Nonlinear dynamics and relaxation of strongly anisotropic ferromagnets

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(Submitted 4 February 1992)
Zh. Eksp. Teor. Fiz. **102**, 618–632 (August 1992)

It is shown that in a strongly anisotropic ferromagnet with spin $S = 1$ the equations for the four parameters describing the spin dynamics can be reduced to two Lorentz-invariant equations for the spin component in the easy plane of the ferromagnet. This simplification is possible for the case of weak anisotropy in the easy plane and near a transition into the quadrupole phase. In this case the macroscopic long-wavelength description of magnetic solitons is adequate. Two types of solitons occurring in such magnets are investigated and it is shown that only one type is stable. The relaxation processes in this system are investigated. In the entire range of parameters, excluding a narrow neighborhood of the transition into the quadrupole phase, magnons are strongly damped. Retardation of solitons of the domain-wall type is also stronger than in weakly anisotropic magnets.

1. INTRODUCTION

It is well known that ferromagnets (FMs) with spin $S > 1/2$ and strong single-ion anisotropy (SA) have a number of special physical properties which Heisenberg magnets do not have. Many of these properties were even noted in the first works on this subject: existence of $2S$ branches of spin waves, i.e., additional branches compared with a single branch in weakly anisotropic ferromagnet[1] or ferromagnets with spin $S = 1/2$ in which SA is impossible, as well as the possibility of quantum reduction of the average value of the spin S, right down to vanishing of $\langle S \rangle$ and transition at $T = 0$ into the so-called quadrupole phase.[1,2] The static and dynamic properties of strongly anisotropic ferromagnets, primarily with spin $S = 1$, have been studied in many works on the basis of a Hamiltonian of the form

$$H=\frac{1}{2}\sum_{l}\Big[-\sum_{l'}I(l-l')\mathbf{S}_l\mathbf{S}_{l'}+BS_{xl}^2-\beta S_{zl}^2\Big]. \tag{1}$$

Here $\mathbf{S}_l$ is the spin operator at the site l, I is an exchange integral, and the constants B and β characterize the single-ion anisotropy. A number of interesting results have been obtained on the basis of this spin model, in which the single-ion anisotropy models the strong spin–orbital splitting of the levels of the ion. However two questions, which in our opinion are fundamental, have obviously not been adequately discussed.

First, nonlinear effects, primarily, nonlinear magnetic-solitons spin excitations, have not been adequately investigated, though the important role of quantum reduction of spin for the soliton thermodynamics of quasi-one-dimensional ferromagnets of the type $CsNiF_3$ was noted in a recent review.[3] Nonlinear waves have been analyzed in only two papers: Ostrovskiĭ's work[4] and the work of Zvezdin and Mukhin,[5] which we discuss below.

Second, the question of the relaxation of elementary excitations, primarily magnons belonging to the additional branches, has not been previously studied. In spite of the fundamental importance of this question, it has not been investigated, evidently because of the enormous computational difficulties which arise in the calculation of the mass operator in the standard Green's function technique for spin-1 operators or Hubbard operators.

We have proposed simple phenomenological equations for describing the linear and nonlinear dynamics of strongly anisotropic ferromagnets and we have analyzed, on the basis of these equations, the two problems indicated above. These equations have also enabled us to study the dissipation of nonlinear excitations in the form of moving domain walls (kink-type solitons).

2. EQUATIONS OF MAGNETIZATION DYNAMICS

The special properties of ferromagnets with strong SA are attributable to the fact that their spin dynamics is not described, as in the case $S = 1/2$, only by the average value of the spin. In the phenomenological approach this is manifested as an increase in the number of variables required to describe the system. In the case $S = 1$ of interest to us Ostrovskiĭ[4] showed that four variables must be used, for which it is convenient to choose the average spin S and the three Euler angles φ, θ, and γ (the analysis was performed both in terms of the equations of motion for the complete set of states of the operator $\hat{\mathbf{S}}$ and by means of the method of generalized coherent states). The angles θ and φ are the standard angles employed for describing the unit (normalized) magnetization vector. The angle γ, describing the rotation of the spin system around the average spin $\langle S \rangle$, must be included because of the importance of the higher-order (quadrupole) averages of the components of the operator $\hat{\mathbf{S}}$.

Ostrovskiĭ's equations for S, γ, θ, and φ are quite unwieldy. For completeness, we present the Lagrangian from which they are derived. This Lagrangian can be represented in the form

$$\mathscr{L}=\int L\,d\mathbf{r}/a^3,$$

where the Lagrangian density is given by

$$L=\hbar S(\dot\gamma+\cos\theta\dot\varphi)-w(\theta,\ \varphi,\ \gamma,\ S), \tag{2}$$

and a^3 is the volume per spin.

Here w/a^3 is the energy density of the ferromagnet. It can be written, by virtue of Ref. 4, in the form

 0038-5646/92/080329-09$05.00

$$w = {}^1/_8 I_0 R_0^2 ((\nabla S)^2 + S^2[(\nabla\theta)^2 + \sin^2\theta(\nabla\varphi)^2]) + {}^1/_4 \sin^2\theta(B\cos^2\varphi + \beta) - {}^1/_2 I_0 S^2 + {}^1/_4(1-S^2)^{1/2}[B(\cos 2\gamma\cos 2\varphi - \cos\theta\sin 2\gamma\sin 2\varphi) - \sin^2\theta\cos 2\gamma(B\cos^2\varphi + \beta)]. \quad (3)$$

In Eq. (3)

$$I_0 = \sum_l I(l), \quad I_0 R_0^2 = 2\sum_l I(l) l^2.$$

The dynamic part of the Lagrangian can be easily reconstructed from the explicit form of the equations. The energy w of a ferromagnet contains both terms which are standard for a Heisenberg ferromagnet (first line) and additional terms, which are related to the variable γ. In the limit $S \to 1$ the last term, which is specific to ferromagnets with SA, drops out. Introducing the angular variables for the magnetization $M_z = |M|\cos\theta$, $M_x + iM_y = |M|\sin\theta \times \exp(i\varphi)$ and redefining the constants

$$|\mathbf{M}|^2\alpha/2 \to I_0 R_0^2/8, \quad B/4 \to B/2, \quad \beta/4 \to \beta/2$$

the formula for the energy is transformed into the standard expression

$$w \to \alpha/2(\nabla\mathbf{M})^2 + {}^1/_2[\beta M_z^2 - BM_x^2], \quad (3')$$

where $\mathbf{M}$ is the magnetization of the ferromagnet, $|\mathbf{M}|2\mu_0/a^3$ is the saturation magnetization with $S = 1$, and a^3 is the volume per spin.

