Radiation by a system of fast charged particles in a dispersive scattering medium

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We study the emission by a system of classically fast non-interacting charged particles which undergo multiple elastic collisions in a scattering medium with temporal dispersion. We find the spectral distribution of the radiation energy of such particles. The obtained spectrum depends significantly both on the parameters of the initial particle beam and on the characteristics of the scattering medium. We study in detail the emission of a single-direction beam of charged particles in a Maxwellian plasma and also under conditions when the Cherenkov effect occurs.

1. INTRODUCTION

Ter-Mikaelyan[1] was the first to consider the effect of polarization on the spectral distribution of the bremsstrahlung (BS) energy of an ultrarelativistic particle in a medium; the suppression of the Bethe–Heitler BS intensity in a plasma medium was indicated there in the case of rather high Langmuir frequencies. In Refs. 2 and 3, quantitatively exact formulas were obtained by a method proposed by Migdal[4] for calculating the BS spectrum in a medium, and a detailed study was made of the BS energy distribution of a classically fast particle which undergoes multiple elastic scattering in a medium with permittivity $\varepsilon(\omega)$ such that $\varepsilon^{1/2}(\omega) < v_0^{-1}$ (ω is the frequency of the radiation, v_0 is the particle velocity, and we put $\hbar = c = 1$). The effect of multiple elastic collisions with the atoms of a material on Cherenkov radiation $[\varepsilon^{1/2}(\omega) > v_0^{-1}]$ was considered in Ref. 5.

However, Refs. 1 to 5 dealt everywhere with emission by an individual particle, whereas in many cases when fast carriers pass through scattering media (with permittivities which are, in general, different from unity) the aggregate of the particles is the radiation source. From a general physics point of view the interest in the problem of the emission by a system of charged particles in a medium with $\varepsilon(\omega) \neq 1$ is connected with the variety of mechanisms (collisional, interference, and in the $\varepsilon^{1/2}(\omega) > v_0^{-1}$ case "Cherenkov" formation of a radiation spectrum in a medium) with coherence times which significantly depend on the parameters characterizing the scattering of particles in the medium and also on the dispersive properties of the medium.

In the present paper we study the radiation by a system of classically fast noninteracting charged particles which undergo multiple elastic scattering by the randomly distributed atoms of a medium with permittivity $\varepsilon(\omega)$. We find for an arbitrary dispersion law $k(\omega) = \varepsilon^{1/2}(\omega)\omega$ [$\mathbf{k}(\omega)$ is the wavevector] the spectral energy distribution of the emission by such particles. The spectrum obtained depends significantly both on the parameters of the initial particle beam and also on the characteristics of the scattering medium. We show that in the range of extremely low frequencies for any (but finite as $\omega \to 0$) $\varepsilon(\omega)$ the mechanism for the formation of radiation is essentially the bremsstrahlung mechanism and the particles emit BS quanta under conditions of complete coherence.

We study in detail the radiation from a single-direction δ—pulse beam of charged particles in a plasma and in coherently radiating media. In the case of the motion of particles in an electrically neutral Maxwellian plasma[6] the presence of polarization in the medium leads under well defined conditions to a radical change in the spectral distribution of the BS of such particles as compared to the situation with $\varepsilon(\omega) = 1$.[7] If, however, $\varepsilon^{1/2}(\omega) > v_0^{-1}$, there is considerable braking in the very long-wavelength range of the radiation spectrum, as in the case where $\varepsilon^{1/2}(\omega) < v_0^{-1}$, whereas in the case of rather high frequencies the BS turns out to be strongly suppressed and the mechanism for the formation of radiation is essentially the interaction of a system of charged particles with a coherently radiating medium (Cherenkov effect).

2. STATEMENT OF THE PROBLEM. SPECTRAL DISTRIBUTION OF THE RADIATION BY A SYSTEM OF CHARGED PARTICLES IN A SCATTERING MEDIUM WITH A PERMITTIVITY $\varepsilon(\omega)$

We consider a system of noninteracting classically fast ($E \gg \omega$), ultrarelativistic ($E \gg m$) charged particles (E, m, and e are the energy, mass, and charge of each particle, ω is the frequency of the radiation, and we use $\hbar = c = 1$) which undergo multiple elastic scattering in a uniform semi-infinite amorphous medium with a permittivity $\varepsilon(\omega)$. Initially, at time $t = 0$, the particles are positioned at points with coordinates $\mathbf{r}_{01}, \mathbf{r}_{02},\ldots,\mathbf{r}_{0N}$ and have velocities $\mathbf{v}_{01}, \mathbf{v}_{02},\ldots,\mathbf{v}_{0N}$ which have the absolute magnitude $v_0 = [1 - (m/E)^2]^{1/2}$ and which are directed at angles $|\Delta_\mu| \ll 1$, $\mu = 1,\ldots,N$ to the $\mathbf{e}_z$ vector (the vector along the outward normal to the boundary of the medium). Let the characteristic longitudinal size of the beam

$$l_B = \max_{\mu,\nu} |\mathbf{r}_{0\nu} - \mathbf{r}_{0\mu}|$$

be such that $l_B v_0^{-1}$ is small compared to the time T when the particles move in the material. The spectral density of the radiation energy of these particles is then given by the formula[7]

$$\frac{d\mathscr{E}_\omega}{d\omega} = \frac{e^2\omega^2}{2\pi^2}\operatorname{Re}\Big\{\varepsilon^{1/2}(\omega)\sum_{\mu,\nu=1}^{N}\int d\Omega_{\mathbf{n}}\int_0^T dt\int_0^{T-t} d\tau$$

$$\times\exp[-i\omega\tau + i\mathbf{k}(\mathbf{r}_{0\mu}-\mathbf{r}_{0\nu}) + i\mathbf{k}(\mathbf{r}_\mu[t+\tau)$$

$$-\mathbf{r}_\nu(t)][\mathbf{n}\mathbf{v}_\mu(t+\tau)][\mathbf{n}\mathbf{v}_\nu(t)]\Big\}, \qquad (1)$$

where N is the number of particles, $\mathbf{k}(\omega)$ is the wavenumber

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of the radiation field, $d\Omega_n$ is an element of solid angle in the direction of $\mathbf{n} = \mathbf{k}/|\mathbf{k}|$, $\mathbf{r}_\mu(t) + \mathbf{r}_{0\mu}$ is the radius vector of the μth particle, $\mathbf{v}_\mu(t)$ is its velocity, τ is the time for the formation of the radiation (coherence time[4]), t is the instant it is emitted, and we have $\tau \ll t \sim T$.

