Nonstationary dynamics of rotating superfluid systems

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The nonstationary dynamics of rotating superfluid systems such as pulsars or a vessel with He-II is considered with allowance for the spatial dependence of the friction of Feynman–Onsager vortices. Solutions are obtained that describe the relaxation of the angular velocity of rotation of the system and its derivative, after an initial jump of these quantities. It is shown that in the superfluid cores of neutron stars the limit of strong coupling between vortices and the normal component is realized. The relaxation times of the superfluid core, of the order of a few days to 10^2 days, are large enough for the core to be responsible for the relaxation of macrojumps of pulsars.

1. INTRODUCTION

The investigation of the dynamics of nonstationary rotation of superfluid systems is of special interest in connection with the observed irregularities in the rotation of neutron stars—objects that contain a superfluid nuclear liquid in their interiors. A rotating vessel with helium-II is also a superfluid system analogous in many respects to a neutron star. The experiments of Tsakadze[1,2] in a study of nonstationary rotation of He-II, which aim to model the dynamics of a pulsar, have demonstrated convincingly that the temporal behaviors of the angular velocity of rotation of neutron stars (pulsars) and of a vessel with superfluid helium have the same nature. To a considerable extent, this is due to the circumstance that in both systems the interaction of the normal component and superfluid component is realized through Feynman–Onsager quantum vortices.

At the same time, in the details there are substantial differences between these systems, these differences being manifested in the dynamics of their nonstationary rotation. For example, the curve arising from Tsakadze's experiment on the relaxation of the rotational velocity of a vessel with He-II that had initially suffered a jump in the rotational angular velocity $\omega(t)$ is parametrized well by the equations obtained by Krasnov.[3] The latter predict an initial fast exponential relaxation of the quantity $\omega(t)$, which then goes over into slow relaxation that is quasilinear in time (see also Refs. 4 and 5).

On the other hand, the observational data on the relaxation of the angular velocities $\Omega(t)$ of rotation of pulsars after so-called macrojumps shows that the time dependence of $\Omega(t)$ has, as a rule, an appreciably more complicated form.[6] In particular, in different stages the processes of relaxation of the jumps are described by different exponents.

Thus, the model of a rotating vessel with He-II,[3] while giving a generally correct picture of the relaxation of a jump, cannot be applied directly for an adequate analysis of the post-jump relaxation of pulsars. In view of this, the need arises to construct a correct model of pulsar dynamics that takes account of the specific properties of pulsars. This is the aim of this article.

The most important difference between the indicated superfluid systems is the following. In the conditions of neutron stars the coefficient of friction between the quantum vortices and the normal component of the system depends in an essential way on the coordinates, and can change by several orders of magnitude as a function of the distance from the center of the star. In the case of the model with He-II, on the other hand, this coefficient of friction is a function that does not depend on the coordinates. We note also that in the process of relaxation of jumps of pulsars the superfluid component is involved with a moment of inertia amounting to ~ 0.1 of the total moment of inertia of the star. This makes it possible to regard the ratio of the moment of inertia of the superfluid component to that of the normal component as a small parameter of the problem. Finally, in the conditions of neutron stars, the role of the normal component becomes that of a superdense plasma, since the density of the latter exceeds by several orders of magnitude the density of the elementary excitations of the superfluid liquid. (In the *n-p-e* phase of neutron stars the normal component is the relativistic electrons, while in the *A-e-n* phase the Coulomb lattice of the nuclei also comes into play.)

Before proceeding to the equations of motion, we note the following circumstance. In neutron stars significant changes of the coefficient of viscous friction of the vortices occur over macroscopic length scales, much greater than the intervortex spacing, and this makes it possible to work with the averaged hydrodynamic equations of a superfluid liquid.

Henceforth we shall assume that the normal component of the system rotates as a solid. In the case of pulsars this condition holds as a consequence of the ultrastrong magnetic field $B \sim 10^{12}$ G of the star, which ties the normal component of the superfluid regions to the solid core of the star with characteristic relaxation times of the order of seconds. We shall also neglect the pinning of vortices to the surface of the core (or vessel), i.e., we shall assume complete slipping of the vortices along these surfaces, and also assume that during their motion the vortices retain their rectilinear form. The analysis is performed for a system with cylindrical symmetry, rotating about its symmetry axis.