The complete analysis of the system of equations for the variables θ, φ, γ, and S is quite complicated, and only some of the particular solutions were indicated in Ref. 4. Moreover, for an arbitrary ratio of the parameters I_0, B, and β with $I_0 \sim B/4 \sim I_0 - B/4 \sim \beta$ the soliton solutions that are obtained are localized in a region of order R_0, i.e., comparable to the interatomic distance a. There is thus no sense in switching to a macroscopic description. The problem can be radically simplified in two limiting cases: the quasiclassical case, when the change in S is small compared with unity, and the "ultraquantum" case, when the quantum reduction of spin is large and $S \ll 1$ holds in both the ground state and the soliton. The case $S \neq \text{const}$ but $1 - S \ll 1$ corresponds to relatively weak anisotropy, $B \ll I_0$, and $(1 - S) \sim B/I_0$. Analysis of this case is important, for example, for describing quantum spin reduction effects in solitons of the quasi-one-dimensional ferromagnet $CsNiF_3$; see Ref. 3. We now examine the ultraquantum case as the most characteristic case.

The situation $S \ll 1$ is realized near transitions from the ferromagnetic phase with $S \neq 0$ into the so-called quadrupole phase, in which $S = 0$, and only averages of the type $\langle S_i S_k - 1/3 S^2 \delta_{ik} \rangle$ are different from zero.

An example of a specific material is nickel fluorosilicate $NiSiF_6 \cdot 6H_2O$, in which we have $S \ll 1$ at normal pressure and S decreases with increasing pressure; see Ref. 6. On the basis of the model (1), such a transition occurs for certain values of the constants of the problem, namely, for $I_0 = B/4$ (for nickel fluorosilicate $B/I_0 \approx 3.54$ at $P = 8.6$ kbar; see Ref. 6). This can also be seen from the macroscopic energy (3). Assuming $0 < \beta < B$ and $I_0 \sim B/4$, we easily find that the minimum of the expression (3), i.e., the ground state of the ferromagnet, corresponds to

$$S = \begin{cases} 0, & I_0 < B/4 \\ S_0 = (1 - B^2/16 I_0^2)^{1/2}, & I_0 > B/4 \end{cases} . \quad (4)$$

In this case, in both phases we have $\gamma = 0$. The angular variables θ and φ in the ground state are determined by the signs of the anisotropy constants. Since we have assumed $B > \beta > 0$, the most difficult axis is the x axis, the yz plane is the anisotropic easy plane, and the easy axis is the z axis, i.e., in the ground state we have $\theta = 0$, $\varphi = \pi/2$.

In order to simplify the Lagrangian (2), (3) we assume that the values of γ and $\psi = \pi/2 - \varphi$ are small (the condition $\psi \ll 1$ even arises for $B \ll I_0$ and $S \approx I$ in the case $\beta \ll B$ (see Ref. 7); we verify below the fact that γ is small). Varying the Lagrangian with respect to γ and ψ gives, in the leading order approximation in these small parameters and $S \ll 1$, the simple expressions

$$\gamma = [\hbar/(B+\beta)][\dot{S} + S\dot{\theta}\cos\theta/\sin\theta], \quad (5)$$

$$\psi = [\hbar/(B + \beta\sin^2\theta)][-(S\dot{\theta}/\sin\theta)(B + \beta\sin^2\theta) + \dot{S}\beta\cos\theta]. \quad (6)$$

The formula for ψ with $S = \text{const}$ is identical to the formula derived in Ref. 7. The singularity of the form $(\dot{\theta}/\sin\theta)$ in the limit $\theta \to 0$ turns out to be insignificant and does not arise in the final expressions for the Lagrangian. It is easy to see that the conditions $\psi \ll 1$ and $\gamma \ll 1$ correspond to smallness of the time derivatives, $\hbar\omega \ll B$, where ω is the characteristic frequency,

$$\omega \sim \max\{(\dot{\theta}\cos\theta/\sin\theta), \dot{S}/S\}.$$

As we shall verify below, the maximum value of the characteristic frequencies of the problem is determined by the expression $\omega \sim \max\{S^2, \beta/I_0\}$, so that for $S \ll 1$ and $\beta \ll B$ the condition $\hbar\omega \ll B$ is satisfied.

Substituting Eq. (5) into the complete Lagrangian of the ferromagnet (2) and (3) we obtain the final expression for the Lagrangian describing the dynamics of the magnetization of a strongly anisotropic ferromagnetic in terms of only two dynamical variables S and θ:

$$L = [\hbar/2B(B+\beta)][B(\dot{S}^2 + S^2\dot{\theta}^2) + \beta\{(S\cos\theta)^{\cdot}\}^2] - w,$$

$$w = {}^1/_8 I_0 R_0^2[(\nabla S)^2 + S^2(\nabla\theta)]^2 - {}^1/_4 I_0 S_0^2 S^2 + \tfrac{1}{8} I_0 S^4 - {}^1/_8 \beta S^2 \sin^2\theta. \quad (7)$$

The quantity $w = w(\theta, S)$ is the energy density of the static distribution of the variables S and θ and is obtained from $w(\theta, \varphi, \gamma, S)$ by setting $\gamma = 0$ and $\varphi = \pi/2$ in the latter and making a series expansion in the small parameter S up to S^4 inclusively. Having found the distribution θ and S in an arbitrary nonlinear wave of magnetization, we can reconstruct the variables γ and $\varphi = \pi/2 - \psi$ according to the formula (5).

We verify below that the condition $\hbar\omega \ll B$ for all types of linear and nonlinear waves is satisfied only for the case $\beta \ll B$, which corresponds to the case when the anisotropy in the easy plane is small compared with the out-of-plane anisotropy. In this case the second term in the kinetic part of Eq. (7) is small, and the kinetic part of the Lagrangian becomes isotropic. Dropping this small term, we arrive at a Lorentz-invariant Lagrangian, in which the derivatives with respect to time and the coordinates appear only in the combination $(\partial f/\partial t)^2 - c^2(\nabla f)^2$. It is convenient to write this Lagrangian in terms of the spin components in the basal plane $S_z = S\cos\theta$ and $S_y = S\sin\theta$:

$$L=(I_0R_0^2/8c^2)((\dot{S}_y)^2+(\dot{S}_z)^2-c^2[(\nabla S_y)^2+(\nabla S_z)^2])$$
$$+{}^1/_4 I_0S_0^2(S_y^2+S_z^2)-{}^1/_8\beta S_y^2-{}^1/_8 I_0(S_y^2+S_z^2)^2, \tag{8}$$

where the characteristic velocity is $c = (R_0/2\hbar)(BI_0)^{1/2} \approx R_0B/\hbar$. Such a Lagrangian was derived in Ref. 8 from qualitative considerations, based on analysis of magnon spectra and the structure of static domain walls (DWs).

A Lorentz-invariant Lagrangian of the form (8) was obtained in Ref. 5 for a strongly anisotropic ferromagnet on the basis of a two-level model. On the other hand, the mathematical properties of a model with the Lagrangian (8), called the complex (two-component) φ^4 model has been investigated, without reference to any physical problem, by many authors; see Refs. 9 and 10. Eleonskiĭ and Kulagin[11] showed, for simple nonlinear waves of the form $S_{y,z} = S_{y,z}(\xi)$, where $\xi = x - vt$, that the corresponding dynamical problem with two degrees of freedom can be integrated exactly; this makes it possible to make an exact analysis of all types of simple waves. Some particular cases, describing, for example, two types of domain walls, were written out in Ref. 9. Thus the approximations adopted have made it possible to simplify significantly the analysis of the problem.

