Two-dimensional system of interacting particles in a strong magnetic field: charge density waves

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This paper studies the instability of a two-dimensional system of interacting particles in a strong magnetic field in relation to the transition to a nonuniform state. The instability is caused by a pole at the zero-sound vertex. In view of this equations are derived for vertices in the "parquet" approximation. Special attention is paid to the case of a Landau level with a small occupation factor, $\nu \ll 1$. Analysis suggests that charge density waves may form when the wave vector is small, and the corresponding criterion is derived.

1. INTRODUCTION

This paper continues the study, started in Refs. 1–3, of properties of a two-dimensional ($2D$) system of interacting particles in a strong magnetic field B by the diagrammatic technique. The model is based on the widely used condition that the Coulomb energy $\varepsilon_c = e^2/\varkappa l_B$, where $\varkappa$ is the dielectric constant, and $l_B = (c\hbar/eB)^{1/2}$ is the magnetic length, be much smaller than the distance between Landau levels; that is, the interaction conserves the number of particles on a Landau level. The system is assumed polarized, so that in what follows all spin indices are omitted.

The analysis carried out in Refs. 1 and 2 has shown that in a given order of perturbation theory some topologically distinct diagrams are equal to each other. This makes it possible to sum an entire class of diagrams by introducing an effective interaction potential. Another important result of Ref. 2 is that the instability with respect to the transition to a nonuniform state (i.e., the formation of charge density waves) is determined by the pole at the zero-sound vertex. The transition of a $2D$ system in a strong magnetic field to a nonuniform state is of interest because rich experimental evidence has appeared in recent years concerning phenomena caused, apparently, by such a transition. This is true of work on transport phenomena,[4,5] radio-frequency radiation absorption,[6] sound absorption,[7] and magneto-optical phenomena.[8–11] Hence the great importance of investigating the conditions for the transition of a $2D$ system to a nonuniform state. Some estimates for this transition were obtained in Ref. 3. The present paper uses the analysis of equations for vertices in the "parquet" approximation to show that when the occupation factor of a Landau level is small the transition of a $2D$ system in a strong magnetic field to a nonuniform state is in principle possible. The corresponding criterion is also derived.

2. "PARQUET" EQUATIONS FOR VERTICES

A previous paper by the present author[2] shows that the instability of a $2D$ system connected with the formation of a charge density wave (CDW) appears in the form of a pole at the zero-sound vertex. If the initial renormalized interaction potential is substituted into zero-sound bubble diagrams (and equivalent diagrams), the resulting criterion for CDW formation agrees fully with the results of Fukuyama, Platzman, and Anderson.[12] An essential feature of this criterion is that it provides the lower bound on the magnitude q of the wave vector for CDW; that is, only waves with $q \geqslant q_0$ can exist ($q_0 l_B \simeq 0.5$ for the Coulomb potential). On the other hand, for a Wigner crystal one should, apparently, expect $q l_B = (2\pi\nu)^{1/2}$ to hold, where the occupation factor for the Landau level $\nu = 2\pi l_B^2 n_s$, with n_s the $2D$-particle density. Hence in studying CDW with fairly low values of q we must go beyond the scope of the "ladder" approximation for the zero-sound vertex considered in Ref. 2. The analysis carried out below shows that when the occupation factor for the Landau level is low, $\nu \ll 1$, we can limit our discussion to the method of successive approximations for "parquet" diagrams.[13,14] Since the "parquet" approximation may in itself be of interest in studies of properties of $2D$ systems when $\nu \lesssim 1$, we will discuss its derivation in detail for the model considered.