2. THE EQUATIONS OF MOTION

Let the system rotate with angular velocity $\omega(t) \gg \Omega_{c1}$, where Ω_{c1} is the critical angular velocity for vortex formation. The quantization of the circulation of the averaged velocity of the superfluid component will have the form

$$\mathrm{rot}\,\mathbf{v}_s(\mathbf{r},\ t) = \boldsymbol{\nu}_0 n(\mathbf{r},\ t), \tag{1}$$

where $n(\mathbf{r}, t)$ is the local density of vortex filaments, $\nu_0 = 2\pi\hbar/m$ is the quantum of circulation, $\boldsymbol{\nu}_0/\nu_0$ is the unit vector along the axis of the vortex, and m is the mass of a

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helium atom, or, in the case of pulsars, the mass of a Cooper pair of neutrons.

We write the continuity equation for the density of vortices in the form

$$\frac{\partial n(\mathbf{r},t)}{\partial t}+\operatorname{div}[n(\mathbf{r},t)\mathbf{v}_L(\mathbf{r},t)]=0, \tag{2}$$

where $\mathbf{v}_L(\mathbf{r},t)$ is the local velocity of the motion of a vortex.

For uniform rotation of a homogeneous system the condition (1) leads to the Feynman formula $n=2\omega/\nu_0$, which relates the equilibrium density of vortices to the angular velocity ω of the rotation of the system. In this case all the components of the system rotate as a solid: $\mathbf{v}_s=\mathbf{v}_L=\mathbf{v}_n=[\boldsymbol{\omega}\mathbf{r}]$ ($\mathbf{v}_n$ is the velocity of the normal component).

Any change in the rotational velocity of the vessel or core will lead to motion of the vortices and to the establishment of a new quasiequilibrium distribution of the vortices. The equation of the dynamics of the vortices is given by the condition that the sum of all the forces acting on each element of a vortex is equal to zero. For straight vortices we have[7]

$$\rho_s(\mathbf{r})[\mathbf{v}_s-\mathbf{v}_L,\ \mathbf{v}_0]-\eta(\mathbf{r})(\mathbf{v}_L-\mathbf{v}_n)+\beta(\mathbf{r})[\mathbf{v}_L-\mathbf{v}_n,\ \mathbf{v}_0]=0, \tag{3}$$

where the first term is the Magnus force and the second and third terms comprise the frictional force between the vortex and the normal component of the liquid. Here, ρ_s is the density of the superfluid component, $\mathbf{v}_s(\mathbf{r},t)$ is the local velocity of the superfluid flux incident on the vortex, and $\eta(\mathbf{r})$ and $\beta(\mathbf{r})$ are the coefficients of longitudinal and transverse mutual friction.

Finally, the equation of motion of the normal component is written as

$$I_n\frac{d\boldsymbol{\omega}(t)}{dt}=\mathbf{K}_{int}(t)+\mathbf{K}_{ext}(t), \tag{4}$$

where I_n and $\boldsymbol{\omega}(t)$ are the moment of inertia and angular velocity of rotation of the normal component, $\mathbf{K}_{\text{int}}$ is the moment of the frictional forces between the superfluid component and normal component of the system, and $\mathbf{K}_{\text{ext}}$ is the external moment of the frictional forces: $K_{\text{ext}}=\text{const}\cdot\omega^k$, where $k\approx3$ for pulsars and $k=1$ for a vessel with He-II in the case of viscous friction of the vessel against an external medium.

Equations (1)–(4) fully determine the dynamics of the superfluid system if the functions $\eta(\mathbf{r})$, $\beta(\mathbf{r})$, and $\rho_s(\mathbf{r})$ and the time-independent parameters of the system itself are known.

