Coherent population trapping associated with double radio-optical resonance

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The phenomenon of coherent trapping of populations in the case of double radio-optical resonance is investigated. Suggestions are made for obtaining ultracold atomic beams in such systems, as well as for prolonged localization of atoms in a periodic potential of a one-dimensional standing light wave.

I. INTRODUCTION

In the last few years investigations of the excitation of three-level systems in connection with double optical resonance have been sharply stepped up. A striking manifestation of these features is coherent population trapping (CPT), as a result of which the common level of the three-level system is completely depleted.[1] This phenomenon occurs both for the Λ-system and for the cascade (Ξ)-system of levels under the following conditions on the frequency mismatch of the light waves:

$$\Omega_1-\Omega_2=\omega_1-\omega_2-(\omega_{31}-\omega_{32})=0 \quad (1a)$$

for the Λ-system and

$$\Omega_1+\Omega_2=\omega_1+\omega_2-(\omega_{31}+\omega_{32})=0 \quad (1b)$$

for the Ξ-system, where ω_m are the frequencies of the optical fields and ω_{3m} are the frequencies of the allowed transitions in the three-level system ($m=1,2$), and $\omega_{3m}=-\omega_{m3}$.

When the conditions (1) hold the system as a whole goes over in a time $t\approx\gamma_0^{-1}$ (γ_0 is the natural width of the level $|3\rangle$) to a coherent superposition state of the lower levels, that is not sensitive to the presence of resonance fields.[2] Actually, the conditions (1) are the conditions for the well-known double resonance in a three-level system and can pertain to any type of double resonance, such as double radio-optical, optical-microwave, microwave-infrared, and double optical.

Thus far, however, the phenomenon of CPT itself has been considered only for the case of double optical resonance, i.e., in the case when the frequencies of the exciting waves fell into the optical region of the spectrum. For this reason, in our opinion, this phenomenon should be examined for other types of double resonance. The most interesting double resonance is, apparently, the case of radio-optical double resonance, which makes it possible, first of all, to observe the phenomenon of CPT using only one laser field, which could be helpful for different physical applications, and, second, to expand the class of objects for observing this phenomenon.

We emphasize that the analysis of the phenomenon of CPT in the case of radio-optical resonance is very important also from the theoretical standpoint because of the existence of strong asymmetry of the relaxation scheme of such systems. As a result of this, the excitation of such systems exhibits unexpected features, which, in our opinion, are of a general character and have not been previously studied.

Finally, the investigation of CPT under radio-optical resonance permits making a number of interesting suggestions for cooling atoms and ions in traps, channeling, and localization of atoms in a standing wave.

II. CPT UNDER DOUBLE OPTICAL RESONANCE

In order to solve the problem posed, we assume that the Hamiltonian describing the interaction of the atom (Fig. 1) with a field can be represented in the form

$$\begin{aligned}\hat H_{int}&=\hat V+\hat U,\\ \hat V&=-\hbar^{-1}\hat{\mathbf d}\mathbf E\exp[-i\omega_1t+ik_1z],\\ \hat U&=-\hbar^{-1}\hat{\boldsymbol\mu}\mathbf H\exp(-i\omega_2t),\end{aligned} \quad (2)$$

where $\hat{\mathbf d}$ and $\hat{\boldsymbol\mu}$ are the electric and magnetic dipole moment operators and **E** and **H** are the electric and magnetic fields with frequencies ω_1 and ω_2, respectively.

We also assume that the transition $|3\rangle-|1\rangle$ is dipole-allowed with optical frequency ω_{31} and lifetime $\sim\gamma_0^{-1}$ in the $|3\rangle$ level, the transition $|2\rangle-|3\rangle$ is a magnetic-dipole transition, and the transition $|1\rangle-|2\rangle$ is strongly forbidden. These conditions are satisfied, for example, by the excitation schemes of Zn, Hg, and Cd atoms, where the intercombinational optical transition corresponds to the blue part of the visible region of the optical spectrum, while the $|2\rangle-|3\rangle$ transition is a radio frequency (rf) transition and lies in the THz range.

The question of the existence of a coherent superpositional state, which does not interact with fields, in systems with radio-optical double resonance (RODR) is not trivial and has not been investigated before.

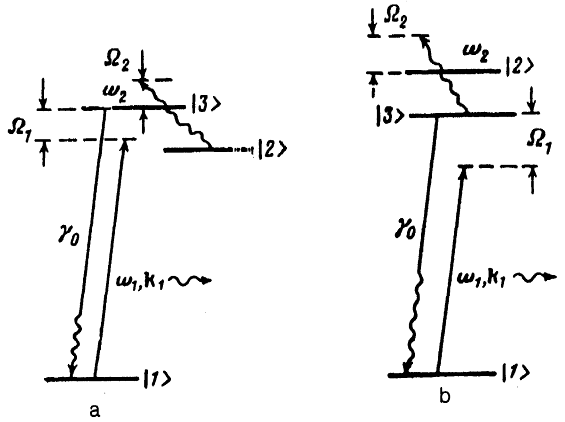

FIG. 1. The system of levels considered: The transition $|1\rangle-|3\rangle$ is an optical transition, $|2\rangle-|3\rangle$ ($|3\rangle-|2\rangle$) is a radio-frequency transition, Ω_m ($m=1,2$) are the corresponding mismatches; a) Λ-system of levels, CPT condition (1a); b) cascade Ξ-system, CPT condition (1b).

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It is known[2] that in systems with relaxation, besides two-photon resonance (1), the intensity of the exciting fields must satisfy a certain condition in order for CPT to arise. The rate of establishment of CPT is determined only by the spontaneous relaxation rate of the common level. A characteristic feature of systems with RODR, which we investigated, is that the relaxational scheme is extremely asymmetric, and in addition radiative decay of the state $|2\rangle$ (not indicated in Fig. 1), which is part of the coherent superposition responsible for CPT, occurs with rate $\gamma_2 \ll \gamma_0$ ($\gamma_2 \approx 10^{-2}$–10^{-3} sec^{-1} for the atoms considered). For this reason, one would think that CPT should be established over times $\sim \gamma_2^{-1}$, and the use of the CPT phenomenon in schemes with RODR would be very problematic. As will be shown below, however, when the intensity of the exciting fields satisfies certain conditions, the main result of Ref. 2 remains in force: CPT is established at a rate determined by the relaxation rate γ_0 of the state $|3\rangle$ (Fig. 1).

