Effect of short-wave temperature modulation on the generation of a quasisteady magnetic field in a laser plasma

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The generation of a quasisteady magnetic field over electron time scales in the presence of a small-scale temperature modulation is analyzed. The modulation results in a strengthening of the large-scale magnetic field. The amplitude of this field, plotted as a function of the modulation amplitude M, reaches a maximum at $M \sim (kL)^{-1}$, where k is the modulation wave vector, and L is a length scale of the average temperature profile.

The generation of a quasisteady magnetic field as laser light is absorbed in a plasma is a well-known effect.[1-3] The fields which are generated do not have any significant effect on the motion of the plasma ions, but they can strongly affect the electron transport. Several mechanisms are known to lead to a strengthening of the quasisteady fields: a deviation of the density gradient and the temperature gradient from a parallel orientation, resonant absorption of light, parametric instabilities, etc.[3-7] The spatial structure and time evolution of the spontaneous magnetic fields in a laser plasma were studied in Refs. 8–12.

Different generation mechanisms correspond to different spatial and temporal scales of the field. In particular, the

$$\mathbf{B}_t \sim [\nabla n,\ \nabla T]$$

mechanism generates a field with a length scale corresponding to the predominant length scale of the plasma motion. Local variations in the density and temperature distributions lead to a strengthening of the field at the corresponding scale. As was shown in Ref. 11, however, variations can also lead to a strengthening of the average field with the maximum (hydrodynamic) length scale. The approach taken in Ref. 11, which is based on perturbation theory, yielded the component of the average field due to the modulation and estimates of the conditions under which this component is comparable to the field generated in the absence of a modulation. Strictly speaking, however, at high modulation amplitudes the results of Ref. 11 should be regarded as approximate.

In the present paper we analyze the generation of a spontaneous magnetic field over times on the order of the time scale of the electron thermal conductivity. We examine the effect of a short-wave modulation of the temperature on the strength and dynamics of the average field. This temperature modulation might be caused by, for example, a modulation of the laser intensity or fluctuations in the intensity due to instabilities. The particular formulation of the problem adopted here reflects certain aspects of experiments with ultrashort laser pulses. In particular, this formulation of the problem corresponds to the experimental situation in which the plasma is produced near a solid target by a low-intensity precursor pulse and is then heated by the intense ultrashort pulse. In this case, a significant strengthening of the magnetic field can occur (saturation may also set in) over a time shorter than the time scale of the hydrodynamic motion, so the usual hydrodynamic convection can be ignored.[13] On the other hand, it is necessary to consider the convective transport of magnetic field by the electron heat flux (Nernst convection). The equation for the field in this can be written[8,14]

$$\frac{\partial \mathbf{B}}{\partial t} = \operatorname{rot}[\mathbf{V}, \mathbf{B}] + \frac{c}{en}[\nabla T, \nabla n]. \tag{1}$$

Here

$$\mathbf{V} = -(\tau_e/m_e)((\beta_1\chi^2 + \beta_0)/\Delta)\nabla T,\ \Delta = \chi^4 + \delta_1\chi^2 + \delta_0,$$

$\chi = \omega_H \tau_e$ is the Hall parameter, τ_e is the electron mean free time, and β_1, β_0, δ_1, and δ_0 are constants whose values are given in Ref. 14. Equation (1) is the standard MHD equation, in which the mass velocity has been replaced by the velocity (**V**) of heat propagation due to electron thermal conductivity.

We begin our analysis with a very simple model. We assume that the density and temperature profiles are given and, for simplicity, are independent of the time. We also assume that the density depends on only the coordinate z, which runs along the normal to the target surface, while the temperature depends on only the coordinate x. In this case the magnetic field has only the one component $B_y = B(x,z,t)$. With a qualitative analysis of the modulation effects in mind, we will ignore the convection associated with the Nernst effect along the z axis. This assumption does not affect the result if the temperature profile is modulated only along the x axis. The transport of magnetic field (associated with Nernst effect) along the direction of the laser beam in the absence of a modulation was studied in Ref. 15.

Under the assumptions made above, we can replace Eq. (1) by the scalar equation

$$\frac{\partial B}{\partial t} + \frac{\partial (BV)}{\partial x} + G\frac{\partial T}{\partial x} = 0, \tag{2}$$

where $G = (c/en)\partial n/\partial z$. If the magnetic field is weak enough that the Hall parameter satisfies $\chi \ll 1$, Eq. (2) becomes linear and comparatively easy to study analytically. We begin our analysis with this case. The general solution of the Cauchy problem for Eq. (2) can be written in the form

$$B(x,t) = \frac{G}{V(x)}[F(\nu(x) - t) - T(x)], \tag{3}$$

where

$$V(x) = -\frac{\tau_e}{m_e}\frac{\beta_0}{\delta_0}\nabla T = -V_0 T^{3/2}\frac{\partial T}{\partial x},$$