In view of the axial symmetry of the problem, the local velocity $\mathbf{v}_s(\mathbf{r},t)$ of the superfluid component and the density $n(\mathbf{r},t)$ of the vortex filaments will not have an azimuthal dependence. Writing the velocity of a vortex filament in cylindrical coordinates in the form $\mathbf{v}_L=[\boldsymbol{\omega}\mathbf{r}]+v_\varphi\mathbf{e}_\varphi+v_r\mathbf{e}_r$, from Eqs. (1)–(3) in components we have

$$\frac{\partial v_s}{\partial r}+\frac{v_s}{r}=\nu_0 n, \tag{5}$$

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial r}(nv_r)=0, \tag{6}$$

$$v_r=(v_s-v_n)\sin\theta\cos\theta, \tag{7}$$

$$v_\varphi=v_r\operatorname{ctg}\theta, \tag{8}$$

where $\tan\theta=\eta/\rho_s\nu_0(1-\beta/\rho_s)$. It can be seen from the relations (7) and (8) that θ is the angle between the azimuthal direction and the direction of motion of the vortex in the rotating coordinate frame. In the limit of weak coupling $[\eta\ll\rho_s\nu_0(1-\beta/\rho_s)$, i.e., $\theta\to0]$, we have $v_r\to0$ and $v_\varphi\to v_s-v_n$, whence $v_L\to v_s$: the vortex moves together with the superfluid component. In the opposite limit $[\eta\gg\rho_s\nu_0(1-\beta/\rho_s)$, i.e., $\theta\to90°]$, we have $v_r\to0$, $v_\varphi\to0$, and $v_L\to\omega r$: the vortex is dragged by the normal component.

Integrating (5) and (6), taking it into account that $v_r=dr/dt$, we have

$$\omega_s(r,\ 0)=\omega_s(r,\ t)r^2(t)/r^2(0), \tag{9}$$

$$n(r,\ 0)=n(r,\ t)r^2(t)/r^2(0), \tag{10}$$

where $\omega_s(r,0)$, $r(0)$ and $\omega_s(r,t)$, $r(t)$ are, respectively, the angular velocity of the superfluid liquid and the distance of the vortex from the axis of rotation at times $t=0$ and $t=t$. Substituting these relations into (7), we obtain

$$\frac{\partial}{\partial t}N(r,t)=a(r)N(r,t)\left[\frac{\omega(t)}{\omega_s(r,0)}-N(r,t)\right], \tag{11}$$

where

$$N(r,\ t)\equiv n(r,\ t)/n(r,\ 0),$$

$$a(r)=2\omega_s(r,\ 0)\sin\theta\cos\theta/(1-\beta/\rho_s).$$

We note that Eq. (11) coincides with the analogous equation of Krasnov,[3] except that in the case under consideration it has a local meaning in view of the spatial dependence of the quantity a. In other words, the function $N(r,t)$ is no longer the universal function, independent of the spatial coordinates of the vortex, that was assumed in Ref. 3. From this, in particular, it follows that when the vortices move through distances of the order of the characteristic length scale L of the variation of the quantity a gradients of the density of vortex filaments arise.

In the case of pulsars, over the time scales that are characteristic for the irregularities of their rotation, the external torque $\mathbf{K}_{\text{ext}}$ is, to high accuracy, constant.

The expression for the internal torque,

$$\mathbf{K}_{int}(t)=\int[\mathbf{F}_{fr}(r)\mathbf{r}]n(r,t)dV, \tag{12}$$

where $\mathbf{F}_{\text{fr}}(r)$ is the force of viscous friction per unit length of the vortex, can be transformed by considering a cylindrical layer of thickness $\sim dr\ll L$. Taking it into account that $\mathbf{F}_{\text{fr}}\cdot\mathbf{e}_\varphi=\eta v_\varphi=\rho_s\nu_0 v_r$, we have

$$K_{int}(t)=2\pi\nu_0\int\rho_s v_r n(r,t)l_v r^2\,dr=-\omega_s(0,r)\frac{d}{dt}\int N(r,t)dI_s, \tag{13}$$

where l_v is the length of a vortex filament and $dI_s=2\pi\rho_s l_v r^3dr$ is the moment of inertia of the superfluid component of the layer under consideration. The equation of motion (4) of the normal component takes the form

$$\frac{d}{dt}\left[I_n\omega(t)+\omega_s(0,r)\int_0^R N(t,r)dI_s\right]=K_{ext}, \tag{14}$$

where R is the radius of the system.