The problem of CPT is most conveniently solved in the density matrix formalism using superposition states as the basis:[3-5]

$$
\begin{aligned}
|r\rangle &= \frac{g_2}{g}\,|1\rangle - \frac{g_1}{g}\,|2\rangle, \\
|s\rangle &= \frac{g_1^*}{g}\,|1\rangle + \frac{g_2^*}{g}\,|2\rangle.
\end{aligned} \tag{3}
$$

Here $g_1 = dE/\hbar$ and $g_2 = \mu H/\hbar$ are the Rabi frequencies of the exciting waves, $g^2 = |g_1|^2 + |g_2|^2$, and $|m\rangle$ ($m = 1, 2, 3$) are the eigenfunctions of the unperturbed Hamiltonian.

The state $|r\rangle$, as first shown in Ref. 1, becomes nonabsorbing at the two-photon resonance and it does not participate in the interaction with the applied fields. For this reason, the element of the density matrix $\rho_{rr} \equiv \langle r|\hat{\rho}|r\rangle$ determines the population trapped in the state $|r\rangle$ at the two-photon resonance. In systems without relaxation ρ_{rr} is an integral of the motion.[3] Switching on spontaneous relaxation can, depending on the decay scheme, either completely empty the absorbing state (for example, the V-system is of this type) or it can transfer a large fraction of the atomic population into the state $|r\rangle$. In the latter case, however, the size of the trapped population will depend on the ratio of the intensity of the applied fields and the relaxation constants of the system.[2]

1. Distribution of stationary populations in systems with RODR

We shall find the condition for total trapping to occur (in this case $\rho_{rr} \approx 1$) in our experimental system, with RODR. For this we write the equations for the elements of the density matrix in the basis of states (3) in the rotating-wave approximation:

$$
\begin{aligned}
\dot{\rho}_{33} &= -g v_{3s} - \gamma_0 \rho_{33}, \\
\dot{\rho}_{rr} &= \left[\frac{g_2^2}{g^2}\gamma_0 - \frac{g_2^2(g_2^2-g_1^2)}{g^4}\gamma_2 - \frac{2g_1^2g_2^2}{g^4}\Gamma \right]\rho_{33} \\
&\quad - \left[\frac{(g_2^2-g_1^2)^2}{g^4}\gamma_2 + \frac{4g_1^2g_2^2}{g^4}\Gamma \right]\rho_{rr} + \frac{g_1g_2(g_1^2-g_2^2)}{g^4}(\gamma_2-\Gamma)u_{rs} \\
&\quad + \left[\frac{g_2^2(g_2^2-g_1^2)}{g^4}\gamma_2 + \frac{2g_1^2g_2^2}{g^4}\Gamma \right], \\
\dot{u}_{3s} &= \Omega v_{3s} - {}^1/_2\gamma_0 u_{3s}, \\
\dot{v}_{3s} &= -\Omega u_{3s} + 2g(2\rho_{33}+\rho_{rr}-1) - {}^1/_2\gamma_0 v_{3s}, \\
\dot{u}_{3r} &= \Omega v_{3r} - g v_{rs} - {}^1/_2\gamma_0 u_{3r}, \\
\dot{v}_{3r} &= -\Omega u_{3r} - g u_{rs} - {}^1/_2\gamma_0 v_{3r}, \\
\dot{v}_{rs} &= g u_{3r} - \Gamma v_{rs}, \\
\dot{u}_{rs} &= g v_{3r} + \frac{2g_1g_2}{g^2}\left[\gamma_0 - \frac{2g_2^2}{g^2}\gamma_2 + \frac{(g_2^2-g_1^2)}{g^2}\Gamma \right]\rho_{33} \\
&\quad + \frac{4g_1g_2}{g^2}(g_1^2-g_2^2)(\gamma_2-\Gamma)\rho_{rr} - \left[\frac{(g_2^2-g_1^2)^2}{g^4}\Gamma + \frac{4g_1^2g_2^2}{g^4}\gamma_2 \right] u_{rs} \\
&\quad + \frac{2g_1g_2}{g^2}\left[\frac{2g_2^2}{g^2}\gamma_2 - \frac{(g_2^2-g_1^2)}{g^2}\Gamma \right], \\
&\rho_{33}+\rho_{ss}+\rho_{rr} = 1,
\end{aligned} \tag{4}
$$

where we have introduced the following notation: $\rho_{ij} = \langle i|\hat{\rho}|j\rangle$, $i, j = r,s,3$, $u_{ij} = 2\,\mathrm{Re}\,\rho_{ij}$, $v_{ij} = 2\,\mathrm{Im}\,\rho_{ij}$, $i \neq j = r,s,3$, Γ is the relaxation rate of the coherence between the levels $|1\rangle$–$|2\rangle$, and $\Omega_m = \omega_m - \omega_{3m}$ ($m = 1,2$) are the frequency mismatches. We also assume that $\Omega_1 = \pm\Omega_2 \equiv \Omega$, the velocity of the atom $\mathbf{v} = 0$, and the Rabi frequencies g_1 and g_2 are assumed to be real.

