Backward rescattering of stimulated Brillouin scattering

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We predict that stimulated Brillouin scattering (SBS) can be rescattered backward at an angle to the backward direction under actual plasma experiment conditions if the SBS has a Stokes component in the almost forward direction.

1. The present paper contains a prediction and a theoretical description of a new effect which shows up in stimulated Mandel'shtam–Brillouin scattering (SBS) even when the scattered radiation has a relatively small intensity. Indeed, we show by the example of a plasma that scattered radiation is transferred at an exactly backward direction into scattered radiation at an "almost backward" angle θ. This effect, which we call here rescattering, arises naturally in a new formulation of the backward SBS problem which differs substantially from the conventional formulation.[1] Together with the backward scattered wave we take into account the possibility of the excitation of yet another backward scattered wave at an "almost backward" angle θ, and also—and this is essential in the present statement—we take into account the forward scattering of the pump wave at an "almost forward" angle θ. The interference picture which then arises of the wavefields due to the nonlinear effect of the pump and the forward scattered electromagnetic wave also leads to a new effect in which the backward travelling and the almost backward scattered waves rescatter on each other.

The rescattering effect described below, in particular, corresponds under well defined conditions to the fact that with an increasing pump intensity the theory predicts not only the usual increase in the backward SBS intensity, but also its decrease. This effect of a decrease in the SBS intensity has been observed experimentally.[2] The theory also predicts an anomalous behavior for almost backward SBS. The effect discussed here will not be observed when the aperture of the lens which collects the forward scattered radiation is large, since in that case the total contribution from both the forward- and the almost forward-scattered waves is recorded. For a convincing experimental demonstration of the angular rescattering of the scattered radiation we predict, it is necessary to study the angular distribution of the SBS radiation with a good angular resolution.

2. In order to introduce the necessary notation and also to formulate the theoretical model we discuss first of all the theoretical position describing SBS in a layer of a uniform transparent plasma of thickness $0 \leqslant x \leqslant L$, electron density n_e, and electron and ion temperatures T_e and T_i, respectively.

The structure of the electromagnetic field in SBS process is usually taken in the form[3]

$$E = {}^1/_2 \exp(-i\omega_0 t)\,[E_{01}\exp(i\mathbf{k}_{01}\mathbf{r}) + E_{0-1}\exp(i\omega t + i\mathbf{k}_{0-1}\mathbf{r})] + \text{c.c.}, \qquad (2.1)$$

where the scattered field (the pump field) is characterized by a field strength E_{01}, a frequency ω_0 and a wavevector $\mathbf{k}_{01} = \mathbf{e}_x k_0$, respectively, and E_{0-1}, $\omega_0 - \omega$, $\mathbf{k}_{0-1} = -\mathbf{e}_x k_0 \cos\theta_0 - \mathbf{e}_y k_0 \sin\theta_0$ characterize the Stokes component—the field scattered at an angle θ_0. Corresponding to Eq. (2.1) the field of the excited sound waves has the following form:

$$\frac{\delta n_e}{n_e} = -i v_0 \exp(-i\omega t + i\mathbf{k}_S \mathbf{r}) + \text{c.c.}, \qquad (2.2)$$

where δn_e is the perturbation of the electron density, we have $\mathbf{k}_S = \mathbf{k}_{01} - \mathbf{k}_{0-1}$, $k_S(\theta_0) = 2k_0\cos(\theta_0/2)$ and according to Ref. 4 we have

$$v_0 = \frac{\Delta(\theta_0)}{\gamma_S(\theta_0) + i(\Delta(\theta_0) - \omega)} \left(1 + \frac{d_S}{k_S^2 l_e^2}\right) \frac{E_{01}E_{0-1}^*}{32\pi n_c k_B T_e}. \qquad (2.3)$$

Here k_B is the Boltzmann constant, we have $\Delta(\theta_0) = k_S(\theta_0) v_S$, $n_c = m_e\omega_0^2/4\pi e^2$ is the critical plasma density, l_e is the electron mean free path with respect to collisions with ions, we have

$$d_S = (Z\cdot 1{,}04 + 0{,}45)/(Z + 5T_i/3T_e),$$

$$v_S(\theta_0) = \frac{ZT_e + f(\theta_0)T_j}{m_j},$$

$$f = \begin{cases} 3, & 2k_0 v_S(\theta_0)\tau_{ii}\cos(\theta_0/2) \gg 1 \\ {}^5/_3, & 2k_0 v_S(\theta_0)\tau_{ii}\cos(\theta_0/2) \ll 1' \end{cases}$$

