Channeling of waves in the whistler frequency range within nonuniform plasma structures

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(Submitted 4 February 1992)
Zh. Eksp. Teor. Fiz. **102**, 1151–1166 (October 1992)

We investigate the channeling of propagating waves with frequencies in the whistler range within density ducts in which the plasma density variation transverse to an external magnetic field is nonmonotonic. Such structures arise near a radiator in a magnetized plasma as a result of thermal nonlinear effects. In such plasmas two independent modes of waveguide propagation are observed: in one, the field is concentrated in the central part of the channel, where the plasma density is lower than the background density, while the field of the other is largest near the central part of an annular layer whose density is higher than the background. We have carried out detailed phase and amplitude measurements of the fields of these waves. Based on experimental data and the results of theoretical calculations we have established that channeling of conical refraction waves occurs in the rarefied plasma region, while channeling of whistlers occurs in the layer with increased density. We show that under certain conditions leakage of energy from the high-density duct is extremely small when the latter has a width comparable to the wavelength of a whistler. We compare the results of our theoretical calculations with the experimental data.

1. INTRODUCTION

A large number of papers (see, e.g., Refs. 1–5) have dealt with the problem of guided propagation of whistler-frequency electromagnetic waves in density ducts of a magnetoactive plasma under the conditions

$$\omega_{LH} \ll \omega \ll \omega_{He} \ll \omega_{pe} \tag{1}$$

(here ω_{LH} is the lower hybrid frequency and ω_{He}, ω_{pe} are the gyrofrequency and plasma frequency of the electrons respectively). As a rule, the ducts under study have widths that are large on the scale of the propagation wavelength, with smooth plasma density profiles N_e that permit investigations of the dispersion properties and eigenmode fields of such channels using either the parabolic equation method[2] or the WKB approximation.[3]

Recently, there has been considerable interest in the distinctive features of guided propagation of whistler waves in ducts with nonmonotonic variations in the plasma density N_e in the transverse directions. Such channels can arise in a magnetized plasma near antenna structures due to thermal nonlinear effects.[4–7] Laboratory experiments have shown[4–7] that when sufficiently high levels of high-frequency power are fed to an antenna, a nonuniform channel forms which extends along the external magnetic field, within which the plasma density falls below the background value. This channel is caused by heating of the electrons in the near field of the antenna, which gives rise to a thermal-diffusion-driven redistribution of plasma near the radiator. As the channel with decreased plasma density N_e forms, it is accompanied by a surrounding annular layer with a plasma density higher than the background. The goal of this paper is to investigate experimentally and theoretically the channeling behavior of whistler waves in such plasma structures.

It is found that two types of guided whistler mode propagation are possible in these structures: localized modes, whose fields are concentrated in the vicinity of the low-density plasma, and quasilocalized (slightly lossy) modes whose fields are concentrated in the vicinity of the annular layer.

Recall that whistler mode energy leaks from a high-density duct as a result of the linear conversion of whistlers into fine-scale waves whose propagation is not supported in such channels.[1,3] Our interest here is primarily in "narrow" channels with widths $a < \lambda_B$ (where λ_B is the wavelength of a whistler propagating in the waveguide), for which the WKB approximation, which is used in the majority of papers dealing with channeling of whistlers in density ducts (see Ref. 3 and the literature cited therein), is not appropriate. We note that the dispersion properties and fields of guided modes in "narrow" ducts with increased plasma concentration form a topic of interest in its own right, which has not received enough attention. In this paper we show that when the condition $\omega_{He}/\omega \gg 1$ holds it is possible for slightly lossy whistler modes to exist in channels with increased density, provided that the characteristic transverse scales of confined and lossy waves are significantly different.

A further feature of the case we treat here is that the scales of longitudinal waves confined to the lower-density channel differs markedly from those of waves confined to the higher-density annular layer. In our experiments the condition $\omega_{He}/\omega \gg 1$ was well satisfied, in contrast to the experiments of Refs. 4 and 5; this allowed us to observe channeling of various types of whistler-frequency waves excited by an antenna in the nonuniform plasma.

2. EXPERIMENTAL RESULTS

Our experiments were carried out in a vacuum chamber 150 cm in length and 80 cm in diameter. An argon plasma was created at a pressure $p_0 = 5 \cdot 10^{-3}$ Torr by a high-frequency pulsed discharge in a uniform magnetic field $H_0 = 240$ G; this plasma was shaped like a quasiuniform column of length 100 cm and diameter 40 cm. Under the conditions of our experiment the electron temperature T_e and ion temperature T_i in the unperturbed plasma column

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coincided and came to $T_e \approx T_i \approx 0.4$ eV. The antenna, which was a loop of radius $a_0 = 2.5$ cm, was placed on the axis of the column, with the plane of the loop perpendicular to the external magnetic field. After switching off the pulse source, during the decay stage of a plasma with density $N_e \approx 4 \cdot 10^{12}$ cm^{-3} (characteristic decay time $\tau_N \approx 2$ msec) a high-frequency voltage was fed to the antenna with fixed frequency $\omega = 3.9 \cdot 10^8$ sec^{-1}, amplitude $U = 2$ to 150 V, and length up to $\tau_u = 2$ msec.

The background plasma density was monitored within the time of the experiments using a microwave interferometer (with wavelength $\lambda_0 = 8$ mm). The density perturbations in the vicinity of the antenna zone were measured by movable double and resonance microwave probes.[8] The spatial distribution of electromagnetic fields was investigated using a movable frame antenna (radius $a_1 = 0.5$ cm). In order to eliminate the dependence of the input impedances of the transmitting and receiving antennas on processes connected with precipitation of charged particles onto their surfaces, the antennas were covered by an insulating layer.[9] To study the spatial spectrum of the electromagnetic waves excited in the plasma, we used an interferometric method in which signals from the receiving antenna and excitation oscillator were fed to a balanced mixer (through a directional coupler) whose output signal $A = A_0 \cos \varphi$ allowed us to study the amplitude A_0 and the phase φ of the propagating wave.