The second term in the square brackets in Eq. (14) should coincide in its meaning with the angular momentum of the superfluid component. We shall prove this:

$$L_s=\frac{1}{2}\rho_s l_v \nu_0 \sum_j (R^2-r_j^2)$$

$$=\frac{1}{2}\rho_s l_v \nu_0 \int_0^{R_i} n(r,t)(R^2-r^2)(2\pi r\,dr),$$

where R_i is the radius of the boundary of the irrotational region ($R_i(t)<R$). Since

$$n(r,t)=\frac{2}{\nu_0}\omega_s(r,t)=2\omega_s(r,0)N(r,t),$$

we have

$$L_s=\rho_s l_v \int_0^{R_i} \omega_s(r,0)N(r,t)(R^2-r^2)(2\pi r\,dr).$$

Taking into account that $\omega_s(r,0)\equiv\omega_s(0)$ and $|R_i-R|\ll R$, and introducing the integration variable $x^2=R^2-r^2$, we have

$$L_s=\rho_s l_v \omega_s(0)\int_R^0 N(t,r)(R^2-r^2)\pi d(R^2-r^2)$$

$$=\rho_s l_v \omega_s(0)\int_0^R N(t,r)x^2(2\pi x\,dx)=\omega_s(0)\int_0^R N(t,r)dI_s.$$

This is what we had to prove.

For a given function $a(r)$ and given time-independent parameters, the system of equations (11) and (14) fully determines the dynamics of the superfluid system.

3. DYNAMICS OF THE RELAXATION OF A JUMP IN THE ANGULAR VELOCITY OF ROTATION OF THE SYSTEM

We shall consider the problem of the determination of the temporal behavior of the angular velocity of rotation of the system after an initial jump $\delta\omega$ of the angular velocity of its normal component. We shall assume that the relative magnitude of the jump is small: $\delta\omega/\omega\ll 1$. If we also assume that the relative moment of inertia of the superfluid region is small and that the external torque is constant (see Secs. 1 and 2), the problem can be solved analytically.

In Eqs. (11) and (14) we shall go over to reduced quantities:

$$\Omega(t)=\omega(t)/\omega(0),\quad dp=(I_s/I_n)(dI_s/I_s)\equiv p_0 dy,$$
$$q=\omega(0)/\omega_s(0),\quad \gamma=K_{ext}/I_n\omega(0).$$

Integrating (14), we have the system of equations

$$\frac{\partial}{\partial t}N(r,t)=-a(r)N(r,t)[N(r,t)-q\Omega(t)],\tag{15}$$

$$\Omega(t)=1+\frac{1}{q}\int dp[1-N(r,t)]-\gamma t\tag{16}$$

with the initial conditions $N(t)=\Omega(t)=1$ at $t=0$. Next, in accordance with what has been said above, considering small deviations from the initial density distribution $N(r,t)=1+\delta n(r,t)$, $\delta n(r,t)\ll 1$, we linearize Eq. (15). We obtain

$$\frac{\partial}{\partial t}\delta n(r,t)+a(r)\delta n(r,t)=-a(r)[1-q\Omega(t)],\tag{17}$$

$$\Omega(t)=1-\frac{1}{q}\int dp\,\delta n(r,t)-\gamma t.\tag{18}$$

Substituting (18) into (17), we seek the solution of the resulting equation in the form of an expansion in powers of the small parameter $p_0=I_s/I_n$:

$$\delta n=\delta n_0+\sum p_0^k\delta n_k,\qquad k=1,2,\ldots,\tag{19}$$

where the coefficients δn_k of the expansion are determined from the equations

$$\frac{\partial}{\partial t}\delta n_0+a\delta n_0=-a(1-q)-qa\gamma t,\tag{20}$$

$$\frac{\partial}{\partial t}\delta n_k+a\delta n_k=-a\int dy\,\delta n_{k-1},\qquad k=1,2,\ldots.\tag{21}$$