To evaluate the absorbed spectral density of the radiation energy of the particles in the material, $dE_\omega/d\omega$, we must average expression (1) over all possible trajectories of the carriers in the scattering medium.[4] It is shown in Ref. 7 that in the case of multiple elastic collisions of ultrarelativistic charged particles with the randomly distributed atoms of the material the averaging procedure reduces to finding the Fourier component of the equal-time distribution function[1)] $F_\mathbf{k}(\mathbf{v}_\mu(\boldsymbol{\eta}),\mathbf{v}_\nu(\boldsymbol{\zeta}),t,\tau)$:

$$\langle [\mathbf{n}\mathbf{v}_\mu(t+\tau)][\mathbf{n}\mathbf{v}_\nu t]\exp\{i\mathbf{k}[\mathbf{r}_\mu(t+\tau)-\mathbf{r}_\nu(t)]\}\rangle$$
$$=v_0^2\int d^2\eta\int d^2\zeta\{\theta_\mathbf{k}^2+\eta\zeta-\eta\theta_\mathbf{k}-\zeta\theta_\mathbf{k}\}F_\mathbf{k}(\mathbf{v}_\mu(\eta),\mathbf{v}_\nu(\zeta),t,\tau), \tag{2}$$

where the angle brackets indicate averaging over the positions of the scatterers in the medium and the quantities $\boldsymbol{\eta}$, $\boldsymbol{\zeta}$, $\boldsymbol{\theta}_\mathbf{k}$ are angular vectors which are connected with the velocities $\mathbf{v}_\mu(\boldsymbol{\eta})$ and $\mathbf{v}_\nu(\boldsymbol{\zeta})$ and the wavevector $\mathbf{k}$ through the standard relations of the theory of small-angle diffusive scattering of fast particles in a material.[9] Integrating in Eqs. (1) and (2) over all possible $\boldsymbol{\eta}$, $\boldsymbol{\zeta}$, and $\boldsymbol{\theta}_\mathbf{k}$ [the explicit form of the function $F_\mathbf{k}(\mathbf{v}_\mu(\boldsymbol{\eta}),\mathbf{v}_\nu(\boldsymbol{\zeta}),t,\tau)$ is given in the Appendix] we find the spectral distribution of the energy of the radiation of charged particles in a material with dielectric permittivity $\varepsilon(\omega)$:

$$\frac{dE_\omega}{d\omega}=N\left(\frac{dE\omega}{d\omega}\right)_1+\frac{e^2\omega^2v_0^2}{2\pi q}\operatorname{Re}\Big\{\varepsilon^{1/2}(\omega)\sum_{\mu\neq\nu=1}^{N}\int_0^T\frac{dt}{t}$$
$$\times\int\frac{ds}{a}\frac{\exp[-(1+i)s\beta\chi]}{A^2(s)\operatorname{ch}^2 s}\Big\{1-\frac{k^2}{4A(s)}\Big[((\mathbf{d}_{\mu\nu})_\perp+v_0t\mathbf{b}_{\mu\nu})^2$$
$$+2i(\mathbf{d}_{\mu\nu})_z((\mathbf{d}_{\mu\nu})_\perp+v_0t\mathbf{b}_{\mu\nu})\frac{\partial}{\partial\mathbf{B}_0}-(\mathbf{d}_{\mu\nu})_z^2\left(\frac{\partial}{\partial A_0}-\frac{\partial^2}{\partial A_0^2}\right)\Big]$$
$$+\frac{ik}{2}\mathbf{b}_{\mu\nu}\Big[(\mathbf{d}_{\mu\nu})_\perp+v_0t\mathbf{b}_{\mu\nu}-i(\mathbf{d}_{\mu\nu})_z\frac{\partial}{\partial\mathbf{B}_0}\Big]\Big\}$$
$$\times A_0^{-1}\exp\Big[-\frac{B_0^2}{4A_0}+C_0\Big]\Big\}. \tag{3}$$

We use in Eq. (3) the following notation:

$$a=(ikqv_0/2)^{1/2},\quad \chi=[\omega/qv_0\varepsilon^{1/2}(\omega)]^{1/2},$$
$$\mathbf{d}_{\mu\nu}\equiv\mathbf{r}_{0\mu}-\mathbf{r}_{0\nu}=(\mathbf{d}_{\mu\nu})_\perp+\mathbf{e}_z(\mathbf{d}_{\mu\nu})_z,$$
$$\mathbf{b}_{\mu\nu}=\boldsymbol{\Delta}_\nu-\boldsymbol{\Delta}_\mu,\quad \beta=1-v_0\varepsilon^{1/2}(\omega),$$
$$A=\frac{ik}{2}(\mathbf{d}_{\mu\nu})_z+\frac{a}{q}\operatorname{th}s,\quad A_0=(qt)^{-1}+\frac{iak\operatorname{th}s}{2A(s)q}(\mathbf{d}_{\mu\nu})_z, \tag{4}$$
$$\mathbf{B}_0=-\frac{ik^2(\mathbf{d}_{\mu\nu})_z}{2A(s)}[(\mathbf{d}_{\mu\nu})_\perp+v_0t\mathbf{b}_{\mu\nu}]$$
$$+k\Big[(\mathbf{d}_{\mu\nu})_\perp+\frac{v_0t}{2}\mathbf{b}_{\mu\nu}\Big]-\frac{2i\boldsymbol{\Delta}_\nu}{qt},$$
$$C_0=-\frac{k^2}{4A(s)}[(\mathbf{d}_{\mu\nu})_\perp+v_0t\mathbf{b}_{\mu\nu}]^2-\frac{qk^2v_0^2t^3\mathbf{b}_{\mu\nu}^2}{48}$$
$$-\frac{\boldsymbol{\Delta}_\nu^2}{qt}-\frac{ikv_0t}{2}\mathbf{b}_{\mu\nu}\boldsymbol{\Delta}_\mu+i(\mathbf{d}_{\mu\nu})_zk.$$

The integration in the second term in Eq. (3) over the variable $s=a\tau$ is performed in the complex plane along the bisectrix of the right angle in the first quadrant, q is the mean square multiple scattering angle per unit path length,[9] and the quantity $(dE_\omega/d\omega)_1$ is the spectral distribution of the energy density of the radiation by an individual particle:[2,3,5]

$$\left(\frac{dE_\omega}{d\omega}\right)_1=\frac{2e^2\omega T\beta}{\pi\varepsilon^{1/2}}\Big\{\int_0^\infty\frac{ds\exp\{-|\beta|\chi s\}\sin(|\beta|\chi s)}{\operatorname{th}s}-\frac{\pi}{4}\Big\}$$
$$+\theta(-\beta)\Big\{-\frac{2e^2\omega T\beta}{\varepsilon^{1/2}}-\frac{4e^2\omega T}{\varepsilon^{1/2}}$$
$$\times\operatorname{Re}[\exp((1-i)\pi|\beta|\chi)-1]^{-1}\Big\}, \tag{5}$$

where $\theta(s)$ is Heaviside's theta-function.[10] We note that in the $\beta<0$ case Eqs. (3) to (5) are, in general, valid for $|\beta|\ll 1$.

If the permittivity $\varepsilon(\omega)$ is equal to unity we get from Eqs. (3) and (4), by displacing the integration contour along the direction of the real semiaxis, the spectral BS energy distribution of a system of ultrarelativistic particles in a scattering medium without dispersion.[7]

In the very long wavelength range of the spectrum, $\omega\ll q|\beta|^{-2}\varepsilon^{1/2}(\omega)$, retaining the main terms in the integrand in Eq. (3) as $\omega\to 0$, we have

$$\frac{dE_\omega}{d\omega}=(Ne)^2\frac{(q\omega)^{1/2}T}{\pi\varepsilon^{1/4}(\omega)}\Big\{1+O\Big[\Big(\frac{\omega}{q|\beta|^{-2}\varepsilon^{1/2}(\omega)}\Big)^{1/2}\Big]\Big\}. \tag{6}$$

From this last equation it follows that in the range of very low frequencies a system of ultrarelativistic charged particles radiates under total coherence conditions ($dE_\omega/d\omega\propto N^2$). The formation mechanism for the radiation spectrum is for any (but finite as $\omega\to 0$) $\varepsilon(\omega)$ essentially a brems mechanism and the Cherenkov emission (which occurs for $\beta<0$) turns out to be strongly suppressed. This is connected with the fact that as $\omega\to 0$ the BS energy decreases proportional to $\omega^{1/2}$ whereas the quantity $dE_\omega/d\omega$ tends for the Cherenkov radiation to zero following the law $dE_\omega/d\omega\propto\omega$.