Figure 1 depicts the simplest "parquet" diagrams. In the Landau gauge the two-dimensional momentum is $P = (p,\omega)$, where $p \equiv p_y$, and the frequency $\omega = \pi T(2n+1)$, with T the temperature and n an integer. A distinctive topological property of this class of diagrams is that they can always be divided into two parts by cutting only the two internal lines. We denote by $\gamma_C(P_1P_2;P_3P_4)$ the set of all such diagrams reducible in relation to the momentum pairs (P_1P_2) and (P_3P_4), that is, can be divided into two parts by the above method, one part containing only the pair of momenta (P_1P_2) and the other the pair (P_3P_4). In the case at hand, the momenta P_1 and P_2 correspond to "in" lines and P_3 and P_4 to "out" lines. The quantity $\gamma_C(P_1P_2;P_3P_4)$ is known as the Cooper "brick." We can also introduce the zero-sound "bricks" $\gamma_z(P_1P_3;P_2P_4)$ and $\gamma_z(P_1P_4;P_2P_3)$, which are reducible in relation to the respective momentum pairs. Note that the Cooper brick $\gamma_C(P_1P_2;P_3P_4)$ is antisymmetric with respect to the interchange within each pair of the in and out momenta. As a result we get a system of nonlinear equations for the various bricks, a situation depicted in Figs. 2 and 3. The internal lines represent the following Green's function:

$$G(K) = (i\omega - \xi)^{-1}, \quad \omega = \pi T(2n+1), \tag{1}$$

where $K = (k,\omega)$, and there is summation over frequency ω and integration over k. The other quantities in Eq. (1) have the following meaning. The initial vertex ($l_B = 1$) is

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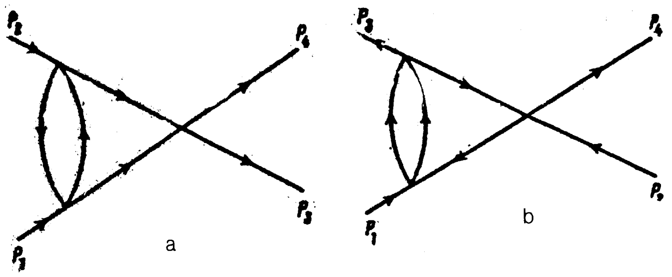

FIG. 1. Diagram a corresponds to the insertion of a zero-sound bubble diagram into a Cooper vertex, and diagram b of a Cooper bubble diagram into a zero-sound vertex.

$$\Gamma^{(0)}(P_1P_2;P_3P_4)$$
$$=-2\int\frac{dq_x}{2\pi}\exp([iq_x(p_1-p_4)])\tilde{U}(q_x,p_1-p_3),\qquad(2)$$

where $P_i=(p_i,\omega_i)$, and the effective interaction potential is[1,2]

$$\tilde{U}(q)=\frac{1}{2}\left[V(q)-\frac{1}{2\pi}\int\exp i(qp)\,V(p)\,dp\right]$$
$$=4\pi\sum_m{}'\exp(-q^2/2)E_mL_m(q^2).\qquad(3)$$

Here the prime on the summation sign indicates that summation is over odd values of the positive integer m, and $L_m(x)$ is the Laguerre polynomial. For the Coulomb interaction, the two-particle energy levels are

$$E_m=\frac{e^2}{2\varkappa l_B}\frac{\Gamma(m+{}^1/_2)}{\Gamma(m+1)},\qquad(4)$$

where $\Gamma(x)$ is the gamma function.

In view of Eqs. (2) and (3), the initial vertex $\Gamma^{(0)}$ is antisymmetric under the interchange of initial (or final) momenta. This makes it necessary to introduce the factor 1/2 into the equation for the Cooper brick (see Fig. 2).