We now find the stationary[1)] ($t \gg \tau$) solution of the system (4). The expression for ρ_{rr} for $\gamma_0 \gg \Gamma$, γ_2 and $\Omega = 0$ has the form

$$
\rho_{rr} = \frac{g_2^2\gamma_0[(2g^2+\Gamma\gamma_0)(g_2^2+{}^1/_4\gamma_0\gamma_2)+g_1^2(2g^2+\gamma_0\gamma_2)]}{g^2(2g^2+\Gamma\gamma_0)(g_2^2\gamma_0+{}^1/_4\gamma_0^2\gamma_2+2g_1^2\gamma_2)}. \tag{5}
$$

It is easy to verify that $\rho_{rr} \approx 1$ for the following set of conditions:

$$g^2 = g_1^2 + g_2^2 \gg \Gamma\gamma_0, \tag{6a}$$

$$g_2^2 \gg \gamma_2\gamma_0, \tag{6b}$$

$$g_2^2\gamma_0 \gg g_1^2\gamma_2. \tag{6c}$$

The conditions (6) are in complete agreement with the results of Ref. 2, which were obtained with $g_1 = g_2$. In our case ($g_1 \neq g_2$), however, they are nontrivial. First, the condition (6a) can be satisfied in several ways. One is $g_2^2 \gg \Gamma\gamma_0$ and $g_1^2 \ll g_2^2$, i.e., the optical field can be arbitrarily weak. If prior to interaction with the fields (at $t = 0$) the entire population of the system was concentrated in the state $|1\rangle$, then for weak optical radiation the populations of the states $|2\rangle$ and $|3\rangle$ will of course, be very small. However, the shape of the line (Fig. 2) due to fluorescence from the level $|3\rangle$ will nonetheless have the form which is characteristic of coherent population trapping.[1] In the case $g_1 = 0$ the atom is simply not excited (or the state $|1\rangle$ is pumped by the *rf* field, if the entire population was initially concentrated in the state $|2\rangle$). In this sense the population will also be trapped in the state $|r\rangle$.

Second, we note that the relaxation $|2\rangle$–$|1\rangle$ with rate γ_2 plays an important role. Namely, the presence of the finite rate γ_2 (let it be as small as desired) allows the system to pass into a stationary solution and results in the appearance of the conditions (6b) and (6c). The condition (6c) is especially exotic. If this condition is not satisfied, i.e., $g_1^2 \gg g_2^2(\gamma_0/\gamma_2)$, then the vibrational contour of the transition $|1\rangle$–$|3\rangle$ is decoupled from the contour of the transition $|2\rangle$–$|3\rangle$ and the population undergoes Rabi oscillations between the states $|1\rangle$ and $|3\rangle$ (Fig. 3), and the shape of the line due to fluorescence from the level $|3\rangle$ acquires the characteristic form for

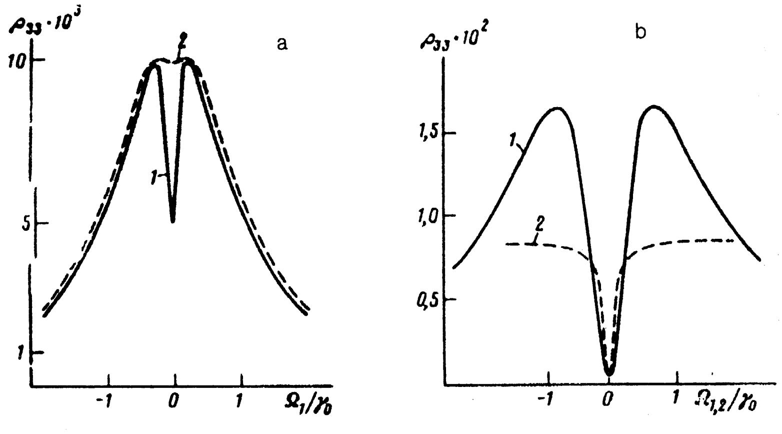

FIG. 2. Dependence of the probability of occupation of the level $|3\rangle$ while scanning a) the frequency of the optical field ($\Omega_2 = 0$, $g_1^2/\gamma_0^2 = 10^{-2}$, $g_2^2/\gamma_0^2 = 10^{-3}$, $\gamma_2/\gamma_0 = 10^{-8}$) for $\Gamma/\gamma_0 = 10^{-2}$ (curve *1*) and 10^{-1} (curve *2*); b) the frequency of the optical field ($\Omega_2 = 0$, $g_1^2/\gamma_0^2 = 10^{-1}$, $g_2^2/\gamma_0^2 = 10^{-2}$, $\gamma_2/\gamma_0 = 10^{-8}$, $\Gamma/\gamma_0 = 10^{-2}$)—curve *1* and the frequency of the rf-field ($\Omega_1 = 0$)—curve *2*.

lines taking into account the Autler-Townes effect (Fig. 4a). For comparison, Fig. 4b shows the populations ρ_{33} and ρ_{rr} versus the frequency of the rf field, when the condition (6b) is known to hold. We note, however, that in our atomic systems $\gamma_2 \approx 10^{-2}$–10^{-3} sec^{-1} and $\gamma_0 \approx 10^6$–10^7 sec^{-1}, so that the condition (6c) holds for all situations which can be realized in practice.

We now examine the population ρ_{33} of the common level $|3\rangle$, the value of which determines the fluorescence of the ensemble of three-level atoms. As is well known, it is the "black line" in the fluorescence spectrum that makes possible different physical applications of CPT. The most general expression for the stationary population ρ_{33} is presented in the Appendix. Here we write this expression when we have $\Omega = 0$, $v = 0$, and $\gamma_0 \gg \Gamma$, γ_2:

$$\rho_{33} = \frac{4g_1^2[g_2^2\Gamma + {}^1/_2\gamma_2(g_1^2 - g_2^2) + {}^1/_4\gamma_0\Gamma\gamma_2]}{(2g^2 + \Gamma\gamma_0)(g_2^2\gamma_0 + {}^1/_4\gamma_0^2\gamma_2 + 2g_1^2\gamma_2)}. \quad (7)$$

One can see from Eq. (7) that when the conditions (6) are not satisfied, the probability of finding an atom in the common level has the form

$$\rho_{33} \approx 4g_1^2/\gamma_0^2, \quad (8)$$

i.e., it is proportional to the intensity of the light field. This character of the population corresponds completely to excitation of a two-level atom with low light intensities (for which the transition is not saturated). The upper level is populated in a different manner if the conditions (6) are satisfied. Then

$$\rho_{33} \approx 2\Gamma g_1^2/\gamma_0 g^2. \quad (9)$$

In practice Γ is determined, for example, by fluctuations of the laser radiation of transit broadening. Here we assume that $\Gamma \ll \gamma_0$, which usually holds when the atom is excited by correlated fields.[6]

Figure 2 illustrates the nonlinear properties of the system which are under discussion. One can see that the appearance of a characteristic resonance of CPT is associated with the transition fields determined by the condition (6a). The magnitude of the base is determined by the value of Γ/γ_0. We underscore the fact that the character of ρ_{33} as a function of one of the frequencies with the other frequency held fixed is completely different for the conditions (6) and depends on which frequency varies. Thus, if the optical radiation is tuned to exact resonance, $\Omega_1 = 0$, then ρ_{33} determines the RODR signal (Fig. 2b). In the case when the frequency of the optical transition is scanned ($\Omega_2 = 0$), however, we obtain an expression for ρ_{33} which determines the population of the upper level under conditions of CPT.