In the very high frequency limit,

$$\omega\gg\max\{q\varepsilon^{1/2}(\omega)|\beta|^{-2};\ (\mathbf{d}_{\mu\nu})_\perp^2/[(\mathbf{b}_{\mu\nu})^2(\mathbf{d}_{\mu\nu})_z\varepsilon^{1/2}(\omega)qT^3]\},$$

we find, expanding the hyperbolic functions in small $s\ll 1$ in the integrand in Eq. (3) and retaining in the factor of the exponential and in its exponent the main terms for $\omega\to\infty$,

$$\frac{dE_\omega}{d\omega}=N\Big\{\frac{e^2qT\theta(\beta)}{3\pi\varepsilon^{1/2}(\omega)\beta}-\theta(-\beta)\frac{2e^2\omega T\beta}{\varepsilon^{1/2}(\omega)}\Big\}$$
$$-\frac{4e^2}{q\varepsilon^{1/2}(\omega)(2\pi)^{1/2}}\operatorname{Re}\Big\{\sum_{\mu\neq\nu=1}^{N}\int_{\tau_{max}}^{T}\frac{idt}{t(\mathbf{d}_{\mu\nu})_z^2}$$
$$\times\Big[\frac{[(\mathbf{d}_{\mu\nu})_\perp+v_0t\mathbf{b}_{\mu\nu}]^2}{(\mathbf{d}_{\mu\nu})_z}+\mathbf{b}_{\mu\nu}((\mathbf{d}_{\mu\nu})_\perp+v_0t\mathbf{b}_{\mu\nu})\Big]$$
$$\times z_0^{-1/2}\exp[iz_0+ik(\mathbf{d}_{\mu\nu})_z-qk^2v_0^2t^3\mathbf{b}_{\mu\nu}^2/48]\Big\}, \tag{7}$$

where $z_0=\omega t|\mathbf{b}_{\mu\nu}|[-v_0\varepsilon^{1/2}(\omega)\beta]^{1/2}$, and $\tau_{max}=q^{-1}\varepsilon^{-1/2}|\beta|$ is the time for the formation of a radiation quantum, a time maximal for the given frequency range.

It follows from Eq. (7) that in the very high frequency range the interference terms decrease as ω increases and $dE_\omega/d\omega$ becomes equal to the energy of the radiation of N independent particles. The BS is then strongly suppressed for $\beta<0$ and $(dE_\omega/d\omega)N^{-1}$ is the Cherenkov radiation energy.[5] If, however, we have $\beta>0$ the quantity $(dE_\omega/d\omega)N^{-1}$ is equal to the Bethe–Heitler radiation energy.[4]

In the case when the characteristic longitudinal (in the direction of the particle motion) size of the beam is such the $\min\{(\mathbf{d}_{\mu\nu})_z\}\gg\max[k^{-1},\tau]$ (but, of course, $\max\{(\mathbf{d}_{\mu\nu})_z\}\sim l_B\ll Tv_0$) the terms in the sum on the right-hand side of (3) are periodic functions of the frequency. The inequality

$$\left|\left(\frac{dE_\omega}{d\omega}\right)_{\text{extr}}-N\left(\frac{dE_\omega}{d\omega}\right)_1\right|\left[N\left(\frac{dE_\omega}{d\omega}\right)_1\right]^{-1}$$

$$\sim\frac{\ln(T\tau_{max}^{-1})}{qTk(\mathbf{d}_{\mu\nu})_z}(N-1)\leqslant(N-1)\frac{\tau_{max}}{T}\ln(T\tau_{max}^{-1})\ll1, \tag{8}$$

is then satisfied for any ω. For a sufficiently extended beam of emitting particles the interference effects thus turn out to be of little importance and the spectral distribution of the radiation energy $dE_\omega/d\omega$ as a function of the frequency ω basically repeats the behavior of the function $(dE_\omega/d\omega)_1$ in its ω-dependence, which occurs for an individual particle.[2,3,5]

In the opposite limiting case of small $(\mathbf{d}_{\mu\nu})_z$ $(\max\{(\mathbf{d}_{\mu\nu})_z\}\ll\tau|\beta|^{-2})$, we have, putting all $(\mathbf{d}_{\mu\nu})_z=0$ in Eqs. (3) and (4), after some simple transformations

$$\frac{dE_\omega}{d\omega}=\frac{2e^2\beta\omega}{\pi}\operatorname{Re}\left\{\frac{i}{\varepsilon^{1/2}(\omega)}\sum_{\mu,\nu=1}^{N}\int_0^T dt\right.$$

$$\times\exp\left[\frac{ikv_0t\mathbf{b}_{\mu\nu}^2}{2}-\frac{qk^2v_0^2t^3\mathbf{b}_{\mu\nu}^2}{48}-\frac{qtk^2}{4}\left[(\mathbf{d}_{\mu\nu})_\perp+\frac{v_0t}{2}\mathbf{b}_{\mu\nu}\right]^2\right.$$

$$\left.+ik(\mathbf{d}_{\mu\nu})_\perp\Delta_\nu\right]\left\{\int_C\frac{ds}{\operatorname{th}s}\exp[-(1+i)s\beta\chi-\gamma\operatorname{cth}s]\right.$$

$$-\frac{a\varepsilon^{1/2}(\omega)}{2\beta\omega}\int_C\frac{ds}{\operatorname{sh}^2 s}\mathbf{b}_{\mu\nu}[(\mathbf{d}_{\mu\nu})_\perp+v_0t\mathbf{b}_{\mu\nu}]$$

$$\left.\left.\times\exp[-(1+i)s\beta\chi-\gamma\operatorname{cth}s\right\}\right\}, \tag{9}$$

where $\gamma=qk^2[(\mathbf{d}_{\mu\nu})_\perp+v_0t\mathbf{b}_{\mu\nu}]^2/4a$ and $a=(qkv_0/2)^{1/2}$; the integration in Eq. (9) is in the complex plane along the bisectrix of the right angle of the first quadrant.

For further study of the radiation spectrum of a system of noninteracting charged particles in a scattering medium with temporal dispersion we must specify either the actual form of the permittivity or the beam geometry. Below we study in detail the spectral distribution of the radiation energy of a single-direction δ-pulse beam ($(\mathbf{d}_{\mu\nu})_z=0$, $|\Delta_\mu|=0$, $\mu=1,2,...,N$) of ultrarelativistic particles. This situation is specially singled out because for $(\mathbf{d}_{\mu\nu})_z=0$, $|\Delta_\mu|=0$ there is in the initial beam neither a spatial distribution of particles in the propagation direction of the radiation, nor a "spread" in velocity for the particles. Interference effects which occur in this case are thus essentially dynamic, i.e., connected with the process of the passage of the particles through the scattering medium.