We shall confine ourselves to the first two terms of the expansion (19); this corresponds to keeping terms of order p_0^2 in the expression for $\Omega(t)$. Equation (20) gives

$$\delta n_0=Q(1-e^{-at})-q\gamma t,\tag{22}$$

where $Q=q(1+\gamma/a)-1$. Substituting the solution (22) into (21) ($k=1$), we find the solution δn_1:

$$\delta n_1=q\gamma t-\frac{q\gamma}{a}(1-e^{-at})-Q\int dy_1\frac{a(1-e^{-a_1t})-a_1(1-e^{-at})}{a-a_1}.\tag{23}$$

Here, $a_1(r_1)$ is a function of y_1, since the variable y_1 itself depends on r_1. Finally, the solution (19) is written in the form

$$\delta n=\delta n_0+p_0\delta n_1=q\gamma t(p_0-1)-\left(\frac{q\gamma p_0}{a}-Q\right)(1-e^{-at})$$
$$-Qp_0\int dy_1\frac{a(1-e^{-a_1t})-a_1(1-e^{-at})}{a-a_1}.\tag{24}$$

For the reduced angular velocity $\Omega(t)$ we obtain

$$\Omega(t)=1-(1-p_0+p_0^2)\gamma t-\left(\frac{Qp_0}{q}-\frac{\gamma}{a}p_0^2\right)\int dy(1-e^{-at})$$
$$+Q\frac{p_0^2}{q}\int dy\int dy_1\frac{a(1-e^{-a_1t})-a_1(1-e^{-at})}{a-a_1}.\tag{25}$$

If we introduce the notation $1+\Delta=q$, then, by definition, $\Delta=\delta\omega(0)/\omega(0)$ is the relative magnitude of the jump. In the approximation under consideration the expression (25) is rewritten in the form

$$\Omega(t,\Delta)=1-(1-p_0+p_0^2)\gamma t$$
$$-p_0\left(\Delta+\frac{\gamma}{a}-\frac{\gamma}{a}p_0\right)\int dy(1-e^{-at})$$
$$+\left(\Delta+\frac{\gamma}{a}\right)\frac{p_0^2}{q}\int dy\int dy_1\frac{a(1-e^{-a_1t})-a_1(1-e^{-a_1t})}{a-a_1}.\tag{26}$$

In the absence of the jump the dynamical behavior of the system is described by Eq. (26) if we set in it $\Delta=0$. For the difference $\Omega_\Delta(t)=\Omega(t,\Delta)-\Omega(t,0)$, where $\Omega(t,\Delta)$ and $\Omega(t,0)$ are, respectively, the angular velocities of the system in the presence of a jump at time $t=0$ and in its absence, we obtain

$$\Omega_\Delta(t)=-\Delta p_0\int dy(1-e^{-at})$$
$$+\Delta p_0^2\int dy\int dy_1\frac{a(1-e^{-a_1t})-a_1(1-e^{-at})}{a-a_1}.\tag{27}$$

Next, we go back from the reduced to the initial quantities and introduce the notation $a^{-1}=\tau$, $a_1^{-1}=\tau_1$, and $\omega=\nu/2\pi$. Finally, in the approximation linear in the jump we have

$$\nu_\Delta(t)=-p_0\delta\nu(0)\int dy(1-e^{-t/\tau})$$
$$+p_0^2\delta\nu(0)\int dy\int dy_1\frac{\tau(1-e^{-t/\tau_1})-\tau_1(1-e^{-t/\tau})}{\tau-\tau_1}. \quad (28)$$

In the analysis of post-jump relaxations of pulsars one usually uses the quantity $\dot\nu\equiv d\nu/dt$. Differentiating (28), we obtain

$$\dot\nu_\Delta(t)=-p_0\delta\nu(0)\int dy\frac{e^{-t/\tau}}{\tau}+p_0^2\delta\nu(0)\int dy\int dy_1\frac{e^{-t/\tau}+e^{-t/\tau_1}}{\tau-\tau_1}. \quad (29)$$

The expression (29) solves the problem posed, if the parameters $\delta\nu(0)$ and p_0 of the problem, and the functional dependence $\tau(y)$, are specified.