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$$\nu(x)=\int^{x}\frac{dx}{V(x)}$$

and $F(\nu-t)$ is an arbitrary function of its argument. Using $B(x,0)=0$ as an initial condition, we find an equation for determining F: $F(\nu(x))=T(x)$. It is convenient to use the solutions of Eq. (3) to calculate average values of the magnetic field over x, i.e., to find the large-scale component of the field. A direct calculation yields

$$\langle B(x_0,t)\rangle=\frac{k}{2\pi N}\int_{x_0}^{x_0+2\pi N/k}B(x,t)\,dx=\frac{Gk}{2\pi N}\int_{\nu^-}^{\nu^+}[F(\nu-t)-F(\nu)]\,d\nu,$$

where $\nu^{-1}=\nu(x_0)$, $\nu^{+}=\nu(x_0+2\pi N/k)$, k is the modulation wave number, and N is a natural number.

We note an obvious property of the solutions of Eq. (2). taking an average of this equation over x between the points a and b, at which the temperature gradient vanishes, we find

$$\langle B(a,b,t)\rangle=\frac{1}{(b-a)}\int_a^b B(x,t)\,dx=G[T(b)-T(a)]\frac{t}{(b-a)}.$$

The field thus does not reach saturation in the "heat-insulated" regions. The field increases linearly with time; this increase may be limited by hydrodynamic convection or dissipative effects. Let us examine the behavior of the solutions of Eq. (3) over a short time. For this purpose we expand $F(\nu-t)$ in a Taylor series in t. After some straightforward manipulations we find

$$B(x,t)=G\frac{\partial}{\partial x}\left\{\sum_{n=0}^{\infty}\frac{(-t)^{n+1}}{(n+1)!}[D^nT(x)]\right\},\tag{4}$$

where the operator D is given by $D=V(x)\,\partial/\partial x$. Retaining only the first two terms, we find

$$B(x,t)\approx Gt\frac{\partial T}{\partial x}+G\frac{t^2}{2}\frac{\partial}{\partial x}\left(V(x)\frac{\partial T}{\partial x}\right).\tag{5}$$

It can be seen from Eq. (5) that over a short time the field increases linearly (independently at each point). The second term on the right side of Eq. (5) is due to convection. Depending on its sign, it determines a tendency toward either a stabilization of the field or a convective strengthening of the field. The coefficient of t^2 in this term is proportional to the gradient of the flux density, $\partial/\partial x(bV)$, calculated in the corresponding approximation.

We now consider a modulated temperature profile $T(x)=T_S(x)\,(1+M\cos(kx))$, where $T_S(x)$ is a slowly varying function with a length scale L. We assume $M\ll 1$ and $kL\gg 1$. Using Eq. (5), and taking an average over the rapidly varying functions, we find

$$\langle B(x,t)\rangle=-Gt\frac{\partial T_s}{\partial x}+\frac{Gt^2}{2}\left[\frac{\partial}{\partial x}\left(V_s\frac{\partial T_s}{\partial x}\right)+\frac{7}{4}V_sT_s(Mk)^2\right].\tag{6}$$

Here $V_S=-V_0T_S^{3/2}\partial T/\partial x$. It follows from Eq. (6) that the modulation always leads to a strengthening of the average field, and the corresponding component is proportional to $(Mk)^2$. This conclusion agrees with a result found in Ref. 11. The method used to derive Eq. (6) is actually equivalent to the perturbation theory which was used in Ref. 11, so the range of applicability of Eq. (6) is limited to small values of (MkL) and to short times t. Since series Eq. (4) converges, however, it could in principle be used for arbitrary t, if the necessary number of terms were taken into account.

At $MkL\gg 1$ the situation changes qualitatively. In this case, points with $\partial T/\partial x=0$ can appear on the modulated temperature profile:

$$\frac{\partial T}{\partial x}\approx\frac{\partial T_s}{\partial x}-MkT_s\sin(kx)\approx\frac{\partial T_s}{\partial x}[1-MkL\sin(kx)].\tag{7}$$

As we mentioned earlier, the magnetic field in the region between these points increases linearly with time, and convection does not affect this growth. If the modulation amplitude remains low ($M\ll 1$), then at $MkL\gg 1$ the average field does not depend on the modulation amplitude and is determined by the average temperature gradient $\partial T_S/\partial x$. The local field is oscillatory and large in magnitude. An estimate of this field on the basis of our model might prove to be overly crude, because we have ignored the dependence of the Nernst velocity on the magnetic field (this dependence is important at $\chi\sim 1$) and also because we have ignored the field diffusion, which plays an important role when the gradients are high.

There is yet another factor which limits the applicability of the simple model described above for calculating the small-scale magnetic field. Numerical estimates show that under the conditions typical of laser experiments kinetic effects may be important. In particular, the nonlocal nature of the heat flux may be important.[16,17] A switch to a kinetic description would seriously complicate the formulation of the problem and the task of solving it. However, by applying the method of Ref. 11 to the equations derived in Ref. 17, one can show that the qualitative picture of the increase in the average field as the result of a small-scale modulation of the temperature profile remains the same in a kinetic description.