3. SPECTRAL DISTRIBUTION OF THE RADIATION ENERGY OF A SINGLE-DIRECTION δ-PULSE BEAM OF ULTRARELATIVISTIC PARTICLES

Since the distances between the particles in the initial beam are, as a rule, random quantities, to find the observed spectral density of the radiation energy it is necessary to average $dE_\omega/d\omega$ over all possible values of the vector $(\mathbf{d}_{\mu\nu})_\perp$. Putting in Eq. (9) all Δ_μ and $(\mathbf{d}_{\mu\nu})_z$ equal to zero and averaging the obtained expression over $(\mathbf{d}_{\mu\nu})_\perp$ along δ-pulse beam cross section (which we assume to be approximately a circle of diameter D) we obtain

$$\frac{dE_\omega}{d\omega}=N\left(\frac{dE_\omega}{d\omega}\right)_1-32N(N-1)\frac{e^2\omega\beta}{\pi D^2}$$

$$\times\operatorname{Im}\left\{\int_0^T\frac{dt}{\varepsilon^{1/2}(\omega)}\int_C\frac{ds}{\operatorname{th}s}\frac{\exp[-(1+i)s\chi\beta]}{(qtk^2+qk^2/a\operatorname{th}s)}\right.$$

$$\left.\times\left(1-\exp\left[-\left(qtk^2+\frac{qk^2}{a\operatorname{th}s}\right)\frac{D^2}{16}\right]\right)\right\}, \tag{10}$$

where the integration contour C is the bisectrix of the right angle of the first quadrant in the complex plane.

In the far long-wavelength range of the spectrum, $\omega\ll q\varepsilon^{1/2}(\omega)|\beta|^{-2}$, we have, putting $\tanh s=1$ in the exponent in Eq. (10) and changing from integration over the contour to integration along the real axis,

$$\frac{dE_\omega}{d\omega}=\left\{\frac{Ne^2(q\omega)^{1/2}T}{\pi\varepsilon^{1/4}(\omega)}+\frac{16e^2N(N-1)}{\pi\varepsilon^{5/4}\omega^{1/2}q^{1/2}D^2}\left[\ln\frac{T}{\tau_q}\right.\right.$$

$$\left.\left.-E_1\left(\frac{q\varepsilon(\omega)\omega^2\tau_qD^2}{16}\right)+E_1\left(\frac{q\varepsilon(\omega)\omega^2TD^2}{16}\right)\right]\right\}$$

$$\times\left\{1+O\left[\left(\frac{\omega|\beta|^2}{q\varepsilon^{1/2}(\omega)}\right)^{1/2}\right]\right\}, \tag{11}$$

where $E_\alpha(x)$ is the exponential integral,[10] $\tau_q=(q\omega)^{-1/2}$ is a characteristic time for the formation of radiation in the material in the low frequency range, $\omega\ll q\varepsilon^{1/2}(\omega)|\beta|^{-2}$.

From Eq. (11) we find in the various frequency ranges

$$\frac{dE_\omega}{d\omega}=\begin{cases}\dfrac{N^2e^2(q\omega)^{1/2}T}{\pi\varepsilon^{1/4}(\omega)}-\dfrac{N(N-1)e^2q^{3/2}}{64\pi}\omega^{5/2}\varepsilon^{3/4}(\omega)T^2D^2,\\ \omega\ll\min\{q\varepsilon^{1/2}|\beta|^{-2};\ (q\varepsilon TD^2)^{-1/2}\}\\ N\dfrac{e^2(q\omega)^{1/2}T}{\pi\varepsilon^{1/4}(\omega)}+\dfrac{16N(N-1)e^2}{\varepsilon^{5/4}(\omega)\omega^{3/2}q^{1/2}\pi D^2}\ln\dfrac{T}{\tau_q},\\ (qD^4)^{-1/3}\ll\omega\ll q\varepsilon^{1/2}|\beta|^{-2}.\end{cases} \tag{12}$$

In the long-wavelength region of the spectrum $dE_\omega/d\omega$ is thus a growing function of the frequency and with increasing ω the interference term in Eq. (11) increases as a function of the time T more slowly than the term corresponding to the "intrinsic" radiation of the particles. The reason is that as the particles pass through the material the coherence of the emission of quanta by the radiating particles is violated due to the scattering in the medium. We note that in the rather low frequency range, $\omega\ll q\varepsilon^{1/2}(\omega)|\beta|^{-2}$, the mechanism for the formation of radiation by a single-direction δ-pulse beam is essentially a brems mechanism, as in the $(\mathbf{d}_{\mu\nu})_z \neq 0$ case.

In the short-wavelength region of the spectrum, $\omega \gg q\varepsilon^{1/2}(\omega)|\beta|^{-2}$, we have, expanding the factor of the exponent and its index in Eq. (10) in terms of small $s \ll 1$ and restricting ourselves to the main terms in $(q\varepsilon^{1/2}(\omega)|\beta|^{-2})/\omega \ll 1$,

$$\frac{dE_\omega}{d\omega} = \Big\{ N\frac{e^2qT}{3\pi\beta} + N(N-1)\frac{4e^2}{3\pi\varepsilon^{1/2}(\omega)}\Big\{\frac{4\ln(T/\tau_{max})}{\varepsilon^{1/2}(\omega)\beta D^2\omega^2} - K_2\Big[\frac{D\omega}{2}(2\beta\varepsilon^{1/2}(\omega))^{1/2}\Big]\Big[E_1\Big(\frac{q\varepsilon(\omega)\omega^2D^2\tau_{max}}{16}\Big) - E_1\Big(\frac{q\varepsilon(\omega)\omega^2D^2T}{16}\Big)\Big]\Big\}\Big\} \times \Big\{1+O\Big[\Big(\frac{q\varepsilon^{1/2}(\omega)}{\omega\beta^2}\Big)^{1/2}\Big]\Big\}, \quad \beta>0, \tag{13}$$

$$\frac{dE_\omega}{d\omega} = \Big\{ N\frac{e^2\omega T}{\varepsilon^{1/2}(\omega)}\Big[1-\frac{1}{\varepsilon(\omega)v_0^2}\Big] - \frac{32e^2\beta N(N-1)}{qD^2\omega\varepsilon^{1/2}(\omega)} \times \Big\{\ln\frac{T}{\tau_{max}} - J_0\Big[\frac{\varepsilon^{1/2}\omega D}{2}\Big(1-\frac{1}{\varepsilon(\omega)v_0^2}\Big)^{1/2}\Big] \times \Big[E_1\Big(\frac{q\varepsilon(\omega)\omega^2D^2\tau_{max}}{16}\Big) - E_1\Big(\frac{q\varepsilon(\omega)\omega^2D^2T}{16}\Big)\Big]\Big\} - \frac{Ne^2qT}{3\pi|\beta|}\Big\}\Big\{1+O\Big[\Big(\frac{q\varepsilon^{1/2}(\omega)}{\omega\beta^2}\Big)^{1/2}\Big]\Big\}, \quad \beta<0, \tag{14}$$

where $J_\alpha(x)$ and $K_\alpha(x)$ are Bessel and modified Bessel functions,[11] $\tau_{\max} = (\omega^{-1}|\beta|^{-1})_{\max}$ is the maximum of the characteristic times for the formation of radiation in the $\omega \gg q|\beta|^{-2}\varepsilon^{1/2}(\omega)$ frequency range considered, and $\tau_{\max} \ll T$. Using the asymptotic expansions of the functions $J_\alpha(x)$, $K_\alpha(x)$, and $E_\alpha(x)$ for $x \gg 1$ and $x \ll 1$,[10,11] we find from Eqs. (13) and (14) the following relations:

1) in the case when $\beta = 1 - v_0\varepsilon^{1/2}(\omega) > 0$ we have

$$\frac{dE_\omega}{d\omega} = \begin{cases} \dfrac{Ne^2qT}{3\pi\beta} + \dfrac{16e^2N(N-1)}{3\pi D^2\omega^2\beta\varepsilon(\omega)}\ln\dfrac{T}{\tau_{max}}, \\ \omega \gg \max\{q\beta^{-2}\varepsilon^{1/2};\ (qTD^2)^{-1/2}\} \\ \dfrac{N^2e^2qT}{3\pi\beta} - \dfrac{N(N-1)(eq\omega DT)^2\varepsilon(\omega)}{192\pi\beta} \\ q\varepsilon^{1/2}\beta^{-2} \ll \omega \ll (qTD^2)^{-1/2} \end{cases}, \tag{15}$$

2) if, however, $\beta < 0$ we have

$$\frac{dE_\omega}{d\omega} = \begin{cases} \dfrac{Ne^2\omega T}{\varepsilon^{1/2}}\Big[1-\dfrac{1}{v_0^2\varepsilon(\omega)}\Big] - \dfrac{32N(N-1)e^2\beta}{q\omega D^2\varepsilon^{1/2}(\omega)}\ln\dfrac{T}{\tau_{max}}, \\ \omega \gg \max\{q|\beta|^{-2}\varepsilon^{1/2}(\omega);\ (qTD^2)^{-1/2}\} \\ \dfrac{N^2e^2\omega T}{\varepsilon^{1/2}}\Big[1-\dfrac{1}{v_0^2\varepsilon(\omega)}\Big] \\ + \dfrac{N(N-1)q\beta(eDT)^2\varepsilon^{1/2}\omega^3}{64\pi} - \dfrac{Ne^2qT}{3\pi|\beta|}, \\ q|\beta|^{-2}\varepsilon^{1/2}(\omega) \ll \omega \ll \min\Big\{(qTD^2)^{-1/2}; \\ \Big[D\Big(1-\dfrac{1}{v_0^2\varepsilon}\Big)^{-1/2}\Big]^{-1}\Big\}. \end{cases} \tag{16}$$

It follows in particular from (15) and (16) that in the case of very high frequencies, $\omega \gg \max\{q|\beta|^{-2}\varepsilon^{1/2}(\omega); (qTD^2)^{-1/2}\}$, the quantity $N^{-1}(dE_\omega/d\omega)$ tends for $\beta > 0$ to the Bethe-Heitler BS energy for a system of charged particles in a scattering medium with permittivity $\varepsilon(\omega)$. If, on the other hand, $\beta < 0$ the BS is suppressed and the quantity $(dE_\omega/d\omega)N^{-1}$ turns out to be equal to the energy of the Cherenkov radiation. We note that as $\omega \to \infty$ the interference term decreases for $\beta > 0$ faster $[(dE_\omega/d\omega)_{\text{int}} \propto \omega^{-2}]$ than in the $\beta < 0$ case when we have $(dE_\omega/d\omega)_{\text{int}} \propto \omega^{-1}$.

4. SPECTRAL DISTRIBUTION OF THE BS ENERGY OF A SINGLE-DIRECTION BEAM IN A PLASMA MEDIUM

We consider the radiation of a single-direction δ-pulse beam of ultrarelativistic charged particles moving in a scattering plasma medium with a permittivity equal to (Maxwellian plasma)

$$\varepsilon(\omega) = 1 - \omega_0^2/\omega^2, \quad \omega \gg \omega_0, \tag{17}$$

where $\omega_0^2 = 4\pi n_e q_e^2/m_e$ is the Langmuir (plasma) frequency and q_e, m_e, and n_e are the charge, mass, and density of the electrons in the plasma.[6] Since the permittivity $\varepsilon(\omega)$ given by Eq. (17) is such that $v_0\varepsilon^{1/2}(\omega) < 1$, the mechanism for the formation of the radiation spectrum is a brems mechanism and the spectral distribution of the BS energy is given by Eqs. (10), (11), and (13).

For sufficiently low Langmuir frequencies, such that $1 \ll (\omega_0/q)^{1/3} \ll \xi^{-1}$, the BS spectrum is then in the intervals

$$\begin{gathered} \omega \lesssim \omega_0, \\ \omega_0(\omega_0/q)^{1/3} \ll \omega \ll \omega_0\xi^{-1}, \\ \omega_0\xi^{-1} \ll \omega \ll q\xi^{-4}, \quad \xi = m/E \end{gathered} \tag{18}$$

[in this case the parameter β occurring in Eq. (10) satisfies the inequality $\omega \ll q\varepsilon^{1/2}(\omega)\beta^{-2}$] given by Eqs. (11) and (12) and the quantity $dE_\omega/d\omega$ is an increasing function of ω (see Fig. 1).

If, however, the frequency ω satisfies the inequalities

$$\omega_0 \ll \omega \ll \omega_0(\omega_0/q)^{1/3}, \tag{19}$$

[the parameter β is such that $\omega \gg q\beta^{-2}\varepsilon^{1/2}(\omega)$] the spectral distribution of the BS is given by Eqs. (13) and (15). In the range $\omega_0 \ll \omega \ll \omega_0(\omega_0/q)^{1/3}$ the radiation energy increases

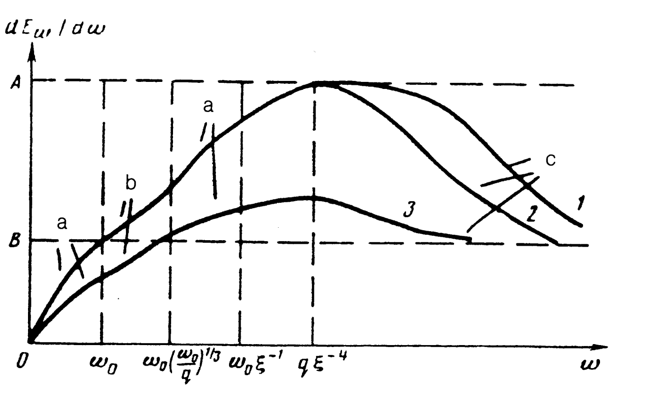

FIG. 1. Spectral distribution of the BS energy of a single-direction beam as function of the frequency of the radiation in a plasma medium for the $\omega_0 \ll q\xi^{-3}$ case. Here we have $A = N^2e^2qT/3\pi\xi^2$, $B = Ne^2qT/3\pi\xi^2$; 1—$qD\xi^{-3} \ll \xi(qT)^{-1/2}$, 2—$qD\xi^{-3} \lesssim \xi(qT)^{-1/2}$, 3—$qD\xi^{-3} \gg 1$; a—$\propto \omega^{1/2}$, b—$\propto \omega^2$, c—$\propto \omega^{-2}$.