In conclusion, we emphasize once again that the solution has been obtained in the following approximation: $\delta\omega/\omega\ll1$, $p_0=I_s/I_n\ll1$, and $\gamma/a\ll1$. The latter condition, which is essential for the solution of the problem, is the requirement that the external torque be constant during the relaxation process. In fact, the ratio $\gamma/a\equiv\tau/\tau_0$ is the ratio of the characteristic relaxation times to the time of the variation of the external torque, which has the order of magnitude of the pulsar lifetime $\tau_0\approx\omega/\dot\omega$. All the above conditions are satisfied for the processes that we have considered—pulsar jumps and their relaxation. Thus, we have

$$\delta\omega/\omega\sim10^{-6}-10^{-8},\quad \tau/\tau_0\sim10^{-6},\quad p_0\sim0{,}15$$

(see, e.g., Ref. 8).

4. RELAXATION TIMES IN THE SUPERFLUID CORE OF A NEUTRON STAR

According to the solutions (28) and (29) obtained above, the relaxation of the angular velocities of pulsars and their derivatives is characterized by relaxation times τ determined by[1)]

$$\tau=\frac{1}{2\omega_s(0)}\frac{\rho_s\nu_0}{\eta}\left[1+\left(\frac{\eta}{\rho_s\nu_0}\right)^2\right]. \quad (30)$$

It is remarkable that the characteristic relaxation time takes on large values in the two limiting cases $\eta/\rho_s\nu_0\gg1$ and $\eta/\rho_s\nu_0\ll1$, i.e., in cases of extremely strong and extremely weak coupling between the vortices and the normal component of the star. The physical meaning of this dependence of τ on η is clear: The relaxation of the superfluid component occurs via a change of the density of the vortex filaments following radial compression and expansion of the vortex lattice. In both limits the radial velocity of the vortices is small, since in the former case the vortex is frozen into the normal component while in the latter case it is dragged by the superfluid liquid.

An example of a system in which, depending on the physical conditions, different limits of the coupling of the vortices and the normal component are realized is the inner core (*A-e-n* phase) of a neutron star. In the regions of the *A-e-n* phase in which pinning of vortices to the lattice of nuclei occurs, so that radial motion of the vortices occurs by quantum tunneling between pinning centers, the strong-coupling limit is realized.[9] In those regions in which pinning is not effective, the vortices interact weakly with the electron-phonon system. We then have the weak-coupling limit.[10]

We shall discuss the dynamics of the superfluid core of a neutron star in more detail and calculate the characteristic times of the relaxation processes. As is well known, the superfluid core or *n-p-e* phase of a neutron star consists principally of a superfluid neutron condensate with a small admixture (a few percent) of superconducting protons and relativistic normal electrons. When considering the dynamics of the *n-p-e* phase it is necessary to take into account that the phenomenon of the drag of the proton condensate by the neutron condensate leads to important changes in the physical properties of the system of neutron vortices: a) The generation of local magnetic fields by the currents arising from the drag of the protons by the neutrons leads to the formation of a dense lattice of proton vortices about each neutron vortex at a radius $r_1<b$ (b_1 is the radius of the neutron vortex); b) the neutron vortex acquires flux $\Phi_1=|k|\Phi_0$ (Φ_0 is the quantum of magnetic flux and $|k|$ is the drag coefficient) and, via the electromagnetic interaction, interacts rigidly with the bundle of proton vortices associated with it.[11,12]

Thus, the process of the dynamical response of the superfluid core of a neutron star involves the participation of clusters of vortex filaments, each of which is a neutron vortex with an induced flux Φ_1, "dressed" by a lattice of proton vortices. The number of proton vortices in a cluster is of the order of $\overline{B}/\Phi_0\sim|k|/4\pi\lambda_*^2\sim10^{20}$, where $\overline{B}$ is the average local magnetic field of the cluster and $\lambda_*=(1+|k|)^{1/2}\lambda$, where λ is the magnetic-field penetration depth.[12]

Next, we note that the interaction of a cluster of vortices with an incident flux of superfluid liquid reduces to the interaction of the central neutron vortex with the incident flux through the Magnus effect. No Lorentz-type force from the interaction of the circulation currents of the proton vortices with the neutron flux arises here, since the interaction of the neutron and proton condensates is exhausted by the drag effect.

