Stabilization of the Rayleigh–Taylor instability by convection in an ablatively accelerated laser plasma

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We use the WKB-approximation to treat the problem of the stabilization by an inhomogeneous convective current of the Rayleigh–Taylor instability developing in the ablation zone when the plasma of laser targets is accelerated by ablation. The problem of the eigenvalues—the instability growth rates—is reduced to the solution of an algebraic equation with coefficients which depend on the structure of the unperturbed profiles of the hydrodynamic variables. We show for the practically important case of subsonic flow of an incompressible plasma that the instability growth rate vanishes for $k = k_0 = \max[2(g|\nabla \ln \rho|^{1/2}/v]$. The condition for the self-consistency of the model is that the local Froude number be small in the region where the instability develops; however, comparison with numerical calculations shows that the model is also applicable in the case of rather steep density gradients when the Froude number is of order unity.

1. INTRODUCTION

The process of the acceleration by ablation of the plasma formed from planar foils and spherical laser targets through the action of strong laser radiation is hydrodynamically unstable due to the development of a Rayleigh–Taylor (RT) instability, for the existence of which the density and pressure gradients must be in opposite directions. Just such a situation is realized in the ablative acceleration of a target near the ablation surface where the laser radiation absorbed in the corona is transferred by thermal conductivity to the deep layers of the target. The RT instability is the subject of intensive numerical and analytical studies, since it is this instability, developing in the inhomogeneous laser plasma, which restricts the maximally obtainable degree of spherically symmetric compression and thus the limits of energy build-up, in particular for the implosion of thin spherical shells in the schemes for laser ICF, considered at the present time.

Experiments carried out in recent years on ablative acceleration of thin foils,[1–4] and also the results of two-dimensional numerical simulations[1,2,4–10] demonstrate that the instability growth rate is much smaller than that predicted by classical theory,[11,12] up to complete stabilization of the instability in the short-wavelength limit. This difference is caused by a large number of physical factors such as the convective efflux of matter from the instability region, the structure of the unperturbed profiles of the hydrodynamic variables, electron thermal conductivity, and the compressibility of the plasma, and in the nonlinear regime also the generation by thermal currents of strong magnetic fields.[13]

According to the results of recent series of numerical calculations[4,7–10] the main stabilizing factor in the linear stage is evidently convection, and the dispersion relation for the growth rate $\sigma(k)$ is well described by an approximate formula proposed by Takabe:[14,15]

$$\sigma=0{,}9\,(gk)^{1/2}-3kv, \tag{1}$$

where k is the wavenumber and v the convective velocity of the plasma particles near the ablation zone in which the unstable modes are localized.

The possibility of an analytical solution of the problem of the growth of the RT instability taking convection[16–20] and thermal conductivity[20] into account has been considered in a number of papers. The great majority of analytical approaches are based on a discontinuity model, in which the unstable region in the ablation zone with large gradients of the hydrodynamic quantities is replaced by a surface of discontinuity separating uniform plasma currents. In a model with a surface of discontinuity attempts to solve the problem encounter a fundamental difficulty first indicated in Ref. 16: the number of boundary conditions on the perturbed discontinuity surface is insufficient to solve the problem uniquely. To obtain the dispersion equation one needs additional boundary conditions (for subsonic flow only one) which are not a consequence of the conservation laws on the surface of discontinuity.

It is pointed out in Ref. 21 that the additional boundary condition needed for an analytical solution of the problem in the discontinuity model follows from the assumption of the evolutionarity of the unperturbed flow. The evolutionarity of a flow with a surface of discontinuity is a necessary and sufficient condition for the existence of a discontinuity structure,[22] which itself is caused by dissipation. The variation of this additional condition guarantees in principle an additional relation for small perturbations, necessary to obtain the dispersion equation, which, however, cannot be written in analytical form in the discontinuity model.