The experiments were carried out with both linear and nonlinear power levels P fed to the antenna. In the linear case ($P \leqslant 2$ W) there was no density perturbation, and electromagnetic waves excited in the uniform plasma ($N_e \lesssim 4 \cdot 10^{12}$ cm^{-3} and $\omega_{pe} a_0 / c \gg 1$, where c is the velocity of light in vacuum) propagated from the source along the external magnetic field $\mathbf{H}_0 = H_0 \mathbf{z}_0$. Results of the corresponding

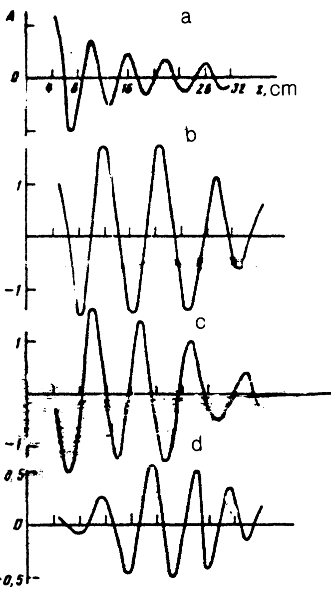

FIG. 1. Results of phase measurements of the field in a channel at time $\tau = 500\ \mu$sec (the amplitude of signal A is proportional to the quantity $|H_z|^2$): a – $r = 0$ cm ($P \leqslant 2$ W), b – $r = 0$ cm, c – $r = 2$ cm, d – $r = 7$ cm (for curves b through d, $P = 80$ W).

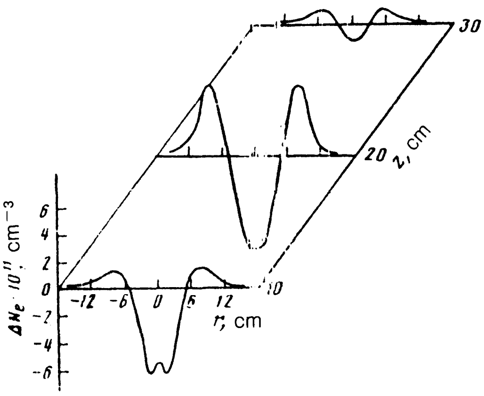

FIG. 2. Spatial distribution of the plasma density perturbations ΔN_e at time $\tau = 500\ \mu$sec ($N_B < 4 \cdot 10^{12}$ cm^{-3}, $P = 80$ W).

phase measurements, which were carried out as the receiving antenna was moved along the system axis $r = 0$ (the actual function was $A(z) = A_0 \cos k_z z$, where k_z was the longitudinal wave number) are shown in Fig. 1a. It is clear from Fig. 1a that in the linear case the wavelength of the wave propagating in the plasma was $\lambda_z \approx 6$ cm, while its amplitude fell off as z^{-1} with distance z from the source.

For sufficiently high power levels fed to the antenna ($P \geqslant 80$ W) the electron temperature near the radiator showed a marked increase due to ohmic heating of the plasma in the quasistatic field of the loop (the same as in the experiments of Refs. 4–7); at $\tau = 500\ \mu$sec after the start of the high-frequency voltage pulse, this temperature reached a value of $T_e \approx 1.5$ eV. As a result of electron heating and the thermal-diffusive redistribution of the plasma connected with it,[7] a perturbed density profile formed near the axis of the system (Fig. 2). It is clear from Fig. 2 that the perturbed density distribution was characterized by formation of a channel with decreased density on the axis of the system and an annular layer with an enhanced density surrounding this channel.

Measurements of the spatial distribution of the fields showed that the picture of electromagnetic field propagation is more complicated in the nonlinear case than it is at linear power levels. Near the channel with reduced plasma density we recorded guided propagation of electromagnetic radiation with a longitudinal wavelength on the order of $\lambda_1 \approx 9.5$ cm. Evidence of this is apparent in the phase measurements shown in Fig. 1b,c, which were carried out with the receiving antenna fairly close ($r \leqslant 2$ cm) to the system axis. Guided propagation of electromagnetic radiation was also observed in the region of enhanced plasma density ($r \approx 7$ cm). The longitudinal wavelength in this case came to $\lambda_2 \lesssim 5.5$ cm (see Fig. 1d, for 20 cm $< z <$ 30 cm).

The creation of waveguide channels at nonlinear power levels was confirmed by the structure of the equiphase lines in the perturbed plasma (Fig. 3). From Fig. 3 it follows that the waves trapped in the region with decreased plasma density had phase velocities that were almost perpendicular to the external magnetic field $\mathbf{H}_0$, while in the channel with increased density the phase velocity of the trapped waves was almost parallel to the field $\mathbf{H}_0$. We note that analogous effects, i.e., spatial separation of various types of waves in the whistler region, were observed experimentally in Ref. 4 at linear power levels within a nonuniform plasma with a specially selected density profile.

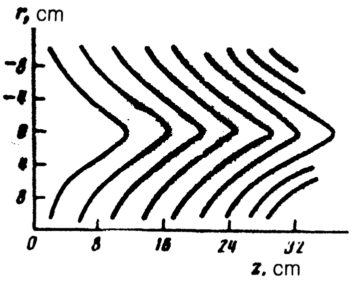

FIG. 3. Equiphase curves in the channel at time $\tau = 500\,\mu$sec for an input power $P = 80$ W (the phase shift between adjacent curves is $\Delta\varphi = \pi$).

The differences in character of the propagating electromagnetic waves in the linear and nonlinear cases are easy to trace by comparing the amplitude distributions of the fields in planes perpendicular to the system axis. In Fig. 4 we show the dependence of the squared amplitude of the longitudinal field component H_z on the transverse coordinate r at various distances z from the source. At linear power levels unchanneled propagation of waves occurs along the external magnetic field and we observe a characteristic distribution of $|H_z|^2$ with a maximum on the axis $r = 0$ (Fig. 4a). At nonlinear power levels additional maxima and minima appear in the transverse distribution $|H_z|^2$, which corresponds to the structure of the fields of the corresponding guided modes (Figs. 4b–4d). Close to the antenna (Figs. 4b, 4c) it is easy to see the field of a mode trapped in the lower-density channel; also noteworthy is the increase of the field at $r \approx 5$ cm, corresponding to a mode trapped in the annular layer (Fig. 4c). As the point of observation moves away from the source the channel "collapses" with decreased plasma density, the maximum of the wave field for the wave trapped in the annular region becomes more noticeable (Fig. 4d: $r \approx 4$ cm). As the distance from the source increases ($z > 30$ cm) the density perturbation gradually decreases and the waveguide propagation ceases.