As one can see from Fig. 2b, the widths of the dips appearing as both the rf and optical radiation frequencies are scanned are virtually identical. Indeed, we obtain from Eq. (A1) under the conditions (6) the width of the resonance for $g_1, g_2 \ll \gamma_0$ both for the case of fixed mismatch $\Omega_1 = 0$

$$\Gamma_1 \approx 2g^2/\gamma_0 \ll \gamma_0,$$

and for the case $\Omega_2 = 0$

$$\Gamma_2 \approx (g_1^4 + 62g_1^2g_2^2 + g_2^4)^{1/2}/2\gamma_0 \ll \gamma_0.$$

Thus under conditions RODR it is possible to obtain an rf signal with widths less than the natural line width of the optical transition for sufficiently high light intensities. The limitation here are the additional broadenings, including the broadening due to the instability of the light field.[6] It is usually assumed that in order to observe the CPT resonance in the case of completely uncorrelated fields the width of the spectrum of the light field must be smaller than the natural width γ_0. However, this condition can be strongly weakened

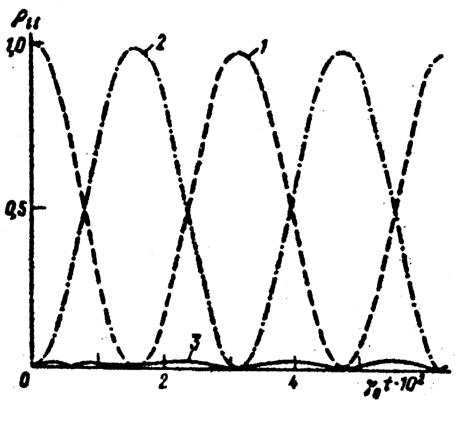

FIG. 3. Temporal evolution of the populations of the states $|1\rangle$, $|2\rangle$, $|3\rangle$, with $g_1/\gamma_0 = 10^3$, $g_2/\gamma_0 = 1$, $\gamma_2/\gamma_0 = 10^{-6}$, $\Gamma/\gamma_0 = 10^{-2}$. Curves *1*—$\rho_{11}$, *2*—$\rho_{22}$, 3—$\rho_{33}$.

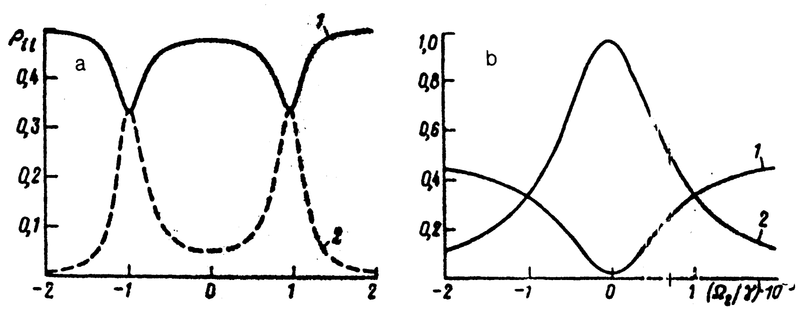

FIG. 4. Dependence of the stationary populations ρ_{33} (curves *1*) and ρ_{rr} (curves *2*) in the case when the frequency of the rf-field is scanned ($\Omega_1 = 0$, $g_1/\gamma_0 = 10^3$, $g_2/\gamma_0 = 1$, $\Gamma/\gamma_0 = 10^{-2}$) with $\gamma_2/\gamma_0 = 10^{-6}$ (a) and 10^{-9} (b).

in the case when the fields acting on the system are correlated and in practice CPT can be observed for significant widths of the spectra.[6]

2. Temporal evolution of the atomic populations in systems with RODR

We now investigate the temporal evolution of the populations. The system of equations (4) is decoupled and admits an analytical solution in three cases: 1) $\gamma_0 = \gamma_2 = \Gamma$; 2) $\gamma_2 = \Gamma$; and, 3) $g_1 = g_2$. The systems under consideration with RODR have significantly different relaxation constants ($\gamma_0 \gg \gamma_2, \Gamma$). For this reason, we first present the solution for $g_1 = g_2 \equiv g_0$ and $\Omega = 0$. This does not limit the generality, since all characteristic features of the temporal behavior of the populations are also manifested here.

Thus, if the conditions (6), which for $g_1 = g_2$ reduce to the condition $g_0^2 \gg \Gamma\gamma_0$, hold, then we obtain

$$\rho_{rr}(t) = 1 - (1-\rho_{rr}^0)\exp(-{}^1/_4\gamma_0 t) \times \left[1 + 2^{-3/2}\frac{\gamma_0}{g_0}\exp(-{}^1/_8\gamma_0 t)\sin(2^{3/2} g_0 t)\right], \tag{10}$$

$$\rho_{33}(t) = \Gamma/\gamma_0 + \exp(-{}^1/_4\gamma_0 t)\Big\{-\Gamma/\gamma_0 + {}^1/_2(1-\rho_{rr}^0) \times \Big[1 - \exp(-{}^1/_8\gamma_0 t)\Big(\cos(2^{3/2} g_0 t) + 3\cdot 2^{-9/2}\frac{\gamma_0}{g_0}\sin(2^{3/2} g_0 t)\Big)\Big]\Big\}, \tag{11}$$

where $\rho_{rr}^0 = \rho_{rr}(t=0)$.