The explicit introduction of a discontinuity structure into our considerations makes it possible to state and uniquely solve the spectral problem, since then the additional variable—the amplitude of the perturbation of the position of the front of the discontinuity—is not introduced. In the static case the RT instability was considered in Refs. 11 and 23 to 25, where the structure of the unperturbed density and pressure profiles was taken into account. The results obtained in those papers demonstrate the saturation of the instability growth rate at perturbation wavelengths on the order of the size of the inhomogeneity, but they cannot explain

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the total suppression of the RT instability in the short-wavelength limit, observed in experiments and numerical calculations. One can describe this behavior of the instability growth rate only by consistently taking into account both the structure of the profiles of the density and of the hydrodynamic acceleration and the inhomogeneous material convection current in the unstable region together with the thermal conductivity and up to the present time this has not been done.[1)]

We show in this work that it is possible to use a semiclassical approximation to obtain a solution of the problem of the convective stabilization of the RT instability for any density profile structure of a stationary plasma and to explain the suppression of the RT instability in the short-wavelength limit of the perturbations: $\lambda = 2\pi k^{-1} \ll L$ (L is the scale of the inhomogeneity).

Let us consider the growth of the RT instability mode in the plasma formed when thin foils are ablatively accelerated, restricting ourselves therefore to planar geometry of the unperturbed flow. We discuss in Sec. 2 the linearized equations of ideal hydrodynamics describing the growth of small perturbations and the conditions for the applicability of the model. In Sec. 3 we obtain in the quasiclassical approximation the instability growth rate—the maximum eigenvalue (for a given wavenumber k)—and also the spectrum $\sigma_n(k)$ ($n = 0,1,2,\ldots$) for a steady inhomogeneous plasma with arbitrary density and acceleration profiles. We consider in Sec. 4 the stabilization of the short-wavelength RT instability by convection and find the values of the wavevector for which the growth rate vanishes, and we make a comparison with the results of a numerical simulation.

2. HYDRODYNAMICAL MODEL. EQUATIONS FOR THE PERTURBATIONS

Under conditions when the laser radiation energy absorbed at the critical surface is distributed over the interior of the target much faster than the plasma expands hydrodynamically, a practically steady regime of motion for the plasma of the target is reached after the shock wave has left the free surface of the target and the rarefaction wave has passed.[26–28] The characteristic features of this steady motion, the shape of the unperturbed density, temperature, velocity, etc., profiles, are determined by the relative role played by such physical processes as convection, inertia, and thermal conductivity in the ablation zone, and also the intensity of the absorbed laser radiation. The main property of the quasisteady plasma flow is, on the one hand, the presence of a localized ablation front and, on the other hand, a maximum density in the transition region which separates the ablation zone from the accelerated part of the target and in which the condition for the development of the RT instability is satisfied, i.e., the hydrodynamic acceleration and the density gradient are in opposite directions.[29]

We assume that the small perturbations of the flow are two-dimensional in the xz-plane, where the z-axis is directed parallel to the gradients of the unperturbed quantities so that $g(z) > 0$. Because the unperturbed flow is one-dimensional and steady we shall look for small deviations of the system from the equilibrium position in the form

$$\Psi(t, x, z) = \Psi(z)\exp(ikx + \sigma t).$$

In the case where the unperturbed flow is unsteady with a characteristic time scale τ we must understand $\sigma = \sigma(t)$ to be the instantaneous growth rate, characterizing the rate of growth of the perturbations at a given time, provided $\sigma\tau \gg 1$. The growth of the perturbations after a finite time t_0 can be estimated to be

$$\exp\left(\int_0^{t_0} \sigma(t)\,dt\right).$$

In the adiabatic approximation the linearized equations of ideal hydrodynamics, describing small perturbations, have the form

$$\sigma\rho_1 + \frac{d}{dz}(\rho_1 v + \rho v_{1z}) + ik\rho v_{1x} = 0, \tag{2}$$