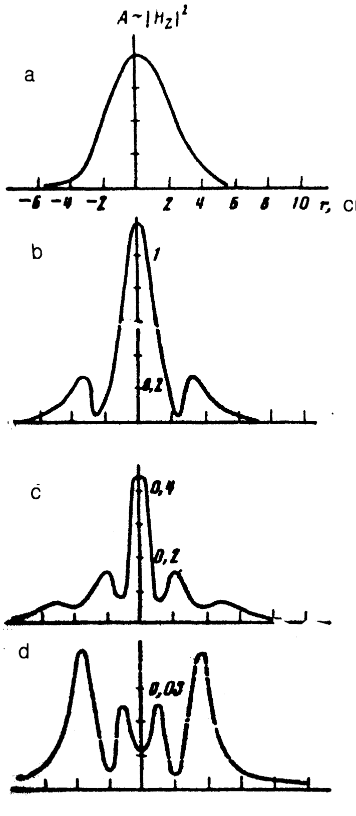

FIG. 4. Spatial distribution of fields in the channel at time $\tau = 500\,\mu$sec at various distances from the source: a – $z = 10$ cm ($P < 2$ W), b – $z = 10$ cm, c – $z = 20$ cm, d – $z = 30$ cm (for curves b to d, $P = 80$ W).

We studied the phenomenon of waveguide channel formation due to nonuniform variation in the plasma density in the transverse direction at various levels of high-frequency power. It is noteworthy that even rather small modulations of the plasma density (less than 10% of the background value) led to the creation of such waveguide channels.

We can find a qualitative explanation of the experimental results by analyzing the form of the whistler wave refractive index surface of a uniform plasma. Recall that in our case the dielectric permittivity tensor of a cold collisionless magnetized plasma $\mathbf{H}_0 \| \mathbf{z}_0$) can be written as follows:[10]

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon & -ig & 0 \\ ig & \varepsilon & 0 \\ 0 & 0 & \eta \end{pmatrix}, \tag{2}$$

where

$$\varepsilon = \frac{v}{u-1}, \quad g = -\frac{vu^{1/2}}{u-1}, \quad \eta = -v,$$

$$v = \omega_{pe}^2/\omega^2, \quad u = \frac{\omega_{He}^2}{\omega^2}.$$

In this case only an extraordinary wave can propagate, whose refractive index surface is described by the expression

$$q_{1,2}^2(p,v) = p^2\left(\frac{u}{2} - 1\right) - v \mp \frac{pu}{2}\left(p^2 - 4\frac{v}{u}\right)^{1/2}. \tag{3}$$

This is shown in Fig. 5 for three values of the density N_e: N_0, N_1, N_B, where $N_0 < N_B < N_1$. Here the quantities q and p denote components of the wave vector $\mathbf{k} = \mathbf{k}_\perp + \mathbf{k}_\| \mathbf{z}_0$ normalized by $k_0 = \omega/c$, i.e., its longitudinal $p = k_\| / k_0$ and transverse $q = k_\perp / k_0$ components, in a uniform plasma with density given by $N_e = m\omega^2 v/(4\pi e^2)$; the branch q_1 corresponds to the whistler wave characteristic, while branch q_2 describes quasielectrostatic waves (for $p^2 \gg 4\,v/u$); the region $q_1 \approx q_2$ ($p^2 \approx \mathcal{P}_c^2 = 4\,v/u$) corresponds to conical refraction waves.

At linear power levels we have $N_e = N_B$ ($v = v_B$) and

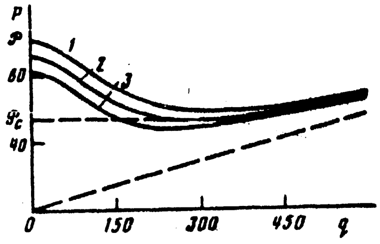

FIG. 5. Refractive index surface for various values of the density. 1—$N_1 = 4.8 \cdot 10^{12}$ cm^{-3}, 2—$N_B = 4 \cdot 10^{12}$ cm^{-3}, 3—$N_0 = 3.2 \cdot 10^{12}$ cm^{-3}; here $\omega = 3.9 \cdot 10^8$ sec^{-1}, $\omega_{He} = 4.2 \cdot 10^9$ sec^{-1} ($H_0 = 240$ Oe).

the condition $k_0 a_0 v_B^{1/2} \gg 1$ holds. Under these conditions, a loop of radius a_0 should be most effective in exciting those whistler waves with propagation constants $p \lesssim \mathscr{P}$, where $\mathscr{P} = [v_B/(u^{1/2}-1)]^{1/2}$ is the dimensionless propagation constant of a whistler along the external magnetic field in the uniform background plasma. The longitudinal wavelength scale observed in experiment $\lambda = 6$ cm ($p = 80.6$) is in good agreement with the theoretical estimate $\mathscr{P} = 90$ ($N_B = 3.8\cdot 10^{12}$ cm^{-3}).

For a plasma with a perturbed density profile, it follows from analysis of the refractive index surface that in the channel with the decreased density $N_e = N_0$ localized (nonlossy) waves can propagate when the condition

$$4v_0/u < p^2 < 4v_B/u \tag{4}$$

holds, while in the annular layer with increased density $N_e = N_1$ propagation of quasilocalized (slightly lossy) waves is possible ($v_{0,1} = v_B N_{0,1}/N_B$) under the condition

$$\max\left\{\frac{v_B}{u^{1/2}-1},\ 4\frac{v_1}{u}\right\} < p^2 < \frac{v_1}{u^{1/2}-1}. \tag{5}$$

The wavelengths measured under our experimental conditions ($N_0 \approx 3.2\cdot 10^{12}$ cm^{-3}, $N_B \approx 3.8\cdot 10^{12}$ cm^{-3}, $N_1 \gtrsim 4.0\cdot 10^{12}$ cm^{-3}), i.e., $\lambda_1 = 9.5$ cm and $\lambda_2 < 5.5$ cm ($p_1 = 50.9$, $p_2 = 91.3$) satisfied relations (4) and (5) respectively. This shows that at these nonlinear power levels we actually were observing the formation of waveguide channels for the two types of wave.