As expected, for high field intensities $\rho_{rr} \to 1$ as $t \to \infty$. If, however, initially the entire population is in the state $|r\rangle$ ($\rho_{rr}^0 = 1$), then temporal evolution virtually ceases. Only small population currents, proportional to Γ/γ_0, remain. The rate at which $\rho_{rr}(t)$ is established, however, is determined by the spontaneous relaxation rate of the common level $|3\rangle$.

Nonzero mismatches Ω as well as $g_1 \neq g_2$ do not change fundamentally the character of the temporal evolution—decaying oscillations of the populations (see the numerical calculations in Ref. 2), if the conditions (6) hold.

For weak fields, $g_0^2 \ll \Gamma\gamma_0$, the stationary population of the nonabsorbing state is established over a time of order Γ^{-1}:

$$\rho_{rr}(t) = {}^1/_2 - ({}^1/_2 - \rho_{rr}^0)\exp(-\Gamma t), \tag{12}$$

$$\rho_{33}(t) = (4g_0^2/\gamma_0^2)\{1 + 2({}^1/_2 - \rho_{rr}^0)\exp(-\Gamma t) - [\Gamma/\gamma_0 - 2(1-\rho_{rr}^0)]\exp(-\gamma_0 t) + [\Gamma/\gamma_0 - 4(1-\rho_{rr}^0)] \times \exp(-{}^1/_2\gamma_0 t)\}. \tag{13}$$

We note that even in this case part (one-half) of the population is trapped, but the interaction with the fields does not cease. Radiation absorption by the atom, determined by the quantity v_{3s} (Ref. 5)

$$v_{3s}(t \gg \Gamma^{-1}) \approx -2^{3/2} g_0/\gamma_0,$$

is proportional to the Rabi frequency of the applied fields, as happens in two-level systems.

It is interesting to determine how the satisfaction of the condition (6c) affects the temporal evolution of the populations. For this, we solve the system (4) with $\Gamma = \gamma_2$ and $g_1 \neq g_2$. The expressions for ρ_{rr} and ρ_{33} in this case will be too complicated. For this reason, we write the solution in the form

$$\rho_{rr} = D + A\exp(\lambda_1 t) + B\exp(\lambda_2 t) + C\exp(\lambda_3 t)$$

and we present here only the roots λ_i ($i = 1, 2, 3$) of the characteristic equation. If the conditions (6a) and (6b) are satisfied, then

$$\lambda_1 = -(3g_2^2\gamma_0 + 2g_1^2\gamma_2)/6g^2, \qquad \lambda_{2,3} = -({}^1/_2 + g_1^2/4g^2)\gamma_0 \pm 2ig. \tag{14}$$

For $g_2^2\gamma_0 \gg g_1^2\gamma_2$ the rate at which the populations are established in systems with RODR under conditions of CPT is determined by the spontaneous relaxation rate γ_0 of the common level. If, however, $g_2^2\gamma_0 \ll g_1^2\gamma_2$, then the stationary solution is established over a time γ_2^{-1} and, analogously to Eq. (12), the stationary state is absorbing, i.e., CPT does not arise in the system.

Thus, as one can see from Eqs. (10)–(14), the rate at which a stationary state is established depends strongly on the relationship of the parameters of the system; this is a characteristic feature of systems with strong asymmetry of the relaxational constants. When the conditions (6) are satisfied the time over which a stationary state is reached $\tau \sim \gamma_0^{-1}$, but if the conditions (6) are not satisfied, then $\tau \sim \Gamma^{-1}$ or $\tau \sim \gamma_2^{-1}$. We note in this connection that in the stationary solution (5), (A1) (obtained under the condition $t \gg \tau$) passage to the limit $\Gamma \to 0$ or $\gamma_2 \to 0$ is possible only if the intensities of the excited fields satisfy the conditions (6).

III. POSSIBLE PHYSICAL APPLICATIONS

We now discuss the new possibilities opened up by the simultaneous use of optical and rf fields in such systems.

1. Possibility of observing the process of establishment of CPT

As we have shown above, the phenomenon of CPT can also be observed in radio-optical resonance, as in the case of double optical resonance. However, systems in which the required radio-optical resonance exists are much more suitable for observation of establishment of CPT in them. Indeed, since CPT is established over a time $\tau \approx \gamma_0^{-1}$, in order for observations to be successful the establishment times cannot be very short.[7] The optimal conditions for investigating this effect exist, in all probability, for the elements Zn (optical transition 4^1S_0–$4^3P_1^0$, with lifetime $\tau_0 \approx 2.5 \cdot 10^{-6}$ sec, and an rf transition $4^3P_0^0$–$4^3P_1^0$, with wavelength $\lambda \approx 5 \cdot 10^{-3}$ cm, and Cd (5^1S_0–$5^3P_1^0$ with $\tau_0 \approx 2.4 \cdot 10^{-6}$ sec, and $5^3P_0^0$–$5^3P_1^0$ with $\lambda = 2 \cdot 10^{-3}$ cm, since, on the one hand, the establishment time $\tau \approx 10^{-6}$ sec, is quite long, while on the other hand, such linewidths suggest the light field will have the relative stability necessary for observing CPT.

2. Possibility of cooling atomic beams under RODR conditions down to temperatures of several μK

It is well known[8] that under the conditions of CPT deep cooling of atomic beams to temperatures of several μK is possible in the case of double optical resonance. Correspondingly, the same possibility can also be observed in the case of RODR. We now show this. According to Ref. 9, the light pressure force F_z and the impulsive diffusion coefficient D_{zz} can be expressed in terms of the population of the common level of the system as

$$F_z = 2\hbar k_1 \gamma_0 \rho_{33}, \quad D_{zz} = 2\hbar^2 k_1^2 \gamma_0 \rho_{33}, \tag{15}$$

and from Eq. (A1) we have for $\Omega_1 = \pm \Omega_2 = \Omega$

$$\begin{gathered}\rho_{33} = g_1^2 a L_0^{-1},\\ a = (k_1 v_z)^2 + 2\Gamma g_1^2/\gamma_0,\\ L_0 = (k_1 v_z)^4 - 2\Omega (k_1 v_z)^3\\ + (\Omega^2 + {}^1/_4 \gamma_0^2 + 2g_1^2 - 2g_2^2)(k_1 v_z)^2 + 2g_2^2 \Omega (k_1 v_z)\\ + (g^4 + 4g_1^2 \Omega^2 \Gamma/\gamma_0).\end{gathered} \tag{16}$$