τ_{ii} is the ion-ion collision frequency, $M_j(Z)$ are the mass and degree of ionization of the ions, and finally we use for the damping rate of ion sound the expression[4]

$$\gamma_S(\theta_0) = [\gamma_{Sa}^{-1}(\theta_0) + (\gamma_{Li}(\theta_0) + \gamma_i(\theta_0))^{-1}]^{-1} + \gamma_{Le}(\theta_0) + \gamma_{et} + \gamma_r(\theta_0), \qquad (2.4)$$

where we have $\gamma_{Sa}(\theta_0) = [\frac{5}{9} + \frac{25}{36} v_{Ti}^2/v_S^2(\theta_0)] \times 4k_0^2 v_{Ti}^2 \tau_{ii}\cos^2(\theta_0/2)$, v_{Ti} is the ion thermal velocity, $\gamma_{Le}(\gamma_{Li})$ is the Landau damping due to electrons (ions), $\gamma_i(\theta_0) = [4v_{Ti}^2/5v_S^2(\theta_0)]\tau_{ii}^{-1}$ is the damping due to ion-ion collisions, $\gamma_r(\theta_0) = [2v_{Ti}^2 m_e/3v_S^2(\theta_0) m_i]\tau_{ei}^{-1}$ is caused by the relaxation of the ion temperature, τ_{ei}^{-1} is the electron-ion collision frequency, and $\gamma_{et} = (Z\cdot 0.26 + 0.11)(m_e/m_i)\tau_{ei}^{-1}$ is the damping rate due to the electron thermal conductivity. In particular, for the experiments described in Ref. 3 we have $\gamma_S \approx \gamma_{Li}$ for $\theta_0 \leqslant 1$, and $\gamma_S(\theta_0) = \gamma_{Sa}(\theta_0) + \gamma_r(\theta_0) + \gamma_{et}$ for $\pi - \theta_0 \ll 1$. We note also that Eq. (2.4) for the damping rate is suitable for moderate values of the scattering angle when we have $|\pi - \theta_0| > (k_0 l_e)^{-1}\sqrt{Zm_e/m_i} \equiv \theta_{ap}$. In particular, for the experiments of Ref. 3 we have $k_0 l_e \approx 45$ so that $\theta_{ap} \approx 0.5\cdot 10^{-3}$. We shall show in what follows that the effects studied here can occur at angles considerably larger than θ_{ap} so that in what follows we shall use just Eq. (2.4) for the damping rate.

Assuming that the backward scattering corresponds to an amplification ε_0 of the fluctuations of the electromagnetic field at the right-hand (rear) side of the plasma layer ($x=L$) we have for the intensity of the Stokes component

$$|E_{0-1}(x,\theta_0,\omega)|^2=|\varepsilon_0|^2\exp\{\varkappa_b(\theta_0,\omega)(L-x)/L\}. \quad (2.5)$$

The spatial amplification coefficient $\varkappa_b$ is here determined by the formula

$$\varkappa_b(\theta_0,\omega)=L\frac{\omega_{Le}^2|E_0|^2\cos(\theta_0/2)v_S(\theta_0)}{16\pi n_c T_e\gamma_S(\theta_0)c^2k_B}$$
$$\times\left[1+\frac{d_S}{4k_0^2l_e^2\cos^2(\theta_0/2)}\right]\left[1+\left(\frac{\Delta(\theta_0)-\omega}{\gamma_S(\theta_0)}\right)^2\right]^{-1}. \quad (2.6)$$

ω_{Le}^2 is the electron Langmuir frequency, and we have $E_0(x)=E_{01}(0)\equiv E_0$. The largest scattering coefficient at a given angle θ_0 corresponds to the resonance frequency $\omega=\Delta(\theta_0)$ and is equal to

$$\varkappa_b(\theta_0,\omega)=L\frac{\omega_{Le}^2|E_0|^2\cos(\theta_0/2)v_S(\theta_0)}{16\pi n_c T_e\gamma_S(\theta_0)c^2k_B}$$
$$\times\left(1+\frac{d_S}{4k_0^2l_e^2\cos^2(\theta_0/2)}\right). \quad (2.7)$$

Equations (2.5) and (2.7) which describe SBS in a wide range of angles turn out to be unsuitable in a small range of almost forward scattering angles when one must take into account the anti-Stokes component of the scattered field as well as the Stokes component. In contrast to (2.1) the structure of the electromagnetic field then has the form

$$E={}^1/_2\exp(-i\omega_0t)\,[E_{01}\exp(i\mathbf{k}_{01}\mathbf{r})+E_S\exp(i\omega t+i\mathbf{k}_S\mathbf{r})+E_a\exp(-i\omega t+i\mathbf{k}_a\mathbf{r})]+\text{c.c.} \quad (2.8)$$

It is convenient in what follows to change to the angle $\theta\equiv\pi-\theta_0$ so that almost forward scattering corresponds to the range of small angles $\theta(\theta\ll1)$.

Using the boundary conditions $E_S(x=0)=\varepsilon_S$, $E_a(x=0)=\varepsilon_a$, we have for the amplitudes E_S and E_a

$$E_S(x)=\frac{\exp(\varkappa_f x/2L)}{\rho_2-\rho_1}\left\{(\varepsilon_S\rho_2-\varepsilon_a)\exp\left(\frac{\varkappa_f}{2L}\rho_1x\right)-(\varepsilon_S\rho_1-\varepsilon_a)\exp\left(\frac{\varkappa_f}{2L}\rho_2x\right)\right\},$$
$$E_a(x)=\frac{\exp(\varkappa_f x/2L)}{\rho_2-\rho_1}\left\{\rho_1(\varepsilon_S\rho_2-\varepsilon_a)\exp\left(\frac{\varkappa_f}{2L}\rho_1x\right)-\rho_2(\varepsilon_S\rho_1-\varepsilon_a)\exp\left(\frac{\varkappa_f}{2L}\rho_2x\right)\right\}, \quad (2.9)$$