The total vertex is

$$\Gamma(P_1P_2;P_3P_4)=\Gamma^{(0)}(P_1P_2;P_3P_4)+\gamma_C(P_1P_2;P_3P_4)$$
$$+\gamma_z(P_1P_3;P_2P_4)-\gamma_z(P_1P_4;P_2P_3).\qquad(5)$$

Using the diagrammatic representation for "parquet" equations, we can easily derive the respective analytical expressions. This gives us the following system of nonlinear equations [allowing for condition (5)]:

$$\gamma_C(P_1P_2;P_3P_4)=\frac{T}{2}\sum_\omega\int\frac{dk}{2\pi}[\Gamma^{(0)}(P_1P_2;P_1+P_2-K,K)$$
$$+\gamma_z(P_1,P_1+P_2-K;$$
$$P_2K)-\gamma_z(P_1K;P_2,P_1+P_2-K)]$$
$$\times G(K)G(P_1+P_2-K)\Gamma(K,P_1+P_2-K;P_3P_4)\qquad(6)$$

and

$$\gamma_z(P_1P_3;P_2P_4)$$
$$=-T\sum_\omega\int\frac{dk}{2\pi}[\Gamma^{(0)}(P_1K;P_3,P_1+K-P_3)$$
$$+\gamma_C(P_1K;P_3,P_1+K-P_3)]\cdot$$
$$\times G(K)G(P_1+K-P_3)\Gamma(P_1+K-P_3,P_2;KP_4),\qquad(7)$$

where $K=(k,\omega)$.

The system of nonlinear equations (5)–(7) cannot be solved exactly. Hence the need to analyze the quantities entering into these equations. According to Ref. 2, the simplest Cooper bubble diagram is proportional to

$$\frac{1-2\nu}{i(\omega_1+\omega_2)-2\xi},\qquad(8)$$

and the simplest zero-sound bubble diagram is proportional to

$$\frac{\nu(1-\nu)}{T}\delta_{\omega_1,\omega_3}.\qquad(9)$$

The quantity ξ, which enters into the Green's function (1) and expression (8), is related to the dimensionless density in the following manner:

$$\xi=T\ln(\nu^{-1}-1).\qquad(10)$$

On the other hand, for the simplest "parquet" diagrams of Fig. 1 we can easily find that diagram *a* is proportional to the expression

$$\nu(1-\nu)G(\omega_1)G(\omega_2),\qquad(11)$$

and diagram *b* to the expression

$$\nu(1-\nu)\left[G(\omega_1)G(\omega_3)-\frac{1-2\nu}{T}G(\omega_1)\delta_{\omega_1,\omega_3}\right].\qquad(12)$$

The expressions obtained (8)–(12) lead to the following conclusion concerning the case $\nu\ll1$. First, from (8) and (9)

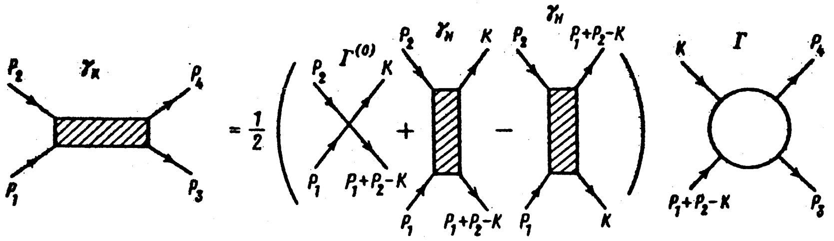

FIG. 2. The diagram equation for a Cooper brick. The constituent zero-sound bricks are $\gamma_z(P_1,P_1+P_2-K;P_2P_4)$ and $\gamma_z(P_1P_4;P_2,P_1+P_2-K)$.

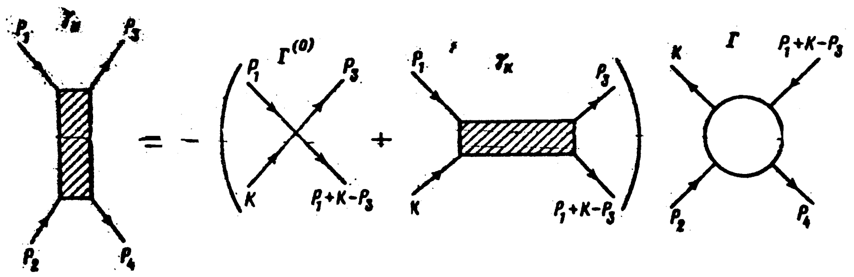

FIG. 3. The diagrammatic equation for a zero-sound brick. The Cooper brick inside the parentheses is $\gamma_C(P_1K;P_3,P_1+K-P_3)$.