Thus, when one considers the dynamics of the *n-p-e* phase of a neutron star, it is necessary in the first term of Eq. (3) to take ρ_s to be the density of the superfluid neutron component, and the quantum of circulation to be $\nu_0=\pi\hbar/m_n$ (m_n is the neutron mass).

We shall determine the magnitude of the viscous friction η. The most effective mechanism of interaction of the normal component of the *n-p-e* phase with the vortex clusters is the process of scattering of relativistic electrons by the nonuniform magnetic field of a cluster. The relaxation time of this process is determined in Ref. 13:

$$\tau_{ej}^{-1}=\frac{(\pi/3)^{1/2}}{32}\frac{\hbar}{m_p}k_e^2\left(\frac{e^2}{m_pc^2}k_e\right)^{1/2}\frac{|k|}{(1+|k|)^{1/2}}\left(\frac{\xi}{\lambda_*}\right)^{2/3|k|}, \quad (31)$$

where k_e is the Fermi wave vector of the electrons, m_p is the proton mass, λ_*/ξ is the Ginzburg–Landau parameter of the undragged proton condensate, and $|k|=1-m_p^*/m_p$.

By writing the force acting on unit volume of the superfluid liquid in the relaxation-time approximation:

$$\mathbf{f}=2\tau_{ej}^{-1}\int n(\mathbf{p},\mathbf{v}_L)\mathbf{p}\frac{d\mathbf{p}}{(2\pi\hbar)^3}, \quad (32)$$

TABLE I. Dependence of the microscopic parameters of the problem and the dynamical-relaxation time on the density of the superfluid core of a neutron star.

ρ, 10^{14} g·cm^{-3}	k_e, fm^{-1}	m_p^*/m_p	τ_{ej}, 10^{-14} sec	η, 10^{21} g·cm^{-1}·sec^{-1}	τ, days
2,31	0,43	0,63	8,11	0,13	10,9
2,85	0,50	0,61	2,13	0,9	61,3
3,33	0,55	0,59	0,85	3,33	194
3,88	0,61	0,57	0,32	13,6	676
4,52	0,63	0,55	0,13	48,8	$2,1\cdot 10^3$

Column 1—density of the core layer; column 2—Fermi wave vector of the electrons; column 3—ratio of the effective proton mass to the "bare" proton mass; column 4—electron-relaxation time at clusters of proton vortices; column 5—coefficient of viscous friction of the vortices; column 6—dynamical-relaxation times.

where $n(\mathbf{p},\mathbf{v}_L)$ is the distribution function of the relativistic electrons, for the viscosity coefficient we have

$$\eta = \frac{\hbar k_e}{c\tau_{ej}} \frac{n_e}{n_v}. \tag{33}$$

Here, n_e and n_v are the electron density and neutron-vortex density, respectively.

In Table I we give the dependences of the microscopic parameters of the problem,[14] the quantity $\tau_{\rm eff}$, the viscosity coefficient η, and the relaxation time τ for the pulsar "Vela" ($\nu = 11.2$ Hz) on the density ρ of the n-p-e phase of the neutron star. As we see, the relaxation times have values that agree in order of magnitude with the observed relaxation times of pulsar jumps. In the case under consideration the limit of strong coupling of the vortices and normal component is realized.

To determine the dynamics of the relaxation of the n-p-e phase of a neutron star it is necessary next to choose a model of the star, i.e., to specify its central density and the equation of state of superdense nuclear matter. Then the relative moment of inertia p_0 of the superfluid region and the profile of the distribution of the moment of inertia of the n-p-e phase are uniquely determined as a function of the density of the star, i.e., we have the dependence of y on ρ. We recall that in (28) and (29) we have $0\leqslant y\leqslant 1$ and the reference surface is the outer boundary of the superfluid core.

As a result, we have the function $\tau(y)$ in parametric form and, according to (28) and (29), the problem of the dynamics of the relaxation of the n-p-e phase is thereby solved if the observational parameter $\delta\nu(0)$ is given.

[1)] Here and below we neglect the longitudinal friction, which is not important in the case of neutron stars, and set $\beta(r) = 0$.

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