More specific and refined conclusions can be drawn based on the results of rigorous theoretical calculations. Here linear theory is sufficient to describe the distinctive features of channeling of whistlers, since the creation of the nonuniform density profile is not associated with the propagation of whistler waves, which have rather small amplitudes, but is rather a result of the nonlinear interaction of the strong quasistatic antenna field with the plasma.[7]

3. THEORETICAL CALCULATIONS

1. Consider a cylindrical plasma column oriented along an external magnetic field $\mathbf{H}_0$. The components of the fields of axially symmetric eigenmodes guided by the plasma column, which is nonuniform in radius, can be obtained from the following system of equations

$$\Delta_\perp E_\varphi - \frac{E_\varphi}{r^2} + \left(\frac{k_0^4 g^2}{h^2 - k_0^2\varepsilon} - h^2 + k_0^2\varepsilon\right)E_\varphi = \frac{k_0^2 g h}{h^2 - k_0^2\varepsilon}\frac{\partial E_z}{\partial r},$$

$$\Delta_\perp E_z + \frac{h^2}{h^2 - k_0^2\varepsilon}\frac{1}{\varepsilon}\frac{d\varepsilon}{dr}\frac{\partial E_z}{\partial r} - \frac{\eta}{\varepsilon}(h^2 - k_0^2\varepsilon)E_z$$

$$= \frac{h}{\varepsilon}(h^2 - k_0^2\varepsilon)\frac{1}{r}\frac{\partial}{\partial r}\frac{rgE_\varphi}{h^2 - k_0^2\varepsilon} \tag{6}$$

$$E_r = \frac{i}{h^2 - k_0^2\varepsilon}\left(h\frac{\partial E_z}{\partial r} - k_0^2 g E_\varphi\right), \qquad \mathbf{H} = ik_0^{-1}\,\mathrm{rot}\,\mathbf{E},$$

where $h = k_0 p$ is the propagation constant of a mode in the channel, r, φ, z are cylindrical coordinates, and

$$\Delta_\perp = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right),$$

we assume that all fields are proportional to $\exp(i\omega t - ihz)$.

Let us limit ourselves to the simplest model of the plasma density profile in the column: $N_e(r) = N_0$, $v(r) = v_0$ for $r < d$, $N_e(r) = N_1$, $v(r) = v_1$ for $d < r < d + 2a$, $N_e(r) = N_B$, $v(r) = v_B$ for $r > d + 2a$; here $N_{0,1}$, N_B, $v_{0,1}$, and v_B are constants such that $N_0 < N_B < N_1$ ($v_0 < v_B < v_1$). It is not difficult to show that when condition (4) and $k_0 a|\mathrm{Im}\, q_{1,2}(p, v_1)| \gg 1$ hold, it is sufficient to consider the simpler density profile $N_e(r) = N_0$, $v(r) = v_0$ for $r < d$, $N(r) = N_1$, $v(r) = v_1$ for $r > d$ to describe the dispersion properties and eigenmode field structure for the modes localized in the region of low-density plasma. In this case the solution to Eq. (6) can be written in the following way

a) $r < d$,

$$E_\varphi = \sum_{m=1}^{2} A_m J_1(k_0 q_m r)\exp(-ihz),$$

$$E_z = -v_0^{-1}\sum_{m=1}^{2} A_m \tilde{n}_m q_m J_0(k_0 q_m r)\exp(-ihz) \tag{7}$$

b) $r > d$,

$$E_\varphi = \sum_{m=1}^{2} B_m K_1(k_0 s_m r)\exp(-ihz),$$

$$E_z = v_1^{-1}\sum_{m=1}^{2} B_m n_m s_m K_0(k_0 s_m r)\exp(-ihz),$$

where

$$q_m^2 = q_m^2(p, v_0),\quad s_m^2 = -q_m^2(p, v_1),$$

$$\tilde{n}_m = n_m(p, v_0),\quad n_m = n_m(p, v_1).$$

$$m = 1, 2,\quad n_{1,2}(p, v) = [p \mp (p^2 - 4v/u)^{1/2}]u^{1/2}/2;$$

here $J_n(\zeta)$ and $K_n(\zeta)$ are the Bessel function and modified Bessel function of the second kind, respectively, and A_m and B_m are constants.

From the condition of continuity of the tangential field components at $r = d$ we can obtain the following dispersion equation for the eigenmodes of the waveguide, which are valid for the case of channels with both increased and decreased plasma density:[11]

$$[J(Q_1)v_0/v_1 + C_1K(S_1) - C_2K(S_2)]J(Q_2) + [C_3J(Q_1) + K(S_1)]K(S_2) = C_4J(Q_1)K(S_1). \tag{8}$$

Here

$$J(Q_m) = \frac{J_1(Q_m)}{Q_m J_0(Q_m)},\qquad K(S_m) = \frac{K_1(S_m)}{S_m K_0(S_m)},$$

$$Q_m = k_0 d q_m,\qquad S_m = k_0 d s_m,$$

$$C_1 = C_0(\tilde{n}_2 - n_1)(n_2 v_0/v_1 - \tilde{n}_1),\ C_2 = C_0(\tilde{n}_2 - n_2)(n_1 v_0/v_1 - \tilde{n}_1),$$
$$C_3 = C_0(\tilde{n}_1 - n_2)(n_1 v_0/v_1 - \tilde{n}_2),\ C_4 = C_0(\tilde{n}_1 - n_1)(n_2 v_0/v_1 - \tilde{n}_2),$$
$$C_0 = (n_1 - n_2)^{-1}(\tilde{n}_1 - \tilde{n}_2)^{-1}.$$

Equation (8) coincides in form with Eq. (17) of Ref. 12, in which an analogous problem was considered. However,

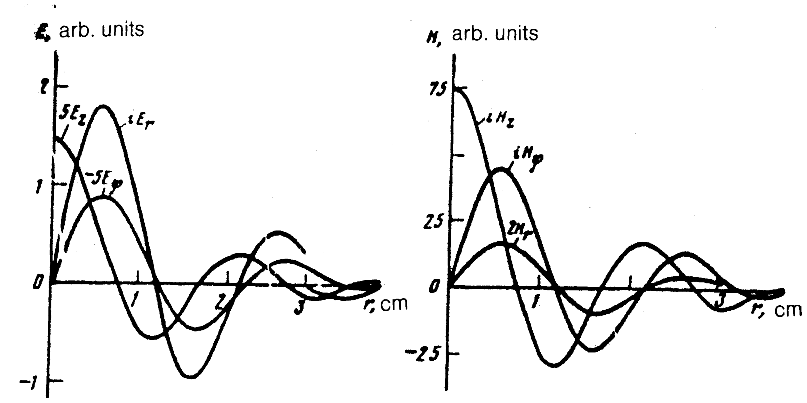

FIG. 6. Distribution of the field components in the transverse direction for the lowest mode in the channel with decreased plasma density: $\omega = 3.9\cdot 10^8$ sec^{-1}, $\omega_{He} = 4.2\cdot 10^9$ sec^{-1}, $d = 3$ cm, $N_0 = 3.2\cdot 10^{12}$ cm^{-3}, $N_1 = 4.8\cdot 10^{12}$ cm^{-3}.

there was some imprecision in the expressions used in Ref. 12 for the coefficients C_i, which greatly affected the results of the solution to the dispersion equation (especially for channels with increased plasma density).