with increasing ω according to the formula $dE_\omega/d\omega \propto \omega^2$ whereas for $\omega \gg q\xi^{-4}$ the spectral distribution $dE_\omega/d\omega$ is a decreasing function of the frequency tending "from above" [see Eq. (15) and Fig. 1] to the value

$$\frac{dE_\omega}{d\omega} = N\frac{e^2qT}{3\pi\beta} + \frac{16N(N-1)e^2\ln(qT/\xi^2)}{3\pi\beta D^2\omega^2\varepsilon(\omega)} \underset{\omega\to\infty}{\longrightarrow} N\left(\frac{dE_\omega}{d\omega}\right)_{\text{B.H.}}\left[1+O\left(\frac{1}{\omega^2}\right)\right], \tag{20}$$

where $(dE_\omega/d\omega)_{\text{B.H.}} = 2e^2qT/3\pi\xi^2$ is the Bethe–Heitler BS energy.[4] For sufficiently low Langmuir frequencies, $\omega_0 \ll q\xi^{-3}$ the spectral distribution of the BS of a single-direction beam has thus, as in the $\varepsilon = 1$ case,[7] always a maximum (the quantity $dE_\omega/d\omega$) increases with increasing ω up to values $\omega \sim q\xi^{-4}$ and for $\omega \gg q\xi^{-4}$ the BS energy is a decreasing function of the frequency). If $qD\xi^{-3} \ll \xi(qT)^{-1/2} \ll 1$, the latter has the form of a plateau with width $\Gamma \sim D^{-1}(qT)^{-1/2}$ and we have

$$\left(\frac{dE_\omega}{d\omega}\right)_{max} \approx \frac{2e^2N^2qT}{3\pi\xi^2}.$$

As the diameter D of the beam increases the plateau changes into a strict maximum ($qD\xi^{-3} \lesssim \xi(qT)^{-1/2}$), and for $qD\xi^{-3} \gg 1$ the quantity $(dE_\omega/d\omega)_{\max}$ becomes of the order of $N(dE_\omega/d\omega)_{\text{B.H.}}$.

In the opposite limiting case of sufficiently high Langmuir frequencies, $\omega_0 \gg q\xi^{-3}$ (for definiteness we assume that $\omega_0 \ll q\xi^{-4}$) the spectral distribution $dE_\omega/d\omega$ is in the range $\omega \lesssim \omega_0$ given by Eqs. (11) and (12); $dE_\omega/d\omega$ then increases with increasing ω according to the formula $dE_\omega/d\omega \propto \omega^{1/2}$. If, however, the frequency of the radiation lies in one of the ranges $\omega_0 \ll \omega \ll \omega_0\xi^{-1}$, $\omega \gtrsim \omega_0\xi^{-1}$, $dE_\omega/d\omega$ as function of ω is given by Eqs. (13) and (15). For a sufficiently compact peak, when $D \lesssim \omega_0^{-1}\xi(qT)^{-1/2}$ (such a situation means that the frequencies $\omega \lesssim \omega_\Gamma \sim D^{-1}(qT)^{-1/2}$ for which interference effects occur are high as compared to the frequency $\omega = \omega_0\xi^{-1}$ for which it is important to take into account the polarization of the medium), the spectral distribution of the BS has a maximum, as in the $\omega \ll q\xi^{-3}$ case (see Fig. 2). If, however, $\omega_\Gamma \sim D^{-1}(qT)^{-1/2} \ll \omega_0\xi^{-1}$ then, depending on the relations between the beam diameter, the number of radiating particles, and the Langmuir frequency, the following situations are possible: for $\omega_0 \gg \xi\omega_\Gamma N^{1/2}$ the energy density $dE_\omega/d\omega$ of the BS is, in contrast to the $\varepsilon = 1$ case,[7] an increasing function of the frequency in the whole of its range $\omega \ll E$; if, however, $\xi\omega_\Gamma \ll \omega_0 \ll N^{1/2}\xi\omega_\Gamma$, the BS spectrum has a maximum at frequencies close to $\omega \sim q\xi^{-4}$ (see Fig. 2).

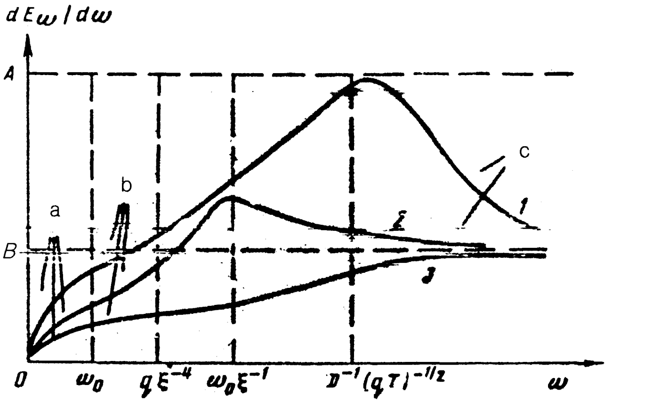

FIG. 2. Spectral distribution of the BS energy of a single-direction beam as function of the frequency of the radiation in a plasma medium for the $\omega_0 \gg q\xi^{-3}$ case. The meaning of A, B, a, b, and c is the same as in Fig. 1; 1—$\omega_\Gamma \approx D^{-1}(qT)^{-1/2} \gg \omega_0\xi^{-1}$, 2—$\xi\omega_\Gamma \ll \omega_0 \ll N^{1/2}\xi\omega_\Gamma$, 3—$\omega_0 \gg N^{1/2}\xi\omega_\Gamma$.

5. RADIATION BY A SINGLE-DIRECTION BEAM OF FAST CHARGED PARTICLES IN A MATERIAL UNDER CHERENKOV-EFFECT CONDITIONS

Since the Cherenkov effect occurs in situations when $\beta = 1 - v_0\varepsilon^{1/2}(\omega) < 0$, the spectral distribution of the radiation energy is in that case given by Eqs. (11) and (14). In the low frequency limit, $\omega \ll q|\beta|^{-2}$ we have from Eqs. (11) and (12)

$$\frac{dE_\omega}{d\omega} = \begin{cases} \dfrac{N^2(q\omega)^{1/2}T}{\pi\varepsilon^{1/4}}, & \omega \ll \min\{q|\beta|^{-2};\ (qTD^2)^{-1/2}\} \\[2ex] \dfrac{N(q\omega)^{1/2}T}{\pi\varepsilon^{1/4}}, & (qD^4)^{-1/3} \ll \omega \ll q|\beta|^{-2}. \end{cases} \tag{21}$$

In the long-wavelength range of the spectrum the mechanism for the formation of the radiation of a single-direction beam, even under Cherenkov effect conditions, comprises multiple collisions of the particles with the atoms of the medium. We note that if the characteristic size of the beam is such that $qD|\beta|^{-3/2} \ll [|\beta|/(qT)]^{1/2}$, a system of particles radiates at low ω under conditions of complete coherence ($dE_\omega/d\omega \propto N^2$). If, however, $qD|\beta|^{-3/2} \gg 1$ the spectral density of the radiation energy is proportional to the number of emitting particles.