Here the velocity of the atom is v_z and the conditions for CPT (6) are satisfied, and as before it is also assumed that $\gamma_2, \Gamma \ll \gamma_0$. Figure 5 shows the typical dependence of the force F_z of the light pressure on the velocity of the atom for different frequency mismatches. The temperature of the cold atoms can then be determined, according to Ref. 9, as the ratio of the diffusion coefficient D_{zz} to the dynamic friction coefficient β, both being calculated at a definite velocity point, for example, for $v_z = 0$. For this, we expand in a series the force F_z up to first order in the velocity of the atom

$$F_z = F_z^0 - M\beta v_z, \tag{17}$$

where

$$F_z^0 = \hbar k_1 \gamma_0 \frac{2g_1^2 g^2 \Gamma/\gamma_0}{g^4 + 4g_1^2 \Omega^2 \Gamma/\gamma_0}, \tag{18a}$$

$$\beta = \frac{\hbar k_1^2}{M} \frac{4g_1^2 g_2^2 g^2 \Gamma \Omega}{(g^4 + 4g_1^2 \Omega^2 \Gamma/\gamma_0)^2}, \tag{18b}$$

where M is the mass of the atom. The first term in the expansion in Eq. (17), F_z^0, determines the motion of the ensemble of cold atoms as a whole, while the second term, βv_z, characterizes the rate of narrowing of the velocity distribution centered on $v_z = 0$.

Calculating, next, the impulsive diffusion coefficient for $v_z = 0$, we find the temperature of the atoms in the form

$$T = T_0 (g^4 + 4g_1^2 \Omega^2 \Gamma/\gamma_0)/g_2^2 \Omega \gamma_0, \tag{19}$$

where $T_0 = \hbar \gamma_0 / 2k_B$ and k_B is Boltzmann's constant. One can see from Eq. (19) that for certain values of g_1^2, g_2^2, Ω, and Γ the temperature of the cold atoms can be made significantly lower than T_0. We now find the minimum temperature that is achievable here for prescribed intensities g_1^2 and g_2^2 of the applied fields and the coherence relaxation rate Γ:

$$T_{min} = 2T_0 g^2 g_1 (\Gamma/\gamma_0)^{1/2}/g_2^2 \gamma_0 \quad \text{for} \quad \Omega_{min} = g^2 (\gamma_0/\Gamma)^{1/2}/2g_1. \tag{20}$$

For example, for Hg atoms cooled on the transition 6^1S_0–$6^3P_1^0$ with radiation width $\gamma_0 \approx 8.5 \cdot 10^6$ sec^{-1}, for $g_1/\gamma_0 = g_2/\gamma_0 = 0.1$ and $\Gamma/\gamma_0 = 10^{-2}$ we have from Eq. (20) $T_{min} \approx 1.2 \cdot 10^{-6}$ K for $\Omega_{min} = \gamma_0$.

Since the rate at which the velocity distribution narrows is determined by the quantity β^{-1}, the change Δv_z in the velocity of the atoms over the time $t \approx \beta^{-1}$ can be estimated. We shall estimate Ω_{min} from Eq. (20). Then Δv_z is determined as

$$\Delta v_z = M F_z^0 \beta^{-1} = \frac{M^2}{k_1} \frac{2g^2 g_1}{g_2^2} \left(\frac{\Gamma}{\gamma_0}\right)^{1/2} \approx 0{,}4 \text{ cm/sec}$$

for the values of the parameters presented above.

At the same time the width δv_z of the velocity distribution, characterized by the temperature T_{min} (20), for Hg atoms is

$$\delta v_z = (2k_B T_{min}/M)^{1/2} \approx 1{,}3 \text{ cm/sec},$$

and over the time β^{-1} the formation in which the narrow velocity peak forms there is not enough time for the force F_z^0 to displace the atoms over a distance δv_z. Figure 6 shows how the velocity distribution of the beam of Hg atoms is deformed after the beam interacts with optical and rf fields according to the RODR scheme during the time $t \gtrsim \beta^{-1}$. The initial velocity distribution was assumed to be Gaussian

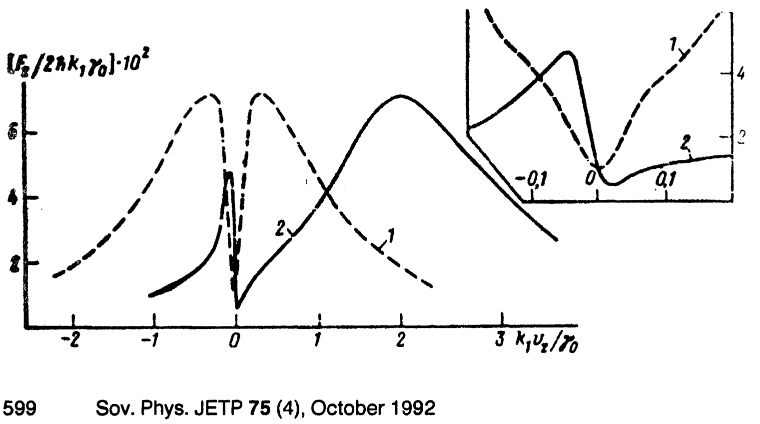

FIG. 5. Light pressure force F_z versus the velocity v_z of the atom for $g_1^2/\gamma_0^2 = 10^{-1}$, $g_2^2/\gamma_0^2 = 10^{-2}$, $\gamma_2/\gamma_0 = 10^{-8}$, $\Gamma/\gamma_0 = 10^{-2}$ and $\Omega = 0$ (curve 1) and $\Omega/\gamma_0 = 2$ (curve 2). Inset: Range of velocities close to zero.