The dimensionless propagation constants p for waveguide eigenmodes are roots of the dispersion relation (8), which in the general case must be investigated numerically. For the fundamental mode of the low-density cylindrical waveguide, at the pre-specified values $\omega = 3.9\cdot 10^8$ sec^{-1}, $\omega_{He} = 4.2\cdot 10^9$ sec^{-1} ($H_0 = 240$ Oe), $N_0 = 3.2\cdot 10^{12}$ cm^{-3}, $N_1 = 4.8\cdot 10^{12}$ cm^{-3}, $d = 3$ cm corresponding to the conditions of our laboratory experiment, numerical calculations give $p = 48.085$ (we also note that the values of the transverse wave numbers corresponding to this value of the propagation constant were $Q_1 = 8.89$, $Q_2 = 11.02$, $S_1 = S_2^* = 7.14 + i\cdot 10.02$, and the wavelength in the channel $\lambda_C = 10.06$ cm).

The distribution of field components for this mode along the transverse coordinate are shown in Fig. 6. It should be noted that the condition $k_0 a|\mathrm{Im} q_{1,2}(p,v_1)| = (a/d)\,\mathrm{Re}\, S_{1,2} \gg 1$ $(a\sim d)$ is satisfied here with a considerable margin, while the propagation constant p lies between the values $\mathscr{P}_0 = 2(v_0/u)^{1/2} = 47.83$ and $\mathscr{P}_1 = 2(v_1/u)^{1/2} = 58.59$, corresponding to conical refraction waves in a uniform plasma for the given values of the density N_0 and N_1 respectively. The condition $p \ll 2(v_B/u)^{1/2}$ [see (4)] also holds when the background plasma density $N_B > 3.3\cdot 10^{12}$ cm^{-3} (in our experiment $N_B \lesssim 4\cdot 10^{12}$ cm^{-3}). As the width of the waveguide d increases the propagation constant of a given eigenmode shifts to the value $\mathscr{P}_0$, while as the channel shrinks it shifts to the value $\mathscr{P}_1$.

2. We now turn to an analysis of the distinctive features of guided whistler propagation in the channel with increased plasma density. We begin our considerations with the simplest case, namely a planar waveguide (layer), since, as we will see later, under certain conditions the dispersion properties of modes guided by such a layer are in many ways analogous to those of modes of an annular layer observed in experiment. We specify the plasma density profile in the plane layer as follows: $N_e(x) = N_1$, $v(x) = v_1$, $|x| < a$, $N_e(x) = N_B$, $v(x) = v_B$, $|x| > a$, $N_B < N_1$ $(v_B < v_1)$.

It is well known that eigenwaves of a planar waveguide propagating along the z axis whose field does not depend on y can be divided into even modes [$E_y(0) \neq 0$] and odd modes [$E_y(0) = 0$]. Let us pause for a more detailed study of the dispersion properties of the even modes (the odd modes are treated analogously).

The components of the field of an individual even mode are written in the following form:

a) $|x| < a$,

$$E_y = \sum_{m=1}^{2} A_m \cos(k_0 q_m x)\exp(-ihz),$$

$$E_z = v_1^{-1}\sum_{m=1}^{2} A_m \tilde{n}_m q_m \sin(k_0 q_m x)\exp(-ihz),$$

b) $|x| > a$, (9)

$$E_y = [B_1\exp(-k_0 s_1|x|) + B_2\exp(ik_0 s_2|x|)]\exp(-ihz),$$

$$E_z = v_B^{-1}\,\mathrm{sign}\, x[B_1 n_1 s_1 \exp(-k_0 s_1|x|) - iB_2 n_2 s_2\exp(ik_0 s_2|x|)]\exp(-ihz),$$

where

$$q_m^2 = q_m^2(p, v_1),\ s_1^2 = -q_1^2(p, v_B),\ s_2^2 = q_2^2(p, v_B),$$
$$\tilde{n}_m = n_m(p, v_1),\ n_m = n_m(p, v_B),\ m = 1, 2, \tag{10}$$

and A_m and B_m are constants. In Eqs. (9), the first term (q_1) corresponds to a large-scale whistler trapped in the channel, while the second term (q_2) corresponds to fine-scale waves that leak from the waveguide. The choice of sign of $\mathrm{Re}S_2$ in (9), i.e., $\mathrm{Re}S_2 > 0$, is determined by the radiation condition corresponding to leakage of energy from the waveguide. The dispersion equation for the even modes can be obtained from Eq. (8) if we make the replacements $J(Q_m) \to (-\mathrm{ctg} Q_m)/Q_m$, $K(S_1) \to S_1^{-1}$, $K(S_2) \to iS_2^{-1}$, $v_0 \to v_1$, $v_1 \to v_B$, and $d \to a$, and use Eq. (10). Due to loss of energy the propagation constant p becomes complex: $p = p' - ip''$, $p'' > 0$.