In the short-wave range of the spectrum, when we have $\omega \gg q|\beta|^{-2}$, we find from Eq. (14)

$$\frac{dE_\omega}{d\omega} = \begin{cases} \dfrac{Ne^2\omega T}{\varepsilon^{1/2}(\omega)}\left(1 - \dfrac{1}{v_0^2\varepsilon(\omega)}\right) - \dfrac{e^2NqT}{3\pi|\beta|} \\ \omega \gg \max\{q|\beta|^{-2};\ (qTD^2)^{-1/2}\} \\ \dfrac{Ne^2\omega T}{\varepsilon^{1/2}(\omega)}\left(1 - \dfrac{1}{v_0^2\varepsilon(\omega)}\right)\left\{1 + (N-1)J_0\right. \\ \left.\times\left[\dfrac{\omega D}{2}\left(\varepsilon^{1/2}(\omega) - \dfrac{1}{v_0^2}\right)^{1/2}\right]\right\}, \\ q|\beta|^{-2} \ll \omega \ll (qTD^2)^{-1/2}. \end{cases} \tag{22}$$

It follows from these relations that in the case of a sufficiently compact peak for which $q|\beta|^{-3/2}D \ll (|\beta|/qT)^{1/2}$ the radiation energy $dE_\omega/d\omega$ is proportional to the square of the number of particles up to frequencies $\omega \lesssim D^{-1}[1 - (v_0^2\varepsilon(\omega))^{-1}]^{-1/2}$. As ω increases, but at $\omega \ll (qTD^2)^{-1/2}$, the value of $dE_\omega/d\omega$ tends to the Cherenkov radiation energy according to the formula

$$\frac{dE_\omega}{d\omega} = \frac{Ne^2\omega T}{\varepsilon^{1/2}(\omega)}\left(1 - \frac{1}{v_0^2\varepsilon(\omega)}\right)\left\{1 + (N-1)\left(\frac{2}{\pi}\right)^{1/2}\right. \times\left[\frac{\omega D}{2}\left(1 - \frac{1}{v_0^2\varepsilon(\omega)}\right)^{1/2}\right]^{-1/2}\cos\left[\frac{\omega D}{2}\left(1 - \frac{1}{v_0^2\varepsilon(\omega)}\right)^{1/2} \left. - \frac{\pi}{4}\right]\right\} - \frac{Ne^2qT}{3\pi|\beta|}.$$

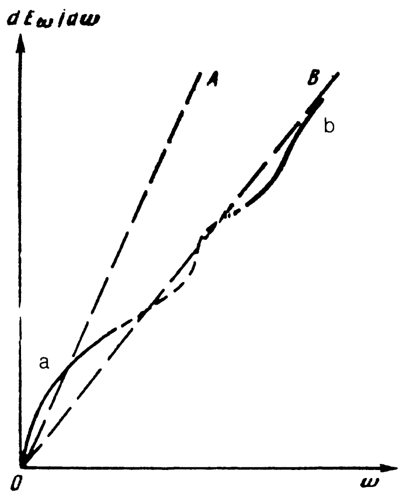

FIG. 3. Frequency dependence of the spectral distribution of the radiation energy under Cherenkov effect conditions for $\beta = 1 - v_0\,\varepsilon^{1/2}(\omega) \approx \text{const} < 0$; $A = e^2 N^2 \omega T (1 - \varepsilon^{-1} v_0^{-2})\,\varepsilon^{-1/2}$; $B = e^2 N \omega T (1 - \varepsilon^{-1} v_0^{-2})\varepsilon^{-1/2}$; a—$\propto \omega^{1/2}$, b—$\propto \omega^{-1}$.

On the other hand, in the very high frequency range, $\omega \gg \max\{q|\beta|^{-2}, (qTD^2)^{-1/2}\}$, the spectral distribution $dE_\omega/d\omega$ of the radiation as a function of ω behaves according to the formula

$$\frac{dE_\omega}{d\omega} = \frac{e^2\omega TN}{\varepsilon^{1/2}(\omega)}\left(1 - \frac{1}{v_0^2\varepsilon(\omega)}\right) - \frac{e^2NqT}{3\pi|\beta|}$$

In Fig. 3 we show the frequency dependence of the spectral distribution of the radiation energy of a single-direction particle beam for the $\beta = 1 - v_0\varepsilon^{1/2}(\omega) \approx \text{const} < 0$ case.

6. CONCLUSION

We have constructed in this paper a consistent theory for the radiation of a system of classically fast noninteracting charged particles which undergo multiple elastic collisions in a scattering medium with temporal dispersion. We have found, for any dispersion law $k(\omega)$, the spectral distribution of the radiation energy from such particles. The obtained spectrum depends strongly both on the parameters $(q, \varepsilon(\omega))$ of the scattering medium, and on the characteristics $(m, E, \mathbf{d}_{\mu\nu}, \Delta_\mu)$ of the initial beam. In the very low frequency limit $(\omega \to 0)$ the mechanism for the formation of the radiation is for arbitrary but finite $\varepsilon(\omega \to 0)$ values essentially a brems mechanism and the particles emit under total coherence conditions $(dE_\omega/d\omega \propto N^2)$. On the other hand, in the case of sufficiently high frequencies,

$$\omega \gg \max\{q|\beta|^{-2}\varepsilon^{1/2}(\omega);\ (\mathbf{d}_{\mu\nu})_\perp^{2}[(\mathbf{b}_{\mu\nu})^2(\mathbf{d}_{\mu\nu})_z\varepsilon^{1/2}qT^3]^{-1}\}$$

the quantity $dE_\omega/d\omega$ is proportional to the number of particles.

We have studied in detail the emission by a single-direction δ-pulse beam of fast charged particles in a dispersive scattering medium. If the latter is an electrically neutral plasma the presence of polarization in the medium leads under well defined conditions to a radical change in the BS spectrum of such a system of particles. In particular, for sufficiently high Langmuir frequencies, $\omega_0 \gg q\xi^{-3}$, and for $D\omega_0 \gg (N\xi^2/qT)^{1/2}$, the maximum in the spectral distribution of the BS radiation is suppressed by the interaction of the radiation field with the plasma medium, and $dE_\omega/d\omega$ becomes a monotonically increasing function of ω in contrast to the $\varepsilon = 1$ case.[7] However, in the case when the permittivity of the material satisfies the inequality $\varepsilon^{1/2}(\omega) > v_0^{-1}$ in the very long-wavelength region of the spectrum, as for $\varepsilon^{1/2}(\omega) < v_0^{-1}$, the radiation turns out to be essentially BS (as before, $dE_\omega/d\omega \propto \omega^{1/2}$ as $\omega \to 0$). If, however, the frequency $\omega \gg q|\beta|^{-2}\varepsilon^{1/2}(\omega)$, the BS is strongly suppressed and the mechanism for the formation of the radiation is the interaction of the system of fast charged particles with the coherently emitting medium.