it follows that the ratio of a zero-sound bubble diagram to a Cooper one is

$$\frac{\nu}{T}\xi\simeq\nu\ln(\nu^{-1})\ll 1.$$

The ratio of the first term inside the square brackets in (12) to the second is of order

$$T/\xi\simeq[\ln(\nu^{-1})]^{-1}\ll 1,\quad \nu\ll 1.$$

On the other hand, comparison of the square of the Cooper bubble diagram ($\lesssim\xi^{-2}$) with (11), which is of the order of $\nu\xi^{-2}$ and corresponds to the substitution of a zero-sound bubble diagram into the Cooper vertex, shows that for $\nu\ll 1$ in the first approximation for the "parquet" equations we can confine our discussion to the summation of ladder diagrams for the Cooper "brick." With the next approximation, in the equation for the zero-bound "brick" (7) with allowance for (5) ($\Gamma_c=\Gamma^{(0)}+\gamma_c$),

$$\begin{aligned}&\gamma_z(P_1P_3;\ P_2P_4)\\&=-T\sum_\omega\int\frac{dk}{2\pi}\,\Gamma_C(P_1K;\ P_3,\ P_1+K-P_3)G(K)G(P_1+K-P_3)\\&\quad\times[\Gamma_C(P_1+K-P_3,\ P_2;\ KP_4)+\gamma_z(P_1+K-P_3,\ K;\ P_2P_4)\\&\quad-\gamma_z(P_1+K-P_3,\ P_4;\ P_2K)]\end{aligned}\qquad(7')$$

we can keep only terms with $\omega_1=\omega_3$, that is, leave the first two terms inside the square brackets. As a result we get the following equation for the zero-sound vertex ($p_1+p_2=p_3+p_4$):

$$\begin{aligned}&\gamma_z(p_1\omega_1,\ p_3\omega_1;\ p_2\omega_2,\ p_4\omega_2)\\&=-T\sum_\omega\int\frac{dk}{2\pi}\Gamma_C(p_1\omega_1,\ k\omega;\ p_3\omega_1,\ p_1+k-p_3,\ \omega)G^2(\omega)\\&\quad\times[\Gamma_C(p_1+k-p_3,\ \omega,\ p_2\omega_2;\ k\omega,\ p_4\omega_2)\\&\quad+\gamma_z(p_1+k-p_3,\ \omega,\ k\omega;\ p_2\omega_2,\ p_4\omega_2)],\end{aligned}\qquad(13)$$

and the Cooper vertex calculated in the ladder approximation is

$$\begin{aligned}\Gamma_C(P_1P_2;\ P_3P_4)=&-2\int\frac{dq_x}{2\pi}\exp[iq_x(p_1-p_4)]\\&\times\tilde\gamma_C(q_x,\ p_1-p_3;\ \omega_1+\omega_2),\end{aligned}\qquad(14)$$

where

$$\begin{aligned}\tilde\gamma_C(q;\Omega)=4\pi&\sum_m{}'\exp(-q^2/2)E_mL_m\\&\times(q^2)\frac{i\Omega-2\xi}{i\Omega-2\xi-(1-2\nu)E_m}.\end{aligned}\qquad(15)$$

from which it follows that in the limit $\nu\to 0$ there is a finite renormalization of the interaction potential (3).