where

$$\rho_{1,2}=\frac{ik_0L}{\varkappa_f(\theta,\omega)}\left\{\theta^2\pm\left[\theta^4+2i\theta^2\frac{\varkappa_f(\theta,\omega)}{k_0L}\right]^{1/2}\right\}-1,$$
$$\varkappa_f(\theta,\omega)=L\frac{\omega_{Le}^2|E_0|^2\sin(\theta/2)v_S(\pi-\theta)}{16\pi n_c T_e\gamma_S(\pi-\theta)c^2k_B}$$
$$\times\left(1+\frac{d_S}{4k_0^2l_e^2\sin^2(\theta/2)}\right)\left[1+\left(\frac{\omega_f(\theta)-\omega}{\gamma_S(\pi-\theta)}\right)^2\right]^{-1},$$
$$\omega_f(\theta)=2k_0v_S(\pi-\theta)\sin(\theta/2). \quad (2.10)$$

Since we have

$$\mathrm{Re}\left(\frac{\varkappa_f}{2L}+\rho_1\right)=\frac{k_0}{2}\,\mathrm{Im}\left[\theta^4+2i\theta^2\frac{\varkappa_f(\theta,\omega)}{k_0L}\right]^{1/2}>0,$$
$$\mathrm{Re}\,\rho_2=-\mathrm{Re}\,\rho_1<0,$$

the terms in Eq. (2.9) proportional to $\exp\rho_1x$ increase exponentially fast with increasing x and those proportional to $\exp\rho_2x$ decrease exponentially fast. Therefore even when one penetrates slightly into the plasma the fields E_S and E_a are described with good accuracy by the formulas

$$E_S(x)=A_1\exp\left[\frac{\varkappa_f}{2L}(\rho_1+1)x\right],$$
$$E_a(x)=\rho_1A_1\exp\left[\frac{\varkappa_f}{2L}(\rho_1+1)x\right]. \quad (2.11)$$

The fields (2.11) correspond to an exponential amplification with penetration into the plasma of the Stokes (E_S) and the anti-Stokes (E_a) forward scattering components. When it leaves the layer at $x=L$ the amplification coefficient has the magnitude

$$\varkappa(\theta,\omega)=\varkappa_f(\mathrm{Re}\,\rho_1+1)=k_0L\left|\mathrm{Im}\left[\theta^4+2i\theta^2\frac{\varkappa_f(\theta,\omega)}{k_0L}\right]^{1/2}\right|. \quad (2.12)$$

One sees easily that for small forward scattering angles, for which $\theta\gg(\varkappa_f/k_0L)^{1/2}$ holds, we have $\varkappa(\theta,\omega)=\varkappa_f(\theta,\omega)$. The amplification coefficient (2.12) is a maximum for a given angle θ at the frequency ω equal to the resonant value $\omega_f(\theta)$. For $\omega=\omega_f(\theta)$ we have from (2.10)

$$\varkappa_f(\theta)=L\frac{\omega_{Le}^2|E_0|^2\sin(\theta/2)v_S(\pi-\theta)}{16\pi n_c T_e\gamma_S(\pi-\theta)c^2k_B}$$
$$\times\left(1+\frac{d_S}{4k_0^2l_e^2\sin^2(\theta/2)}\right), \quad (2.13)$$

so that the amplification which is a maximum as function of the frequency occurs with an amplification coefficient

$$\varkappa(\theta)=k_0L\left|\mathrm{Im}\left[\theta^4+2i\theta^2\frac{\varkappa_f(\theta)}{k_0L}\right]^{1/2}\right|. \quad (2.14)$$

For small scattering angles, $\theta\ll1$, we have for $\varkappa_f(\theta)$

$$\varkappa_f(\theta)=L\frac{k_0v_S}{\gamma_{Sa}^{(0)}}\frac{|E_0|^2\omega_{Le}^2}{32\pi n_cT_ek_0c^2k_B}\frac{\theta_t^2+\theta^2}{\theta_v^2+\theta^2}\frac{1}{\theta}. \quad (2.15)$$

where we have used the fact that for small angles we have $\gamma_S(\theta)=\gamma_{et}+\gamma_r+\gamma_{Sa}^{(0)}\theta^2$,

$$\theta_v=[(\gamma_{et}+\gamma_r)/\gamma_{Sa}^{(0)}]^{1/2},\quad \theta_t=d_S^{1/2}(k_0l_e)^{-1},$$
$$\gamma_{Sa}^{(0)}=({}^5/_9+{}^{25}/_{36}\,v_{Ti}^2/v_S^2)k_0^2v_{Ti}^2\tau_{ii}.$$

In accordance with what we have said above about the conditions for the applicability of Eq. (2.4), Eq. (2.15) is valid for $\theta\gg\theta_{ap}$.