$$\rho\left(\sigma v_{1x} + v\frac{d}{dz}v_{1x}\right) = -ikP_1, \tag{3}$$

$$\rho\left[\sigma v_{1z} + \frac{d}{dz}(v v_{1z})\right] = -\frac{d}{dz}P_1 + \rho_1\left(g - v\frac{d}{dz}v\right), \tag{4}$$

$$\sigma P_1 + \gamma P_1\frac{d}{dz}v + \gamma P\left(\frac{d}{dz}v_{1z} + ikv_{1x}\right) + v\frac{d}{dz}P_1 = -\rho g v_{1z}, \tag{5}$$

where ρ, P, v, and g are, respectively, the unperturbed values of the density, the pressure, the hydrodynamic velocity, and the acceleration in a frame of reference fixed to the rear of the foil, the value of the adiabatic index $\gamma = (\partial \ln P/\partial \ln \rho)_S$ is assumed to be constant, and the small perturbations are indicated by the index 1.

The adiabaticity condition means that the wavelengths of the perturbations cannot be too small. Assuming that the effects connected with thermal conductivity are small we obtain a lower limit for the wavelength of the perturbations in the form

$$\lambda/L \gg (l_e/LM)^{1/2},$$

where l_e is the electron mean free path and M the Mach number.

In what follows (except in Sec. 3) we shall consider the development of perturbations in the limit of an incompressible medium: $\gamma \to \infty$. This assumption, which is valid for the plasma of targets irradiated by sufficiently strong and long-wavelength laser radiation when the plasma flow in the ablation region is strongly subsonic ($M \approx 0.1$–0.2), is not fundamental for the solution considered here, but it permits considerable simplification of the calculations. It was shown in Ref. 20 that the compressibility of the plasma in the present problem can be neglected when the following conditions are satisfied:

$$v \ll c_T, \quad k^{-1} \ll c_T^2/g, \quad \sigma \ll kc_T,$$

where $c_T = (P/\rho)^{1/2}$ is the isothermal sound velocity. We note that in the WKB approximation one can also consider the development of RT perturbations taking the finite compressibility of the plasma into account in the limit of a supersonic flow in the unstable region; this can occur in the case of short-wavelength low-power laser radiation when the critical surface is close to the ablation zone.[30]

In the $\gamma\to\infty$ limit the adiabaticity equation takes the form

$$P_1\frac{d}{dz}v+P\left(\frac{d}{dz}v_{1z}+ikv_{1x}\right)=0. \quad (6)$$

The instability growth rate is an eigenvalue of the boundary value problem consisting of Eqs. (2)–(4) and (6) and the boundary conditions corresponding to the actual physical statement of the problem. As is usually done, we shall assume that the inhomogeneous ablation region in which the instability develops separates two plasma flows for which the flow may be assumed to be uniform. In that case the boundary conditions reduce to the evanescence of the perturbations as $z\to\pm\infty$. That in the quasi-classical limit considered here, where the eigenfunctions are strongly localized in the inhomogeneous region, the actual form of the boundary conditions has little effect on the eigenvalues.

3. SPECTRUM OF THE RT INSTABILITY GROWTH RATE FOR A STATIONARY PLASMA

To illustrate the WKB approximation we consider the case of a plasma which is stationary in the unperturbed state.[2)] In that case the complete set of equations (2)–(5) for the perturbations can be reduced to a single equation for v_{1z}:

$$\frac{d}{dz}\left(\frac{\rho\sigma^2\gamma P}{\rho\sigma^2+k^2\gamma P}\frac{d}{dz}v_{1z}\right)-\left[\rho\sigma^2+g\frac{d\rho}{dz}-\frac{k^2\rho^2g^2}{\rho\sigma^2+k^2\gamma P}-\frac{d}{dz}\left(\frac{\sigma^2\rho^2g}{\rho\sigma^2+k^2\gamma P}\right)\right]v_{1z}=0. \quad (7)$$