The region of applicability of the results obtained in this paper is limited by a number of approximations. The condition $E \gg \omega, m$ is the usual one for the discussion of this kind of problems and is sufficiently accurately satisfied, for instance, in situations where the radiation spectrum is formed by the fast charged particles which make up the cosmic rays.[12] As to the neglect of the interaction between the emitting particles, this is legitimate if the deformation of the particle trajectory caused by them (by the interaction) is small as compared to the effect which the multiple elastic collisions with the atoms of the medium have on their motion:[2)]

$$(\Delta v_e)^2 \ll (\Delta v_q)^2,$$

where $(\Delta v_q)^2$ and $(\Delta v_e)^2$ are the squares of the transverse (with regard to the initial direction) velocities of a particle acquired by it due to multiple scattering $[(\Delta v_q)^2 \sim v_0^2 qT]$ and the interaction with the other emitting particles $[(\Delta v_e)^2]$. Estimating $(\Delta v_e)^2$ to be

$$(\Delta v_e)^2 \sim e^2 (m\varepsilon_0 |(\mathbf{d}_{\mu\nu})_\perp|_{min})^{-1},$$

from the last inequality, in the case of Coulomb collisions of the particles with the atoms of the medium, when the mean square multiple scattering angle is equal to[4]

$$q = 4\pi n_0 z^2 (e^2/E)^2 \lg(180 z^{-1/3}),$$

we get

$$T \gg \{4\pi n_0 m \varepsilon_0 (e^2/E^2) |(\mathbf{d}_{\mu\nu})_\perp|_{min} z^2 \lg(180 z^{-1/3})\}^{-1},$$

where n_0 is the density of the scattering centers, ze the charge of each of them, and ε_0 the characteristic magnitude of the permittivity of the medium.

According to Ref. 7 the double-time distribution function $F_\mathbf{k}(\mathbf{v}_\mu, \mathbf{v}'_\nu, t, \tau)$ satisfies the equations

$$\frac{\partial F_\mathbf{k}(\mathbf{v}_\mu, \mathbf{v}_\nu', t, \tau)}{\partial \tau} - i\mathbf{k}\mathbf{v}_\mu F_\mathbf{k}(\mathbf{v}_\mu, \mathbf{v}_\nu', t, \tau) = \frac{q}{4}\frac{\partial^2 F_\mathbf{k}(\mathbf{v}_\mu, \mathbf{v}_\nu, t, \tau)}{\partial \eta^2}, \tag{24}$$

$$\frac{\partial F_\mathbf{k}(\mathbf{v}_\mu, \mathbf{v}_\nu', t, 0)}{\partial t} - i\mathbf{k}(\mathbf{v}_\mu - \mathbf{v}_\nu')F_\mathbf{k}(\mathbf{v}_\mu, \mathbf{v}_\nu', t, 0) = \frac{q}{4}\left(\frac{\partial}{\partial \eta} + \frac{\partial}{\partial \zeta}\right)^2 F_\mathbf{k}(\mathbf{v}_\mu, \mathbf{v}_\nu, t, 0), \tag{25}$$

where

$$\mathbf{v}_\mu = \mathbf{v}_\mu(\boldsymbol{\eta}) = v_0\mathbf{e}_z(1 - \boldsymbol{\eta}^2/2) + v_0\boldsymbol{\eta},\quad |\boldsymbol{\eta}| \ll 1,\quad \mathbf{e}_z\boldsymbol{\eta} = 0,$$
$$\mathbf{v}_\nu' = \mathbf{v}_\nu'(\boldsymbol{\zeta}) = v_0\mathbf{e}_z(1 - \boldsymbol{\zeta}^2/2) + v_0\boldsymbol{\zeta},\quad |\boldsymbol{\zeta}| \ll 1,\quad \mathbf{e}_z\boldsymbol{\zeta} = 0,$$
$$\mathbf{k} = k\mathbf{e}_z(1 - \boldsymbol{\theta}_k^2/2) + k\boldsymbol{\theta}_k;\quad |\boldsymbol{\theta}_k| \ll 1;\quad \mathbf{e}_z\boldsymbol{\theta}_k = 0.$$

The solution of Eqs. (24) and (25) with the initial condition

$$F_{\mathbf{k}}(\mathbf{v}_\mu, \mathbf{v}_\nu', 0, 0)=\delta(\boldsymbol{\eta}-\boldsymbol{\Delta}_\mu)\delta(\boldsymbol{\zeta}-\boldsymbol{\Delta}_\nu)\exp(i\mathbf{d}_{\mu\nu}\mathbf{k})$$

has the form

$$F_{\mathbf{k}}(\boldsymbol{\eta}, \boldsymbol{\zeta}, t, \tau)=(\pi q t)^{-1}\exp\{iv_0 t\mathbf{k}\mathbf{b}_{\mu\nu} -{}^1/_2 ikv_0 t(\boldsymbol{\Delta}_\nu^2-\boldsymbol{\Delta}_\mu\boldsymbol{\Delta}_\nu)-qk^2v_0^2t^3\mathbf{b}_{\mu\nu}^2/48 -(\boldsymbol{\zeta}-\boldsymbol{\Delta}_\mu)^2/qt-{}^1/_2 ikv_0 t\mathbf{b}_{\mu\nu}\boldsymbol{\zeta}\}G(\boldsymbol{\eta}-\boldsymbol{\zeta}-\mathbf{b}_{\mu\nu}, \tau),$$

where $|\mathbf{d}_{\mu\nu}|=|\mathbf{r}_{0\mu}-\mathbf{r}_{0\nu}|$ and $|\mathbf{b}_{\mu\nu}|=|\Delta_\nu-\Delta_\mu|$ are the distances between the particles and the angles of their flight at time $t=\tau=0$ and $G(\mathbf{x},\tau)$ is the Green function of Eq. (24) which was first found in Ref. 4:

$$G(\mathbf{x},\tau)=\frac{a}{\pi\,\mathrm{sh}(a\tau)}\exp\left\{-x^2\frac{a}{q}\mathrm{cth}(a\tau)+\frac{a}{q}\boldsymbol{\theta}_k\mathbf{x}\,\mathrm{th}\frac{a\tau}{2} -\frac{a}{q}\boldsymbol{\theta}_k^2\,\mathrm{th}\frac{a\tau}{2}+ikv_0\tau\right\},\quad a=\left(ikv_0\frac{q}{2}\right)^{1/2}.$$

[1)] The physical meaning of $F_{\mathbf{k}}(\mathbf{v}_\mu,\mathbf{v}_\nu,t,\tau)$ in the coordinate representation is the following:[8] $F_{\mathbf{k}}(\mathbf{r}'_\mu,\mathbf{v}'_\mu,t+\tau;\mathbf{r}_\nu,\mathbf{v}_\nu,t)$ is the probability that the μth particle has coordinates $\mathbf{r}'_\mu$ and a velocity $\mathbf{v}'_\mu$ at time $t+\tau$ under the condition that these quantities for the νth (or μth, if $\mu=\nu$) particle at time t were equal to $\mathbf{r}_\nu$ and $\mathbf{v}_\nu$ (or $\mathbf{r}_\mu$ and $\mathbf{v}_\mu$), respectively.

[2)] In the analogous inequality in Ref. 7 and also in the formula for the quantity q we must read d instead of d^2 and $(e^2/E)^2$ instead of (e^2/m).

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