To determine how the nonlinearity of the Nernst effect and the finite conductivity of the plasma affect the generation of the magnetic field, we integrated Eq. (2) numerically. The dependence of the Nernst velocity on the magnetic field was taken into account. A Gaussian temperature profile with a small periodic modulation was assumed for these calculations:

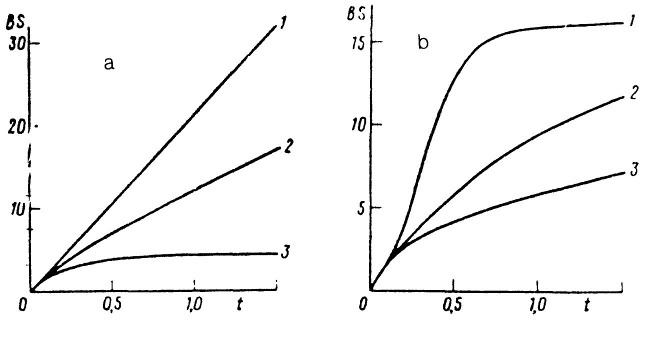

FIG. 1. a—Time evolution of the average magnetic field BS in the absence of field diffusion; b—The same, with field diffusion. *1*—$M=0.01$; *2*—0.005; *3*—0. $T_0=6$.

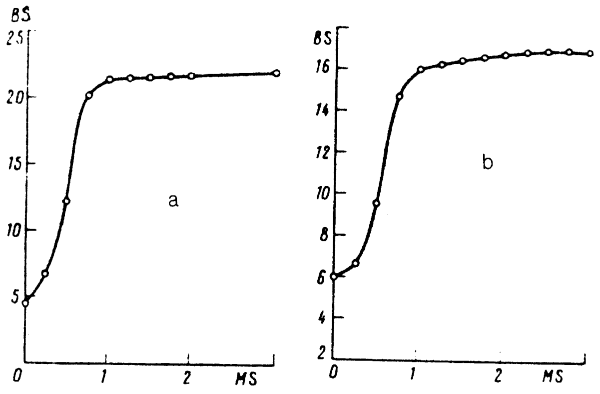

FIG. 2. a—Average magnetic field BS versus the modulation amplitude M ($MS = 100\,M$) in the absence of field diffusion; b—the same, with field diffusion. $T_0 = 6$, $t = 1$.

$$T(x) = T_0 \exp(-x^2/L^2)\,(1 + M\cos(kx)).$$

The following parameter values were adopted: $T_0 = 600$ eV, $L = 25\ \mu$m, $kL = 40\pi$, $\lambda = 2\pi/k = 1.25\ \mu$m, and $G = 2$. The charge of the ions was taken to be 4 (the components β_i and δ_i depend on this charge).

The results of these calculations are shown in Figs. 1 and 2. The fields are given in units of 10^4 G, the time in units of 10^{-10} s, and the temperature in units of 100 eV ($MS = 100M$). Figure 1a shows the time dependence of the magnetic field, averaged over the interval $(0, L/2)$. In the absence of a modulation, the field exhibits a tendency toward saturation, in agreement with a calculation from Eq. (4). A modulation accelerates the increase in the average field over the interval $(0, L/2)$: At $M = 0.008$ ($MkL \simeq 1$), the growth becomes essentially linear. Figure 2a shows the dependence of the field averaged over the interval $(0, L/2)$ on the modulation amplitude M at the time $t = 10^{-10}$ s. The amplitude of the average field reaches saturation at $MkL \sim 1$, in agreement with arguments above.

We have already mentioned that incorporating the field diffusion even at high temperatures (i.e., even at small diffusion coefficients) can have an important effect on the evolution of the local magnetic fields and thus on the magnitude of the average field. To take the diffusion into account in the numerical calculations, we added a diffusion term

$$\frac{\partial}{\partial x}\left[D\frac{\partial B}{\partial x}\right]$$

to the right side of Eq. (1), with a diffusion coefficient

$$D(x) = \frac{m_e c^2}{\tau_e 4\pi n_e e^2}\left(1 - \frac{\alpha_1 \chi^2 + \alpha_0}{\Delta}\right).$$

Here α_1 and α_0 are constants.[14] At a sufficiently high temperature (at $T_0 = 600$ eV), there is no qualitative change in the picture of the average field as a function of the modulation parameter M (i.e., the field increases and then reaches saturation as M is increased). There are, on the other hand, a substantial change in the strength of the field and a temporal growth of this field at $MkL \gg 1$. Figures 1b and 2b show the results of the numerical calculations incorporating field diffusion. These calculations show that the long-wave component of the magnetic field reaches a maximum at $MkL \sim 1$.

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