3. ELEMENTARY EXCITATIONS: SPIN WAVES AND DOMAIN-WALL SOLITONS

Elementary excitations of ferromagnets can be studied in the quasiclassical limit on the basis of the Lagrangian (8): both linear excitations (magnons) and nonlinear excitations (topological kink solitons (domain walls), which are important for the analysis of the thermodynamics of quasi-one-dimensional magnets). We begin with the analysis of the spin waves in the linear theory. In order to analyze them we write

$$S_y=S_y^{(0)}+\sigma_y, \quad S_z=S_z^{(0)}+\sigma_z \tag{9}$$

and linearize the equations of motion derived from Eq. (8) with respect to σ_y and σ_z. The following two independent equations are obtained for σ_y and σ_z with a uniform ground state $S_y^{(0)} = 0$ and $S_z^{(0)} = S_0$:

$$\ddot{\sigma}_y+\omega_0^2\sigma_y-c^2\nabla^2\sigma_y=0,$$
$$\ddot{\sigma}_z+4\Omega_0^2\sigma_z-c^2\nabla^2\sigma_z=0,$$

where $\hbar\omega_0 = \frac{1}{2}(\beta B)^{1/2}$ and $\hbar\Omega_0 = (BI_0S_0^2)^{1/2}$.

Hence it follows that superposed on the uniform ground state there exist two modes of small oscillations of the magnetization with linear polarization and frequencies

$$\omega_t=\omega(k)=(\omega_0^2+c^2k^2)^{1/2}, \quad \omega_l=\Omega(k)=(4\Omega_0^2+c^2k^2)^{1/2}. \tag{10}$$

In the case of a Heisenberg magnet the wave with $\boldsymbol{\sigma}\|\mathbf{e}_y$ (transverse branch) corresponds to the standard spin waves. For $\beta = 0$, i.e., in the limiting case of a weakly anisotropic ferromagnet with an isotropic easy plane, it has a nonactivational dispersion relation: $\omega = c|\mathbf{k}|$. The second branch with $\boldsymbol{\sigma}\|\mathbf{e}_z$ and frequency $\omega = \Omega(k)$ is specific to strongly anisotropic magnets with a branch of longitudinal spin waves. In the Heisenberg limit ($B \ll I_0$, $S_0 \approx 1$) its frequency is much higher than the frequency of transverse spin waves. As we shall verify below, in this limit it is strongly damped and there is no need to take this mode into account. In the ultraquantum limit, $S_0 \ll 1$, of interest to us the frequencies of the longitudinal and transverse magnons will be comparable and $\Omega(k) < \omega(k)$ with $S_0^2 < \beta/I_0$. The longitudinal mode is a soft mode for the transition from the ferromagnetic into the quadrupole phase as $B \to 4I_0$ ($S_0 \to 0$). In the limit $\beta \ll B$, $S_0 \ll 1$, and $kR_0 \ll 1$ the Lorentz-invariant dispersion laws (10) are identical to the magnon dispersion laws obtained previously on the basis of an analysis of the quantum problem (see Refs. 8 and 12).

In quasi-one-dimensional magnets the nonlinear excitations, primarily topological kink-type solitons (domain walls), must be taken into account together with magnons. The question of the structure of DWs is also of interest for ordinary three-dimensional magnets. Domain walls in strongly anisotropic magnets have been studied by many authors (see Refs. 4 and 13). When the Lagrangian (8) is used, however, this analysis simplifies greatly and a much more complete analysis is possible. In particular, the question of the structure of both types of DWs and their stability can be solved exactly.

We now examine the one-dimensional solutions of the equations which follow from the Lagrangian (8) and which describe stationary DWs. We assume that the magnetization is a function of x only and that it satisfies the boundary conditions $S_y \to 0$, $S_z \to \pm S_0$ as $x \to \pm\infty$.

Stationary DWs with energy (8) were studied by Bulaevskiĭ and Ginzburg.[14] They noted that there exists two types of DWs. One type (a "linear" DW) corresponds to variable **S**:

$$S_y=0, \quad S_z=S_0\,\mathrm{th}(S_0x/R_0). \tag{11}$$

In the other type, a "rotating" DW, both the magnitude and direction of the spin are variable. This case corresponds to the solution

$$S_y=S_0(1-\beta/I_0S_0^2)^{1/2}/\mathrm{ch}(x(\beta^{1/2}/R_0)I_0^{1/2}),$$
$$S_z=S_0\,\mathrm{th}(x(\beta^{1/2}/R_0)I_0^{1/2}). \tag{12}$$

A rotating DW exists, as one can easily see from Eq. (12), only if $S_0^2 > \beta/I_0$ holds. A solution of this type for the complex φ^4 model was written down in Ref. 9. The solutions describing moving DWs are easily obtained from Eqs. (11) and (12) by a Lorentz transformation: $x \to x' = (x - vt)/(1 - v^2/c^2)^{1/2}$. The Lorentz-invariant dynamics of DWs in strongly anisotropic magnets was studied in Refs. 4 and 5.

The thicknesses of rotating and linear DWs Δ_R and Δ_L, respectively, are determined by the expressions

$$\Delta_R=R_0(I_0/\beta)^{1/2}, \quad \Delta_L=R_0/S_0.$$

In the range of values of the parameters which we have chosen ($S_0 \ll 1$, $\beta \ll I_0 \approx B/4$) we have $\Delta_{R,L} \gg R_0$. This indicates that the long-wavelength approximation is applicable. The conditions of the long-wavelength approximation break down only in a narrow interval of velocities of DWs near the limiting velocity, i.e., for $[(c-v)/c] < (\beta/I_0)^{1/2}$ or $< S_0$ for the rotating or linear DW, respectively.