Substituting (2.15) into (2.14) we get the following expression for the amplification coefficient:

$$\varkappa(\theta)=k_0L\left|\mathrm{Im}\left[\theta^4+2i\theta\frac{\theta_t^2+\theta^2}{\theta_v^2+\theta^2}\theta_p^3\right]^{1/2}\right|, \quad (2.16)$$

where

$$\theta_p=\left[\frac{k_0v_S}{\gamma_{Sa}^{(0)}}\frac{|E_0|^2\omega_{Le}^2}{32\pi n_cT_ek_0^2c^2k_B}\right]^{1/3}.$$

The amplification coefficient $\varkappa(\theta)$ has a maximum as function of the angle. For low pumping strengths, when $\theta_p < \theta_v, \theta_t$, we have for $\varkappa(\theta)$

$$\varkappa(\theta)=k_0L\left|\mathrm{Im}\left[\theta^4+2i\theta\theta_p^3\left(\frac{\theta_t}{\theta_v}\right)^2\right]^{1/2}\right|$$

and the maximum of $\varkappa(\theta)$ is reached at

$$\theta=2^{-1/6}\theta_p\left(\frac{\theta_t}{\theta_v}\right)^{2/3}. \tag{2.17}$$

when

$$\varkappa_{max}=k_0L\frac{\theta_p^2\theta_t^{4/3}}{2^{1/3}\theta^{4/3}}3^{1/2}\arctan 2^{1/2}.$$

When we increase $|E_0^2|$ we have for $\theta_p > \theta_v, \theta_t$

$$\varkappa(\theta)=k_0L|\mathrm{Im}[\theta^4+2i\theta\theta_p^3]^{1/2}|,$$

and the maximum is reached for

$$\theta=2^{-1/6}\theta_p, \tag{2.18}$$

when

$$\varkappa_{max}=k_0L\frac{\theta_p^2}{2^{1/3}}3^{1/2}\arctan 2^{1/2}.$$

Under typical experimental conditions[3] it turns out that we have $\theta_f \approx \theta_v$ and $\theta_p \geqslant 10^{-2}$, which is considerably larger than θ_{ap}. Under these conditions we can therefore use the prediction of our simple model. We note here also that according to Eq. (2.11) we have $|E_a(x)/E_S(x)|^2 = |\rho_1(\theta)|^2$. This ratio turns out to be small for the optimal angles (2.17) and (2.18), as in those cases $|\rho_1(\theta)|^2 \ll 1$. Therefore when considering forward scattering at the optimal angles we can neglect the anti-Stokes component.

The discussion given here enables us to formulate a theoretical model of the effect we are describing, for which the decisive fact is the interrelationship between the forward and the backward scattering. In fact, in what follows we restrict ourselves when describing the forward scattering field to wave angles corresponding to the optimal scattering angle for which the amplification coefficient of the forward scattering has its maximum value. In accordance with what we have said above we neglect the anti-Stokes forward scattering component. Bearing in mind that the optimal forward scattering angle is not small we use the results of the collisional description of SBS from Ref. 4 corresponding to Eqs. (2.3) and (2.4). In the forward scattering process at an angle θ [scattered wave amplitude E_f, wavevector $\mathbf{k}_f = (\cos\theta\mathbf{e}_x + \sin\theta\mathbf{e}_y$, and frequency $\omega_0 - \omega_f(\theta)$] a sound wave is formed with amplitude ν_f, wavevector $\mathbf{k}_{fS} = \mathbf{k}_{01} - \mathbf{k}_f$ ($k_{fS} = 2k_0\sin(\theta/2)$, and frequency $\omega_f(\theta)$. The excitation of this sound wave means that one must take into account in the backward-scattered radiation not only the angular component with field amplitude E_{0-1}, wavevector $\mathbf{k}_{0-1} - \mathbf{k}_{01}$, and frequency $\omega_0 - \Delta$, which is scattered exactly backward, but also the scattering at an angle θ to the direction $\mathbf{k}_{0-1}$ with amplitude E_b, wavevector $\mathbf{k}_b = -\mathbf{k}_f = -k_0(\cos\theta\mathbf{e}_x + \sin\theta\mathbf{e}_y)$, and frequency $\omega_0 - (\Delta - \omega_f(\theta))$. Such "almost backward" emission arises as the result of the rescattering of the E_{0-1} wave by the sound wave ν_f.

Bearing this fact in mind we can describe the structure of the electromagnetic field, taking into account forward and

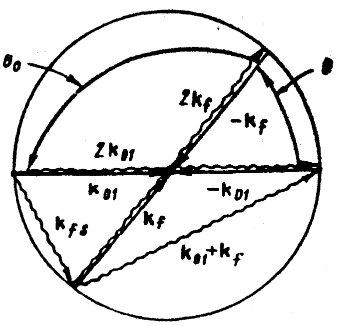

FIG. 1. Diagram of wavevectors corresponding to the electromagnetic field (2.19) (→: electromagnetic waves) and the sound field (2.20) (⇝: sound waves).

backward scattering simultaneously, in the following form:

$$\begin{aligned}E={}&{}^1/_2\exp(-i\omega_0t)\{E_{01}\exp(i\mathbf{k}_{01}\mathbf{r})\\&+E_{0-1}\exp[i\Delta(0)t-i\mathbf{k}_{01}\mathbf{r}]+E_f\exp[i\mathbf{k}_f\mathbf{r}+i\omega_f(\theta)t]\\&+E_f\exp[i(\Delta(0)-\omega_f(\theta))t-i\mathbf{k}_f\mathbf{r}]\}+\text{c.c.}\end{aligned} \tag{2.19}$$

The ion-sound density perturbation corresponding to this structure of the electromagnetic wave has the form

$$\begin{aligned}\frac{\delta n_e}{n_e}={}&-i\{\nu_f\exp[i\mathbf{k}_{fS}\mathbf{r}-i\omega_f(\theta)t]+\nu_b\exp[i(\mathbf{k}_{01}+\mathbf{k}_f)\mathbf{r}\\&-i(\Delta(0)-\omega_f(\theta))t]\\&+\nu_0\exp[2i\mathbf{k}_{01}\mathbf{r}-i\Delta(0)t]+\nu_2\exp[2i\mathbf{k}_f\mathbf{r}-i(\Delta(0)\\&-2\omega_f(\theta))t]\}+\text{c.c.}\end{aligned} \tag{2.20}$$

The wavevector diagram corresponding to the fields (2.19) and (2.20) is shown in Fig. 1.