Considering perturbations with wavelengths smaller than the gradient length of $L\sim c^2/g$, $c=(\gamma P/\rho)^{1/2}$, we can drop in Eq. (7) terms which are small in the parameter $(kL)^{-2}$:

$$\frac{d^2}{dz^2}V_{1z}-k^2\left[1-\frac{g\alpha(z)}{\sigma^2}-\frac{g^2}{\sigma^2c^2}\right]v_{1z}=0 \quad (7a)$$

[$\alpha(z)=-d\ln\rho/dz$ is the slope of the density profile] and thus arrive at a Schrödinger equation for the problem of stationary states of a particle with zero energy in a one-dimensional potential well:

$$U(z)=k^2(1-g\alpha/\sigma^2-g^2/\sigma^2c^2).$$

We find the eigenvalues $\sigma_n(k)$ from the condition that there exist stationary localized states.

Expanding the potential $U(z)$ near its minimum $z=z_m$ in a Taylor series and assuming $d^2U(z_m)/dz^2\neq0$ we can find the eigenvalues σ_n using the well known expression for the energy levels of an harmonic oscillator:[31]

$$\sigma_n=\left(g\alpha+\frac{g^2}{c^2}\right)(z_m)-\frac{2n+1}{4k}\left[-2\frac{d^2[g\alpha+g^2/c^2]}{dz^2}(z_m)\right]^{1/2},\quad n=0,1,2,\ldots, \quad (8)$$

where we have taken into account the terms of zeroth and first order in $(kL)^{-1}$. Equation (8) is valid for $n\ll kL$. The corresponding eigenfunctions have the form

$$v_{1z}=\exp(-\omega_nz^2/2)H_n(\omega_n^{1/2}z),$$

$$\omega_n=\left[-\frac{k^2}{2\sigma_n^2}\frac{d^2(g\alpha+g^2/c^2)}{dz^2}(z_m)\right]^{1/2},$$

where the $H_n(x)$ are Hermite polynomials.

To zeroth order in the small parameter $(kL)^{-1}$ we can neglect the splitting of the levels in the spectrum (8) and thus conclude that the RT instability growth rate—the maximum eigenvalue for a given k—is

$$\sigma_0=\max[(g\alpha+g^2/c^2)^{1/2}].$$

We find the spectrum of the eigenvalues σ_n for $n\gg kL$ by using the Bohr–Sommerfeld quantization rule:

$$k\int_{z_1}^{z_2}\left(\frac{g\alpha}{\sigma_n^2}+\frac{g^2}{\sigma_n^2c^2}-1\right)^{1/2}dz=\pi(n+1/2), \quad (9)$$

where z_1 and z_2 are the turning points, i.e., the points for which $U(z)=0$.

Note that including the terms which were dropped in Eq. (7) would lead to corrections of higher than second order in $(kL)^{-1}$ in the spectra (8) and (9).

To first order in $(kL)^{-1}$ the eigenfunctions corresponding to the eigenvalues (9) have the form

$$v_{1z}=\left(1-\frac{g\alpha}{\sigma_n^2}-\frac{g^2}{\sigma_n^2c^2}\right)^{-1/4}\times\exp\left[k\int_z^{z_1}\left(1-\frac{g\alpha}{\sigma_n^2}-\frac{g^2}{\sigma_n^2c^2}\right)^{1/2}dz'\right],\quad z<z_1,$$

$$v_{1z}=\left(1-\frac{g\alpha}{\sigma_n^2}-\frac{g^2}{\sigma_n^2c^2}\right)^{-1/4}\times\exp\left[k\int_{z_2}^{z}\left(1-\frac{g\alpha}{\sigma_n^2}-\frac{g^2}{\sigma_n^2c^2}\right)^{1/2}dz'\right],\quad z>z_2,$$

$$v_{1z}=2\left(\frac{g\alpha}{\sigma_n^2}+\frac{g^2}{\sigma_n^2c^2}-1\right)^{-1/4}\times\cos\left[k\int_{z_1}^{z}\left(\frac{g^2}{\sigma_n^2c^2}+\frac{g\alpha}{\sigma_n^2}-1\right)^{1/2}dz'-\frac{\pi}{4}\right],\quad z_1<z<z_2. \quad (10)$$