The existence of two topologically equivalent DWs raises the question of the stability of each type of wall against a transition into a wall of the other type. Since the problem is Lorentz-invariant, it is sufficient to study the stability of a stationary DW. In order to analyze the stability, as well as for further applications to the problem of retardation of DWs, we analyze the spectra of small oscillations of the magnetization (spin waves) against the DW background. For this we represent S_y and S_z in the form (9) with $S_{y,z}^{(0)}(x)$ determining a DW at rest [see Eq. (11) or (12)], and we write down the linearized equations of motion for $\sigma_{y,z}$. These equations for both types of DWs can be represented in the form

$$(\hat{L}_1+2C/\mathrm{ch}^2\,\xi+A)\sigma_y+2(C(C+2))^{1/2}(\mathrm{th}\,\xi/\mathrm{ch}\,\xi)\sigma_z+\tilde{\omega}^2\sigma_y=0,$$

$$(\hat{L}_2+2C\,\mathrm{th}^2\,\xi)\sigma_z+2(C(C+2))^{1/2}(\mathrm{th}\,\xi/\mathrm{ch}\,\xi)\sigma_y+\tilde{\omega}^2\sigma_z=0. \quad (13)$$

Here the following notation was introduced: $\xi = x/\Delta$, $\Delta = \Delta_L$ or Δ_R, $\tilde{\omega} = \Omega_0$ or ω_0 for a linear or rotating DW, respectively. The parameter C is proportional to the maximum value of the component S_y in the DW: $C = 0$ for a linear DW and $C = 2(I_0 S_0^2/\beta - 1)$ for a rotating DW and $A = 0$ for the rotating DW and $A = \beta/I_0 S_0^2 - 1$ for the linear DW. The operators $\hat{L}_1$ and $\hat{L}_2$ are the well-known Schroedinger operators with a nonreflecting potential and with a zero lowest eigenvalue:

$$\hat{L}_1=-\Delta^2\nabla^2+1-2/\mathrm{ch}^2\,\xi,\quad \hat{L}_2=-\Delta^2\nabla^2+4-6/\mathrm{ch}^2\,\xi. \quad (14)$$

The frequencies and wave functions of the spin waves are determined by the eigenvalues and eigenfunctions of the operators $\hat{L}_{1,2}$, which are well known (see, for example, Ref. 15). The operator $\hat{L}_1$ has a single localized state φ_0 and a continuous spectrum of states φ_k,

$$\hat{L}_1\varphi_0=0,\quad \varphi_0=1/(2^{1/2}\,\mathrm{ch}\,\xi),\quad \hat{L}_1\varphi_k=(1+k^2\Delta^2)\varphi_k, \quad (15)$$

$$\varphi_k=(\mathrm{th}\,\xi-ik)\exp(ikx)/(1+k^2\Delta^2)^{1/2}.$$

The operator $\hat{L}_2$ has two localized states ψ_0 and ψ_1 given by

$$\hat{L}_2\psi_0=0,\quad \psi_0=3^{1/2}/(2\,\mathrm{ch}^2\,\xi).$$

$$\hat{L}_2\psi_1=3\psi_1,\quad \psi_1=(3/2)^{1/2}\,\mathrm{sh}\,\xi/\mathrm{ch}^2\,\xi, \quad (16)$$

and its states ψ_k in the continuous spectrum correspond to $\hat{L}_2\psi_k = (4+k^2)\psi_k$; the formula for ψ_k is quite unwieldy (see Ref. 15). For simplicity we have written out φ_i and ψ_j ($i = 0$ or k and $j = 0$, 1, or k) for the one-dimensional case. In the three-dimensional case the following obvious substitutions must be made:

$$(\varphi_i,\ \psi_j)\rightarrow(\varphi_i,\ \psi_j)\exp(i\mathbf{k}_\perp\mathbf{r}_\perp)/S^{1/2},$$

and the quantity $k_\perp^2\Delta^2$ must be added to the eigenvalues. Here $\mathbf{k}_\perp = (0, k_y, k_z)$ is the component of $\mathbf{k}$ that lies in the plane of the DW; $\Delta = \Delta_L$ or Δ_R, respectively; and, S is the area of the DW.

The frequencies of the magnons in the continuous spectrum for both types of DWs is determined, naturally, by the formulas (10). Localized magnon states are of a specific nature. It is these states that determine the stability of DWs of each type.

We begin with the case of a linear DW. Here we have $C = 0$, the equations for σ_y and σ_z are independent, and the magnon spectrum contains two independent branches, on each of which σ_y or σ_z oscillates. The mode with $\sigma_z \neq 0$ in the presence of a DW corresponds to two localized states with wave functions ψ_0 and ψ_1 (16). The frequency of the lower state is equal to zero in the one-dimensional case and this state corresponds to a translational mode. In the three-dimensional case it describes bending oscillations of the DW with the dispersion relation

$$\Omega^{(0)}(\mathbf{k}_\perp)=c|\mathbf{k}_\perp|.$$

The frequency of the next localized state is $\Omega^{(1)} = 3^{1/2}\Omega_0$ or, correspondingly,

$$\Omega^{(1)}(\mathbf{k}_\perp)=(3\Omega_0^2+c^2\mathbf{k}_\perp^2)^{1/2}$$

It corresponds to a spin wave in the DW without displacement of the wall.

The wave with $\sigma_y \neq 0$ has only one localized state with wave function $\varphi = \varphi_0$ (15) and frequency $\omega^{(0)}$, where

$$\omega^{(0)}=\omega_0(\beta/S_0^2I_0-1)^{1/2}\ \text{or}\ \omega^{(0)}(\mathbf{k}_\perp)=(\omega_0^2[\beta/S_0^2I_0-1]+c^2\mathbf{k}_\perp^2)^{1/2}$$

in the one- and three-dimensional cases, respectively. It is easy to see that this mode describes instability of the linear DW for $S_0 > (\beta/I_0)^{1/2}$.

The spectrum of spin waves is more difficult to analyze for a rotating DW. This analysis cannot be performed exactly. It is possible to find the wave function of the translational mode [with frequency $\omega = 0$ or $\omega(\mathbf{k}_\perp) = c|\mathbf{k}_\perp|$]. It corresponds to $\sigma_y \sim -(1-\beta S_0^2/I_0)^{1/2}\sinh\xi/\cosh^2\xi$, and $\sigma_z \sim 1/\cosh^2\xi$. The other magnon states can be sought by expanding σ_y and σ_z in eigenfunctions of the operators $\hat{L}_1$ and $\hat{L}_2$, respectively,

$$\sigma_y=\Big(a_0\varphi_0+\sum_k a_k\varphi_k\Big)\exp(i\omega t),$$

$$\sigma_z=\Big(b_0\psi_0+b_1\psi_1+\sum_k b_k\psi_k\Big)\exp(i\omega t).$$

This gives an unwieldy coupled system of equations for the coefficients a and b. In the case $C \ll 1$ (which, as we shall show, is most important for the problem of the stability of a rotating DW) this system can be solved by means of perturbation theory. The stability of the DW is determined by the mode which in the limit $C\rightarrow 0$ goes into $\sigma_y \sim \varphi_0$, $\sigma_z = 0$. For the frequency of this mode $\Omega^{(0)}$ with $C \neq 0$ but $C \ll 1$ the following expression is obtained in the linear approximation in C:

$$\Omega^{(0)}=\Big(2C[\xi_1-4\xi_2^2/3-(2/\pi)\int_{-\infty}^{+\infty}dk|\xi_k|^2/(4+k^2)]\Big)^{1/2}$$