The equations describing the evolution in space of the amplitudes of the electromagnetic fields,

$$\begin{aligned}\frac{\partial E_f}{\partial x}&=\frac{\omega_{Le}^2}{2k_0c^2}(E_{01}\nu_f^*-E_{01}\nu_b-E_b\nu_2),\\\frac{\partial E_b}{\partial x}&=\frac{\omega_{Le}^2}{2k_0c^2}(-E_{01}\nu_b^*+E_{0-1}\nu_f-E_f\nu_2^*),\\\frac{\partial E_{0-1}}{\partial x}&=-\frac{\omega_{Le}^2}{2k_0c^2}(E_{01}\nu_0^*+E_f\nu_b^*+E_b\nu_f^*),\\\frac{\partial E_{01}}{\partial x}&=-\frac{\omega_{Le}^2}{2k_0c^2}(E_{0-1}\nu_0+E_f\nu_f+E_b\nu_b)\end{aligned} \tag{2.21}$$

assume $\theta \ll 1$, so that the derivatives along the propagation direction of the corresponding waves are replaced by the derivative along x. The boundary conditions to Eqs. (2.21) have the form

$$\begin{aligned}&E_{01}(x=0)=E_0,\quad E_{01}(x=L)=\varepsilon_0,\\&E_f(x=0)=\varepsilon_f,\quad E_b(x=L)=\varepsilon_b,\end{aligned} \tag{2.22}$$

where ε_0, ε_f, and ε_b are the levels of the thermal fluctuations of the corresponding angular components of the scattered waves.

The amplitudes of the ion-sound waves in Eqs. (2.21) are determined by the beats of the electromagnetic waves as follows:

$$\nu_f=\frac{2k_0\sin(\theta/2)v_S(\pi-\theta)}{\gamma_S(\pi-\theta)}\left(1+\frac{d_S}{4k_0^2l_e^2\sin^2(\theta/2)}\right)\times\frac{E_{01}E_f^*+E_{0-1}^*E_b}{32\pi n_eT_ek_B},$$

$$\nu_b=\frac{2k_0\cos(\theta/2)v_s(\theta)}{\gamma_s(\theta)}\left(1+\frac{d_s}{4k_0^2l_e^2\cos^2(\theta_0/2)}\right)$$
$$\times\frac{E_{01}E_b^*+E_{0-1}^*E_f}{32\pi n_cT_ek_B},\tag{2.23}$$
$$\nu_0=\frac{2k_0v_s(0)}{\gamma_s(0)}\left(1+\frac{d_s}{4k_0^2l_e^2}\right)\frac{E_{01}E_{0-1}^*}{32\pi n_cT_ek_B},$$
$$\nu_2=\frac{2k_0v_s(0)}{\gamma_s(0)}\left(1+\frac{d_s}{4k_0^2l_e^2}\right)\frac{E_fE_b^*}{32\pi n_cT_ek_B}.$$

3. When considering Eqs. (2.21) one checks easily that indpendent of the form of the amplitudes of the ion-sound waves the set (2.21) has the following first integral:

$$|E_{01}(x)|^2+|E_f(x)|^2-|E_b(x)|^2-|E_{0-1}(x)|^2=\text{const}.$$

In what follows we shall use the relation between the forward and backward scattering for low levels of the backward scattering ($|E_b(0)|,|E_{0-1}(0)|\ll|E_0|$). We show below that the rescattering effect appears even at low levels of backward scattering. Under those conditions we have approximately from the last relation

$$|E_{01}(x)|^2+|E_f(x)|^2=\text{const}\approx|E_0|^2.$$

For the further analysis of the consequences of the system of Eqs. (2.21) we bear in mind that for small forward scattering angles the amplification coefficient of the forward scattering turns out to be considerably larger than the amplification coefficient for the backward scattering. Simplifying Eqs. (2.21), we then get

$$\frac{\partial E_f}{\partial x}=\frac{\omega_{Le}^2}{2k_0c^2}E_{01}\nu_f^*,\qquad \frac{\partial E_{01}}{\partial x}=-\frac{\omega_{Le}^2}{2k_0c^2}E_f\nu_f,$$
$$\frac{\partial B_b}{\partial x}=\frac{\omega_{Le}^2}{2k_0c^2}B_{0-1}\nu_f,\qquad \frac{\partial B_{0-1}}{\partial x}=-\frac{\omega_{Le}^2}{2k_0c^2}B_b\nu_f^*,\tag{3.1}$$

where $B_{b(0-1)}(x)=E_{b(0-1)}(x)\exp[(\varkappa_b/2L)x]$. Solving the boundary value problem (2.22) and (3.1) we use the function $u(x)\equiv E_f^*(x)E_0/(E_{01}(x)\cdot E_0^*)$. We have