Let us show that in certain special cases the losses of these waves due to radiation from the waveguide can become extremely small, so that $p'' \ll p'$. Let us consider a region of parameter values

$$v_1 \gtrsim v_B,\ u^{1/2} \gg 4v_1/v_B \tag{11}$$

for which the corresponding transverse characteristic scales of whistlers trapped in the channel differ considerably from those of the fine-scale waves leaking from the waveguide ($|q_2| \gg |q_1|$) for a rather small drop in the plasma density at the boundary $|x| = a$. In this case the dispersion equation takes the following form:

$$\left(\operatorname{ctg} Q_1 - \frac{Q_1}{S_1} + i\lambda(p)\frac{Q_1}{S_1}\right)\operatorname{ctg} Q_2$$

$$= i\frac{v_B}{v_1}\frac{Q_2}{S_2}\left(\operatorname{ctg} Q_1 - \frac{Q_1}{S_1}\right) - \mu(p)\operatorname{ctg} Q_1, \qquad (12)$$

where

$$\lambda(p) = \left(\frac{v_B}{p^2 u}\right)^3 \frac{S_1}{S_2}\left(\frac{v_1}{v_B} - 1\right)^2,$$

$$\mu(p) = \frac{v_B}{v_1}\frac{v_B}{p^2 u}\frac{Q_2}{S_1}\left(\frac{v_1}{v_B} - 1\right)^2.$$

A solution to Eq. (12) is conveniently sought in the form $p = \mathcal{P} + \Delta p$, where $\mathcal{P}$ is the propagation constant of a whistler along the external magnetic field in a uniform plasma with density N_B When condition (11) holds the quantities $\lambda(p)$ and $\mu(p)$ are small parameters, and

$$q_1^2 \approx q_1^2(\mathcal{P}, v_1) - \frac{1}{2}s_1^2\left(\frac{v_1^2}{v_B^2} + 1\right), \quad q_2^2 \approx \mathcal{P}^2 u - 2v_1,$$
$$s_1^2 \approx 4\Delta p\mathcal{P}, \; s_2^2 \approx \mathcal{P}^2 u - 2v_B. \qquad (13)$$

If Δp is not too small, so that we have

$$|\mu(p)| \approx \frac{1}{2}\left(\frac{\mathcal{P}}{\Delta p}\right)^{1/2}\left(\frac{v_1}{v_B} - 1\right)^2 \ll 1,$$

Eq. (12) can be solved quite easily by perturbation methods to determine the propagation constant $p'(\omega)$, along with the upper $p''_{\max}(\omega)$ and lower $p''_{\min}(\omega)$ boundaries of the region in which the whistler damping rate $p''(\omega)$ for the waveguide lies, i.e., $p''_{\min}(\omega) \leqslant p''(\omega) \leqslant p''_{\max}(\omega)$. It turns out that for even modes the maxima and minima of the function $p''(\omega)$ correspond to $\operatorname{Re} Q_2 = \pi(k - 1/2)$ and $\operatorname{Re} Q_2 = \pi k$ respectively, where $k = 1, 2, \ldots$.

Let us begin by calculating the real part p' of the complex propagation constant p. For $|\lambda(p)| \ll 1$, $|\mu(p)| \ll 1$, in zero-order perturbation theory we obtain from (12) the following equation for the quantity p':

$$\operatorname{ctg} Q_1 = Q_1/S_1. \qquad (14)$$

It is easy to show that Eq. (14) has a solution for any width of the channel.

For sufficiently wide waveguides, when

$$(k_0 a\mathcal{P})^2 \gg \frac{\pi^2(1+2n)^2}{4(v_1 - v_B)} v_B, \quad n = 0, 1, 2, \ldots,$$

the solution $p^{(0)}$ of Eq. (14) can be written in the form

$$p_n^{(0)} = \tilde{\mathcal{P}} - \frac{\pi^2(1+2n)^2}{16(k_0 a)^2\tilde{\mathcal{P}}}, \qquad (15)$$

where $\tilde{\mathcal{P}}$ is the dimensionless propagation constant of a whistler parallel to the external magnetic field in a uniform plasma with density N_1.

For a narrow channel, when

$$\frac{v_1}{v_B} - 1 \ll 2k_0 a\mathcal{P} \ll 2^{1/2}\left(\frac{v_1}{v_B} - 1\right)^{-1/2},$$

only the fundamental mode ($n = 0$) can propagate, for which we have

$$p_0^{(0)} = \mathcal{P} + \frac{1}{4}(k_0 a)^2\mathcal{P}^3\left(\frac{v_1^2}{v_B^2} - 1\right)^2. \qquad (16)$$

In order to calculate the quantities $p''_{\min}(\omega)$, $p''_{\max}(\omega)$ we make a series expansion of the left and right sides of Eq. (12) in powers of $\Delta p^{(0)} = p - p^{(0)}$ at the point $p = p^{(0)}$ such that $\operatorname{Re} Q_2(p^{(0)}) = \pi k$ and $\operatorname{Re} Q_2(p^{(0)}) = \pi(k - 1/2)$, assuming that $p'' \ll [k_0 a u^{1/2}]^{-1}$. Neglecting small terms of order $\lambda\Delta p^{(0)}$ and retaining terms proportional to the first power of $\Delta p^{(0)}$, we find (taking into account the relations $|\lambda| \ll 1$, $|\mu| \ll 1$) that $\Delta p^{(0)} = -ip''$, and

$$p''_{\min}(\omega) = \frac{1}{k_0 a u^{1/2}\beta(p^{(0)})}\left(\frac{v_B}{p^{(0)2}u}\right)^3\left(\frac{v_1}{v_\Phi} - 1\right)^2, \qquad (17)$$

$$p''_{\max}(\omega) = \frac{1}{2k_0 a\gamma(p^{(0)})}\frac{\mathcal{P}^2}{p^{(0)2}}\left(\frac{v_1}{v_\Phi} - 1\right)^2, \qquad (18)$$

where

$$\beta(p) = \left(1 + \frac{1}{S_1} + \frac{Q_1^2}{S_1^2}\right)\left[1 + 2\left(\frac{k_0 a p}{Q_1}\right)^2\right]$$
$$+ \frac{k_0 a p}{S_1}\left[\mu(p)u^{1/2} + \frac{k_0 a p}{S_1^2}\left(1 + \frac{\mathcal{P}^4}{p^4}\right)\right].$$

$$\gamma(p) = \left(1 + \frac{S_1}{Q_1^2} + \frac{S_1^2}{Q_1^2}\right)\left[1 + \frac{1}{2}\left(\frac{Q_1}{k_0 a p}\right)^2\right]$$
$$+ \frac{1}{2S_1}\left(1 + \frac{\mathcal{P}^4}{p^4}\right).$$

Equation (17) implies the inequalities $p''_{\min} \ll p'$, $p''_{\min} \ll [k_0 a u^{1/2}]^{-1}$. From (18), when $(v_1/v_B - 1)^2 \ll 2\gamma(p^{(0)}/u^{1/2}$ we find that $p''_{\max} \ll p'$, $p''_{\max} \ll [k_0 a u^{1/2}]^{-1}$, and