$$\approx(0{,}87C)^{1/2},$$

where

$$\xi_1=\int_{-\infty}^{+\infty}d\xi\,\varphi_0^2/\mathrm{ch}^2\,\xi=2/3,\quad \xi_2=\int_{-\infty}^{+\infty}d\xi\,\varphi_0\psi_1\,\mathrm{sh}\,\xi/\mathrm{ch}^2\,\xi=3^{1/2}\pi/16,$$

$$\ _k=\int_{-\infty}^{+\infty} d\xi\varphi_0\psi_k \operatorname{sh}\xi/\operatorname{ch}^2\xi$$

$$=(\pi i/8)[(4+k^2)/(1+k^2)]^{1/2}k^2/\operatorname{sh}(\pi k/2).$$

The squared frequencies of the other modes superposed on the DW are also positive for all $C>0$. Therefore a rotating DW is stable in its entire region of existence $C>0$, or $I_0S_0^2>\beta$. Writing the condition for stability of a linear DW $(\omega^{(0)})^2>0$ in the form $I_0S_0^2<\beta$, we find that the transition between the linear and rotating DWs proceeds as a unique second-order phase transition with $S_0^2=\beta/I_0$. A linear DW of higher symmetry exists in a narrow range of values of S_0: $0<S_0^2<\beta/I_0\ll 1$. The existence of a kink in the structure for some values of the parameters of the ferromagnetic phase transition and of the soft mode associated with this transition results in the appearance of singularities in the soliton contribution to the thermodynamic quantities of the quasi-one-dimensional magnet for these values of the parameters.[16]

4. MAGNON DAMPING

The nonlinear effects which determine the interaction of magnons with one another can also be easily described on the basis of the Lagrangian (8). In order to analyze them it is convenient to perform quasiclassical quantization of the fields S_y and S_z and to introduce creation and annihilation operators for the magnons of the corresponding branches. For this, we introduce the small deviations σ_y and σ_z [see Eqs. (9)] of these fields from the equilibrium field (uniform and nonuniform, is a DW is present), and we write the Lagrangian as an expansion in powers of these variables and their derivatives:

$$L\{\sigma_y,\ \sigma_z\}=L_2+L_3+L_4+\dots$$

Here L_n is proportional to σ_y and σ_z or their derivatives with a total power of n. We have $L_1\equiv 0$ by virtue of the equations of motion. We now switch to the Hamiltonian formalism, for which we introduce the canonical momenta π_y and π_z which are conjugate to the variables σ_y and σ_z:

$$\pi_y=(I_0R_0^2/4c^2)\dot{\sigma}_y,\quad \pi_z=(I_0R_0^2/4c^2)\dot{\sigma}_z\ . \tag{17}$$

In terms of these variables the system Hamiltonian, describing the dynamics of small oscillations of the magnetization, has the form of an expansion in powers of π_i, σ_i, and $\nabla\sigma_i$. Retaining in the expansion terms up to σ^4 inclusively, we represent the Hamiltonian in the following form:

$$\begin{aligned}H=\int dV\{&(2c^2/I_0R_0^2)(\pi_y^2+\pi_z^2)\\ &+{}^1/_8I_0(\sigma_z[2(S_y^{(0)})^2+6(S_z^{(0)})^2-2S_0^2-R_0^2\nabla^2]\sigma_z\\ &+\sigma_y[\beta/I_0+6(S_y^{(0)})^2+2(S_z^{(0)})^2-2S_0^2-R_0^2\nabla^2]\sigma_y+8S_y^{(0)}S_z^{(0)}\sigma_y\sigma_z\\ &+4[S_y^{(0)}(\sigma_y^3+\sigma_y\sigma_z^2)+S_z^{(0)}(\sigma_z^3+\sigma_z\sigma_y^2)]+(\sigma_y^2+\sigma_z^2)^2)\}.\end{aligned} \tag{18}$$

Here $dV=(dx/a)$ for the one-dimensional case and $dV=d^3x/a^3$ for the three-dimensional case, where a is the distance between the spins. In the expression for H the ground state, determined by the values of $S_y^{(0)}$ and $S_z^{(0)}$, was not specified. In particular, in the uniform case we must assume $S_y^{(0)}=0$ and $S_z^{(0)}=S_0$, and in the presence of a DW $S_y^{(0)}$ and $S_z^{(0)}$ are functions of the coordinates and are determined by the formulas (11) and (12). In the analysis of magnons superposed on a linear DW the operators $S_0^2\hat{L}_2$ and $S_0^2\hat{L}_1+\beta/I_0$, respectively, are obtained in the brackets in the first and second lines of the formula (18), respectively. The last line in Eq. (18) describes nonlinear effects (three- and four-magnon effects).

We expand σ_z and σ_y in some complete set of states $\{\psi_1\}$ and $\{\varphi_1\}$:

$$\sigma_z=\sum_1\sigma_{z1}\psi_1,\quad \sigma_y=\sum_1\sigma_{y1}\varphi_1,$$

where the subscript 1 indicates the collection of eigenvalues. [For the uniform phase φ and ψ are plane waves: φ_k, $\psi_k\sim\exp(i\mathbf{kr})$; for magnons superposed on a DW φ and ψ are the eigenfunctions of the operators $\hat{L}_2$ and $\hat{L}_1$ in Eqs. (15) and (16).] The Hamiltonian describing magnons against the background consisting of a uniform ground state can be written in the following form in terms of the amplitudes σ_1:

$$H=H_0+H_3+H_4+\dots.$$

The quadratic part of H_0 is diagonal:

$$\begin{aligned}H_0=&({}^1/_2m)\sum_k[\pi_y(\mathbf{k})\pi_y(-\mathbf{k})+\pi_z(\mathbf{k})\pi_z(-\mathbf{k})]\\ &+(m/2)\sum_k[\omega_k^2\sigma_z(\mathbf{k})\sigma_z(-\mathbf{k})+\Omega_k^2\sigma_y(\mathbf{k})\sigma_y(-\mathbf{k})],\end{aligned} \tag{19}$$

where $m=4c^2/I_0R_0^2$ and ω_k and Ω_k are the spin-wave frequencies introduced above. We note that the same form is obtained for H_0 for magnons superposed on a linear DW, except that the summation over $\mathbf{k}$ extends over the eigenvalues of the operators $\hat{L}_1$ (for π_y, σ_y) and $\hat{L}_2$ (for π_z, σ_z) and ω_k and Ω_k must be replaced by the corresponding frequencies of magnons against the background of the DW. For magnons superposed on a rotating DW, however, off-diagonal terms of the form $\sigma_y(1)\sigma_z(2)$ or $\sigma_y(1)\sigma_y(2)$ and $\sigma_z(1)\sigma_z(2)$ appear in H_0 in Eq. (19). The role of these terms is discussed in the next section.