Expressions (10) for the eigenfunctions are valid everywhere except in small regions near the turning points ($|z-z_{1,2}|\ll kL$), where the eigenfunctions are given in terms of Airy functions.[31]

In particular, for an exponential unperturbed density profile $\rho(z)\propto\exp(-\alpha_0z)$ $(0<z<L)$ and a constant acceleration $g=g_0$ in the limit $\gamma\to\infty$ Eq. (9) gives a spectrum of the form

$$\sigma_n=\left\{g_0\alpha_0\Big/\left[1+\frac{[\pi(n+1/2)]^2}{(kL)^2}\right]\right\}^{1/2},$$

which for $n\gg kL$ is the same as the exact result obtained in Ref. 24.

4. SUPPRESSION OF THE RT INSTABILITY BY CONVECTION

We consider the behavior of the perturbations in the case where there is inhomogeneous convective flow of material from the unstable region. In the WKB approximation we look for a solution of the linearized system of equations for the perturbations (2) to (4) and (6) in the incompressible limit in the form

$$\Psi(z)=\sum_j \psi_j \exp\left[k\int^z \varphi_j(z')dz'\right], \tag{11}$$

where $\Psi = v_{1z}, v_{1x}, \rho_1$, and P_1 and the amplitudes ψ_j are functions of z which change slowly compared to the exponential terms. Substituting solutions of the form (11) into Eqs. (2) to (4) and (6) and equating to zero the determinant of the resulting homogeneous algebraic system, we get a characteristic fourth-order equation in the φ_j ($j = 1,2,3,4$):

$$\mathcal{R}_4(\varphi)\equiv(\Sigma+\varphi)^2(\varphi^2-1)-(\Sigma+\varphi)\left(g-v\frac{dv}{dz}\right)\times\frac{M^2}{v^2k^2}\frac{d\ln v}{dz}\varphi+\frac{G}{4}=0. \tag{12}$$

We have introduced here the following dimensionless parameters: the dimensionless growth rate $\Sigma(z) = \sigma/vk$, the dimensionless acceleration

$$G(z)=-\frac{4}{(vk)^2}\left(g-v\frac{dv}{dz}\right)\frac{d\ln\rho}{dz}$$

and the Mach number $M(z) = v/c_T$. We have already mentioned that we restrict ourselves to the case of a subsonic flow, $M\ll 1$ and hence we neglect the second term in the characteristic equation (12).

The roots φ_j depend on the z coordinate parametrically. In the regions where we can consider the flow to be uniform we have

$$\varphi_1(z\to\pm\infty)=1,\quad \varphi_2(z\to\pm\infty)=-1,\quad \varphi_{3,4}(z\to\pm\infty)=-\Sigma,$$

and the roots $\varphi_{1,2}$ then describe acoustic modes and $\varphi_{3,4}$ entropic and rotational modes. A perturbation localized in the gradient region, i.e., evanescent as $z\to\pm\infty$ must for $\sigma>0$ have the asymptotic forms