Thus, under the conditions (11) and $\operatorname{Re} Q_2 \geqslant \pi/2$ the leakage of energy from a waveguide with increased density turns out to be very slight (even in the case of a rather narrow channel, i.e., $k_0 a\mathcal{P} \lesssim 1$), while the dependence of the damping rate $p''(\omega)$ on frequency has a pronounced resonant character. This character of the function $p''(\omega)$ may be interpreted as a resonance of the fine-scale waves caused by their multiple reflections from the boundaries of the waveguide $|x| < a$. If we choose a smooth profile $N_e(x)$, the resonances of the function $p''(\omega)$ are smoothed out. Note that the behavior of the function $p''(\omega)$ for odd modes of a planar waveguide is analogous to the function $p''(\omega)$ for even modes, with the only difference that in the case of odd modes the maxima and minima of the quantity $p''(\omega)$ correspond to the conditions

$$\operatorname{Re} Q_2 = \pi k \text{ and } \operatorname{Re} Q_2 = \pi(k - 1/2), \; k = 1, 2, \ldots.$$

There is one important difference between the dispersion properties of even and odd modes of a planar waveguide. For even modes, in contrast to odd modes, there is no minimum critical value for the channel width. This implies that even as $a \to 0$ (in this case $\Delta p \to 0$, $S_1 \to 0$) a single slightly lossy mode can still propagate in the planar layer. In fact, as $a \to 0$ ($|Q_1| < |Q_2| \ll 1$) Eq. (12) has the solution

$$p_0 = \mathcal{P} + \frac{1}{4}(k_0 a)^2 \mathcal{P}^3 \left(\frac{v_1^2}{v_B^2} - 1\right)^2 (1-u_{cr}^{1/2}) \times\left[1-u_{cr}^{1/2}\left(1+2i\frac{v_B}{v_1}k_0 a \mathcal{P} u^{1/2}\right)\right], \qquad (19)$$

where

$$u_{cr} = \frac{\omega_{cr}^2}{\omega^2}, \quad \omega_{cr} = \omega_{He}\frac{v_B}{v_1}\frac{v_1 - v_B}{v_1 + v_B} \approx \omega_{He}\frac{N_1 - N_B}{N_1 + N_B}.$$

Solution (19) is valid under the condition

$$N_B < N_1 < N_B \frac{u^{1/2}+1}{u^{1/2}-1}, \qquad (20)$$

which [see (1)] can be fulfilled even for a rather small dropoff in plasma density in the channel ($v_1 > v_B$). Additional analysis shows that Eq. (20) is the condition for the existence of a weakly damped mode in a planar layer with thickness $a \to 0$ for all frequencies ω that satisfy the inequalities $\omega_{LH} \ll \omega < \omega_{He}/2 \ll \omega_{pe}$ [compare with (1)]. The solution (19) matches the results of Ref. 1, whose authors were the first to investigate energy leakage from narrow waveguides with increased density. In contrast to our paper, only the special case $|Q_{1,2}| \ll 1$ was investigated in Ref. 1, for which the whistler damping rate was determined numerically.[1)]

The results obtained above apply to the case $v_1 \gtrsim v_B$. However, it can be shown that slightly lossy whistler modes also exist in channels that are narrow on the scale of the propagation wavelength ($k_0 a p' \ll 1$) with an appreciable decrease in density ($v_1 \gg v_B$) when $u \gg 1$, $p' > 2(v_1/u)^{1/2}$.

We now present the results of our numerical calculations, which were carried out for a planar layer using the exact dispersion relation. For the fundamental even mode and values of the parameters $N_1 = 4.8 \cdot 10^{12}$ cm^{-3}, $N_B = 4 \cdot 10^{12}$ cm^{-3}, $a = 2$ cm, corresponding to the conditions of our experiment (for which the quantities ω, ω_{He} are the same as in Fig. 5), our numerical calculations give $p' = 98.436$, $p'' = 1.171 \cdot 10^{-2}$ (note that in this case the wavelength in the channel $\lambda_W = 4.92$ cm). The structure of the field for this mode is shown in Fig. 7. On these plots it is easy to see the different spatial scales in the field distributions in the transverse direction corresponding to large-scale whistlers and fine-scale leaking waves. The analogous functions for the components Re E_x, Im $E_{y,z}$, Im H_x, Re $H_{y,z}$, which are appreciably smaller in absolute value than the corresponding field components Im E_x, Re $E_{y,z}$, Re H_x, Im H_{yz}, are not shown in this figure. Note that decreasing the plasma density has the same effect as making the waveguide smaller; i.e. the propagation constant p' of a fixed mode decreases. For example, for $N_1 = 3.4 \cdot 10^{12}$ cm^{-3}, $N_B = 4 \cdot 10^{12}$ cm^{-3} (with values of ω, ω_{He}, a the same as in Fig. 7) we find that $p' = 89.596$, $p'' = 3.158 \cdot 10^{-2}$, and $\lambda_W = 5.40$ cm for the even fundamental mode.

3. These calculations make the analysis of guided propagation of whistlers in an annular layer with increased plasma density much easier. When conditions (5) and (11) hold, the leakage of energy from an annular layer is small as before, and it is sufficient to investigate the dispersion equation in zero-order perturbation theory [compare with (14)] in order to determine the propagation constant p':

$$J_0(\tilde{Q}_1)Y_0(Q_1)[Y(Q_1)+K(S_1)][J(\tilde{Q}_1)-I(\tilde{S}_1)] = J_0(Q_1)Y_0(\tilde{Q}_1)[Y(\tilde{Q}_1)-I(\tilde{S}_1)][J(Q_1)+K(S_1)]. \qquad (21)$$

Here

$$\tilde{Q}_1 = k_0 d q_1, \quad Q_1 = k_0(d+2a)q_1, \quad \tilde{S}_1 = k_0 d s_1,$$
$$S_1 = k_0(d+2a)s_1, \quad Y(Q) = \frac{Y_1(Q)}{QY_0(Q)}, \quad I(S) = \frac{I_1(S)}{SI_0(S)},$$

where $Y_n(\zeta)$ is a Neumann function and $I_n(\zeta)$ is a modified Bessel function; the remaining notation is the same as in (12). In expression (21) we have used a simplifying assumption $N_0 = N_B$, which in the present case has negligible influence on the results of the analysis of the dispersion equation.