The nonlinear terms in the Hamiltonian have the form

$$\begin{aligned}H_3=&2m\omega_0^2S_0\sum_{123}(\sigma_z(1)\sigma_z(2)\sigma_z(3)+\sigma_z(1)\sigma_y(2)\sigma_y(3))\Delta\\ &\times(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3),\\ H_4=&(m\omega_0^2/2)\sum_{1234}(\sigma_z(1)\sigma_z(2)\sigma_z(3)\sigma_z(4)+2\sigma_z(1)\\ &\times\sigma_z(2)\sigma_y(3)\sigma_y(4)\\ &+\sigma_y(1)\sigma_y(2)\sigma_y(3)\sigma_y(4))\Delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4).\end{aligned} \tag{20}$$

It is interesting to note that the cubic Hamiltonian H_3 contains the small parameter S_0 and not all anharmonicities are possible (there are no terms of the form σ_y^3 and $\sigma_y\sigma_z^2$). For this reason, the contribution of the fourth-order Hamiltonian H_4 can compete with the contribution of H_3.

Quasiclassical quantization of small oscillations superposed on a classical ground state is performed in the standard manner. It is sufficient to replace the canonical variables σ and π by operators which satisfy Bose commutation relations. For the variables introduced above, $\pi_i(1)$ and $\sigma_j(1)$, $i,j = y$ or z and $1 \equiv k_1$, these relations have the following form for a uniform state:

$$[\sigma_i(1),\ \pi_j(2)] = i\hbar\delta_{ij}\delta_{12}, \quad [\sigma_i(1),\ \sigma_j(2)] = [\pi_i(1),\ \pi_j(2)] = 0.$$

It is convenient to replace π and σ by creation and annihilation operators for magnons of two types: α_1 and α_1^+ for longitudinal magnons and b_1 and b_1^+ for transverse magnons:

$$\pi_y = i(\hbar m\Omega_k/2)^{1/2}(b_k^+ - b_{-k}), \quad \sigma_y = (\hbar/2m\Omega_k)^{1/2}(b_k^+ + b_{-k}),$$
$$\pi_z = i(\hbar m\omega_k/2)^{1/2}(a_k^+ - a_{-k}), \quad \sigma_z = (\hbar/2m\omega_k)^{1/2}(a_k^+ + a_{-k}). \tag{21}$$

Substituting these expressions into Eqs. (19) and (20), we obtain the magnon Hamiltonian $H = H_0 + H_3 + H_4 = H_0 + H_{\text{int}}$ in the standard form for perturbation-theory analysis of magnon interactions. The zeroth-order Hamiltonian H_0 has a diagonal canonical form

$$H_0 = \sum_k (\hbar\omega_k b_k^+ b_k + \hbar\Omega_k a_k^+ a_k). \tag{22}$$

For magnons superposed on a linear DW H_0 also has the form (22). Only the set of magnon states over which the summation extends changes.

The three- and four-magnon Hamiltonians H_3 and H_4 contain a large number of terms describing processes involving longitudinal as well as transverse magnons. We do not give the explicit form of H_3 and H_4, because it can be easily obtained from Eqs. (20) and (21).

The interaction of magnons in a strongly anisotropic ferromagnet was investigated by Val'kov and Val'kova.[12] They started from the quantum problem formulated in terms of Hubbard operators; they represented these operators in terms of Bose operators and they took into account the contribution of the projection operators, which necessarily appear with such a transformation. Then the Hamiltonian of the interacting magnons, employed in Ref. 12 for analysis of anharmonic corrections to the magnon frequencies, was derived by a quite complicated procedure. It is important to note that both the structure of the Hamiltonians H_3 and H_4 and the form of the amplitudes in the long-wavelength approximation in our work are identical to those employed in Ref. 12. This shows that the simple phenomenological approach proposed in the present work is adequate.

We now analyze the magnon damping. The damping rate $\gamma(k)$ can be calculated by the standard method as the imaginary part of the mass operator of a magnon on a given branch. The calculations can be performed in the spin-wave approximation, which corresponds to low temperatures. The condition for applicability of the low-temperature approximation for the ferromagnetic phase corresponds to the inequality $T \ll T_c$, where T_c is the temperature at which the average spin vanishes. It can be shown that as $S_0 \to 0$ we have

$$T_c \approx I_0/\ln(6/S_0^2) \ll I_0$$

but in this case T_c is higher than the characteristic temperatures $T_l = \hbar\Omega_0 \approx I_0 S_0$ and $T_t = \hbar\omega_0 \approx (\beta I_0)^{1/2}$, which correspond to the activation energies of longitudinal and transverse magnons, respectively.

We now calculate the damping rates in the long-wavelength approximation ($k \to 0$). In this case, the rates γ_l and γ_t are found to be finite. We begin with the contribution of three-magnon processes. Analysis of the conservation laws shows that processes involving three longitudinal magnons are forbidden, and only processes in which one longitudinal and two transverse magnons participate [the term with $\sigma_z\sigma_y\sigma_y$ in Eq. (20] are important. Among them, the processes in which a longitudinal magnon decays into two transverse magnons contribute to the damping of a longitudinal magnon, while the process with the same vertex, describing the merging of two transverse magnons into a longitudinal magnon, contributes to the damping of a transverse magnon. All these three-magnon processes are allowed for $\Omega_0 > 2\omega_0$, i.e., $\beta < I_0 S_0^2$. For longitudinal magnons, when this condition is satisfied, the three-magnon damping rate is equal to zero at $T = 0$ and increases with increasing temperature. We give the expression for $\gamma_l^{(3)}$ for $T_l \sim T_t$ and in the limiting cases of high and low temperature:

$$\gamma_l^{(3)} = 2\Omega_0/\pi\,(a/R_0)^3(1-\beta/I_0S_0^2)^{1/2}\begin{cases} 1/2, & T \ll T_l \\ T/T_l, & T_l \ll T \end{cases}.$$

As for the damping of transverse magnons, the contribution of three-magnon processes to γ_t is exponentially small at low temperatures, while at high temperatures ($T \gg T_t \sim T_l$) it increases linearly with temperature:

$$\gamma_t^{(3)} = 2\omega_0/\pi\,(a/R_0)^3(1-\beta/I_0S_0^2)^{1/2}$$
$$\times\begin{cases} S_0^2(T_l/T_t)^3(T/T_t)\exp\left(\dfrac{T_t^2 - T_l^2}{T_t T}\right), & T \ll T_t \\ (T/8T_t)(1-\beta/2I_0S_0^2)^{-1}, & T_l \sim T_t \ll T \end{cases}.$$

These formulas demonstrate a basic specific property of magnons in a strongly anisotropic orthorhombic ferromagnet: The ratio of the damping rate to the frequency is not necessarily small. A natural small parameter arises only in a narrow range of values of the parameters where $1 - \beta/I_0S_0^2 \ll 1$. However, for $S_0 \approx (2-3)(\beta/I_0)^{1/2}$ even in the case of extremely low temperatures, the ratio $\gamma_l^{(3)}/\Omega_l$ is small only if the quantity a/R_0 is the small parameter. For transverse magnons the situation is somewhat more favorable, since at low temperatures γ_t is exponentially small. Even at temperatures $T \sim T_l$, T_t, however, we have $\gamma_t^{(3)}/\omega_t \sim S_0(I_0/\beta)^{1/2}(a/R_0)^3$, and for $S_0^2 \sim \beta/I_0$ the quantity $(a/R_0)^3$ is the only parameter that formally ensures that the damping is small.