$$\Psi(z)=\psi\exp(k\varphi_1 z),\quad z\to-\infty, \tag{13}$$

$$\Psi(z)=\sum_{j=2}^{4}\psi_j\exp(k\varphi_j z),\quad z\to\infty, \tag{14}$$

which corresponds to a sound wave in the limit $z\to-\infty$ and a superposition of entropy, vortex, and sound waves in the limit $z\to+\infty$. The characteristic behavior of the roots φ_j of the dispersion relation (12) in the complex φ-plane when one moves along the unperturbed profile in the direction of increasing z is shown in Fig. 1. One can find at most two roots simultaneously in the half-plane $\mathrm{Re}\,\varphi>0$. The roots may intersect for some $z=z_*$, becoming multiple. In order to select a localized perturbation, i.e., one which has the asymptotic behavior (13) and (14), it is necessary that there exist points on the profile in which a multiple root of Eq. (12) is formed through the intersection of the root $\varphi_1(z)$ and one of the other roots $\varphi_j(z)$ in the half-plane $\mathrm{Re}\,\varphi>0$. This is possible if the value of the parameter σ is less than a threshold value $\sigma_0(k)$ (Fig. 1a). In the limiting case $\sigma=\sigma_0(k)$ there is one such point. The value σ_0 will then be the maximum eigenvalue for a given k (i.e., the instability growth rate) in zeroth order in the small parameter $(kL)^{-1}$.

If we write down the condition for the multiplicity of the root:

$$\mathcal{R}_4(\varphi_*)=0,\quad d\mathcal{R}_4/d\varphi=0,$$

we find the values φ_* and Σ_* corresponding to them:

$$\varphi_*=\left[1-\frac{G^{1/3}}{(s+1)^{2/3}+(s-1)^{2/3}+{}^1/_3G^{1/3}}\right]^{1/2}, \tag{15}$$

$$\Sigma_*=\frac{1-2\varphi_*^2}{\varphi_*}, \tag{16}$$

where $s=(1+G/27)^{1/2}$.

The maximum eigenvalue σ_0 for a given k is determined by the solution of a set of algebraic equations:

$$\Sigma(z)=\Sigma_*(z), \tag{17}$$

$$\frac{d}{dz}\Sigma(z)=\frac{d}{dz}\Sigma_*(z). \tag{18}$$

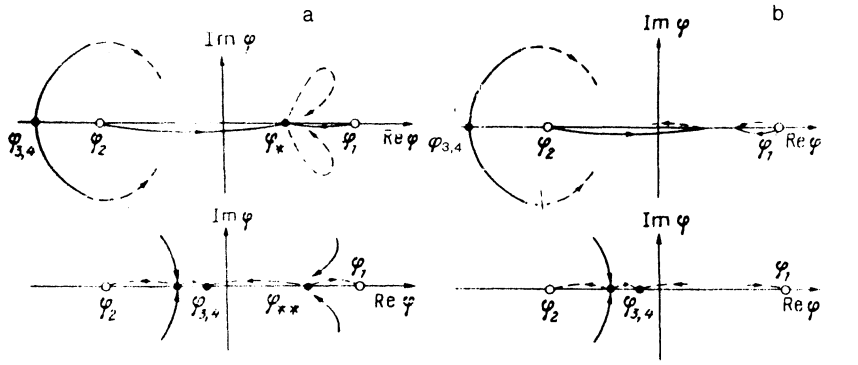

FIG. 1. Trajectories of the roots φ_j ($j=1,2,3,4$) of the characteristic equation (12) in the complex plane found when moving along the profile in the direction of increasing z (a) for $\sigma(k)<\sigma_0(k)$; the roots intersect, becoming multiple in the $\mathrm{Re}\,\varphi>0$ half-plane; and (b) for $\sigma(k)>\sigma_0(k)$.

Using the explicit form of the functions $\Sigma(z)$ and $\Sigma_*(z)$ we can eliminate the eigenvalue from the set (17) and (18) and find for the coordinate z_* of the contact point the equation

$$\frac{d}{dz}G=\frac{1-2\varphi_*^2}{1+2\varphi_*^2}[12(1-\varphi_*^2)^+ + G]\frac{d}{dz}\ln v, \tag{19}$$

where $\varphi_*=\varphi_*(G(z_*))$ is given by Eq. (15). The required growth rate σ_0 will then be

$$\sigma_0=kv(z_*)\Sigma_*(z_*). \tag{20}$$

The problem of finding the instability growth rate as an implicit function of the wavenumber for given profiles of the hydrodynamic variables is solved in principle by Eqs. (19) and (20).