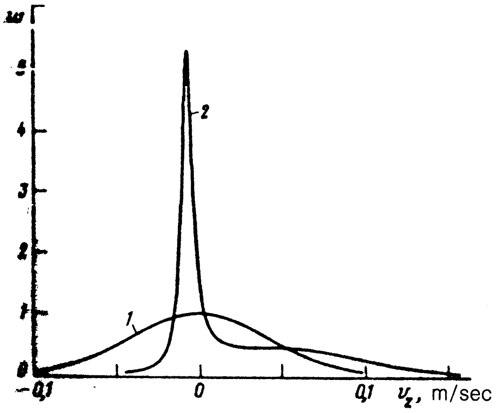

FIG. 6. Evolution of the velocity distribution w of the beam of Hg atoms interacting with optical and rf fields. The offset $\Omega = 2\gamma_0$, the Rabi frequencies of the fields $g_1/\gamma_0 = g_2/\gamma_0 = 0.1$, and $\Gamma/\gamma_0 = 10^{-2}$. $t = 0$ (curve *1*) and $4\cdot 10^{-5}$ sec (curve *2*).

with width 0.1 m/sec, which corresponds to the preliminary cooling to the Doppler limit $T_0 \approx \hbar\gamma_0/2k_B$.

We note two other interesting possibilities: 1) transverse cooling of the beam of atoms by only one laser field down to the temperature $T_{\rm min}$ (20) and 2) longitudinal moderation of an atomic beam in the scheme of adjustment of the frequency of the exciting fields as the atoms cool down and leave resonance.[10]

3. Possibility of channeling of atoms in a standing light wave

It is well known that an atom in the field of a standing light wave is subjected to a force oscillating at the wavelength of the light field.[11] This force creates a periodic potential for atoms in the field of the standing wave. Recently, experiments were performed showing that the atoms indeed "feel" the field of the periodic potential generated by a standing light wave.[12] Since, however, the potential wells are shallow, with depth $U_0 \approx 2\hbar\gamma_0$, and the minimum kinetic energy of the trapped atoms is also near $\hbar\gamma_0$, the atoms cannot be localized in potential wells for a long time.[11]

The possibility of deep cooling of atoms to temperatures below $\hbar\gamma_0/2k_B$ can significantly increase the channeling time of atoms in the periodic potential of a standing wave. A schematic of such an experiment is shown in Fig. 7. The atomic beam *1* is irradiated transversely by a laser beam *2* applied on the $|1\rangle$–$|3\rangle$ transition (Fig. 1), and at the same time a resonance rf field is applied in the interaction region.

Then, according to Eq. (20), the atomic beam is cooled in the transverse direction to temperatures significantly below $\hbar\gamma_0$ ($k_B T \ll \hbar\gamma_0$). After such a cooling cycle, the shutter *3* is opened and a standing light wave is formed with the help of the mirror *4*. As a result, the atoms, whose kinetic energy is significantly less than $\hbar\gamma_0$, are efficiently trapped in the wells of the periodic potential with depth $U_0 \approx 2\hbar\gamma_0$. In Fig. 7, curve *6* corresponds to a periodic potential, and the peaks *7* correspond to the spatial distribution of the trapped atoms. Since the localization time increases sharply in such a scheme, the atoms can channel in the standing wave over quite large distances.

We note that the frictional force existing in such a scheme only improves the situation in the standing wave, and diffusion requires a significant time in order to drive atoms away from the potential barrier.

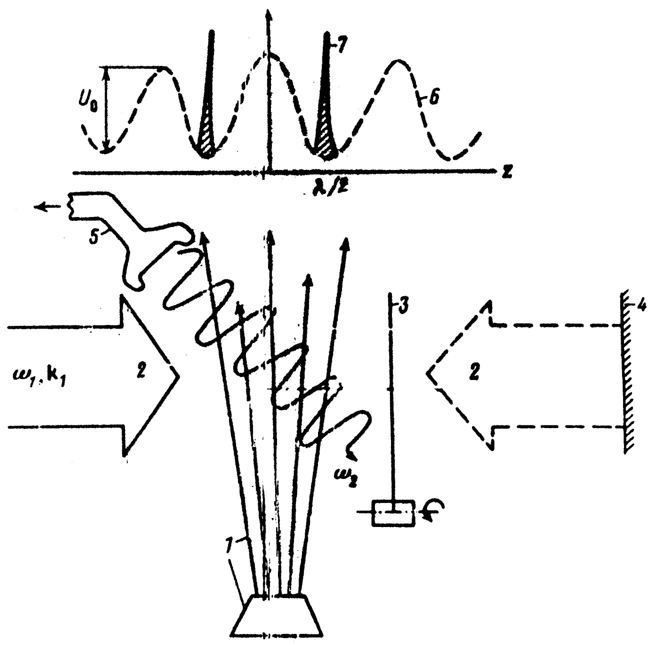

FIG. 7. Channeling of atoms in the standing light wave: *1*) source of atoms and atomic beam, *2*) optical radiation with frequency ω_1, *3*) shutter, *4*) mirror with whose help the standing wave is formed, *5*) system for feeding the rf field into the interaction region, *6*) periodic potential in the standing light wave, 7) spatial distribution of cold atoms.

4. One-dimensional localization of atoms

We examined above an example of how cooling of atoms below $T_0 \approx \hbar\gamma_0/2k_B$ significantly increases the channeling time of atoms through a light wave. Such deep cooling, however, can also solve the problem of localization of atoms in the periodic potential of a standing wave.[13,14]

Figure 8 shows the excitation scheme of a four-level atom, which makes it possible to solve this problem. Here a standing light wave, which generates the periodic potential for cold atoms, acts on the strong optical $|1\rangle - |4\rangle$ transition, while the levels $|1\rangle$, $|2\rangle$, and $|3\rangle$ form the Λ-system studied above with the rf field at the $|2\rangle$–$|3\rangle$ transition and optical excitation of the transition $|1\rangle$–$|3\rangle$. We also assume that the splitting between the levels $|3\rangle$ and $|4\rangle$ is significantly greater than their field-induced broadening, caused by the