Three-magnon damping is equal to zero in a narrow region of the parameters $0 < S_0^2 < \beta/I_0$, but here four-magnon scattering of magnons belonging to different branches leads to similar answers. A large number of different processes contribute to four-magnon damping $\gamma_t^{(4)}$ and $\gamma_l^{(4)}$. This contribution can be described schematically by the formula

$$\gamma_l^{(4)} \sim \omega (a/R_0)^6 (T/T_l)^2 \begin{cases} \exp(-4\tilde{T}_0/T), & T \ll \tilde{T}_0 \\ 1, & T \gg \tilde{T}_0 \end{cases};$$

where $\omega = \Omega_0$ or ω_0 and $\tilde{T}_0 = T_l$ or T_t for longitudinal or transverse magnons, respectively.

Here the same law operates: damping is small only in the case of extremely low temperatures, $T \ll T_l$ or $T \ll T_t$. If, however, the temperature is comparable to the activation of magnons, then the damping is small only for $a/R_0 \ll 1$. The parameter a/R_0, i.e., the ratio of the interatomic distance to the interaction radius R_0, is the formal small parameter of the mean-field theory. In real magnets, however, a/R_0 can hardly be expected to be small. Although this parameter enters in $\gamma^{(3)}$ and especially in $\gamma^{(4)}$ to a high power, the damping of magnons on both branches at finite temperatures $T \sim T_l$, T_t (and longitudinal magnons in the case $S_0^2 > (\beta / I_0)$ at any temperatures right down to zero) cannot be considered to be small. Here the situation differs fundamentally from the situation in Heisenberg magnets, where $\gamma(k)/\omega(k) \to 0$ as $k \to 0$. This fact is quite interesting in and of itself, since magnons in a strongly anisotropic ferromagnet, in contrast to a Heisenberg ferromagnet, are not Goldstone excitations.

The formulas obtained for $\gamma_l^{(3)}$ for $S_0^2 \ll 1$ are also qualitatively applicable for the case $S_0^2 \approx 1$. It is found that magnons of an additional (compared with a Heisenberg ferromagnet) longitudinal mode are strongly damped at any temperature and are not observed in resonance experiments. Our calculation actually showed that the longitudinal magnons are well-determined elementary excitations only if $S_0^2 \leqslant \beta / I_0 \ll 1$ holds, when $\gamma_l^{(3)} = 0$ (it is interesting to note that this region is also the region where linear DWs are stable), and only at low temperatures T, much lower than their activation temperature, when $\gamma_l^{(4)}$ is small. Outside these narrow regions of the parameters the longitudinal-magnon mode is purely dissipative (if it is not assumed that the parameter $(a/R_0)^3$ is small).

5. RETARDATION OF DOMAIN WALLS

The dissipative properties of nonlinear excitations (domain walls, kink-type solitons in quasi-one-dimensional ferromagnets) can also be investigated on the basis of the Lagrangian (8) and the Hamiltonian (18). The main parameter here is the viscosity η, which determines the frictional force $F = -\eta v$ acting on a domain wall moving with velocity v. For quasi-one-dimensional ferromagnets the viscosity and the diffusion coefficient $D = T/\eta$, related to it, describe the width $\Gamma_q = Dq^2$ of the central peak in the response functions in the region of viscous motion of solitons (see the review Ref. 3 and the latest works Refs. 16 and 17). In standard three-dimensional ferromagnets the quantity η describes magnetic losses accompanying magnetic reversal due to displacement of DWs.

In order to make a microscopic calculation of η it is necessary to study energy transfer from a moving domain wall to a magnon gas. The method of analysis is described in detail in Ref. 15. Here we give only a schematic description of the method. Any off-diagonal term in the Hamiltonian describing magnons against the background of a domain wall gives rise to inelastic scattering of the magnons. By expanding the field variables σ_y, π_y, σ_z and π_z in complete sets of eigenfunctions of the operators $\hat{L}_1$ and $\hat{L}_2$ with nonreflective potentials it is easy to verify that the Hamiltonian describing magnons against the background of both DWs contains three-magnon off-diagonal terms of the form

$$\sum_{123} \Psi(1,2,3)\,(a_1^+ b_2 b_3 + 2a_1^+ b_{-2}^+ b_3),$$

where the indices 1, 2, ... label magnon states. Off-diagonal terms of the type

$$\sum_{12} \Psi(1,2) a_1^+ b_2, \qquad \sum_{12} \Phi(1,2) a_1^+ a_2, \qquad \sum_{12} \Phi'(1,2) b_1^+ b_2.$$

also arise in the analysis of a rotating DW. They are produced both by terms of the type $S_y^{(0)} S_z^{(0)} \sigma_y \sigma_z$ [the term with $\Psi(1,2)$] and because the potentials in the quadratic part of the Hamiltonian (18) differ from nonreflecting potentials. The amplitudes of the inelastic processes are determined by integrals of products of the corresponding components of the magnetization in the DW, which engender the nonuniformity for magnons, by the wave functions of the magnons. For example,

$$\Psi(k_1, k_2) = I_0 \Delta \int S_y^{(0)} S_z^{(0)} \psi(k_1) \varphi(k_2)\, d\xi,$$

where $\xi = (x - vt)/\Delta$. Because the domain wall is not stationary the amplitudes are time dependent. This dependence is universal. If all indices 1, 2, ... correspond to volume magnons, then

$$\Psi(1,2,3) \sim \exp(iQ_{123} vt), \quad \Psi(1,2), \quad \Phi(1,2),$$

$$\Phi'(1,2) \sim \exp(iQ_{12} vt),$$

where $Q_{123} = k_{1x} - k_{2x} - k_{3x}$ and $Q_{12} = k_{1x} - k_{2x}$ is the component of the momentum, along the normal to the DW, that is transferred by the DW in this process. If, however, one of the states is localized, then the corresponding term $ik_{ix} vt$ does not occur in the exponential. In the three-dimensional case all amplitudes contain a delta function expressing the conservation of the total momentum of the magnons in the plane of the DW. For example,

$$\Psi(1,2,3) \sim \Delta(\mathbf{k}_{1\perp} - \mathbf{k}_{2\perp} - \mathbf{k}_{3\perp}), \quad \Psi(1,2) \sim \Delta(\mathbf{k}_{1\perp} - \mathbf{k}_{2\perp}).$$