$$E_f(x)=\varepsilon_f\exp\left\{\int_0^x\frac{d\ln u^*}{dx'}\frac{1}{1+|u(x')|^2}dx'\right\},$$
$$E_{01}(x)=E_0\exp\left\{-\int_0^x\frac{d\ln u}{dx'}\frac{|u(x')|^2}{1+|u(x')|^2}dx'\right\},\tag{3.2}$$

$$E_{0-1}(x)=\left[r_0\frac{E_0}{E_0^*}E_{01}^*(x)-r_bE_f(x)\right]\exp\left(-\frac{\varkappa_b}{2L}x\right),$$
$$E_b(x)=\left[r_0\frac{E_0}{E_0^*}E_f^*(x)+r_bE_{01}(x)\right]\exp\left(-\frac{\varkappa_b}{2L}x\right),\tag{3.3}$$

where $r_0\equiv E_{0-1}(x=0)/E_0$, $r_b\equiv E_b(x=0)/E_0$. In turn we get for $u(x)$ the following boundary value problem:

$$\frac{du}{dx}=\frac{\varkappa}{2L}\left[u+(r_br_0^*-r_b^*r_0u^2)\exp\left(-\frac{\varkappa_b}{2L}x\right)\right],$$
$$u(0)=\frac{\varepsilon_f^*}{E_f^*},\qquad u(L)=\frac{\varepsilon_b(r_0E_0)^*-\varepsilon_0^*r_bE_0}{\varepsilon_b(r_bE_0)^*+\varepsilon_0^*r_0E_0}.\tag{3.4}$$

We use the relations

$$r_0=(\varepsilon_0/E_0)\exp(\varkappa_b/2)R_0,\ r_b=(\varepsilon_b/E_0)\times\exp(\varkappa_b/2)R_b$$

to introduce the backward scattering normalization coefficients R_0 and R_b. Using Eqs. (3.2) and (3.3) we can write the relations determining R_0 and R_b in terms of the function $u(x)$

$$R_0=X+\frac{\varepsilon_b}{\varepsilon_0}u^*(L)X^*,\tag{3.6}$$
$$R_b=X^*-\frac{\varepsilon_0}{\varepsilon_b}u(L)X,$$

where

$$X=\exp\left\{-\int_0^L\frac{d\ln u}{dx}\frac{|u(x)|^2}{1+|u(x)|^2}dx\right\}.$$

It follows in particular from these relations that

$$|\varepsilon_0|^2|R_0|^2+|\varepsilon_b|^2|R_b|^2=|\varepsilon_0|^2+|\varepsilon_b|^2.\tag{3.7}$$

Since we assume that $r_0\ll1$, $r_b\ll1$ we can neglect the quadratic term with u^2 in Eq. (3.4). If we neglect that term, Eq. (3.4) has the solution

$$u(x)=\left(\frac{\varepsilon_f^*}{E_0^*}+R_bR_0^*\frac{\varepsilon_b\varepsilon_0^*}{|E_0|^2}\exp\varkappa_b\right)$$
$$\times\exp\left(\frac{\varkappa}{2L}x\right)-R_bR_0^*\frac{\varepsilon_b\varepsilon_0^*}{|E_0|^2}\exp[\varkappa_b(L-x)/L].\tag{3.8}$$

Substituting (3.8) into (3.6) we get

$$R_0=\frac{1+(\varepsilon_b/\varepsilon_0)t^*}{(1+|t|^2)^{1/2}},\qquad R_b=\frac{1-(\varepsilon_0/\varepsilon_b)t}{(1+|t|^2)^{1/2}},\tag{3.9}$$

where

$$t=\frac{E_f^*(L)E_0}{E_{01}(L)E_0^*}\approx\left(\frac{\varepsilon_f^*}{E_0^*}+R_bR_0^*\frac{\varepsilon_b\varepsilon_0^*}{|E_0|^2}\exp\varkappa_b\right)\exp\frac{\varkappa}{2}.\tag{3.10}$$

Formula (3.8) together with Eqs. (3.2) and (3.3) determines the spatial behavior of the electromagnetic waves involved in the scattering. The scattering coefficients are determined by the system of Eqs. (3.9) and (3.10).

4. To demonstrate the characteristic effect of the rescattering in the backward SBS we discuss the consequences following from the system (3.9) and (3.10). Note that when the wave interaction we are considering is absent we have

$$R_0=1,\quad R_b=1.\tag{4.1}$$

To better understand the conditions for the occurrence of the rescattering we point out that the total backward (and almost backward) scattering intensity according to (3.7) is equal to

$$(|\varepsilon_0|^2|R_0|^2+|\varepsilon_b|^2|R_b|^2)\exp\varkappa_b$$
$$=(|\varepsilon_0|^2+|\varepsilon_b|^2)\exp\varkappa_b.\tag{4.2}$$

We assume $\varkappa_b\gg1$, which corresponds to the backward SBS intensity being considerably higher than the thermal level. Moreover, we shall concentrate on the situation which is realized experimentally when (4.2) can reach a few percent, and even more, of the intensity of the incident radiation. One can then confirm that since the spatial forward scattering amplification coefficient is much larger than the backward scattering coefficient ($\varkappa \gg \varkappa_b$) the inequality

$$(\varepsilon_f^*/E_0^*)\exp(\varkappa/2) \gg 1. \tag{4.3}$$

can easily be realized. One can therefore realize conditions for which $t \gg 1$ holds, when we have according to (3.9)

$$R_0 = -\frac{|\varepsilon_b|}{|\varepsilon_0|}, \quad R_b = \frac{|\varepsilon_0|}{|\varepsilon_b|}. \tag{4.4}$$

A comparison of these expressions with (4.1) shows, firstly, that the phase of the wave which is scattered in the forward direction is changed and, secondly, that the exactly backward scattering coefficient is determined by the fluctuations producing the wave E_b which is scattered at a small angle to the backward direction, and correspondingly for the wave E_b it is determined by the fluctuations producing the wave $E_{0\text{-}1}$.