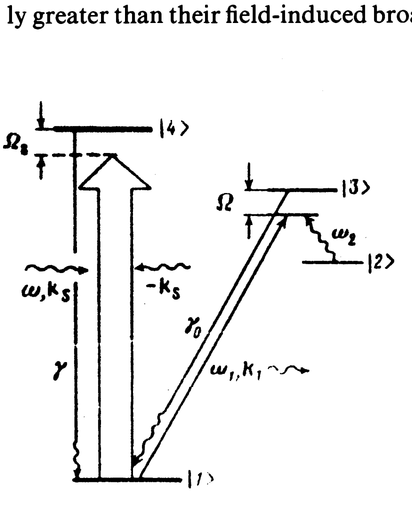

FIG. 8. Excitation scheme for a four-level atom for obtaining one-dimensional localization in the periodic potential of a standing wave with frequency ω.

optical fields at the $|1\rangle-|4\rangle$ and $|1\rangle-|3\rangle$ transitions. In practice, the atoms Zn, Cd, and Hg, for example, have the required energy level schemes.

We note that this excitation scheme successfully combines localization of atoms due to the periodic potential of the standing wave with simultaneous cooling of the atoms to the temperature $T_{\min} \ll \hbar\gamma_0/2k_B$ (20). This makes it possible to confine the atoms for a long time in the potential wells.

In what follows, we confine our attention to estimates based on perturbation theory in the field. Such estimates are adequate for analyzing the problem. Since the depth of the potential wells in the case of the standing wave is

$$U_0 \approx 2\hbar\gamma_0 g_S^2/|\Omega_S|\gamma_0 = \hbar\gamma_0 g_S (\Gamma/\gamma_0)^{1/2}/\gamma_0, \tag{21}$$

and the kinetic energy W of the atoms is determined by the temperature $T_{\min}$ (20), the lifetime of the atoms in the potential wells can be written as

$$\tau = \nu^{-1}\exp(U_0/W) \approx \nu^{-1}\exp(g_S g_2^2/2g_1 g^2), \tag{22}$$

where, for simplicity, it is assumed that the optical characteristics of the transitions $|1\rangle-|3\rangle$ and $|1\rangle-|4\rangle$ are identical, $|\Omega_s| \gg \gamma_0$, g_s is the Rabi frequency for the standing wave, and ν is the vibrational frequency of the atoms in the potential wells[14] and is given by

$$\nu^2 = (4\hbar k_S^2/M) g_S (\Gamma/\gamma_0)^{1/2}. \tag{23}$$

In deriving Eqs. (21)–(23) we also assume $\Omega = \Omega_{\min}$, and the condition

$$\Omega_S = 2g_S(\gamma_0/\Gamma)^{1/2}. \tag{24}$$

is imposed on the mismatch Ω_s of the standing wave. Assuming that $g_s \gg g_1$, we have from Eq. (22) $U_0 \gg W$, which corresponds to prolonged confinement of the atoms. In this case, the cold atoms are concentrated near the bottom of the potential well in a region with half-width

$$\Delta r = (2W/M\nu^2)^{1/2} = 2^{-1/2}\lambda\!\!\!^{-}\,(g_1 g^2/g_S g_2^2)^{1/2}. \tag{25}$$

For example, for $g_s = 10\gamma_0$, $g_1 = g_2 = 0.1\gamma_0$, and $\Gamma/\gamma_0 = 10^{-3}$ the mismatches of the light waves, according to Eqs. (20) and (24), are equal to $\Omega_{\min} = 3.3\gamma_0$ and $\Omega_S \approx 6.6\cdot 10^2\gamma_0$, and for Hg atoms cooled on the transition 6^1S_0–$6^3P_1^0$ ($\gamma_0 \approx 10^7$ sec^{-1} and the frequency of the vibrations of the atoms in the well is $\nu \approx 10^6$ Hz) the temperature of the atoms is $T_{\min} \approx 2\cdot 10^{-6}$ K, the atoms are localized in a region $2\Delta r \approx \lambda/60 \approx 70$ Å, and the localization time is $\tau_L \approx 10^6$ sec.

Thus our estimates show that multilevel atoms can be localized for a long time at the antinodes of a standing wave, if the atoms are additionally cooled on transitions which are not excited by this wave.

APPENDIX

We present here the expression for the population of the level $|3\rangle$ for systems with RODR (Fig. 1):

$$\rho_{33} = [g_1^2 g_2^2 \gamma_0 a^2 + (g_1^2 a_2 - g_1 g_2 b)\gamma_2 a] L_0^{-1}, \tag{A1}$$

where

$$L_0 = d_0\gamma_2 + [2g_1^2 a_2\gamma_2 + g_1 g_2 b(\gamma_0 - 3\gamma_2) + g_2^2 a_1(\gamma_0+\gamma_2)]a + 3g_1^2 g_2^2 \gamma_0 a^2,$$

$$d_0 = a_1 a_2 - b^2,$$

$$a_1 = a(\tfrac14\gamma_0^2 + \alpha_1^2) + g_2^2[g^2 + \varepsilon(\tfrac14\gamma_0^2 - \alpha_1^2) - 2\alpha\alpha_1],$$

$$a_2 = a(\tfrac14\gamma_0^2 + \alpha_2^2) + g_1^2[g^2 + \varepsilon(\tfrac14\gamma_0^2 - \alpha_2^2) + 2\alpha\alpha_2],$$

$$b = g_1 g_2[\varepsilon(\tfrac14\gamma_0^2 + \alpha_1\alpha_2) + g^2 - \alpha^2],$$

$$a = \Gamma^2 + \varepsilon g^2 + \alpha^2, \quad \varepsilon = 2\Gamma/\gamma_0, \quad g^2 = g_1^2 + g_2^2,$$

$$\alpha = \alpha_1 - \alpha_2, \quad \alpha_1 = \Omega_1 - \mathbf{k}_1\mathbf{v}, \quad \alpha_2 = \pm\Omega_2.$$

Here, as previously, the plus and minus signs refer to the Λ and Ξ systems, respectively.

[1)] Here τ is the time needed to reach steady state. This time is discussed in detail in Sec. 2 below.

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