The eigenfunction corresponding to the maximum eigenvalue has the form

$$\Psi(z)=\begin{cases}\exp\left[k\int\limits_{z_*}^{z}\varphi_+(z')dz'\right], & z>z_*,\\ \exp\left[k\int\limits_{z_*}^{z}\varphi_-(z')dz'\right], & z<z_*,\end{cases} \tag{21}$$

where φ_+ is that root of the characteristic Eq. (12) which for $z>z_*$ is shifted along the $\operatorname{Im}\varphi=0$ axis from the point $\varphi=\varphi_*$ in the direction of negative φ while φ_- is the root moving for $z<z_*$ from the point $\varphi=\varphi_*$ to the point $\varphi=1$ corresponding to a sound wave in the flow in front of the ablation front. The root φ_+, which is the asymptotic eigenfunction in a low-density plasma behind the ablation front, can correspond to either a sound wave [for $\sigma_0(k)>kv(\infty)$] or an entropic mode [for $\sigma_0(k)<kv(\infty)$, the case shown in Fig. 1a].

For $G_{\max}<1$, we have $\Sigma_*(z)<0$, and in this case there are no positive eigenvalues σ and the growth rate vanishes at once for the condition $G_{\max}=1$ when $\Sigma_*(G_{\max})=0$. We emphasize that this result is exact in the quasiclassical approximation and for a given form of the profile enables us to find the wavenumber k_0 for which the growth rate vanishes:

$$k_0=[\max(-4gv^{-2}d\ln\rho/dz)]^{1/2}. \tag{22}$$

Of course, it only makes sense to consider the vanishing of $\sigma_0(k)$ for stationary unperturbed solutions. If the unperturbed solutions are quasistationary with characteristic times τ we must, as we noted already in Sec. 2, consider only growth rates which are larger than τ^{-1}.

Note that Eq. (22) gives a somewhat different scaling for the values k_0 than the approximate Eq. (1), although their numerical values for characteristic parameters of the dynamics of the plasma of the targets,

$$g\sim10^{15}\ \text{cm/s}^2,\quad v\sim10^4-10^5\ \text{cm/s},$$

$$(d\ln\rho/dz)^{-1}\sim(1-3)\cdot10^{-4}\ \text{cm},$$

are rather close to one another.

One shows easily that in the limit $k\ll(g/L)^{1/2}/v$ Eqs. (19) and (20) reproduce the result of Sec. 3 for an incompressible fluid: $\sigma_0=\max(-gd\ln\rho/dz)$. In accordance with (22) the effective suppression of the RT instability occurs for $k\sim(g/L)^{1/2}/v$. This means that the condition for the applicability of the quasi-classical approximation is that the local Froude number $\mathrm{Fr}(z)$ be small in the region where the instability develops:

$$\mathrm{Fr}(z)=v^2/gL\ll1. \tag{23}$$

In Fig. 2 we compare the dispersion relations obtained in Ref. 21 as a result of a numerical simulation of the growth of the RT instability in the plasma of an aluminum foil of thickness $10\,\mu$m irradiated by the light from a neodymium laser of wavelength $\lambda_0=1.06\,\mu$m with an intensity $I=10^{13}$ W/cm^2 and results obtained in the quasiclassical approximation from Eqs. (19) and (20) for the same unperturbed solutions. For the numerical solution of the problem the boundary conditions on the eigenvalues for Eqs. (2) to (5) taken in the form of evanescence of the perturbations at $z\to\infty$ and leaving the point $z=0$, corresponding to the backside of the foil, were implemented using the analytically calculated