Assuming that the zero-sound vertex has a form similar to (14) (with appropriate replacement of indices), we finally find that Eq. (13) transforms into

$$\begin{aligned}\tilde\gamma_z(q;\ \omega_1,\omega_2)=&-\frac{2T}{\pi}\sum_\omega\tilde\gamma_C(q:\ \omega+\omega_1)G^2(\omega)\tilde\gamma_C(q;\ \omega+\omega_2)\\&+\frac{T}{\pi}\sum_\omega\tilde\gamma_C(q;\ \omega+\omega_1)G^2(\omega)\tilde\gamma_z(q;\ \omega+\omega_2)\end{aligned}\qquad(16)$$

where ($q=(q_x^2+q_y^2)^{1/2}$) is the wave vector of the CDW. The homogeneous equation that follows from (16),

$$\tilde\gamma_z(q,\omega)=\frac{T}{\pi}\sum_{\omega_1}\tilde\gamma_C(q,\omega+\omega_1)G^2(\omega_1)\tilde\gamma_z(q,\omega_1)\qquad(17)$$

determines the instability in connection with the formation of a CDW with a given wave vector q. Note that when we go over to Eq. (17), the frequency ω_2 can be considered a parameter and omitted. From Eq. (17) it follows that if in (15) the renormalization of the interaction potential caused by summation of Cooper diagrams is ignored (i.e., if formally we set $E_m=0$ in the denominator), the frequency dependence vanishes and the resulting condition that Eq. (17) have a solution is

$$1=\frac{\nu(1-\nu)}{T}\frac{1}{2\pi}\left[\frac{1}{2\pi}\int e^{iqp}V(p)dp-V(q)\right]\qquad(18)$$

which coincides with the results obtained in Ref. 12, that is, forbids the existence of a CDW with $q\leqslant q_0$. The analysis done below shows that the renormalized potential in the form (15) allows for the formation of a CDW with an arbitrarily small wave vector q.

3. CDW FOR SMALL OCCUPATION FACTORS

Since it is impossible to find an exact criterion for solvability of Eq. (17), an attempt was made[3] to qualitatively establish the criterion for the formation of a nonuniform state. Hence the importance of first studying the general properties of Eq. (17). We can represent this equation differently by introducing a new function

$$f_z(q,\omega) = G(\omega)\tilde{\gamma}_z(q,\omega) \quad (19)$$

instead of $\tilde{\gamma}_z(\omega)$. As a result we get the equation

$$f_z(q,\omega)=\frac{T}{\pi}\sum_{\omega_1} G(\omega)\tilde{\gamma}_C(q,\omega+\omega_1)G(\omega_1)f_z(q,\omega_1). \quad (20)$$

whose kernel is a symmetric function of the variables ω and ω_1. Another important property of $f_z(q,\omega)$ that follows from Eq. (20) is the condition

$$f_z^*(q,\omega) = f_z(q,-\omega). \quad (21)$$

If we write Eq. (20) in the form

$$f_z(q,\omega) = \sum_{\omega_1} K(q;\omega,\omega_1) f_z(q,\omega_1), \quad (22)$$

then, allowing for (21), we can easily verify that the quadratic form

$$\sum_{\omega,\omega_1} f_z^{(1)}(q,\omega)K(q;\omega,\omega_1) f_z^{(2)}(q,\omega_1) \quad (23)$$

is Hermitian. This implies that the eigenvalues of Eq. (20) are real and the criterion for CDW formation can be found from the condition

$$\det[1-K(q;\omega,\omega_1)]=0. \quad (24)$$

Since the frequencies ω and ω_1 are expressed in terms of integers [see Eq. (1)], the kernel $K(q;\omega_1,\omega_1)$ is a symmetric matrix. Note that the left-hand side of this equation is a function of the variables T, ν, and q. Equation (24) can serve as a basis for numerical calculations of the criterion for CDW formation, and (23) for applications of the variational principle. On the other hand, we can make direct use of the fact that Eq. (17) has been obtained for the case $\nu \ll 1$. In view of this we write this equation explicitly:

$$\tilde{\gamma}_z(q,\omega)=T\sum_m{}' \sum_{\omega_1}\varphi_m(q)\left[1+\frac{E_m}{i(\omega+\omega_1)-\xi_m}\right] \times G^2(\omega_1)\tilde{\gamma}_z(q,\omega_1), \quad (25)$$

where $\xi_m = 2\xi + E_m$, and [see Eq. (15)]

$$\varphi_m(q)=4\exp((-q^2/2)E_m L_m(q^2). \quad (26)$$

It is convenient to shift from the frequency representation to the complex-time representation.[15] To this end we introduce the following function of variable τ:

$$\gamma(q,\tau)=T\sum_\omega \frac{e^{-i\omega\tau}}{(i\omega-\xi)^2}\tilde{\gamma}_z(q,\omega). \quad (27)$$