It is not difficult to show that for $|Q_2| \gg |Q_1|$ Eq. (21) has solutions that correspond to individual slightly lossy whistler modes. For the special cases $k_0(d+a)q_1 = \pi(m + 1/4)$, $k_0(d+a)q_1 = \pi(m+3/4)$, $m = 0,1,2,\ldots$, and the additional condition $\tilde{S}_1 \gg 1$, Eq. (21) separates into a pair of zero-order equations for the even and odd modes of a planar waveguide with width $2a$ [compare with (14)]:

$$(\mathrm{tg}(k_0 a q_1) + q_1/s_1)(\mathrm{ctg}(k_0 a q_1) - q_1/s_1) = 0.$$

In this case the dispersion properties of the waveguide formed by the annular channel are practically the same as those properties of the planar waveguide investigated above.

The distribution of field components of the fundamental mode of an annular layer are analogous to the functions shown in Fig. 7: the radial and azimuthal components $E_{r,\varphi}$,

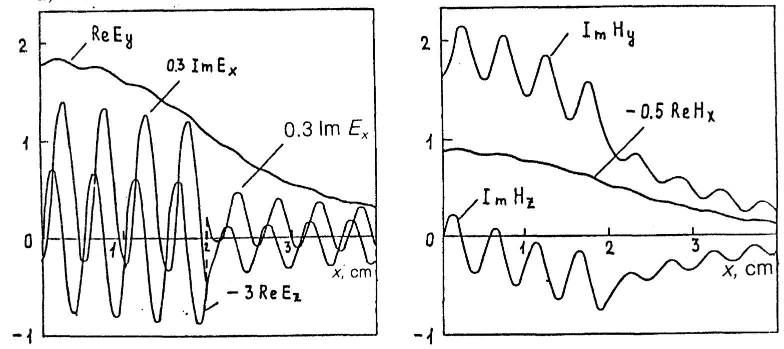

FIG. 7. Distribution of the field components in the transverse direction for the lowest mode in the layer with increased plasma density: $\omega = 3.9 \cdot 10^8$ sec^{-1}, $\omega_{He} = 4.2 \cdot 10^9$ sec^{-1}, $a = 2$ cm, $N_1 = 4.8 \cdot 10^{12}$ cm^{-3}, $N_B = 4 \cdot 10^{12}$ cm^{-3}.

$H_{r,\varphi}$ reach their maximum values within the layer and vanish on the axis $r = 0$, while the longitudinal components E_z, H_z reach their maximum values for $r_{\pm} \simeq d + a \pm a$ and vanish within the layer.

We will not analyze Eq. (21) in detail, since the results of solving the model problem investigated above are completely adequate to explain all the essential features of channeling of whistlers observed in the experiment.

4. DISCUSSION OF RESULTS

We now compare the experimental and theoretical results. However, let us note at the outset that in comparing the transverse distributions of $|H_z|^2$ found in experiment (Fig. 4) with the corresponding theoretical functions (Figs. 6 and 7), it must be kept in mind that in the experiments we measured the total field, i.e., the sum of the field of the discrete waveguide modes and the "background" field of the fine-scale quasipotential waves which were also excited by the antenna.

A comparison of the results of theoretical calculations of the field H_z in the channel with decreased plasma density (Fig. 6) with the experimental data (Figs. 4b and 4c) shows good qualitative agreement between them (and even quantitative agreement: compare Figs. 4b with Fig. 6). The experimental value $\lambda_1 \approx 9.5$ cm for the wavelength for a wave propagating in the near-axis part of the channel also corresponds to the theoretical value $\lambda_c = 10.06$ cm.

There is also satisfactory agreement between the value of the wavelength measured in experiment ($\lambda_2 \lesssim 5.5$ cm) and the results of the theoretical calculations ($\lambda_W = 5$ to 5.4 cm) for the layer with increased plasma density as well. As for the radial distribution of the field in the layer, we clearly discern a maximum of $|H_z|$ on the plots of $|H_z|^2$ (Figs. 4c and 4d, at the points $z = 20$ cm, $r \approx 5$ cm and $z = 30$ cm, $r \approx 4$ cm, respectively), which are located in the vicinity of the inner boundary $r = r_-$ of the annular layer, in agreement with theoretical predictions. The maximum of $|H_z|$ near the external boundary of the layer $r_+ \approx 10$ cm is small and cannot be seen in Fig. 4. Note that according to the theoretical calculations, for the case $p' \gtrsim \mathscr{P}$, $N_1 \gtrsim N_B > N_0$ (see Fig. 2), the magnitudes of the fields E_z, H_z of the fundamental axially symmetric mode of the layer at the maximum $r_- \simeq d$ should exceed the corresponding magnitude of this field at $r_+ \simeq d + 2a$ in absolute value. The fine-scale oscillations of the whistler field in the layer are averaged during the measurement (the oscillation scale $\lambda_\perp = 2\pi/(k_0 q_2) \sim 0.5$ cm, and the diameter of the receiver antenna $2a_1 = 1$ cm), and therefore are not visible on the experimental plots (compare Figs. 4c and 4d with Fig. 7).

The slow dependence of the field amplitude in the channel on the longitudinal coordinate z (see Figs. 1b–1d) is explained by the variation of the channel parameters in space and time during the thermal diffusion of the plasma.

From the discussion given here we can draw a number of conclusions:

1. As a result of electron heating by the quasistatic field of the antenna and thermal-diffusion-driven redistribution of the plasma it produces, it is possible for a nonuniform channel to form near the radiator which acts as a waveguide in the whistler-frequency region (1) for two types of independent modes.

2. Guided propagation of localized conical refraction waves occurs in the central part of the low-density channel, while the high-density annular layer that surrounds this central part supports guided propagation of quasilocalized whistlers.

3. When the high-density channel has a width comparable to the characteristic wavelength of whistler waves, leakage of quasilocalized whistler modes is small under the condition $\omega_{He} \gg \omega$, for which the transverse scales of the whistler modes trapped in the channel and the leaking waves differ significantly.

In conclusion, we note that the waveguide structures we have discussed here allow us to vary the coefficients of excitation of quasilongitudinal whistler waves and conical refraction waves, as well as the directivity diagrams of radiation with respect to a uniform (background) plasma.

[1)] Unfortunately, there are some erroneous assertions in Ref. 1, in particular regarding the frequency region $\omega > \omega_{He}/2$.

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Translated by Frank J. Crowne