilibrium phase transition) of the system from the bottom stable branch of $v(H)$ directly into an upper stable branch, bypassing the intermediate branch or with a relatively short delay determined by Maxwell's rule, in an intermediate state. When Maxwell's rule is taken into account, the computed field dependence $v(H)$ (Fig. 1) is qualitatively very close to the experimentally observed dependence (see, for example, Refs. 11 and 21).

As concerns the quantitative comparison of theory and experiment, it is observed that in cases when a planar DW moves strictly along rational crystallographic axes the lengths of the "shelves" at $v \gtrsim s_t$, calculated from the known values of the magnetoelastic constants and attenuation parameters of sound, have a tendency to be smaller than is observed. This indicates, in our opinion, that inhomogeneous distortions (possibly, unsteady) appear in the profile of the DW as the velocity of sound is approached. In this case the magnetoelastic anomalies increase significantly (see curves *1* and *2*, *3* in Figs. 1 and 2) as a result of the strong orientational dependence of the magnetoelastic retarding force (the Rayleigh dissipative function).

This situation is most pronounced when a flat Bloch DW moves along the b axis. The magnetoelastic anomaly at $v \gtrsim s_t$, according to Eqs. (28) and (30a), formally vanishes and at the same time strong dissipation arises with small variations of the normal to the DW relative to the b axis (for example, as a result of the curvature of the DW). The anomalies in $v(H)$ for $v \| b$, which were observed in Refs. 3 and 37, are probably related to this.

A comparison of phonon amplitudes and the singular points of the curves $v(H)$ indicates the same thing (see Fig. 4 from Ref. 21). Indeed, it follows from them that the amplitude of the deformation field increases significantly only when the velocity reaches the "shelf" at $v = s_t$, and in addition a distinct break is observed at this point in $v(H)$. It is here that the retarding force increases sharply, possibly because of spontaneous perturbations of the profile of the DW.

Analysis of the nonstationary motion of a DW and the numerical experiments showed that when a DW moving with a velocity close to the sound velocity ($v \sim s_{t,l}$) decelerates, a localized elastic deformation (elastic soliton), propagating in the form of a wave packet with the sound velocity ($v \sim s_{t,l}$), detaches from the wall. The distribution of the transverse deformation then has the form of a narrow wave packet whose width is of the order of several widths of the DW. The distribution of the longitudinal deformation, however, is characterized by a significantly larger width and weaker attenuation and the shape remains unchanged for a significant time. The distribution of deformation, observed in Brillouin light scattering experiments,[19–21] in a wave packet propagating with the transverse sound velocity has a shape characteristic not of transverse deformation, but rather of longitudinal deformation. This could possibly indicate that under real conditions the deformation accompanying a DW is not a purely shear or longitudinal deformation, but rather it is a linear superposition of the two, probably, as discussed above, due to the inhomogeneous distortions of the DW profile which grow at $v \sim s_t$.

An interesting problem arises in connection with the longitudinal phonons observed in Ref. 21, which were generated by a DW in stationary motion with velocities which, on the whole, do not exceed s_t. This unexpected effect can be explained on the basis of the same picture of the perturbed (multidimensional) motion of a DW in the critical region $v \gtrsim s_t$. Indeed, as discussed above, unsteady perturbations of the front of the DW, which propagate along the DW with velocities up to the maximum velocity $c = 20$ km/sec and can excite both transverse and longitudinal acoustic phonons, are possible. In particular, nonlinear kink perturbations, which can be excited with rapid deceleration and a change in the direction of motion of DW are interesting. They are especially likely to be excited if the sign of the curvature of the profile of the DW changes when the direction of motion of the DW reverses. Such fast kinks with significant amplitude have been observed in nonstationary dynamics of DW in $YFeO_3$.[39] We also note that kinks moving with supersonic velocities can be a source of "true" Cherenkov emission of transverse and longitudinal phonons, in contrast to a strictly planar DW for which Cherenkov emission is forbidden by the laws of conservation of energy and momentum of the phonons.

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[1)] Solitary magnetoelastic solitons were studied in Ref. 17 and then in a number of other works (see Ref. 18 and the detailed bibliography cited there). In contrast to the topological solitons studied in the present work, magnetoelastic solitons should be classified as "envelope solitons."

[2)] A situation which is, in a certain sense, similar is examined in a recent work (Ref. 28) on resonant interaction of solitons with impurity centers.

[3)] The effect of magnetoelastic interaction on DW dynamics in the case of a ferromagnet was analyzed recently in Ref. 30 on the basis of soliton perturbation theory.[31,32]

[4)] In deriving Eq. (34) we neglected the distortions of the plane of the DW. Such distortions require a separate analysis.

[5)] The effects discussed in this section are related to the appearance of a magnetoelastic gap and quasilocal magnetoelastic oscillations in a moving DW, which were examined in Refs. 35–36.

[6)] We note that this term indeed includes two wave packets, corresponding to the motion of a free elastic wave parallel and antiparallel to the direction of the DW. The latter wave, however, has a small amplitude and can be neglected.

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