As a result of very simple transformations Eq. (25) becomes

$$\left(\frac{d}{d\tau}+\xi\right)^2\gamma(q,\tau)=\frac{1}{\pi}\tilde{U}(q)\delta(\tau)\gamma(0) -\sum_m{}' \varphi_m(q)E_m D_m(\tau)\gamma(q,-\tau), \quad (28)$$

where

$$D_m(\tau)=\exp(-\xi_m\tau)\begin{cases}1+n\left(\frac{\xi_m}{T}\right), & \tau>0\\ n\left(\frac{\xi_m}{T}\right), & \tau<0,\end{cases} \quad (29)$$

with $-1/T \leqslant \tau \leqslant 1/T$, and the function

$$n(x)=(e^x-1)^{-1} \quad (30)$$

determines the number of particle pairs.

The structure of Eq. (28) makes it possible to assert that the problem does not allow for an exact analytical solution. The idea of an approximate solution is based on the fact that the number of particle pairs is very low, since

$$n(\xi_m/T)\simeq\exp(-\xi_m/T)\simeq\nu^2\exp(-E_m/T)\ll 1.$$

The approximation discussed below assumes that particle pairs can be ignored; that is, they contribute very little to the criterion of CDW formation. In view of this we ignore the terms containing ν^2 in Eq. (28); that is, we put $n(\xi_m/T)=0$. This means that for $\tau<0$ the solution to Eq. (28) is

$$\gamma(\tau)=e^{-\xi\tau}(1+\alpha\tau). \quad (31)$$

Substituting this expression into Eq. (28) for $\tau>0$ makes it possible to find $\gamma(\tau)$ in this range of variation of ν, too. Subsequent calculations are fairly simple but cumbersome. Hence, we will discuss the transformations only in principle. First, the term with $\delta(\tau)$ in Eq. (28) makes it possible to partially match $\gamma(\tau)$ for positive and negative τ. Another very important relation follows from the definition of $\gamma(\tau)$ in the form (27), which implies (see Ref. 15) that the following condition must be met:

$$\gamma(\tau<0)=-\gamma\left(\tau+\frac{1}{T}\right). \quad (32)$$

This functional relation allows for complete solution of the problem of finding the criterion of CDW formation. As a result of fairly involved calculations we find the criterion of the following form:

$$\left[1-\nu\sum_m{}'\varphi_m(q)\frac{\exp(-E_m/T)-1}{E_m}\right]^2 =\frac{\nu}{T}\sum_m{}'\varphi_m(q)\exp(-E_m/T) \times\left[1+2\nu T\sum_m{}'\varphi_m(q)\frac{\exp(-E_m/T)-1}{E_m^2}+2\nu\sum_m{}'\varphi_m(q)E_m^{-1}\right]. \quad (33)$$

We note immediately that as $E_m\to 0$, Eq. (33) transforms into condition (18), where the term with ν^2 must be ignored. Also, the terms with ν^2 that appear in criterion (33) are not related to the number of particle pairs and must be retained, since in this expression the smallness of the occupation factor ν is compensated for by the condition that $q\to 0$ (see below).