The contributions of different two- and three-magnon processes to the stopping power are described in the Born approximation by formulas of the form

$$F^{(2)} = (\pi\zeta/\hbar) \sum_{12} Q_{12} |\Psi(1,2)|^2 [n_1 - n_2]\, \delta(\omega_1 - \omega_2 - Q_{12} v),$$

$$F^{(3)} = (\pi\zeta/\hbar) \sum_{123} Q_{123} |\Psi(1,2,3)|^2 [n_1 (n_2+1)(n_3+1)]$$

$$\times\, \delta(\omega_1 - \omega_2 - \omega_3 - Q_{123} v),$$

where ζ is a combinatorial factor and $n_1 = [\exp(\varepsilon_1/T) - 1]^{-1}$ is the occupation number of the state 1 with energy $\varepsilon_1 = \hbar\omega_1$. (The structure of the formulas for the frictional force is studied in greater detail in, for example, Ref. 15.) For low domain-wall velocities the factors containing the occupation numbers and enclosed in square brackets are proportional to v because of the delta function. Hence, as $v \to 0$ the force satisfies $F \sim v$ and it is easy to write down a formula for the viscosity. Switching in these formulas from summation to integration and calculating the corresponding integrals, we obtain expressions for the two- and three-magnon coefficients of viscosity η_2 and η_3; the total coefficient of viscosity in $\eta = \eta_2 + \eta_3$. We present the results of the calculation of η_2 and η_3 in the one- and two-dimensional strongly anisotropic ferromagnets.

In a one-dimensional ferromagnet the contribution of three-magnon processes for both types of solitons of the domain-wall type is given by the formula

$$\eta_{1d}^{(3)} = \pi^2(\hbar/R_0^2)\begin{cases} f(x)\exp(-T_0/T), & T \ll T_0 \\ g(x)(T/T_0)^2, & T \gg T_0 \end{cases} \quad (23)$$

where $x = \beta/I_0S_0^2$ and $T_0 = (\beta I_0)^{1/2}$ for a linear DW ($x > 1$) and $x = C + 1$ and $T_0 = I_0S_0^2$ for a rotating DW; $f(x)$ and $g(x)$ are complicated functions,

$$f(x) = \frac{k_0^4(8+5k_0^2)^2}{576[3(x-1)(x+k_0^2)]^{1/2}\,\mathrm{sh}^2(\pi k_0/2)},$$

$$k_0^2 = 2\{1+[3(x-1)]^{1/2}\}$$

and $f(x)$ is exponentially small for $x \gg 1$. The function $g(x) \sim 1$ and is virtually independent of its argument for all values of x, except $x \approx 1$. For $x = 1$ ($\beta = I_0S_0^2$), i.e., at the point where both DWs become unstable, the functions f and g are proportional to $|x-1|^{-1/2}$. This singularity is caused by the contribution of localized magnons, whose frequencies soften near the transition.

As concerns the two-magnon viscosity, for a linear DW it is equal to zero $\eta_{1d}^{(2)} = 0$. For a rotating DW

$$\eta_{1d}^{(2)} = \frac{C\hbar}{4\pi^3\Delta_R^2}\begin{cases} C\exp(-T_0/T), & T \ll T_0 \\ (T/T_0)(C\zeta_1+\zeta_2), & T \gg T_0 \end{cases} \quad (24)$$

where $\zeta_1 \approx 13.7$ and $\zeta_2 \approx 5.3$ are constants.

Comparing Eqs. (23) and (24) shows that, with the exception of extremely low temperatures, the contribution of two-magnon processes to the coefficient of viscosity is always small compared with three-magnon processes. In the case $T \ll T_0$, however, these contributions can compete only when the factor $f(x)$ in Eq. (23) is anomalously small, $f(x) < (R_0/x_0)^2 \sim S_0^2$, which happens in the "Heisenberg" region $C \gg 1$. Thus in strongly anisotropic ferromagnets there is no competition between two- and three-magnon contributions, which are characteristic for Heisenberg magnets, to the stopping of DWs (see Refs. 15–17).

From the Einstein relation the temperature dependence for the diffusion coefficient D at high temperatures $D \sim 1/T$ is found to be the same as in a number of scalar models and in Heisenberg magnets; see Refs. 15–19. On the whole, the diffusion coefficient in strongly anisotropic ferromgnets is smaller than in Heisenberg ferromagnets; in addition, D decreases rapidly as $C \to 1$. This means that the width $\Gamma_q = Dq^2$ of the central peak of the correlation functions in the viscous state of soliton motion is smaller in this model than for a Heisenberg ferromagnet.

The calculation was performed similarly in the three-dimensional case. The expression for the three-magnon coefficient of viscosity per unit area of the DW is obtained from Eq. (23) by replacing ($\hbar R_0^2$) by ($\hbar/R_0^4$), changing some numerical factors, and by replacing the additional temperature factor at high temperatures, $\eta_{3d}^{(3)} \sim (T/T_0)^3$. As in the one-dimensional case, the contribution of two-magnon processes is small.

CONCLUSIONS

The foregoing analysis has demonstrated a number of unusual properties of orthorhombic ferromagnets with $S = 1$ and strong single-ion anisotropy. The main difference from Heisenberg ferromagnets is the existence of two different types of linear and nonlinear elementary excitations which relax by means of processes that are different from those occurring in standard ferromagnets (a general property is that the damping is stronger than usual). We now discuss other physical models that can be studied on the basis of the proposed method and the conclusions that can be drawn.

For the present model of ferromagnets the case of an isotropic easy plane ($\beta = 0$) is special. In this case the ground state of the system is characterized by continuous degeneracy (with respect to the direction of magnetization in the easy plane), and the transverse magnons are a Goldstone mode and have a linear dispersion relation $\omega_t = c|\mathbf{k}|$. Preliminary analysis has shown that their damping is weak in the long-wavelength limit: $(\gamma_k/\omega_k) \sim k$ and $k \to 0$, i.e., these magnons are weakly damped. This model is also characterized by special soliton excitations—magnetic vortices at whose center the magnetization vanishes.

Although in deriving Eq. (8) we started from the existence of magnetic order, this Lagrangian is also valid for the quadrupole phase (QP). (Solitons in the QP were studied in Ref. 5.) The dynamical and relaxational properties of the QP, including also nonlinear properties, can be described on the basis of the Lagrangian (8).

The dynamical properties of Heisenberg ferromagnets are fundamentally different from those of antiferromagnets (AFM). It is interesting to discuss the properties of strongly anisotropic AFM. In Ref. 8 it was pointed out that for strongly anisotropic AFM with $S = 1$, on the basis of the lagrangian (8), both the structure of the static DWs in the field of the antiferromagnetism vector $\mathbf{L}$ and the low-lying branches of magnons with $\hbar\omega \ll I_0, B$ near the transition from the AFM to the quadrupole phase ($\mathbf{L} = 0$) are found to be identical to those in ferromagnets. It is easy to verify that this makes it possible to reconstruct uniquely the Lagrangian of the problem, which is identical to Eq. (8), if the substitutions $S_y \to L_y$ and $S_z \to L_z$ are made in it. Hence it can be concluded that our results for the structure and stability of moving DWs and relaxation of DWs as well as magnon damping are also valid for AFMs.

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Translated by M. E. Alferieff