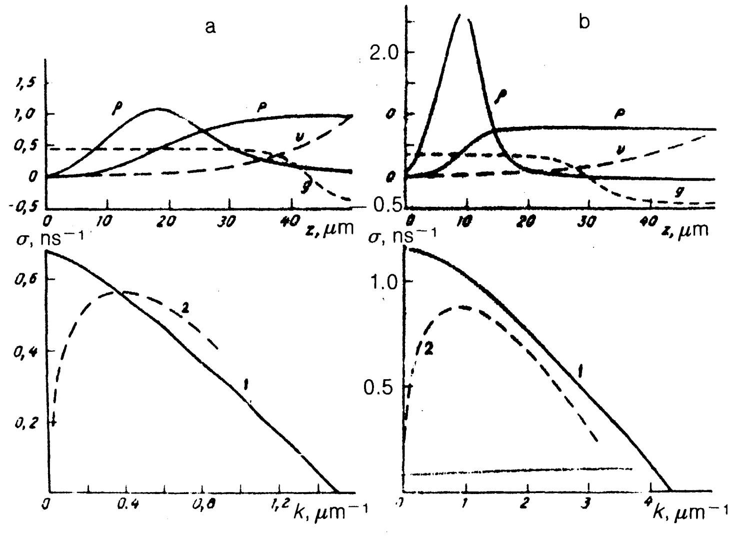

FIG. 2. Profiles (from Ref. 21) of ρ, P (Mbar), v (10^6 cm/s), and g (10^{15} cm/s^2) in the quasisteady stage of the expansion of an aluminum foil of $10\,\mu$m thickness irradiated by light from a neodymium laser ($\lambda_0=1.06\,\mu$m, $I=10^{13}$ W/cm^2) in the rest system of the backside of the target, and the dispersion curves of the RT instability growth rates obtained in the quasi-classical approximation using Eqs. (19) and (20) (curves *1*) and as the result of solving the eigenvalue problem in Ref. 21 (curves *2*); with (a) and without (b) including the x-ray emission from the corona.

asymptotic form.[21] Here, however, we show the characteristic quasistationary unperturbed profiles of the hydrodynamic variables, established after the passage of the shock wave and the discharge wave, taking into account the x-ray emission from the corona (Fig. 2a) and also in the case when the x-ray emission is neglected (Fig. 2b). The one-dimensional motion of the foil was simulated using hydrodynamic Lagrangian "Impul's" code described in Ref. 32, taking into account the real equation of state, the separation of the electron and the ion temperatures, and the ionization kinetics.[3)] The simulated suppression of the x-ray emission from the plasma corona leads to an increase in the Froude number in the unstable region (due to the large slope of the profile). For the flow parameters of the flow in Fig. 2b we have $Fr \sim 1$, i.e., condition (23) is not satisfied, but nevertheless it is clear from a comparison of the dispersion curves obtained in the WKB approximation and the exact solution that the agreement is very satisfactory even outside the formal region of applicability of the WKB approximation.

5. CONCLUSION

In the present paper we have solved in the quasiclassical approximation the problem of the convective stabilization of the RT instability in laser target plasma accelerated by the ablation pressure, taking into account the structure of the density, acceleration, and convective velocity profiles. The instability growth rate, which in the general case is an eigenvalue of the boundary value problem, is found in the form of an implicit algebraic function of the wavenumber and the structure of the unperturbed hydrodynamic profiles.

The formal self-consistency condition of the proposed model is that the local Froude number be small, but comparison with the results of a numerical simulation demonstrates that the RT instability growth rates obtained in the WKB approximation agree very satisfactorily with the exact values obtained as the result of a numerical solution of the problem of the eigenvalues even for rather steep density gradients when the Froude number is of order unity.

The calculations were performed in the subsonic-flow limit of an incompressible plasma, but the proposed method allows a direct generalization and makes it possible to include in the considerations the finite compressibility of the plasma and also to calculate the effect of the electron thermal conductivity and other dissipative processes on the growth rate of the RT modes.

[1)]Note that Ref. 5 reaches a qualitatively incorrect conclusion about the saturation of the growth rate of the RT instability in the short-wavelength limit when convection is present.

[2)]Fradkin[23] was the first to use the WKB approximation for the problem of the hydrodynamic instability of an inhomogeneous quasi-isothermal plasma; he obtained spectra which are similar to the ones obtained by us in the present section.

[3)]For details about the one-dimensional numerical simulation of the foil dynamics see Ref. 21.

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