It can be shown that in the case of a short-range interaction potential Eq. (17) has no solution for $ql_B \ll 1$, as expected. In view of this the remaining calculations are performed for the Coulomb potential. A fairly simple analysis of the sums entering into (33) suggests that in the limit $ql_B \ll 1$ the terms with large values of m play the main role, and summation over m can be replaced with integration. To explain the nature of the following calculations we employ, by way of an example, one of the terms in Eq. (33). For one thing, we find that the following approximations hold:

$$\frac{\nu}{T}\sum_m{}' \varphi_m \exp(-E_m/T) \simeq \frac{\nu}{2T}\sum_m \varphi_m \exp(-E_m/T)$$

$$\simeq \frac{\nu}{2T}\int_0^\infty \varphi_m(q)\exp(-E_m/T)\,dm$$

$$\simeq \frac{\nu\varepsilon_c}{T}\int_0^\infty J_0(2qm^{1/2})\exp\left(-\frac{\varepsilon_c}{2Tm^{1/2}}\right)\frac{dm}{m^{1/2}}, \quad (34)$$

where $J_n(x)$ is a Bessel function. A few more words about this transition. The approximation

$$\sum_m{}' a_m(q) = \frac{1}{2}\sum_{m=0}^{\infty} a_m(q)$$

$$-\frac{1}{2}\sum_{m=0}^{\infty}(-1)^m a_m(q) \simeq \frac{1}{2}\sum_{m=0}^{\infty} a_m(q)$$

follows from the fact that the terms in the sum that contain the factor $(-1)^m$ converge well and the sum acquires no singularity as $q \to 0$. On the other hand, in (34) we employed (26), where one must go over to the asymptotic values of the Laguerre polynomials[16] and the energy levels E_m [Eq. (4)] for $m \gg 1$. The last integral in (34) can be evaluated if we write it as

$$\frac{\nu\varepsilon_c}{Tq}\int_0^\infty J_0(x)\exp(-x_0/x)\,dx = \frac{\nu\varepsilon_c}{Tq}\int_{x_0}^\infty dy\int_0^\infty J_0(x)\frac{\exp(-y/x)}{x}\,dx, \quad (35)$$

with $x_0 = \varepsilon_c q/T$. Using well-known formulas for integrals containing Bessel functions,[16] we find that the final expression for (35) has the following form:

$$\frac{\nu\varepsilon_c}{Tq}x[J_0(x)K_1(x) - J_1(x)K_0(x)], \quad (36)$$

where $x = (2x_0)^{1/2}$, and $K_n(x)$ is a modified Bessel function. All the other sums in Eq. (33) can be represented in a similar manner. As a result this equation assumes the form

$$1 = -\left(\frac{\nu}{q^2}\right)^2 x^4[J_1(x)K_1(x)]^2$$

$$-\frac{\nu}{q^2}\frac{x^3}{2}[J_1(x)K_2(x) + J_2(x)K_1(x)]$$

$$+2\left(\frac{\nu}{q^2}\right)^2 x[J_0(x)K_1(x) - J_1(x)K_0(x)]\int_0^x y^3K_1(y)J_1(y)\,dy, \quad (37)$$

where $x = (2\varepsilon_c q/T)^{1/2}$.

Analyzing Eq. (37), we see that in the limit $x \ll 1$ it becomes

$$1 + \frac{\nu x^2}{2q^2} = 0,$$

which means that in the range of x considered here there is no instability related to CDW formation. This condition corresponds to criterion (18) for $ql_B \ll 1$. On the other hand, we see that there is such an instability for $x \gg 1$. In this range it suffices to limit oneself to the third term on the right-hand side of Eq. (37), which term may be of both signs (this is easily verified). But in this range the CDW wave vector satisfies the condition $ql_B \ll \nu^{1/2}$.