A considerably clearer manifestation of the rescattering effect can be seen when

$$t \ll 1. \tag{4.5}$$

By virtue of the inequality (4.3) condition (4.5) can be realized when the approximate relation

$$\frac{\varepsilon_0^*\varepsilon_b}{|E_0|^2} R_0^* R_b \exp \varkappa_b = -\frac{\varepsilon_f^*}{E_0^*} \tag{4.6}$$

is satisfied with good accuracy. Bearing in mind that the conservation law (3.7) holds at the same time, we find two solutions to supplement the solution of Eq. (3.9) found above, (4.3):

$$|R_0|^2 = \frac{|\varepsilon_0|^2 + |\varepsilon_b|^2}{2|\varepsilon_0|^2} \times \left\{ 1 \mp \left[1 - \frac{4|\varepsilon_f|^2 |E_0|^2}{(|\varepsilon_0|^2 + |\varepsilon_b|^2)^2} \exp(-2\varkappa_b) \right]^{1/2} \right\}, \tag{4.7}$$

$$|R_b|^2 = \frac{|\varepsilon_0|^2 + |\varepsilon_b|^2}{2|\varepsilon_0|^2} \times \left\{ 1 \pm \left[1 - \frac{4|\varepsilon_f|^2 |E_0|^2}{(|\varepsilon_0|^2 + |\varepsilon_b|^2)^2} \exp(-2\varkappa_b) \right]^{1/2} \right\}. \tag{4.8}$$

The nature of the SBS described by Eqs. (4.7) and (4.8) is, firstly, that such a regime occurs only for sufficiently large pumping intensities or, what amounts to the same thing, for sufficiently large $\varkappa_b$:

$$\exp \varkappa_b > \frac{|\varepsilon_0|^2 + |\varepsilon_b|^2}{2|\varepsilon_f|\,|E_0|}. \tag{4.9}$$

Secondly, when condition (4.9) is satisfied we have bistability of the scattering according to (4.7) and (4.8). Thirdly, Eqs. (4.7) and (4.8) describing the rescattering effect correspond to a dependence of the scattered fields on the pumping intensity such that one of the scattered components increases and the other decreases. Indeed, one sees easily in the particular case $|\varepsilon_0| = |\varepsilon_b| = |\varepsilon_f| = |\varepsilon|$ when, for instance,

$$|R_0|^2 = 1 - \left[1 - \frac{|E_0|^2}{|\varepsilon|^2} \exp(-2\varkappa_b) \right]^{1/2},$$
$$|R_b|^2 = 1 + \left[1 - \frac{|E_0|^2}{|\varepsilon|^2} \exp(-2\varkappa_b) \right]^{1/2}, \tag{4.10}$$

that as a function of pump strength $|R_0|^2$ decreases rather than increases as occurs in the SBS theory which does not take into account the rescattering effect, which corresponds to the weak pumping limit. According to (4.10) $|R_b|^2$ grows at the expense of $|R_0|^2$. Equations (4.10) thus establish a possible reason for the nonmonotonic behavior of the backward SBS as function of the pumping intensity.

We note that the occurrence of bistability, described by Eqs. (4.7) and (4.8) when (4.10) holds takes place for

$$\frac{|\varepsilon|^2}{|E_0|^2} \exp(2\varkappa_b) = 1, \tag{4.11}$$

when we have $|R_0|^2 = |R_b|^2 = 1$. In agreement with Eqs. (3.5) we then have for the actual scattering coefficients $|r_b|^2 = |r_0|^2 = \exp(-\varkappa_b)$. In other words the manifestation of the rescattering effect is important at a level of a low backward SBS intensity, i.e., for not very high pumping wave intensity. One reason why this effect can shift into the range of higher intensities is that the pumping pulse is unsteady (compare Refs. 5 and 6). One reason why this effect might shift to higher intensities is an unsteadiness of the pumping pulse (cf. Refs. 5 and 6). We should also expect this unsteadiness to have an effect (like that discussed in Refs. 5 and 6) on the backward stimulated Brillouin rescattering discovered in the present study. To study this effect will require deriving a time-dependent theory for the rescattering. In both that dynamic treatment and the steady-state treatment of the present paper, rescattering will be seen only if there is a simultaneous excitation of the long-wave density perturbations which lead to the forward scattering and the short-wave density perturbations which lead to the backward scattering. The relaxation times for both the long- and short-wave perturbations of the plasma density, taken as the reciprocal of the damping rate corresponding to a perturbation, γ_S, should therefore be no greater than the laser pulse length τ_{imp}.