4. CONCLUSION

We have thus studied the instability of a two-dimensional system of particles in a strong magnetic field with respect to the transition to a nonuniform state. The case considered referred to a Landau level with a small occupation factor, $\nu \ll 1$. The analysis shows that such a transition is possible for small values of the wave vector q, and the criterion for this transition is possible for small values of the wave vector q, and the criterion for this transition is Eq. (37). An important feature of this criterion is that it contains two independent parameters, $\nu(ql_B)^{-2}$ and $\varepsilon_c ql_B/T$. A detailed analysis of the resulting criterion (37) requires numerical calculations. If we assume that the instability has to do with the formation of a Wigner crystal, it is natural to assume $ql_B \sim \nu^{1/2}$, that is, to fix one of the parameters in the criterion. Then the second parameter becomes a function of the dimensionless density ν and temperature. As a result, the fact that the right-hand side of (37) changes its sign shows that there are certain ranges of variation of parameter $\varepsilon_c\nu^{1/2}/T$ where a transition to a nonuniform state does occur; that is, for a fixed occupation factor (or fixed temperature) the nonuniform state first appears and then disappears under temperature (or density) variations. This fact has been observed in experiments.[3,11] At the same time experiments state in Refs. 4–11 that the observed phenomena are related to the transition of a 2*D* system to the crystalline state (the Wigner crystal). The results of the present study do not permit the unambiguous conclusion that the instability considered corresponds to the formation of a Wigner crystal, since the structure of the nonuniform state that forms as a result of the instability has yet to be studied. A promising fact, however, is the qualitative agreement with the experimental data; note above. On the whole, the nature of the instability investigated here is still an open question.

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[1] Yu. A. Bychkov, Fiz. Tverd. Tela **31**(7), 56 (1989) [Sov. Phys. Solid State **31**, 1130 (1989)].
[2] A. V. Andreev and Yu. A. Bychkov, Zh. Eksp. Teor. Fiz. **100**, 725 (1991) [Sov. Phys. JETP **73**, 404 (1991)].
[3] Yu. A. Bychkov, Pis'ma Zh. Eksp. Teor. Fiz. **54**, 147 (1991) [JETP Lett. **54**, 142 (1991)].
[4] H. W. Jiang, R. L. Willet, H. L. Stormer *et al.*, Phys. Rev. Lett. **65**, 633 (1990).

[5]V. J. Goldman, M. Santos, M. Shaegyan, and J. E. Cunningham, Phys. Rev. Lett. **65**, 2189 (1990).
[6]E. Y. Andrei, G. Deville, D. C. Glattli *et al.*, Phys. Rev. Lett. **60**, 2765 (1988).
[7]R. L. Willet, M. A. Paalanen, L. N. Pfeifer *et al.*, in *Proc. of Int. Conf. AHMFSP-7*, Würzburg (1990) p. 111.
[8]H. Buhman, W. Joss, K. v. Klitzing *et al.*, Phys. Rev. Lett. **66**, 926 (1991).
[9]R. G. Clark, R. A. Ford, S. R. Haynes *et al.*, in *Proc. of Int. Conf. AHMFSP-7*, Würzburg (1990) p. 39.
[10]B. B. Goldberg, D. Heiman, A. Pinzuk *et al.*, in *Proc. of Int. Conf. AHMFSP-7*, Würzburg (1990) p. 49.
[11]I. V. Kukushkin, N. J. Pulsford, K. von Klitzing *et al.*, Phys. Rev. B **45**, 4532 (1992).
[12]H. Fukuyama, P. M. Platzman, and P. W. Anderson, Phys. Rev. B **19**, 5211 (1979).
[13]I. T. Dyatlov, V. V. Sudakov, and K. A. Ter-Martirosyan, Zh. Eksp. Teor. Phys. **32**, 767 (1957) [Sov. Phys. JETP **5**, 631 (1957)].
[14]Yu. A. Bychkov, L. P. Gor'kov, I. E. Dzyaloshinskiĭ, Zh. Eksp. Teor. Phys. **50**, 738 (1966) [Sov. Phys. JETP **23**, 489 (1966)].
[15]A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, *Quantum Field Theoretical Methods in Statistical Physics*, Pergamon, New York (1965).
[15]A. Erdélyi, *Higher Transcendental Functions*, Vol. 2 (Bateman Project), McGraw-Hill, New York (1953).

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