The relaxation time of short-wave perturbations in a plasma, $\gamma_S^{-1}(\pi)$ [see (2.4)], is usually shorter than that for long-wave perturbations, $\gamma_S^{-1}(\theta)$. It is thus sufficient to compare τ_{imp} and $\gamma_S^{-1}(\theta)$. To do this, we estimate $\gamma_S^{-1}(\theta)$ for the experimental conditions of Refs. 5 and 6. We first need to estimate the optimum angle θ for forward SBS. Under the experimental conditions of Refs. 5 and 6, the laser pulse length was $\tau_{\text{imp}} \approx 3$ ns, and the temperatures near the peak of the laser pulse were $T_e \approx 90$ and $T_i \approx 30$ eV. According to (2.17) and (2.18), the condition $\theta > 0.1$ rad thus held. From (2.4) we then find $\gamma_S^{-1}(\theta) \approx 0.3$ ns $\ll \tau_{\text{imp}}$. This relation indicates that a rescattering could in principle be manifested, in particular, under the conditions of a real plasma experiment.[5,6]

We have thus shown by the example of actual plasma experiment conditions that because SBS in an almost forward direction is more efficient in a plasma than backward SBS, backward scattering is appreciably changed under the influence of the forward scattering Stokes component. Since the optimum almost-forward SBS scattering direction differs little from the pumping wave direction the backward SBS can accordingly be converted into scattering at a small angle to the backward direction. Note also that the mechanism exhibited here of a change in the directionality diagram of backward scattering can serve as the basis for understanding the behavior in the experiment of Ref. 7, where switching of SBS with a backward direction to scattering at an angle was observed when the pumping intensity increased.

We emphasize that with the aim of demonstrating the possibility in principle of the rescattering effect for SBS in a plasma we have presented above a new analytical nonlinear solution of the four-wave problem describing a nonlinear state of interacting waves: the pumping wave, one wave scattered almost forward at the optimum scattering angle, one wave scattered backward, and one wave scattered almost backward. The simplification of the theoretical model which then occurs is primarily connected with the growth of the forward SBS when the scattering angle is decreased, with a subsequent suppression thanks to the appearance of the anti-Stokes components.

In conclusion we attempt to see somewhat more generally that one can connect the effect predicted by us with the effect, already well known in nonlinear optics, of photo-induced scattering,[8] called "foehning" in Ref. 9. Just when a wave is reflected in the backward direction with wavefront conjugation from a photorefractive crystal there appears parasitic "foehning" scattering in various lateral directions which is, according to Ref. 8, holographic in nature. A theoretical model was proposed in Ref. 10 for such lateral scattering using a ring resonator representation when the usual four-wave interaction process which leads to wavefront conjugation is suppressed by a set of four additional waves which may be said to lead to "foehning." The fact of such a suppression in the model of Ref. 10 was established in the numerical solution of a set of equations of eight electromagnetic waves simultaneously with the equations describing the four nonlinear lattices of the photorefractive crystal.

Of course, the nature of the effect described by us and the nature of the description of "foehning" using the model of Ref. 10 are very different. In this connection one should note, firstly, the frequency shift of the SBS, secondly, the nature of our effect as being a ponderomotive as well as a thermal nonlinearity, and, thirdly, the small number of interacting waves in our case. From the point of view of nonlinear optics the last is, perhaps, the most important. At the same time in the present effect one is dealing with "lateral" scattering arising as in the model of Ref. 10 thanks to nonlinear wave interactions. However, in our case such an interaction is a four-wave one since that is just for us the elementary one in the theoretical rescattering model. This last important conclusion follows from a comparison of our considerations with the model of Ref. 10 which we carried out following a suggestion by V. T. Tikhonchuk, to whom we express our gratitude.

[1]D. W. Forslund, J. M. Kindel, and E. L. Lindman, Phys. Fluids **18**, 1002 (1975).
[2]R. Giles and A. A. Offenberger, Laser and Particle Beams **7**, 597 (1989).
[3]K. Henkel and B. Kronast, Appl. Phys. B **7**, p. 29.
[4]A. V. Maksimov and V. P. Silin, Proc. Inv. Papers of 1989 Intern. Conf. on Plasma Phys., ed. by A. F. Sen and P. K. Kaw, Bungalor, Ind. Acad. Sci., p. 315 (1991).
[5]M. V. Chegotov, K. Henkel, B. Kronast, and V. P. Silin, J. Moscow Phys. Soc. **1**, 253 (1991).
[6]M. V. Chegotov, K. Henkel, B. Kronast, and V. P. Silin, Kratk. Soobshch, Fiz. No. 9, p. 36 (1991).
[7]R. P. Drake, Preprint UCRL-95981, Lawrence Livermore National Laboratory, July 1987.
[8]V. V. Voronov, I. P. Dorosh, Yu. S. Kuz'minov, and N. B. Tkachenko, Kvantovaya Elektron. (Moscow) **7**, 2313 (1980) [Sov. J. Quantum Electron. **10**, 1346 (1980)].
[9]M. Cronin-Golomb, B. Fisher, J. O. White, and A. Yariv, IEEE J. Quantum Electron. **QE-20**, 12 (1984).
[10]N. V. Bogodaev, A. A. Zozulya, L. I. Ivleva, A. S. Korshunov, A. V. Mamaev, and N. M. Polozkov, Kvantovaya Elektron. (Moscow) **19**, No. 4 (1992) [Sov. J. Quantum Electron. **22**, No. 4 (1992)].

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